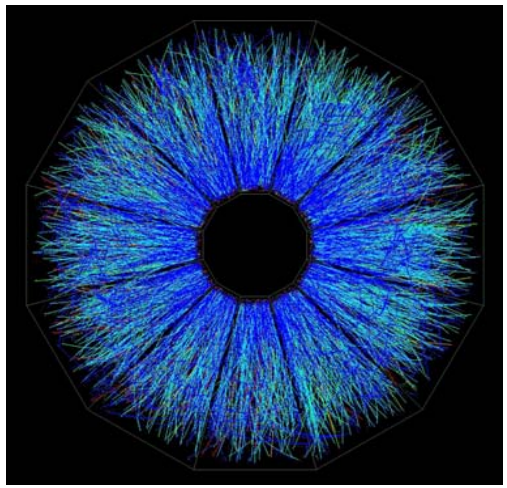
“Cosmological implications”, BNL, June 20, 2005

From Quantum Black Holes to Relativistic Heavy Ions

D. Kharzeev



BROOKHAVEN NATIONAL LABORATORY

Based on work with K. Tuchin:
hep-ph/0501234

Outline

- Why black holes cannot be produced at RHIC
- Why black holes may anyway teach us about the properties of QCD matter produced at RHIC and in the Early Universe

What is a black hole?

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

A. Einstein, 1915: $$G_{\mu\nu} = 8\pi T_{\mu\nu}$$

Einstein tensor:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

Ricci tensor

energy-momentum tensor

K. Schwarzschild, 1916:

A solution for a static isotropic gravitational field

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

$$ds^2 = c^2 dt^2 \left(1 - \frac{2GM}{rc^2}\right) - \frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2 d^2\Omega$$

singularity at r = 2GM !

Do we see black holes in the Universe?

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Cygnus X-1 binary system:

super-giant star orbiting around a black hole?

Primordial black holes formation during the QCD phase transition in the Early Universe ?

QCD phase transition in the Early Universe:

sQGP: small pressure, high energy density =>

gravitational collapse into a black hole is likely(?)

K. Jedamzik, PRD55(97) 5871

The primordial black holes, with a peak in the mass distribution at the QCD horizon mass scale (~ 1 M_{sun}):

dark matter? MACHO's? (MAssive Compact Halo Objects)

Can black holes be produced at RHIC?

$$ds^2 = c^2dt^2(1 - \frac{2GM}{rc^2}) - \frac{dr^2}{1 - \frac{2GM}{rc^2}} - r^2d^2\Omega$$

RHIC:
M < 10^4 GeV
R > 10^{-2} fm
E = 200 GeV

No role for classical or quantum gravity:

$$k_{\rm cl} \equiv \frac{2GM}{Rc^2} < 10^{-22}$$

$$k_{\rm qu} \equiv \frac{GE^2}{\hbar c^5} < 10^{-34}$$

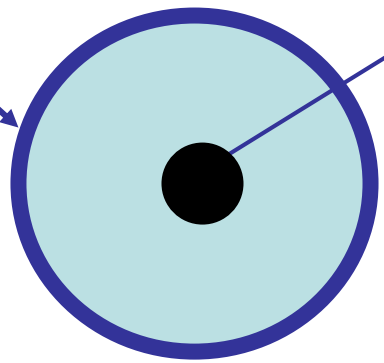
W. Busza, R.L. Jaffe, J. Sandweiss, F. Wilczek, hep-ph/9910333

Can black holes anyway help us to understand the physics at RHIC?

One new idea: use a mathematical correspondence between a gauge theory and gravity in AdS space

Maldacena

Gauge theory ⟺ Gravity



deconfinement - black hole formation in $AdS_5 x \underline{S_3 x T}$ space

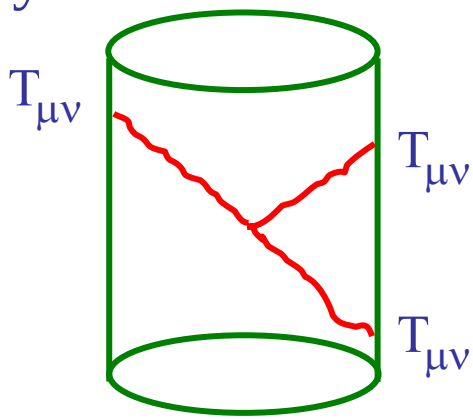
boundary

Anti de Sitter space - solution of Einstein's equations with a negative cosmological constant Λ
(de Sitter space - solution with a positive Λ (inflation))

Witten; Polyakov; Gubser, Klebanov; Son, Starinets, Kovtun; Nastase; …

What is the origin of gravitational effects?

