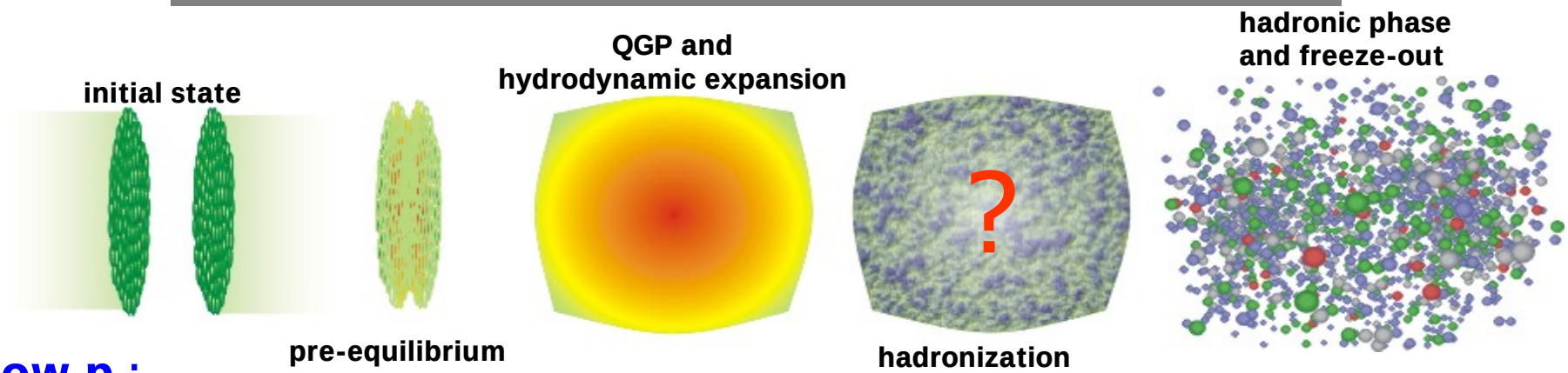
Hadronization at RHIC: Interplay of Fragmentation and Recombination

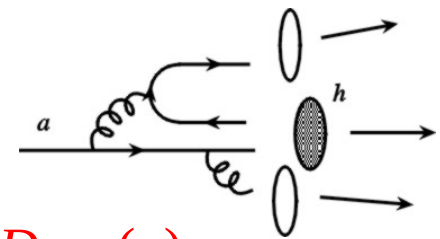
Steffen A. Bass
Duke University
& RIKEN-BNL Research Center

- The baryon puzzle at RHIC
- Recombination + Fragmentation Model
- Results: spectra, ratios and elliptic flow
- Challenges: correlations, entropy balance & gluons

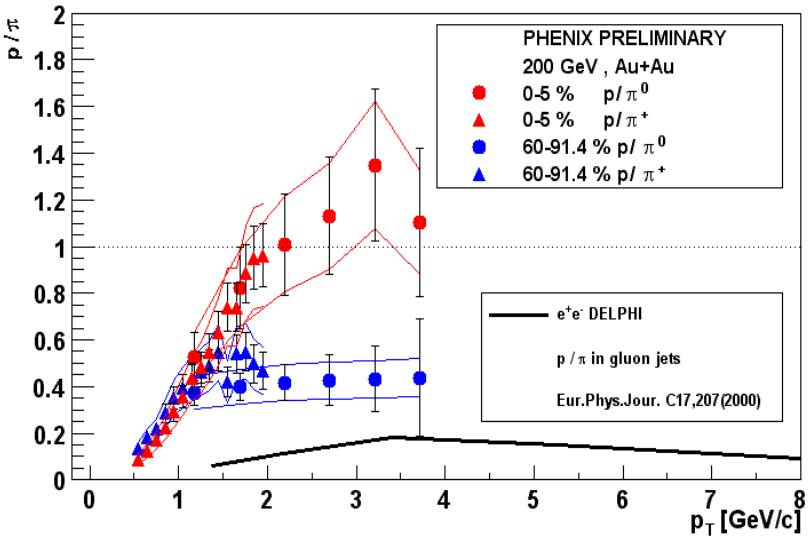
Standard Model of Hadronization

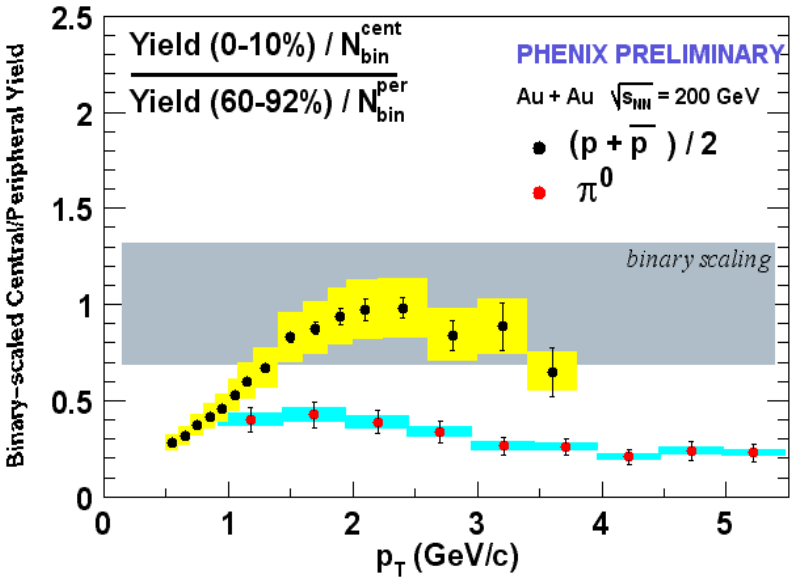


- **low p_t:**
  - hadron yields & ratios fit with a SM: $$n_i(T,\mu) = \frac{g_i}{2\pi^2}\int_0^\infty \frac{p^2 dp}{e^{(E_i-\mu_i B_i-\mu_s S_i)/T} \pm 1}$$
  - spectra via Hydro: $$\frac{dN}{m_T\, dm_T} \propto \int_0^R r\, dr\, m_T\, \mathrm{K}_1\left(\frac{m_T \cosh(\rho)}{T_{fo}}\right) \mathrm{I}_0\left(\frac{p_T \sinh(\rho)}{T_{fo}}\right)$$
- **high p_t:**
  - pQCD is applicable, hadronization via **fragmentation**
  - a fast parton fragments via a color string: a $\rightarrow$ h+X
  - hadron spectrum is given by: $$E\frac{dN_h}{d^3P} = \int_0^1 \frac{dz}{z^2}\frac{E}{z}\frac{dN_a}{d^3(P/z)} D_{\alpha\rightarrow h}(z)$$



