Event-by-Event Fluctuations from PHENIX

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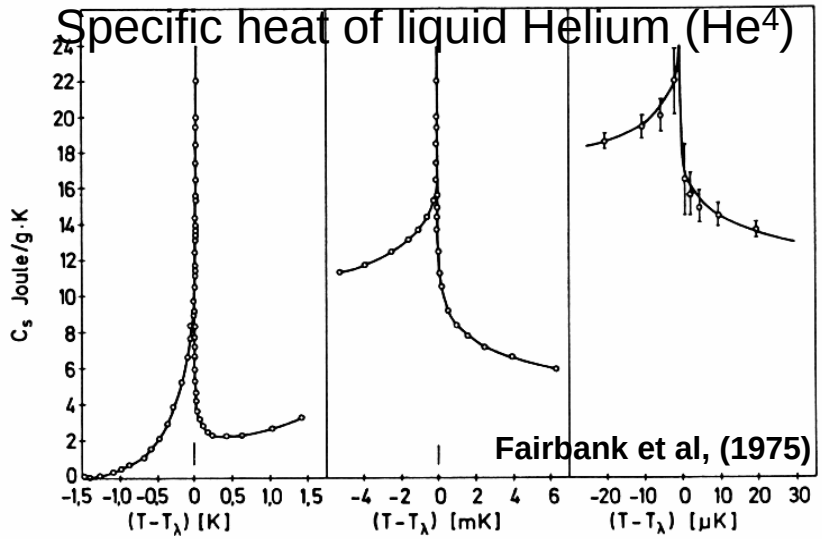
for the PHENIX collaboration

Hiroshima University

Correlations & Fluctuations Workshop at

2005 RHIC & AGS Annual User's Meeting

Phase transition



- According to the classical classification of the phase transition by Ehrenfest, the order of phase transition is defined by the discontinuity in the derivative of free energy.
- In this context, discontinuity in the variables of statistical mechanics or order parameter as a function of the temperature or time evolution is useful to search the critical point of phase transition.
- In particular, the second order phase transitions are often accompanied by the divergence with respect to variables of statistical mechanics or order parameters as critical phenomena.

Variables of statistical mechanics as order parameters

$$\Phi(h) = -\left(\frac{\partial G}{\partial h}\right)_h$$

Φ : extensive variables (G, V, S, N, $\chi \cdots$)

h : intensive variables (T, P, $\cdots$)

G : Gibbs free energy

First order susceptiblity :

$$\chi = -\left(\frac{\partial G}{\partial h}\right)_{h'}$$

voulume : $V = \left(\frac{\partial G}{\partial P}\right)_T$

entropy : $S = -\left(\frac{\partial G}{\partial T}\right)_P$

compressibility : $\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_{T,S}$

Secondorder susceptiblity :

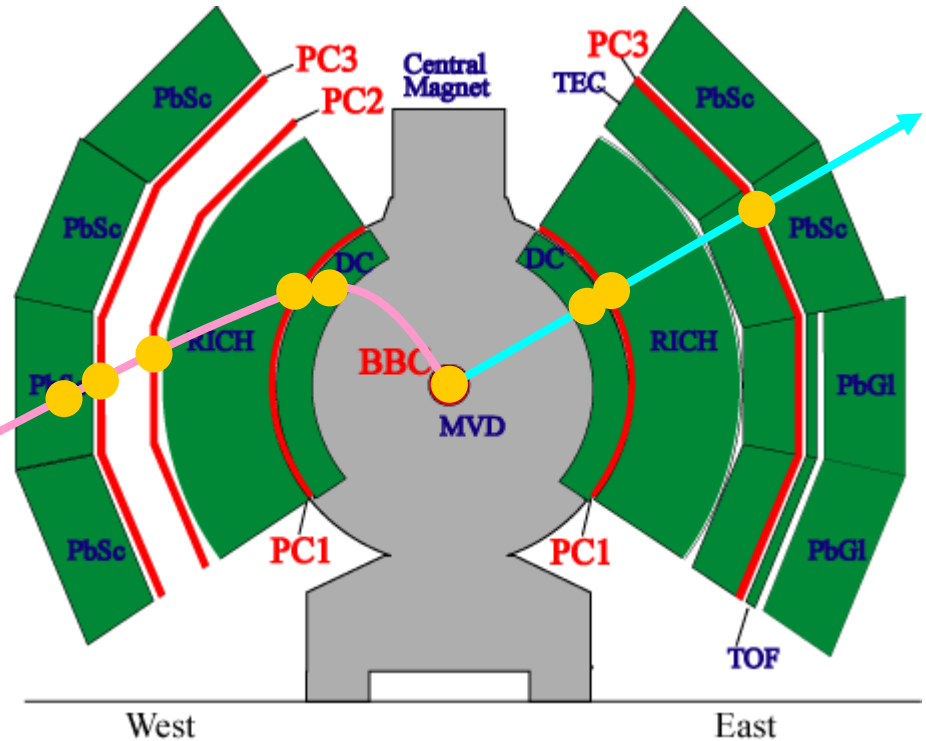
$$\chi = -\left(\frac{\partial^2 G}{\partial h^2}\right)_{h'}$$