In the AdS/CFT correspondence, gravity as described in the Anti - de Sitter space emerges as a consequence of the conformal invariance of the gauge theory (e.g., N=4 SUSY Yang-Mills)

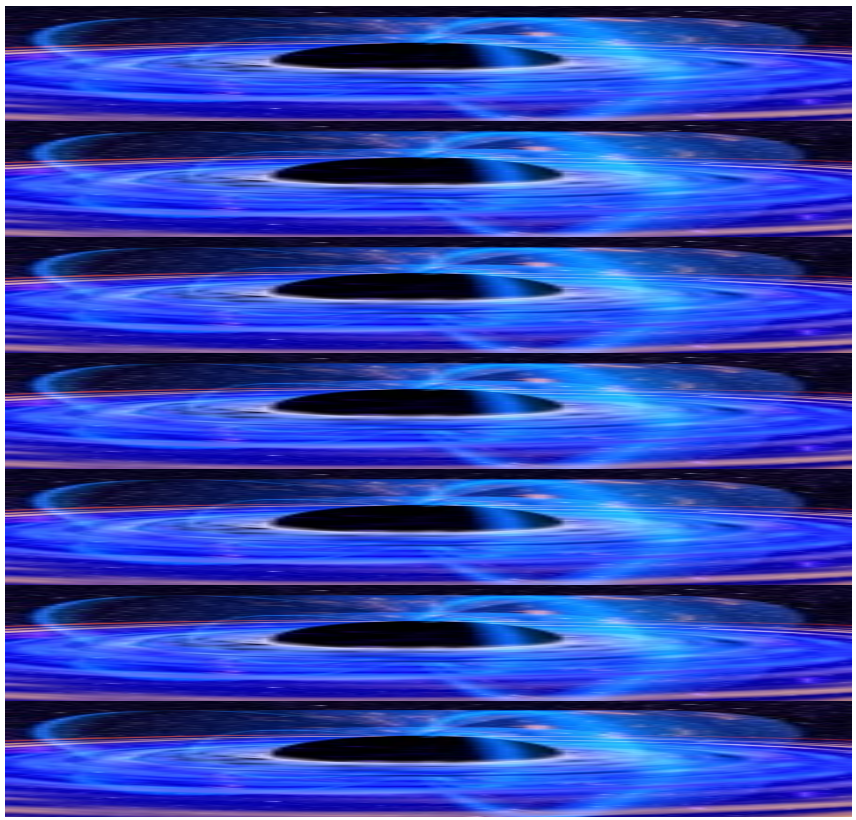


Isometry group = SO(2,d-1) for AdS_d
For AdS_5 boundary is S_3x Time;
SO(2,4) maps the boundary on itself and acts as a conformal group in 3+1 dimensions - hence the gauge theory defined on the boundary is conformal (no asymptotic freedom, no confinement)

Are gravitational-like effects possible in QCD?

Black holes radiate

S.Hawking ‘74



Black holes emit thermal radiation with temperature

$$T = \frac{\kappa}{2\pi}$$

acceleration of gravity at the surface, $(4GM)^{-1}$

Similar things happen in non-inertial frames

Einstein's Equivalence Principle:

Gravity ⟷ Acceleration in a non-inertial frame

↓

An observer moving with an acceleration a detects a thermal radiation with temperature

$$T = \frac{a}{2\pi}$$

W.Unruh '76

In both cases the radiation is due to the presence of event horizon

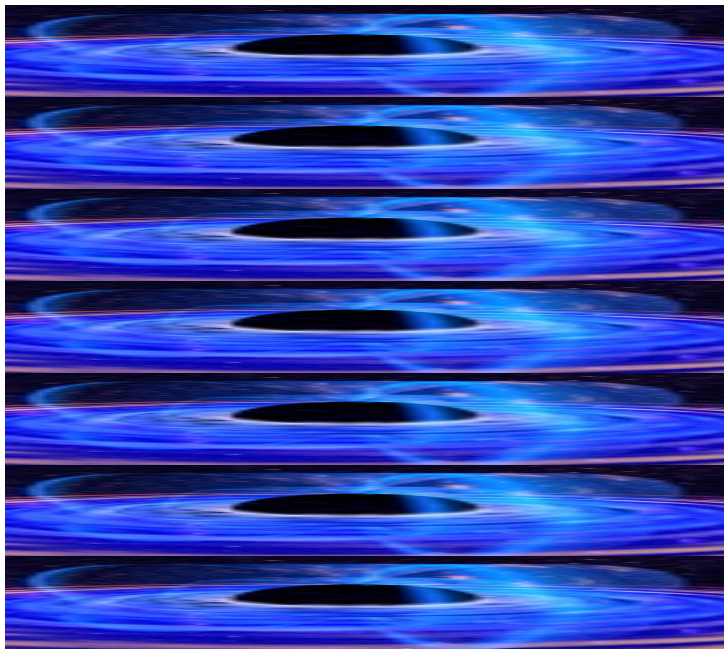
Black hole: the interior is hidden from an outside observer; Schwarzschild metric

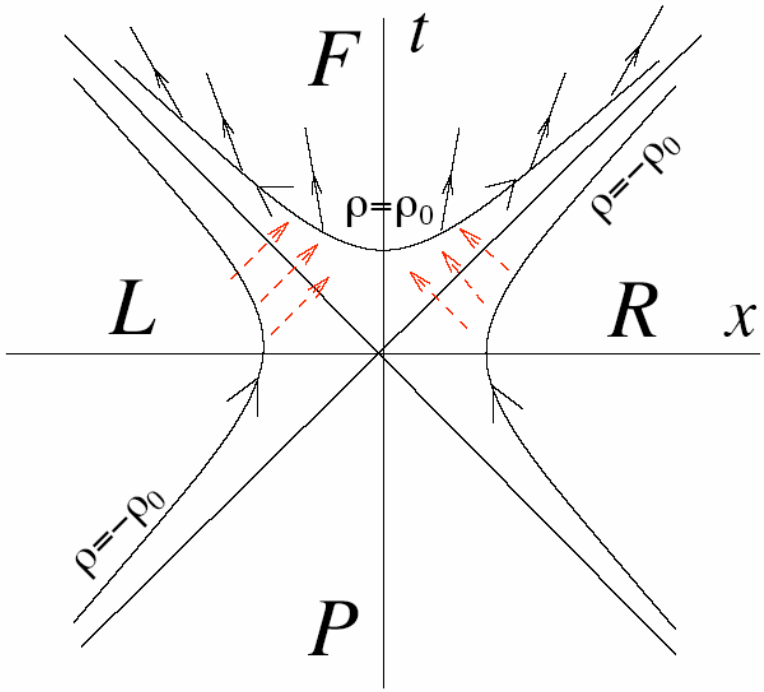
Accelerated frame: part of space-time is hidden (causally disconnected) from an accelerating observer; Rindler metric