The baryon puzzle @ RHIC

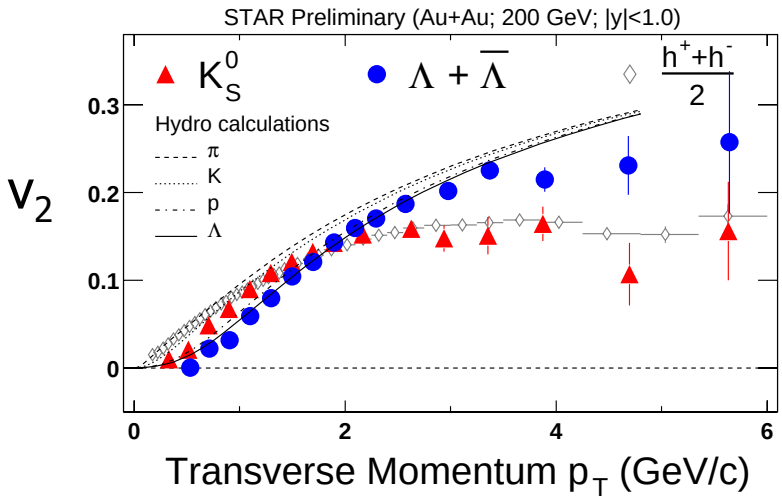




- where does the large proton over pion ratio at high p_t come from?
- why do protons not exhibit the same jet- suppression as pions?
- species dependence of v_2 saturation?

➢ fragmentation yields $N_p/N_\pi << 1$

➢ fragmentation starts with a single fast parton: energy loss affects pions and protons in the same way!



Recombination+Fragmentation Model

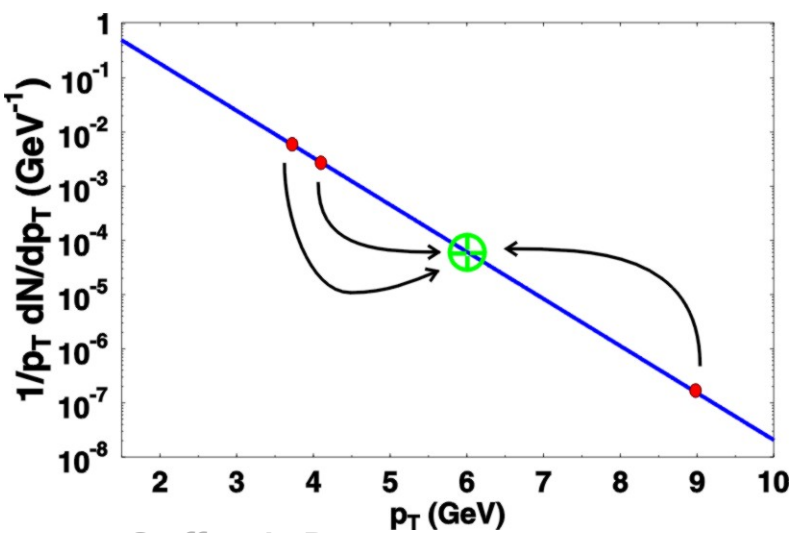
basic assumptions:

- at low p_t, the quarks and antiquark spectrum is thermal and they recombine into hadrons locally "at an instant":

$$q\bar{q} \rightarrow M \qquad qqq \rightarrow B$$

  - ➢ features of the parton spectrum are shifted to higher p_t in the hadron spectrum

- at high p_t, the parton spectrum is given by a pQCD power law, partons suffer jet energy loss and hadrons are formed via fragmentation of quarks and gluons



- for exponential parton spectrum, recombination is more effective than fragmentation
- baryons are shifted to higher p_t than mesons, for same quark distribution

➢ understand behavior of baryons!

Recombination: nonrelativistic formalism

- use thermal quark spectrum given by: w(p) = exp(-p/T)
- for a Gaussian meson wave function with momentum width Λ_M, the meson spectrum is obtained as:

$$\frac{dN_M}{d^3P} = C_M \frac{V}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} w\left(\tfrac{1}{2}P - q\right) w\left(\tfrac{1}{2}P + q\right) \left|\hat{\varphi}_M(q)\right|^2$$

$$= C_M \frac{V}{(2\pi)^3} \left[w\left(\tfrac{1}{2}P\right)\right]^2 \left(1 - \frac{2\Lambda_M^2}{TP} + \cdots\right)$$

- similarly for baryons:

$$\frac{dN_B}{d^3P} = C_B \frac{V}{(2\pi)^3} \left[w\left(\tfrac{1}{3}P\right)\right]^3 \left(1 - O(\Lambda_B^2 / TP)\right)$$

Recombination vs. Fragmentation

Fragmentation…

$$E\frac{dN_{\mathrm{h}}}{d^3P} = \int d\Sigma \frac{P\cdot u}{(2\pi)^3}\int_0^1 \frac{dz}{z^2}\sum_{\alpha} w_{\alpha}(R,\tfrac{1}{z}P)D_{\alpha\to\mathrm{h}}(z)$$

… *never* competes with recombination for a thermal (exponential) spectrum:

$$\left[w\left(P/n\right)\right]^n = \exp(-P\cdot u/T) > \exp(-P\cdot u/zT) = w\left(P/z\right)$$

… but it wins out at large p_T, when the spectrum is a power law ~ $(p_T)^{-b}$:

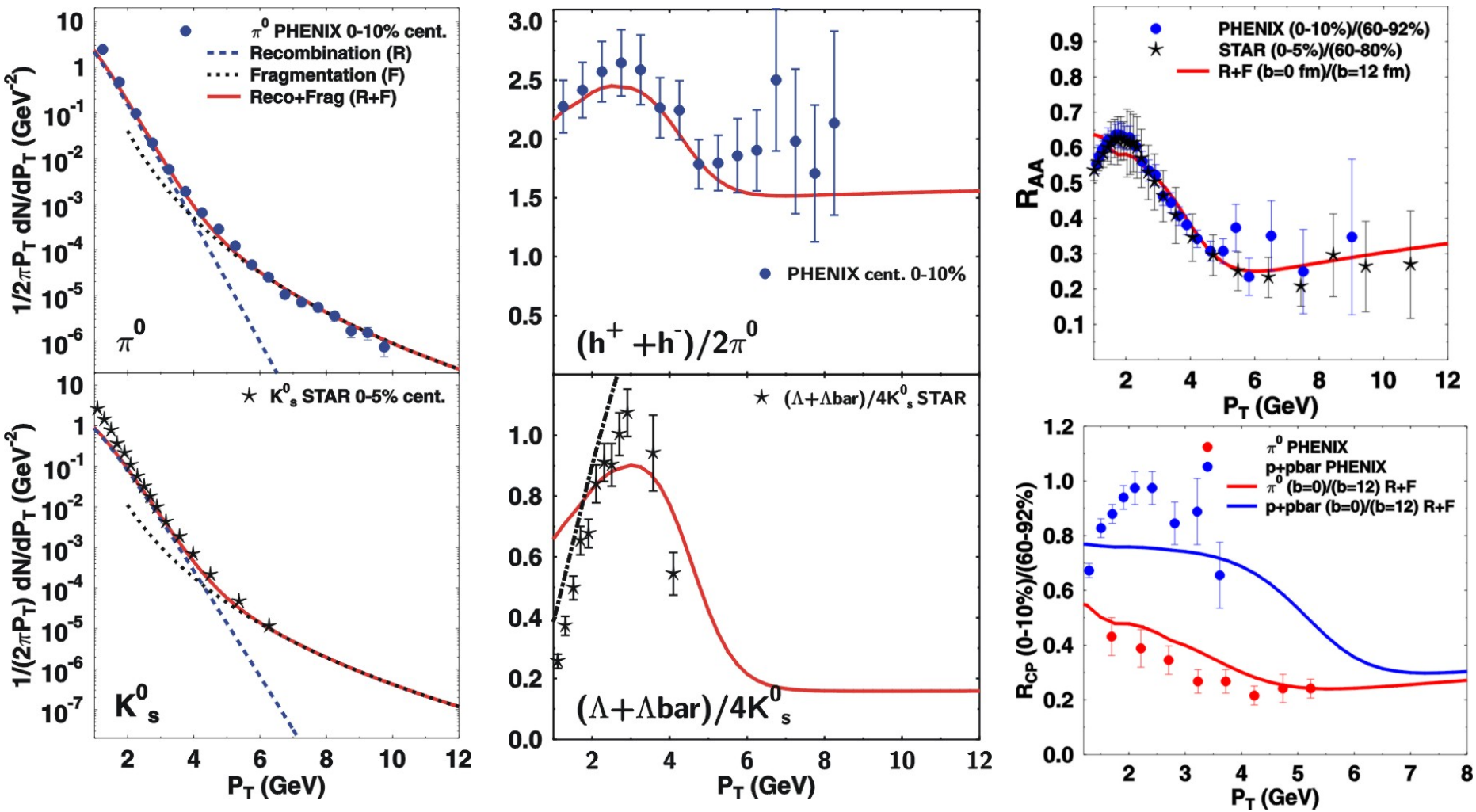
$$dN_{\pi}^{\mathrm{frag}} \propto P^{-b} \qquad dN_{\pi}^{\mathrm{rec}} \propto P^{-2b}$$

Recombination: single particle observables

- hadron spectra
- hadron ratios
- R_{AA}
- elliptic flow

- R.J. Fries, C. Nonaka, B. Mueller & S.A. Bass, PRL 90 202303 (2003)
- R.J. Fries, C. Nonaka, B. Mueller & S.A. Bass, PRC 68 044902 (2003)
- C. Nonaka, R.J. Fries & S.A. Bass, Phys. Lett. B583 73-78 (2004)
- C. Nonaka, B. Mueller, M. Asakawa, S.A. Bass & R.J. Fries, PRC 69 031902 (2004)