specific heat : $C_h = -T\left(\frac{\partial^2 G}{\partial T^2}\right)_h$

correlation length: $\chi_k = \frac{\chi(T)}{1 + k^2 \xi^2}$

Event-by-event fluctuations in heavy-ion collisions measured by PHENIX

- Some variables of statistical mechanics and order parameters of phase can be obtained from event-by-event fluctuations.
- Correlation length can be extracted from the event-by-event multiplicity fluctuations by scanning the phase-space.
- Specific heat can be calculated from the results of event-by-event average p_T fluctuation.
- We have performed measurements of these fluctuations to explore the QCD phase transition accompanied by critical phenomena using the PHENIX spectrometer at RHIC.



Geometrical acceptance

$\Delta\eta < 0.7$

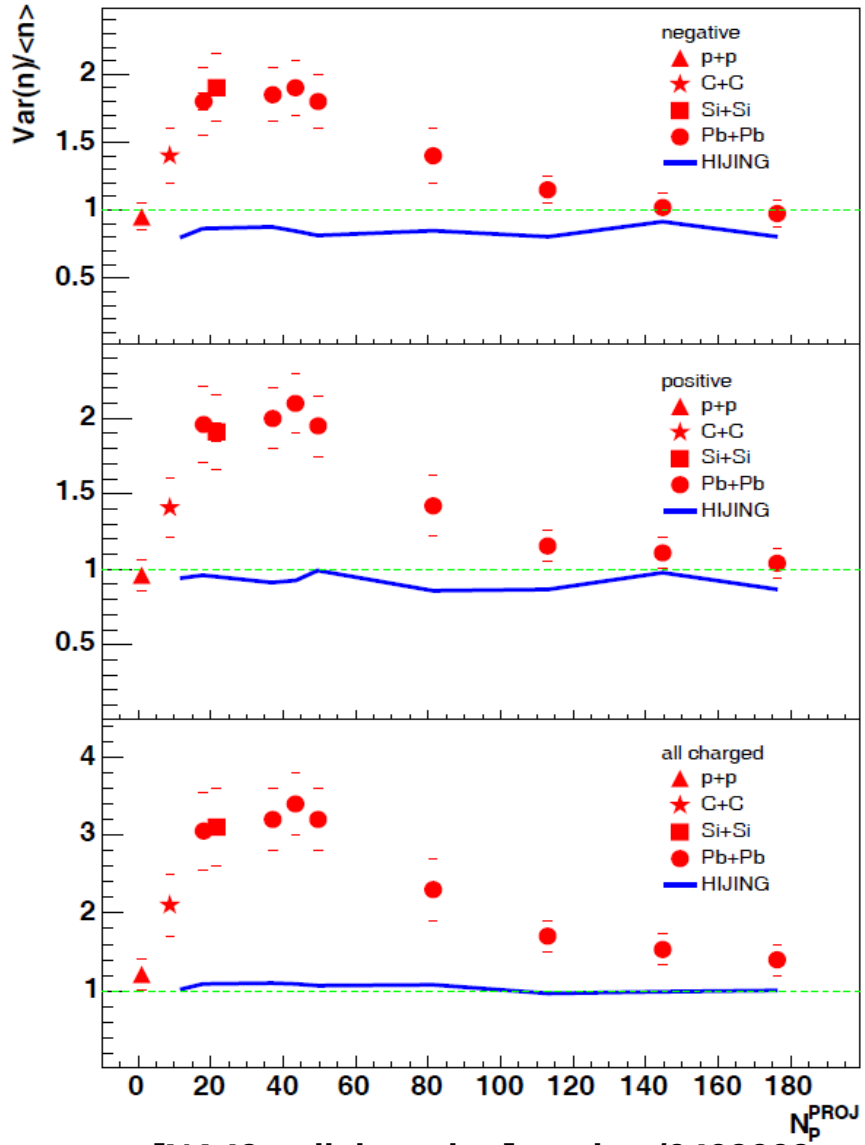
$\Delta\phi < \pi$

Momentum range

$0.2 < p_T < 2.0\, GeV/c$

Multiplicity fluctuations by variance

NA49: p+p, C+C, Si+Si, Pb+Pb 158 A GeV at SPS



[NA49 collaboration] nucl-ex/0409009

Variance of the multiplicity distribution is defined as;