$$\rho^2 = x^2 - t^2, \quad \eta = \frac{1}{2}\ln\left|\frac{t+x}{t-x}\right|$$

$$ds^2 = \rho^2 d\eta^2 - d\rho^2 - dx_\perp^{\ 2}$$

Pure and mixed states: the event horizons





$$|O_{Out}\rangle\langle O_{Out}| = \sum_{Inside} |O_M\rangle\langle O_M|$$

mixed state pure state

$$|O_L\rangle\langle O_L| = \sum_{R} |O_M\rangle\langle O_M|$$

mixed state pure state

Accelerating detector

- Positive frequency Green's function (m=0):

$$G^+(x,x') = \langle \phi(x)\phi(x')\rangle = -\frac{1}{4\pi^2\left[(t-t'-i\epsilon)^2 - |\vec{x}-\vec{x}|^2\right]}$$

- Along an inertial trajectory

$$\vec{x} = \vec{x}_0 + \vec{v}t$$

$$G^+(\Delta\tau) = -\frac{1}{4\pi^2(\Delta\tau - i\epsilon)^2}$$

- Along a uniformly accelerated trajectory

$$x = y = 0, z = (t^2 + a^{-2})^{1/2}$$

$$G^+(\Delta\tau) = -\frac{1}{4\pi^2}\sum_{n=-\infty}^{\infty}(\Delta\tau - 2\pi i\epsilon + in2\pi/a)^{-2}$$

Unruh,76

Accelerated detector is effectively immersed into a heat bath at temperature $T_U = a/2\pi$

Acceleration in a constant electric field

- Consider motion of a charged particle in a *constant electric field E*. Action is given by

$$S = \int (-m\, ds - e\, \phi\, dt)$$

Equations of motion yield the trajectory

$$x(t) = a^{-1}(\sqrt{1 + a^2 t^2} - 1)$$

where `a=eE/m` is the acceleration

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

Classically forbidden trajectory $t \to -it_E$

- Action along the classical trajectory:

$$S(t) = -\frac{m}{a} arcsh(at) + \frac{eE}{2a^2}\left(at(\sqrt{1+a^2t^2}-2) + arcsh(at)\right)$$

- In Quantum Mechanics S(t) is an analytical function of t

- Classically forbidden paths contribute to $\mathrm{Im}\, S(t) = \frac{m\pi}{a} - \frac{eE\pi}{2a^2} = \frac{\pi m^2}{2eE}$

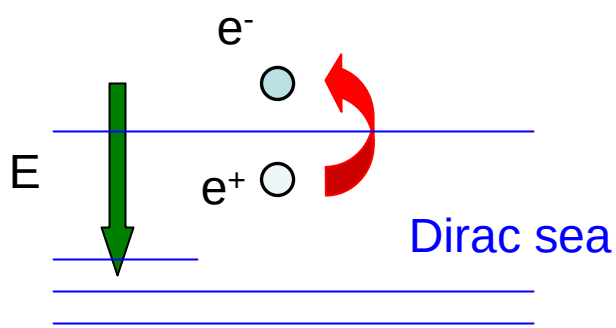
- Vacuum decays with probability

$$\Gamma_{V\to m} = 1 - \exp\left(-e^{-2\,\mathrm{Im}\,S}\right) \approx e^{-2\,\mathrm{Im}\,S} = e^{-\pi m^2/eE}$$

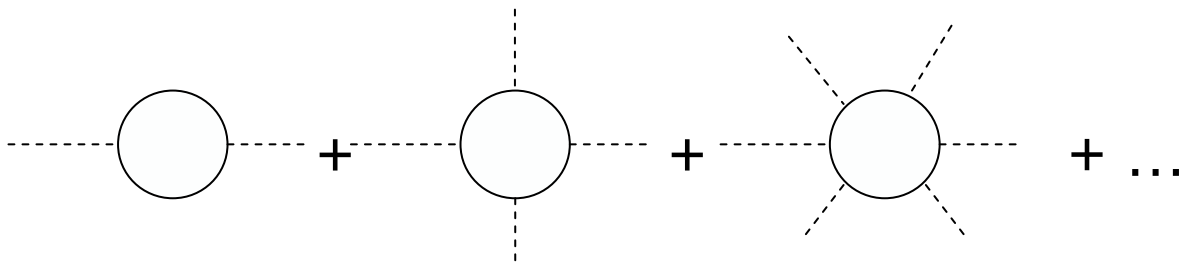
Sauter,31
Weisskopf,36
Schwinger,51

- Note, this expression can not be expanded in powers of the coupling - non-perturbative QED!