Reco Success: Single Particle Observables



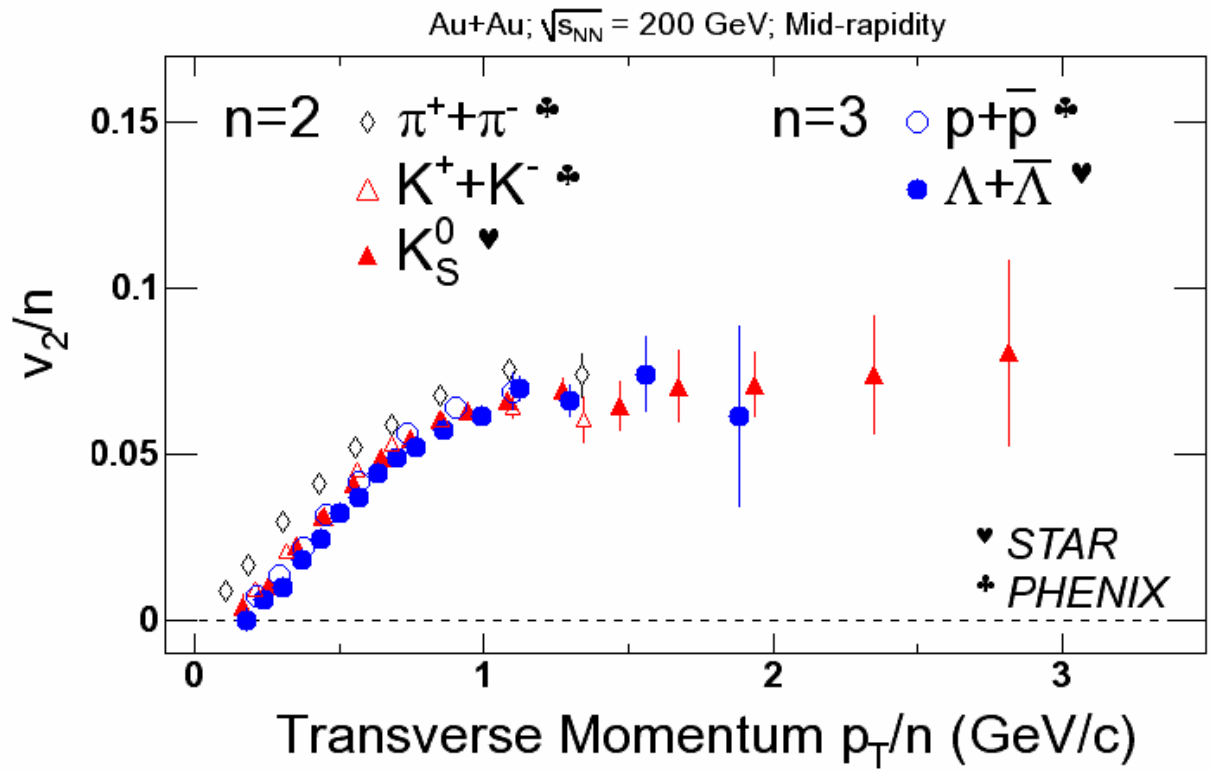
- consistent description of spectra, ratios and R_{AA}

Parton Number Scaling of v_2

- in leading order of v_2, recombination predicts:

$$v_2^M(p_t) \approx 2 v_2^p\left(\frac{p_t}{2}\right)$$

$$v_2^B(p_t) \approx 3 v_2^p\left(\frac{p_t}{3}\right)$$



- smoking gun for recombination
- measurement of partonic v_2 !

Resonance v_2: scaling violations

- QGP resonances:
  hadronizing QGP, no rescattering

  K_0^* $d, \bar{s}$ quarks n=2 scaling

- HG resonances:
  hadronic phase, h-h rescattering

  K_0^* $K^+ + \pi^- \to K_0^*$ n=4 scaling

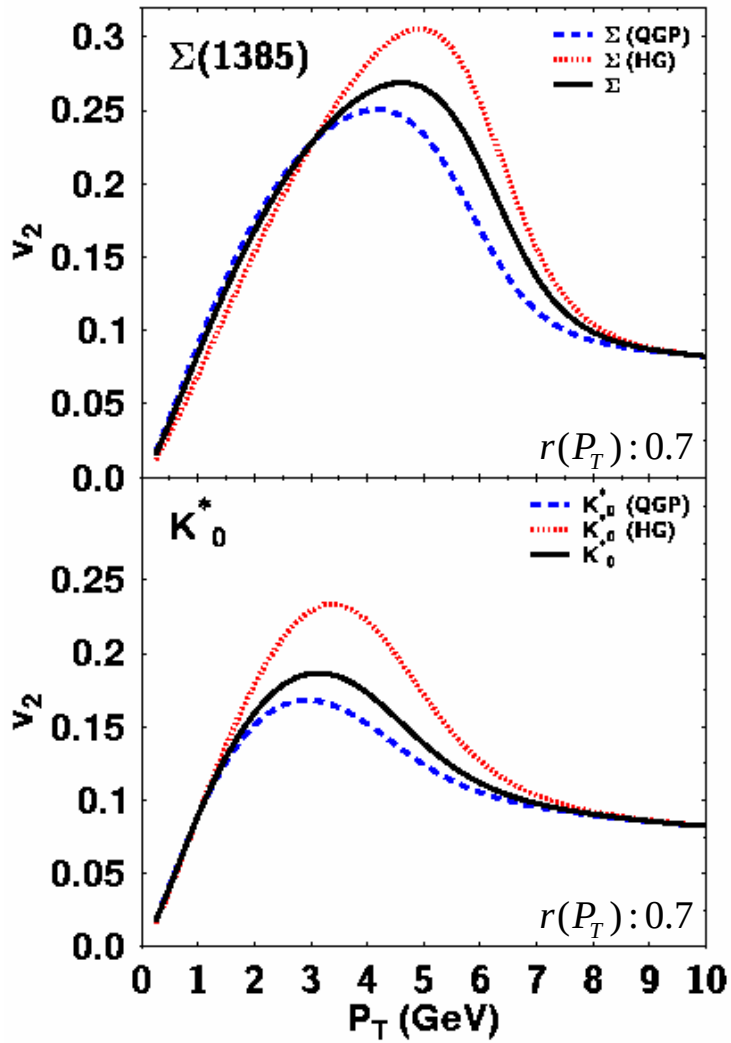
$$v_2^h(P_T) \approx n v_2\left(\frac{1}{n} P_T\right)$$

Key:
v_2 is additive for composite particles

Total

$$v_2^{\text{full}} = r(P_T) v_2^{\text{QGP}} + (1 - r(P_T)) v_2^{\text{HG}}$$

$r(P_T)$ is determined by experiments and related to width of particles and cross section in the hadronic medium.



Challenges:

- dynamical two-particle correlations
- balancing the entropy
- treatment of gluons & sea-quarks

- R.J. Fries, S.A. Bass & B. Mueller, PRL 94 122301 (2005)
- C. Nonaka, B. Mueller, S.A. Bass & M. Asakawa, PRC Rapid Communication in print, nucl-th/0501028
- B. Mueller, S.A. Bass & R.J. Fries, Phys. Lett. B in print, nucl-th/0503003

Two-Particle Correlations: a Challenge?