$$< N >= \sum N \cdot P(N)$$

$$Var(N) = \sum (N - < N >)^2 P(N)$$

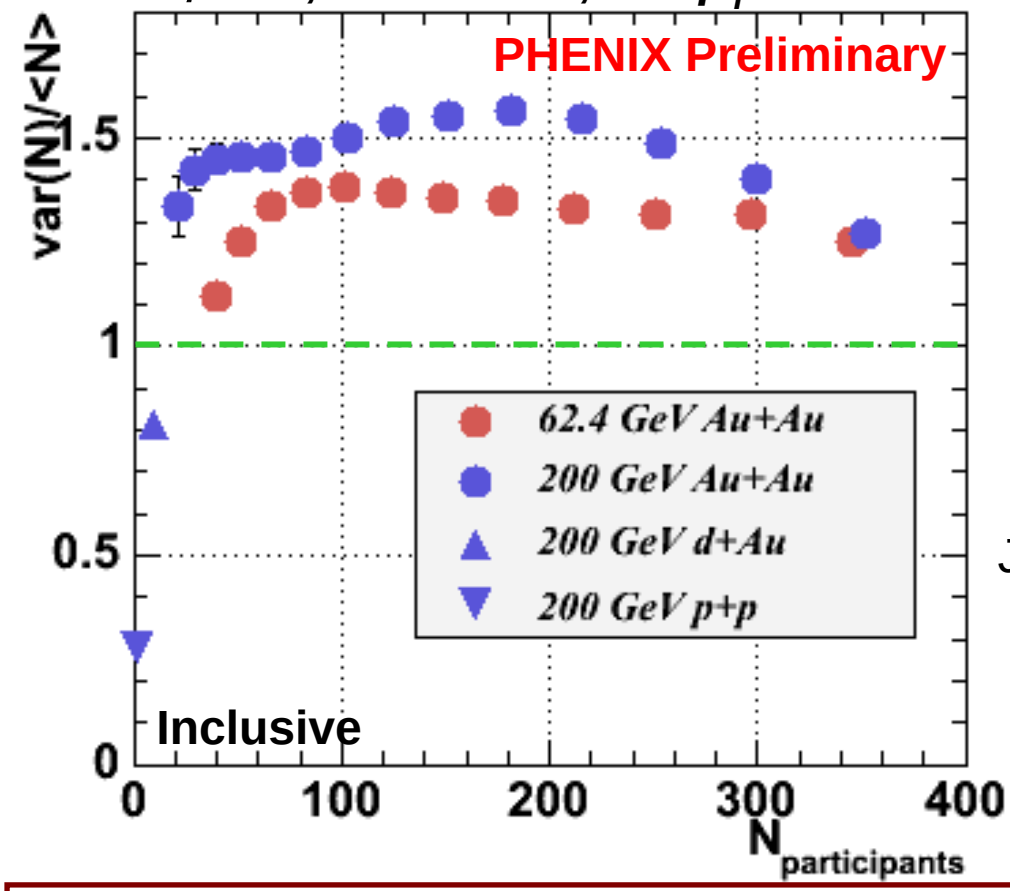
$$= < N^2 > - < N >^2$$

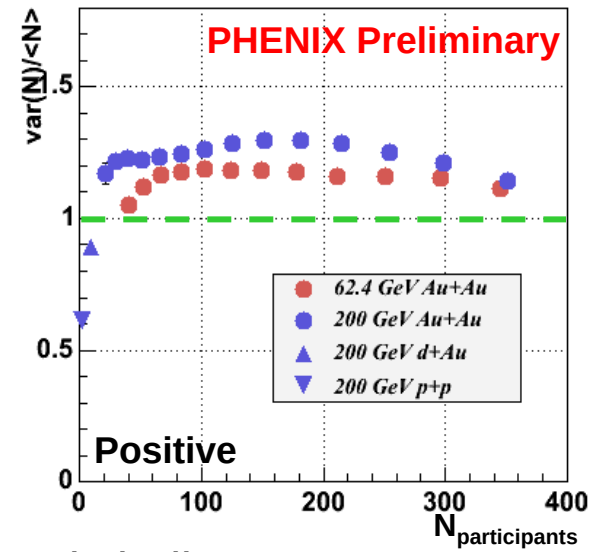
Normalized variance, *Var(N)/<N>,* is used as an observable of multiplicity fluctuation. In the case of Poisonnian distribution, the variance equals mean value and this observable indicates 1.