The Schwinger formula



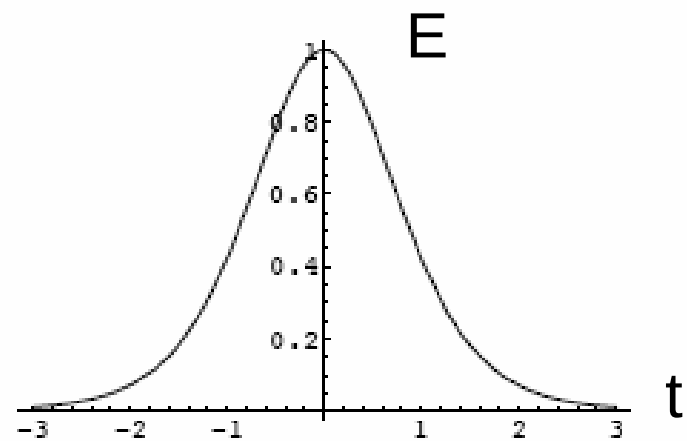
$$R \sim \exp\left(-\frac{\pi m^2}{eE}\right)$$



Pair production by a pulse

Consider a time dependent field

$$A^{\mu} = \left(0,0,0,-\frac{E}{k_0}\tanh(k_0 t)\right)$$



- Constant field limit $k_0 \to 0$
- Short pulse limit $k_0 \to \infty$

$$\Gamma \sim \exp\left[-\frac{2\pi m_{\perp}}{k_0}\right]$$

a thermal spectrum with $T = \dfrac{k_0}{2\pi}$

Chromo-electric field: Wong equations

- Classical motion of a particle in the external non-Abelian field:

$$m\ddot{x}^{\mu} = gF^{a\mu\nu}\dot{x}^{\nu}I_a - g\dot{x}^{\mu}A_{\mu}{}^{b}I^{c} = 0$$

The constant chromo-electric field is described by

$$A^{0}{}_{a} = -Ez\delta^{a3}, A^{i}{}_{a} = 0$$

Solution: vector I_3 precesses about 3-axis with I_3=const

$$\ddot{x} = \ddot{y} = 0, m\ddot{z} = gE\dot{x}^{0}I_3$$

Effective Lagrangian: Brown, Duff, 75; Batalin, Matinian,Savvidy,77; Nayak, van Nieuwenhuizen, 2005

but this is all purely academic (?)

Can one study the Hawking radiation on Earth?

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Gravity?

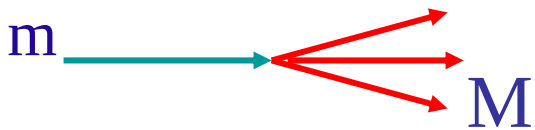
Take g = 9.8 m/s^2;
the temperature is only

$$T = \frac{g}{2\pi} \sim 10^{-20} \quad \mathrm{K}$$

Strong interactions?

Consider a dissociation of a high energy hadron of mass m into a final hadronic state of mass M;

The probability of transition:



$$P(m \to M) = 2\pi |\mathcal{T}(m \to M)|^2 \rho(M)$$

Transition amplitude: $|\mathcal{T}(m \to M)|^2 \sim \exp(-2\pi M/a)$

In dual resonance model: $\rho(M) \sim \exp(4\pi\sqrt{b}M/\sqrt{6})$

b=1/2πσ – universal slope

Unitarity: $\sum_{\bullet} P(m \to M) = const,$

$$\frac{a}{2\pi} \equiv T \leq \frac{\sqrt{6}}{4\pi\sqrt{b}}$$

limiting acceleration

Hagedorn temperature!

Fermi - Landau statistical model of multi-particle production

Enrico Fermi
1901-1954

Lev D. Landau
1908-1968

Hadron production
at high energies
is driven by
statistical mechanics;
universal temperature

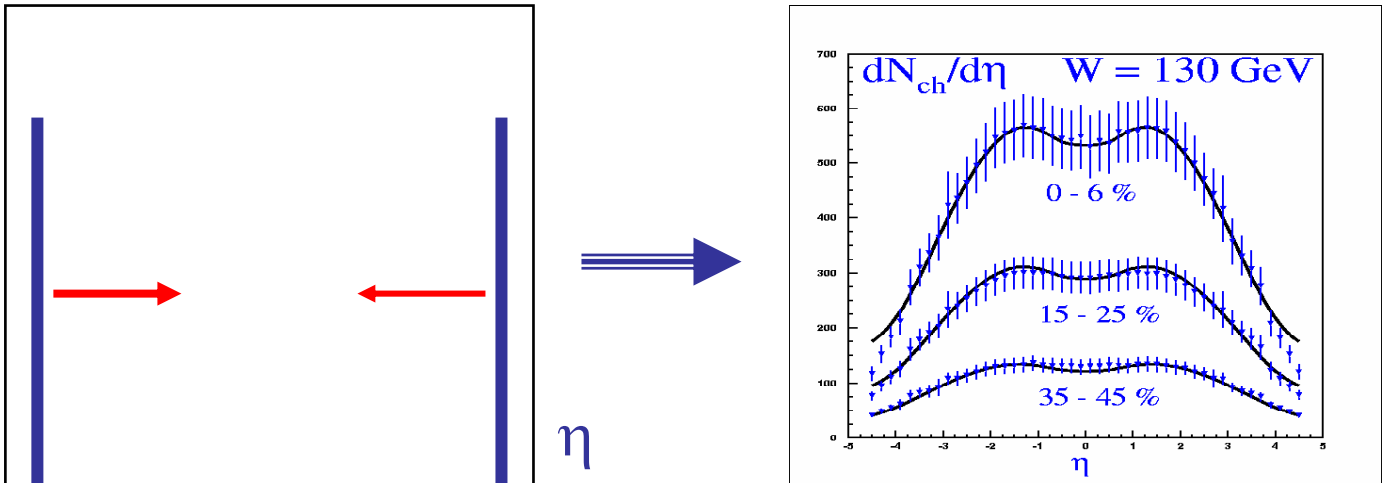
Where on Earth can one achieve the largest acceleration (deceleration) ?