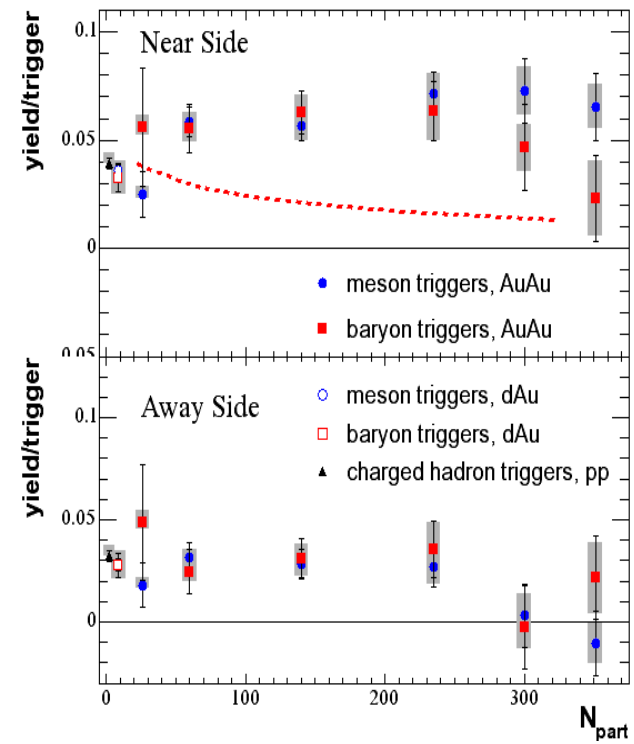
- PHENIX & STAR measure associated yields in p_T windows of a few GeV/c.
- trigger hadron A, associated hadron B: associated yield as a function of relative azimuthal angle

$$Y_{AB}(\Delta\phi) = \frac{1}{N_A}\left(\frac{dN_{AB}}{d(\Delta\phi)} - \frac{d(N_A N_B)}{d(\Delta\phi)}\right)$$

- ➢ clear jet-like structure observed at intermediate p_T
- ➢ very similar to p+p; jet fragmentation?
- analyze as function of integrated yield:

$$Y_{AB}^{\text{cone}} = \int_0^{0.94} d(\Delta\phi)\, Y_{AB}(\Delta\phi)$$

- ➢ simple recombination of uncorrelated thermal quarks cannot reproduce two particle correlations



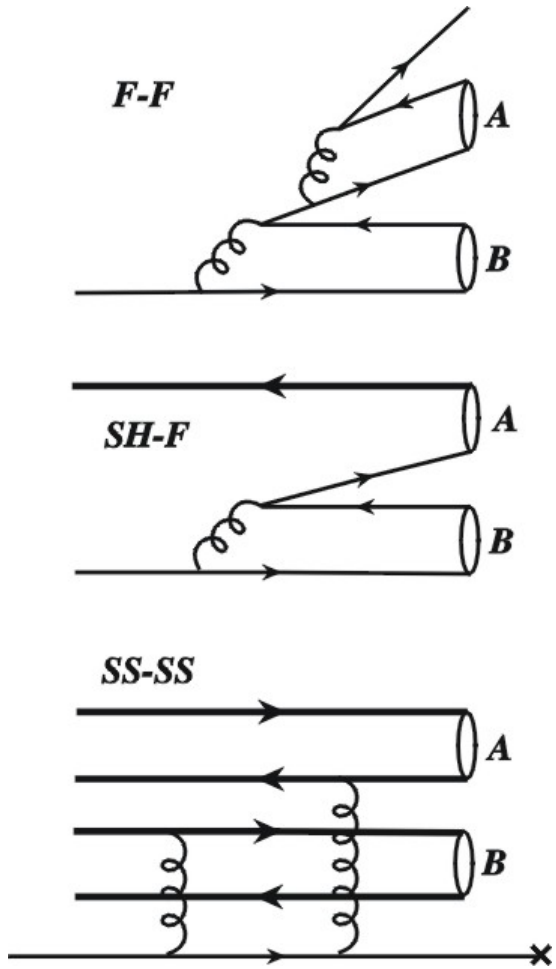
Recombination: Inclusion of Correlations

- Recombination approach allows for two particle correlations, provided they are contained in the parton source distributions
- Three distinct types are conceivable: F-F, SH-F and SS-SS
- Ansatz for SS-SS: for two mesons, use product of correlated parton distributions:

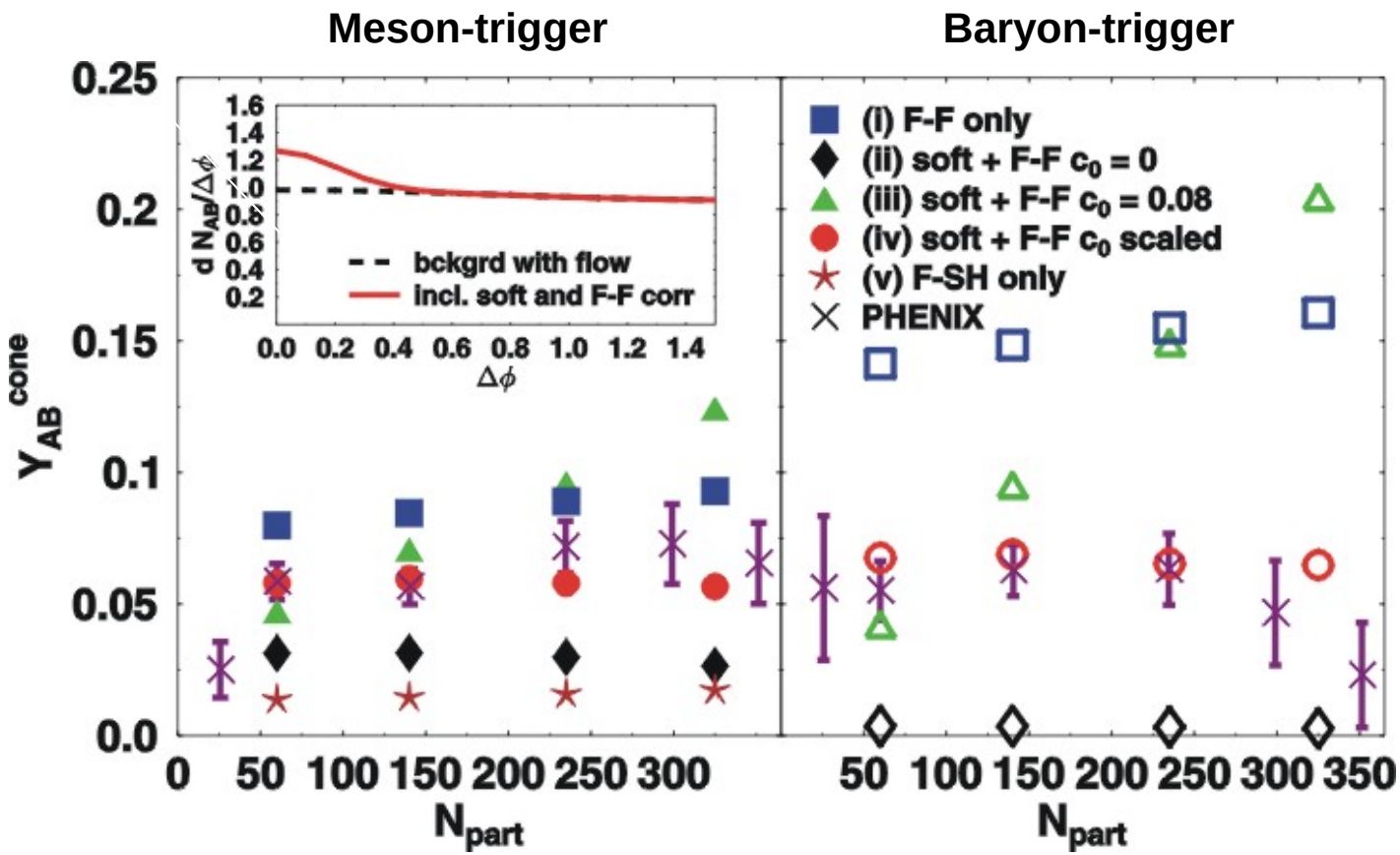
$$W_{1234} = w_1 w_2 w_3 w_4 \left(1 + \sum_{i<j} C_{ij} \right)$$