Deviations of multiplicity fluctuation from Poisonnian distribution was reported in the middle range of the number of projectile participant nucleon at SPS energy.

Multiplicity fluctuations in PHENIX

$\Delta\eta$ <0.7, $\Delta\Phi < \pi$ rad, 0.2<p_T<2.0 GeV/c





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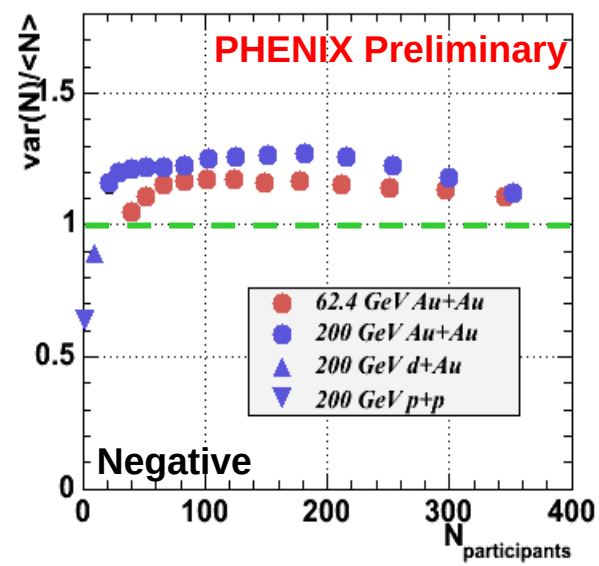
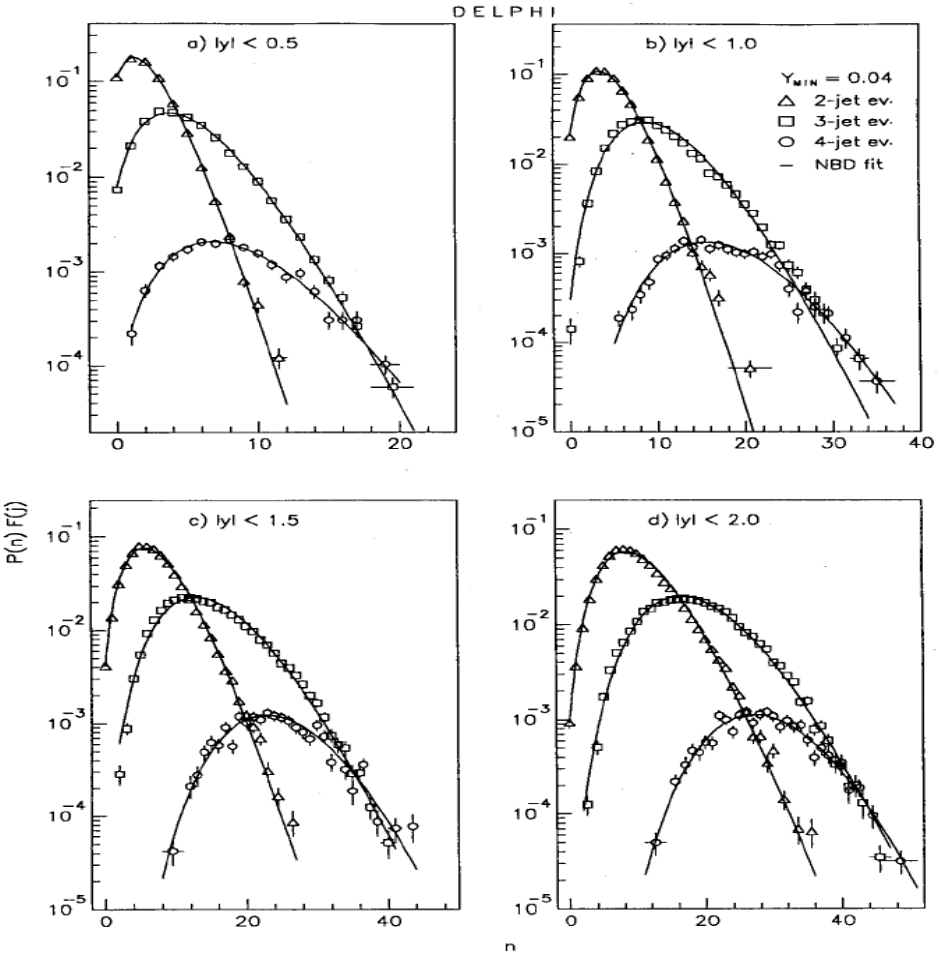