Relativistic heavy ion collisions! -

stronger color fields: $E \sim \sigma \quad \rightarrow \quad E \sim \frac{Q_s^2}{g}$

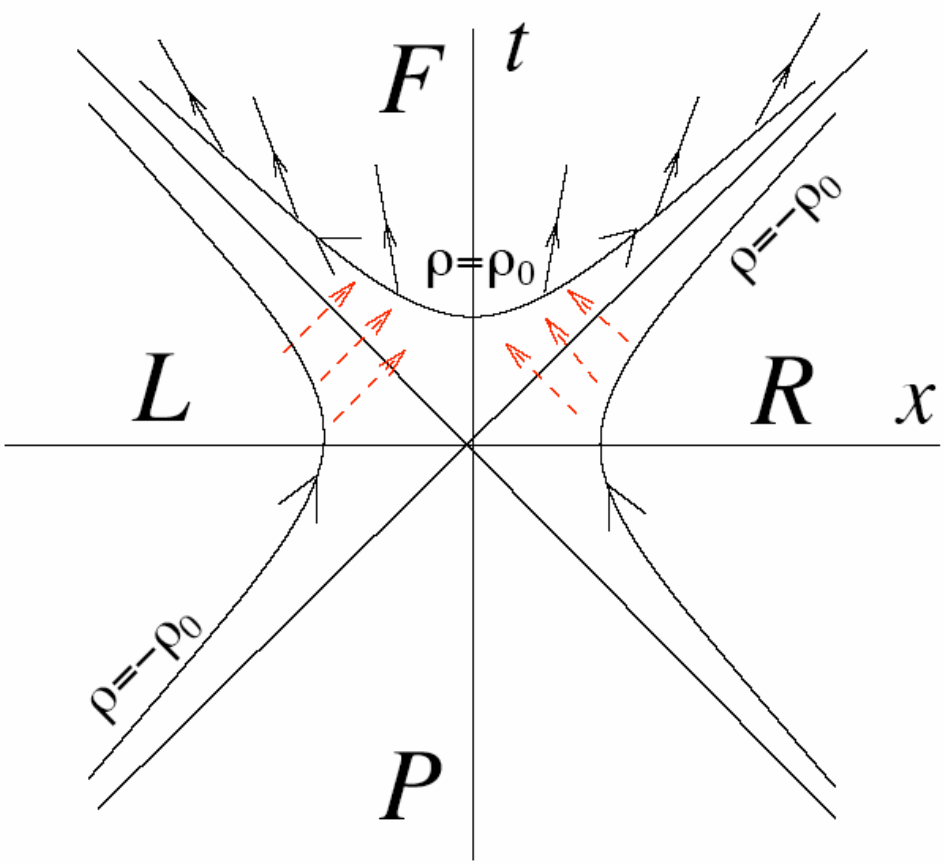
$$v_{initial} \simeq c; \quad v_{final} \simeq 0; \quad \Delta t \simeq 1/Q_s$$

$$a \simeq Q_s \sim 1 \text{ GeV}; \quad T = \frac{a}{2\pi} \sim 160 \text{ MeV}$$