➢ Which results in a correlated two hadron yield:

$$\frac{d^6 N_{AB}}{d^3 P_A \, d^3 P_B} = C_{AB} \int_{\Sigma} d\sigma_A \, d\sigma_B \, \varphi_A \otimes \varphi_B \otimes W_{1234}$$

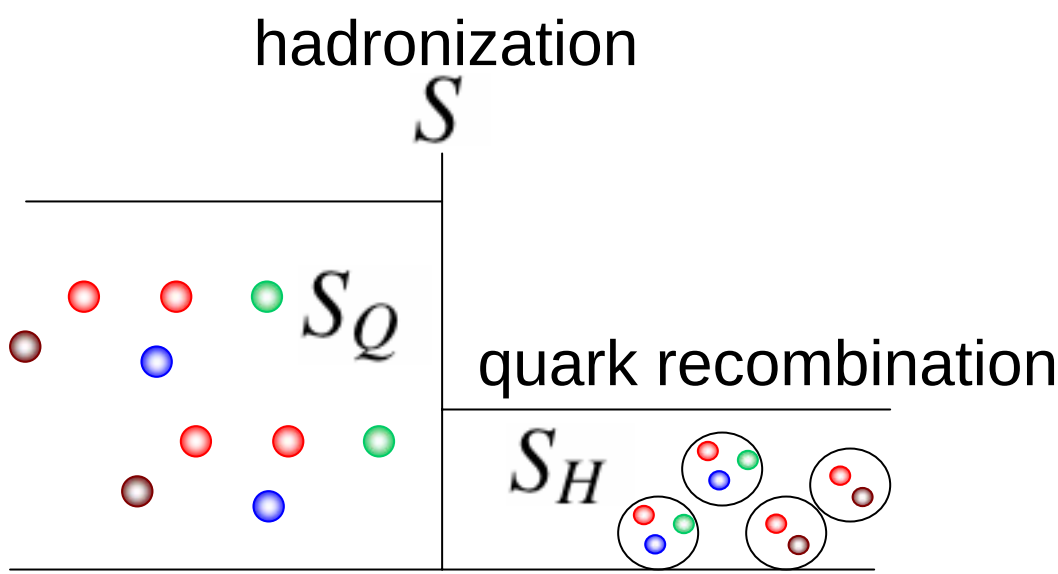


Correlations: Proof of Principle



- strong correlations from fragmentation, but suppressed by soft triggers
- combination of hard fragmentation and soft recombination correlations with a fixed correlation volume is compatible with data

Recombination: Entropy Puzzle



- Does recombination violate the 2nd law of thermodynamics ?
  - particle number decreases drastically in hadronization via reco…
  - ➢ restrict reco approach to intermediate momenta, ignore bulk…
  - ➢ decay of hadronic resonances as possible solution (Greco et al.)

➢ need estimate of entropy at hadronization

Entropy in the Hadronic Phase

1) resonance gas model:

- massless particles:

$$\frac{S}{N} = \begin{cases} 3.6 & \text{: bosons} \\ 4.2 & \text{: fermions} \end{cases}$$

- massive particles:

$$\frac{S}{N} = 3.5 + \frac{m}{T} \quad \left(\frac{m}{T} > 3\right)$$

$$T = T_c = 170 \text{ MeV}$$

$$\frac{S}{N} = \begin{cases} 4.72 & \text{: ground state} \\ 5.15 & \text{: resonances} \end{cases}$$

2) final state entropy from data:

$$\frac{dS}{dy} = 4450 \pm 400$$

Pal & Pratt, PLB578,310 (2004)

$$\frac{dN_{ch}}{dy} = 526 \pm 2(stat) \pm 36(sys)$$

STAR: PRC68, 044905 (2003)