Quantum thermal radiation at RHIC

$$T = \frac{a}{2\pi} \simeq \frac{Q_s}{2\pi}$$

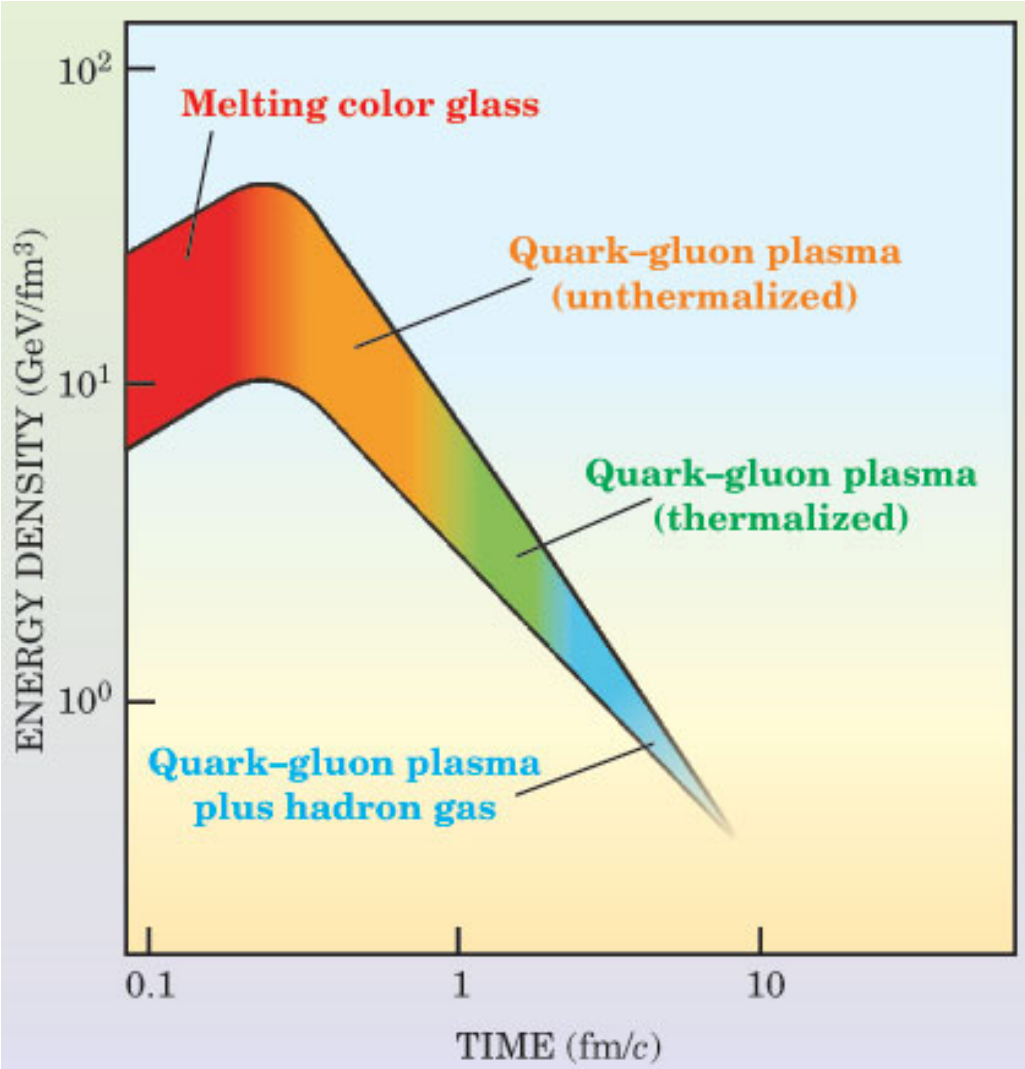


Rindler and Minkowski spaces

The event horizon emerges due to the fast decceleration $a \simeq Q_s$ of the colliding nuclei in strong color fields;

Tunneling through the event horizon leads to the thermal spectrum

The emerging picture



Big question:

How does the produced matter thermalize so fast?

Perturbation theory + Kinetic equations

→ $\tau_{therm} \sim 50\ fm$

T. Ludlam, L. McLerran, Physics Today '03

Fast thermalization

horizons

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

collision point

$\rho_0 \sim Q_s$

Rindler coordinates: $\rho = x^2 - t^2 \quad \eta = \frac{1}{2} \ln \left| \frac{t+x}{t-x} \right|$

Gluons tunneling through the event horizons have thermal distribution. They get on mass-shell in $\Delta t = 2\pi/Q_s$

(period of Euclidean motion)

Deceleration-induced phase transitions?

- Consider Nambu-Jona-Lasinio model in Rindler space

$$L = \overline{\psi} i \gamma^{\nu}(x) \nabla_{\nu} \psi(x) + \frac{\lambda}{2N} \left[(\overline{\psi}(x)\psi(x))^2 + (\overline{\psi}(x) i \gamma_5 \psi(x))^2 \right]$$

e.g.,Ohsaku,04

- Commutation relations:

$$\left\{ \gamma_{\mu}(x), \gamma_{\nu}(x) \right\} = 2 g_{\mu\nu}(x)$$

- Rindler space:

$$\rho^2 = x^2 - t^2, \qquad \eta = \frac{1}{2} \ln \left| \frac{t+x}{t-x} \right|$$

$$ds^2 = \rho^2 d\eta^2 - d\rho^2 - dx_{\perp}{}^2$$

Gap equation in an accelerated frame

- Introduce the scalar and pseudo-scalar fields

$$\sigma(x) = -\frac{\lambda}{N}\overline{\psi}(x)\psi(x) \qquad \pi(x) = -\frac{\lambda}{N}\overline{\psi}(x)i\gamma_5\psi(x)$$

- Effective action (at large N):

$$S_{eff} = \int d^4x\sqrt{-g}\left(-\frac{\sigma^2+\pi^2}{2\lambda}\right) - i\ln\det(i\gamma^\nu\nabla_\nu - \sigma - i\gamma_5\pi)$$

- Gap equation:

$$\sigma = -\frac{2i\lambda\sigma}{a}\int\frac{d^2k}{(2\pi)^2}\int_{-\infty}^{\infty}d\omega\frac{\sinh(\pi\omega/a)}{\pi^2}\left\{(K_{i\omega/a+1/2}(\alpha/a))^2 - (K_{i\omega/a-1/2}(\alpha/a))^2)\right\}$$