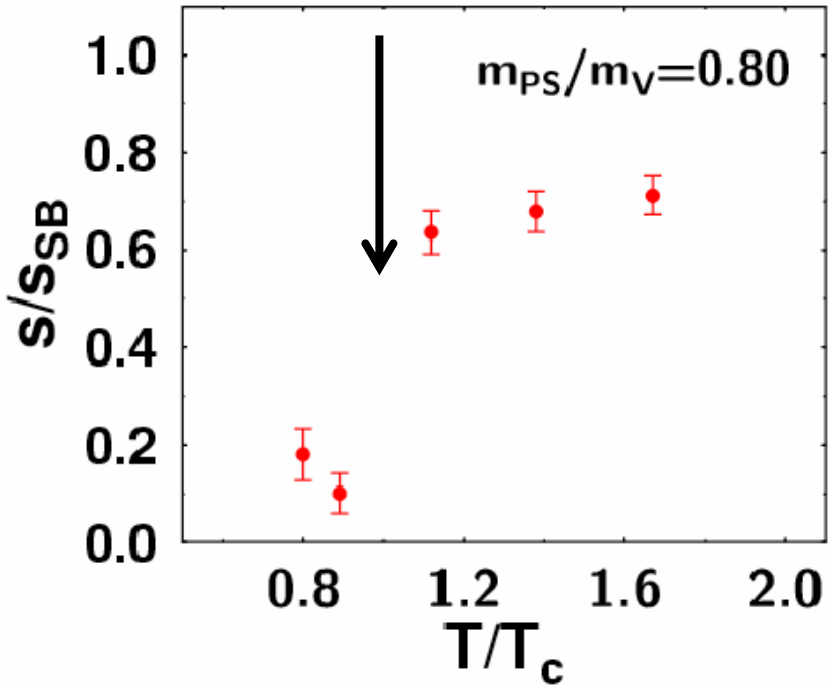
$$\frac{S}{N} \sim \left(\frac{dS/dy}{1.5 dN_{ch}/dy}\right) \sim 5.64 \pm 0.6$$

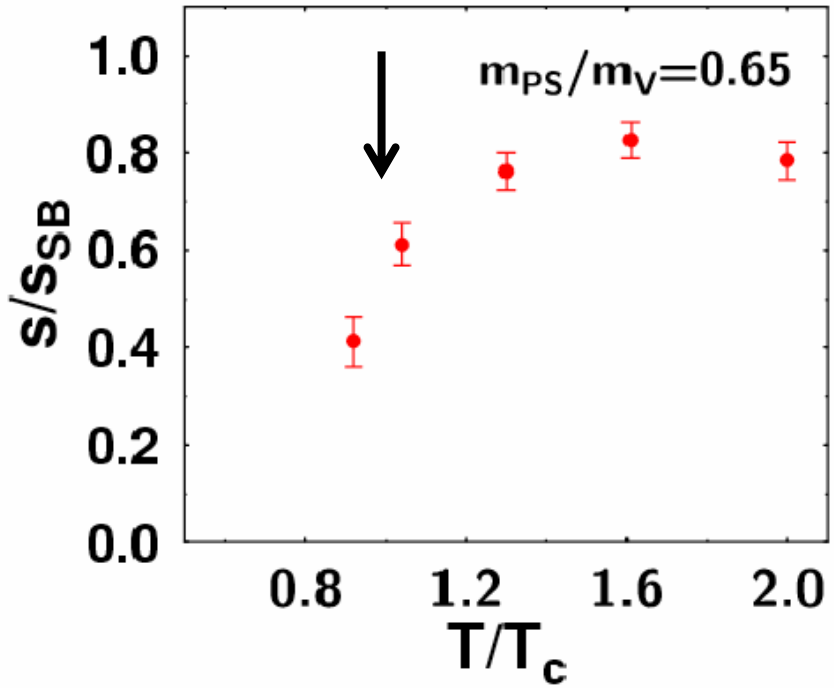
- Hydro+UrQMD calculations indicate a 5% increase in multiplicity due to rescattering in the hadronic phase

➢ entropy content is larger than often assumed!

Entropy in the Deconfined Phase

- Lattice QCD [CP-PACS with N_t=6 & N_f=2]





➢ entropy content of the deconfined phase near T_C is strongly reduced due to interactions!

systematic uncertainties include: thermodynamic limit, continuum limit, unphysically large quark mass…

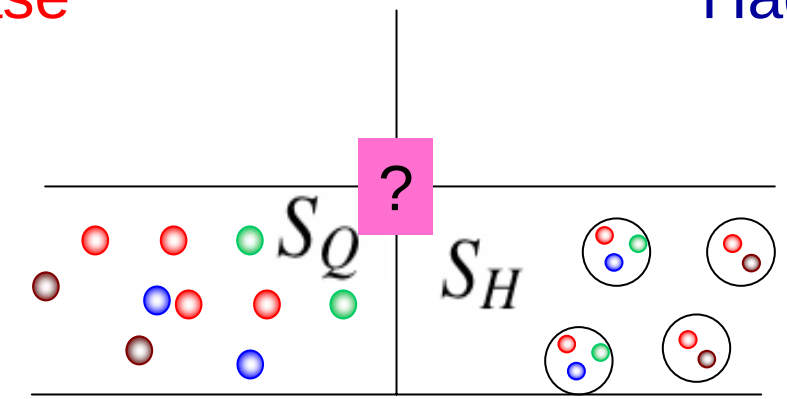
Entropy Puzzle resolved?

Quark Phase

s/s_{SB}

Hadron Phase

S/N



- S_Q near T_C is strongly reduced due to interactions.

- S_H is larger than often assumed.

Recombination may be compatible with the entropy constraint after all!

- No direct comparison of the entropy content of both phases:
  - Volume at hadronization ?
  - Number of quarks on the lattice ?