where $\alpha^2 = k_\perp{}^2 + \sigma^2$

Rapid deceleration induces phase transitions

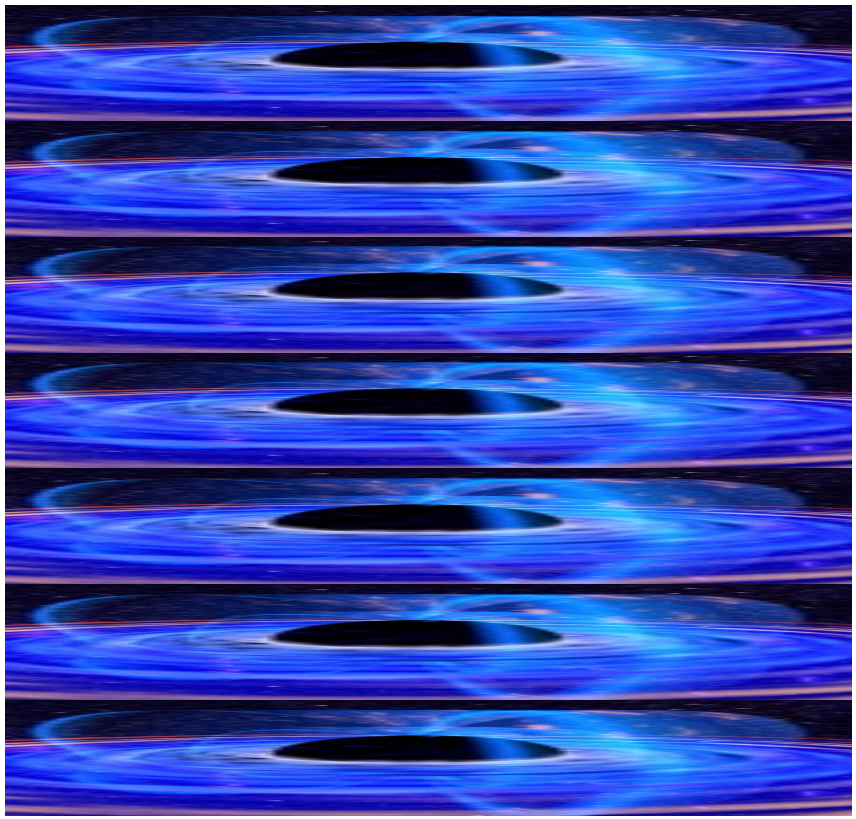
QuickTime™ and a
TIFF (LZW) decompressor
are needed to see this picture.

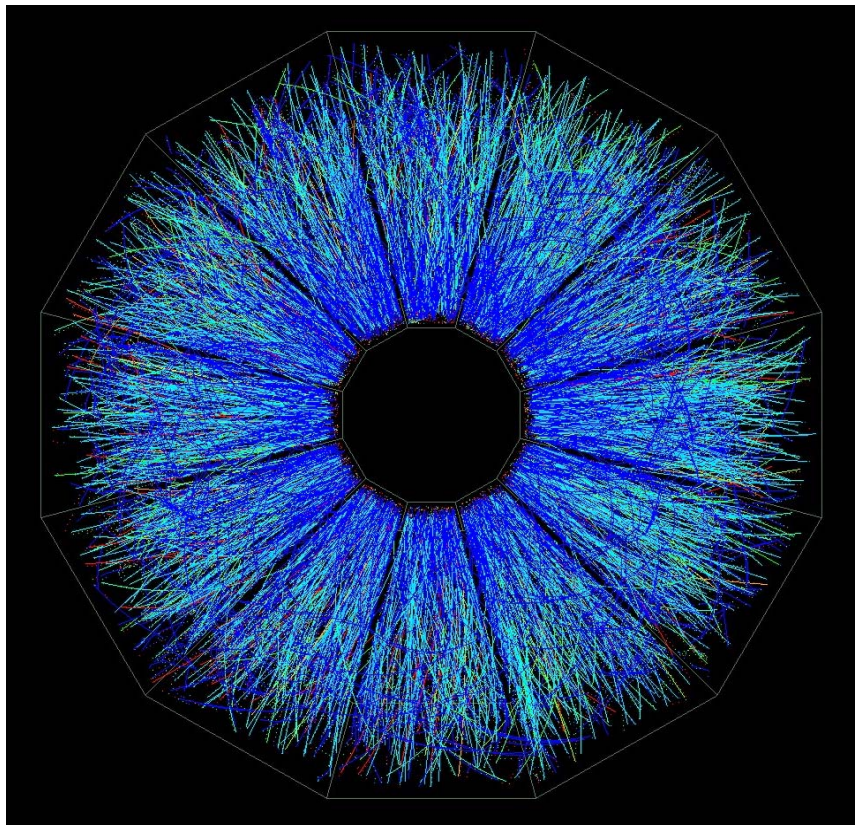
Nambu-
Jona-Lasinio model
(BCS - type)

Similar to phenomena in the vicinity of a large black hole:
Rindler space $\approx$ Schwarzschild metric

A link between General Relativity and QCD?
solution to some of the RHIC puzzles?

Black holes ⟷ RHIC collisions





Additional slides

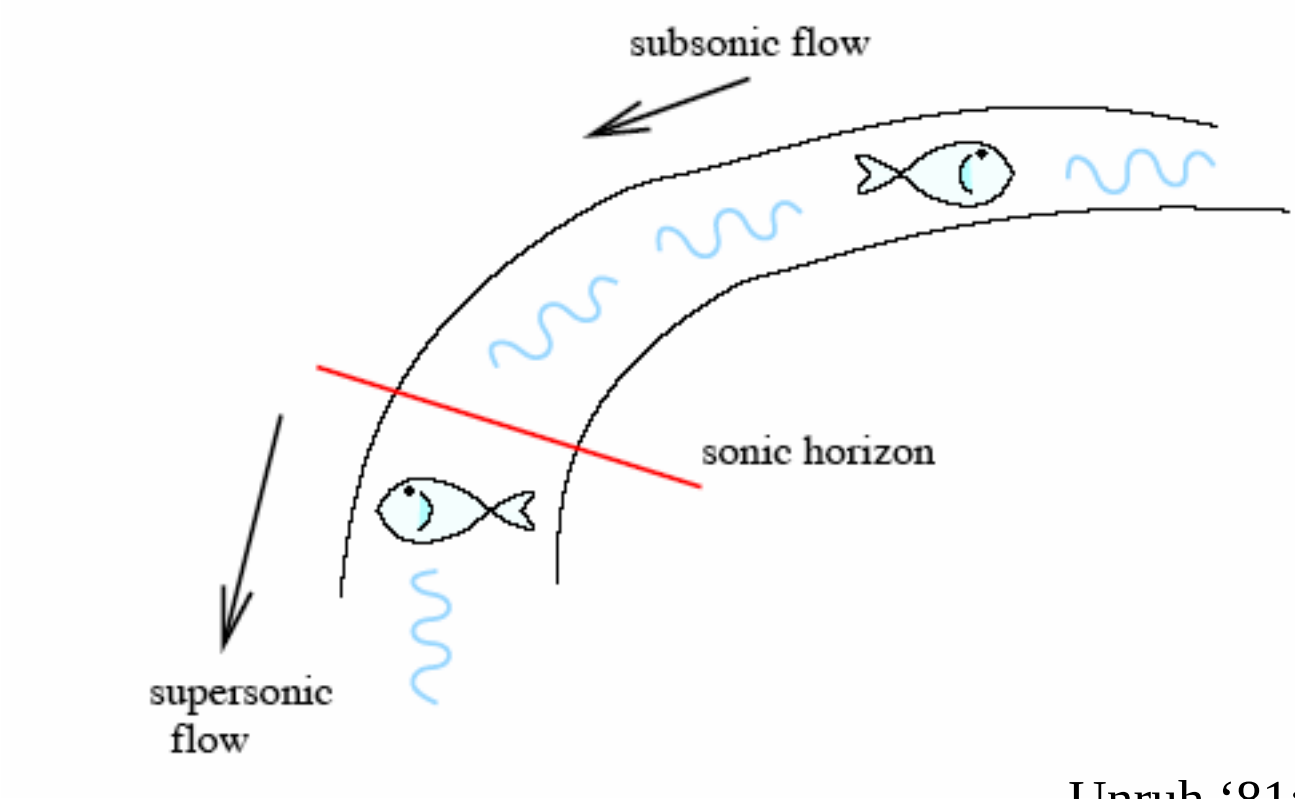
Electric fields? Lasers?

Laser Parameters					
		Optical [43]	**X-ray FEL**		
		Focus: Diffraction limit	Design [33]	Focus: Available [44]	Focus: Goal [37]
Wavelength	λ	$1\ \mu$m	0.4 nm	0.4 nm	0.15 nm
Photon energy	$\hbar\omega = \frac{hc}{\lambda}$	1.2 eV	3.1 keV	3.1 keV	8.3 keV
Peak power	P	1 PW	110 GW	1.1 GW	5 TW
Spot radius (rms)	σ	$1\ \mu$m	$26\ \mu$m	21 nm	0.15 nm
Coherent spike length (rms)	$\triangle t$	500 fs $\div$ 20 ps	0.04 fs	0.04 fs	0.08 ps
Derived Quantities					
Peak power density	$S = \frac{P}{\pi\sigma^2}$	$3 \times 10^{26}\ \frac{\mathrm{W}}{\mathrm{m}^2}$	$5 \times 10^{19}\ \frac{\mathrm{W}}{\mathrm{m}^2}$	$8 \times 10^{23}\ \frac{\mathrm{W}}{\mathrm{m}^2}$	$7 \times 10^{31}\ \frac{\mathrm{W}}{\mathrm{m}^2}$
Peak electric field	$\mathcal{E} = \sqrt{\mu_0\, c\, S}$	$4 \times 10^{14}\ \frac{\mathrm{V}}{\mathrm{m}}$	$1 \times 10^{11}\ \frac{\mathrm{V}}{\mathrm{m}}$	$2 \times 10^{13}\ \frac{\mathrm{V}}{\mathrm{m}}$	$2 \times 10^{17}\ \frac{\mathrm{V}}{\mathrm{m}}$
Peak electric field/critical field	$\mathcal{E}/\mathcal{E}_c$	3×10^{-4}	1×10^{-7}	1×10^{-5}	0.1
Photon energy/e rest energy	$\frac{\hbar\omega}{m_e c^2}$	2×10^{-6}	0.006	0.006	0.02
Adiabaticity parameter	$\gamma = \frac{\hbar\omega}{e\,\mathcal{E}\,\lambda\!\!\!^{-}_e}$	9×10^{-3}	6×10^{4}	5×10^{2}	0.1

Table 1: Laser parameters and derived quantities relevant for estimates of the rate of spontaneous e^+e^- pair production. The column labeled "Optical" lists parameters which are typical for a

compilation by A.Ringwald

Condensed matter black holes?



Unruh '81;
e.g. T. Vachaspati,
cond-mat/0404480

"slow light"?