Thermal Recombination beyond the Valence Quark Approximation

- investigate effects of more sophisticated internal hadron structure
- use light-cone frame
- write hadron wavefunction as expansion in terms of Fock-States:

$$
\begin{aligned}
|M\rangle = & \int_0^1 dx_a dx_b\, \delta(x_a + x_b - 1)\, c_1(x_a, x_b) \left| q_\alpha(x_a)\, \bar{q}_\beta(x_b) \right\rangle \\
& + \int_0^1 dx_a dx_b dx_c\, \delta(x_a + x_b + x_c - 1)\, c_2(x_a, x_b, x_c) \left| q_\alpha(x_a)\, \bar{q}_\beta(x_b)\, g(x_c) \right\rangle \\
& + \int_0^1 dx_a dx_b dx_c dx_d\, \delta(x_a + x_b + x_c + x_d - 1)\, c_3(x_a, x_b, x_c, x_d) \left| q_\alpha(x_a)\, \bar{q}_\beta(x_b)\, q_\gamma(x_c)\, \bar{q}_\gamma(x_d) \right\rangle + \ldots
\end{aligned}
$$

Effects of dynamical quark mass $m(Q^2)$:

- assumption of quasi-free partons with current quark masses ~10 MeV valid for resolution scale $Q^2 > 1\ \mathrm{GeV}^2$
- for $Q^2 \sim (\pi T_C)^2 \approx 0.5\ \mathrm{GeV}^2$ degrees of freedom likely dominated by lowest Fock state (i.e. valence quark state)

Egalitarian Nature of Thermal Recombination

Boltzmann approximation:

- probability of finding a quark with momentum k=xP and energy E_k in a thermal medium is given by: $w_q(x) = \langle q(x) | \hat{\rho} | q(x) \rangle = e^{-E_k/T} = e^{-xP/T}$

  with ρ the thermal density matrix
- for a state with n partons one obtains:

$$\langle q(x_a)\, q(x_b) \ldots | \hat{\rho} | q(x_a)\, q(x_b) \ldots \rangle = w_q(x_a)\, w_q(x_b) \ldots = e^{-(x_a + x_b + \ldots)P/T}$$

- emission probability for a single Fock state reads:

$$W_{q\bar{q}} = \int_0^1 dx_a dx_b\, \delta(x_a + x_b - 1) \left| c_1(x_a, x_b) \right|^2 \langle q(x_a)\, \bar{q}(x_b) | \hat{\rho} | q(x_a)\, \bar{q}(x_b) \rangle = C_1\, e^{-P/T}$$

$$W_{q\bar{q}g} = \int_0^1 dx_a dx_b dx_c\, \delta(x_a + x_b + x_c - 1) \left| c_2(x_a, x_b, x_c) \right|^2 \langle q(x_a)\, \bar{q}(x_b)\, g(x_c) | \hat{\rho} | q(x_a)\, \bar{q}(x_b)\, g(x_c) \rangle = C_2\, e^{-P/T}$$

- combining all contributions from all Fock states, one obtains:

$$W(P) = W_{q\bar{q}} + W_{q\bar{q}g} + W_{q\bar{q}q\bar{q}} + \ldots = \left(C_1 + C_2 + C_3 + \ldots \right) e^{-P/T} = e^{-P/T}$$

- emission probability of complex state from a thermal ensemble is independent of degree of complexity of the structure of the state

Higher Fock States: v_2 Scaling Violations

Generalization of scaling law to higher Fock states:

- assume all partons carry roughly equal momentum $x_i \approx 1/n_v$ with n_v the number of partons in the Fock state

$$v_2^{(H)}(P) \approx \sum_\nu C_\nu \, n_\nu \, v_2(P/n_\nu)$$

- valence quark approximation: v=1, n_1=2,3 and C_1=1

➢ scaled v_2 for mesons and baryons:

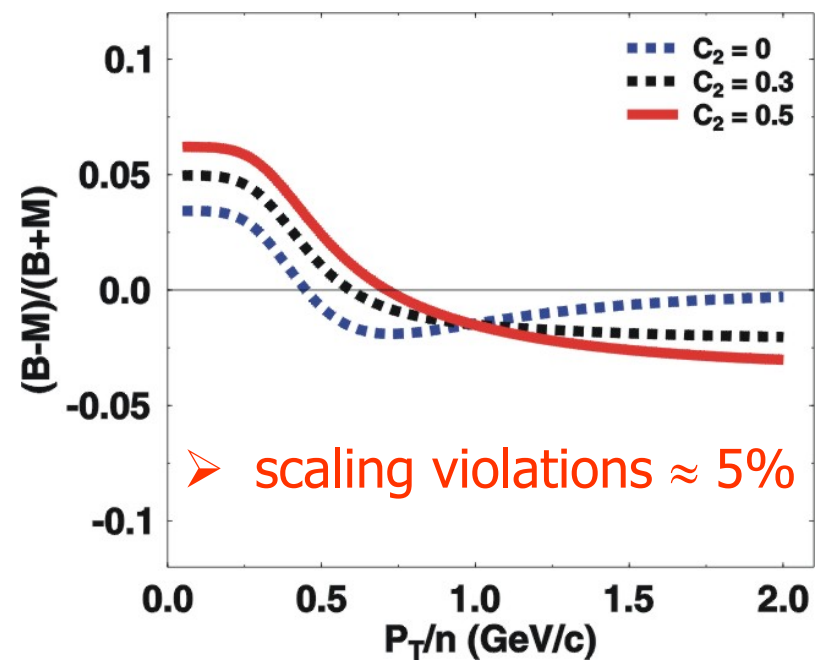
$$\tilde{v}_2^{(M)}(p) = \sum_\nu C_\nu^{(M)} \frac{n_\nu^{(M)}}{2} v_2\left(2p/n_\nu^{(M)}\right)$$

$$\tilde{v}_2^{(B)}(p) = \sum_\nu C_\nu^{(B)} \frac{n_\nu^{(B)}}{3} v_2\left(3p/n_\nu^{(B)}\right)$$

- for valence quark approx:

$$\tilde{v}_2^{(M)}(p) = \tilde{v}_2^{(B)}(p) = v_2(p)$$

(scaled v_2 identical to parton v_2)



➢ scaling violations ≈ 5%

Summary & Outlook

The Recombination + Fragmentation Model:

- provides a natural solution to the baryon puzzle at RHIC
- describes the intermediate and high p_t range of
  - hadron ratios & spectra
  - jet-quenching phenomena
  - elliptic flow
  - leading / next-to-leading particle correlations (work in progress)
  - treatment of gluons & higher Fock states

issues to be addressed in the future:

- realistic space-time dynamics of parton source
- need improved data of identified hadrons at high p_t

The End