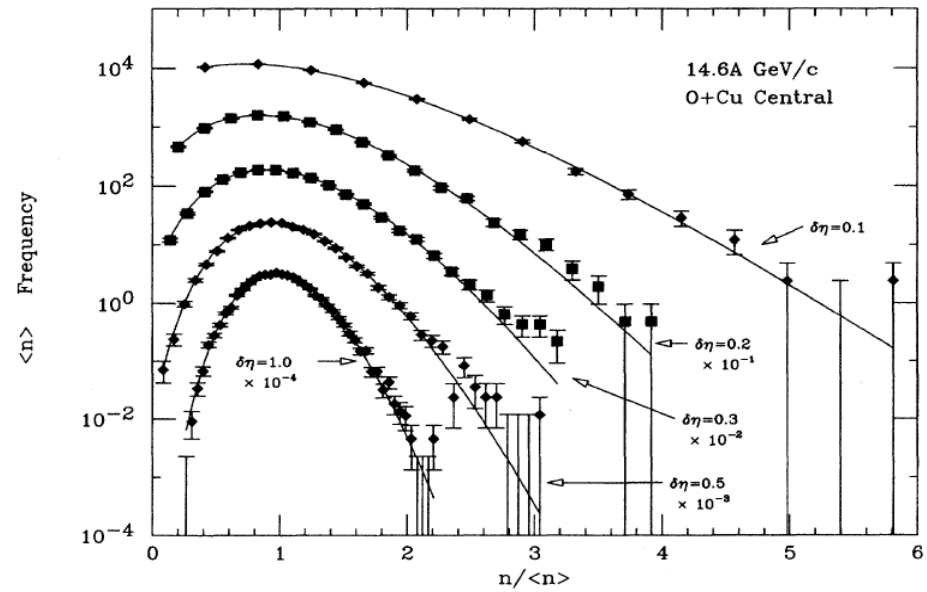
A different behavior of the multiplicity fluctuations as function of number of participants is observed at RHIC energies as compared to SPS. There are no difference about the multiplicity fluctuation between positive and negative charge.

Charged particle multiplicity distribution and negative binomial distribution (NBD)

DELPHI: Z^0 hadronic Decay at LEP
2,3,4-jets events



E802: ^{16}O+Cu 16.4 A GeV at AGS
most central events



[DELPHI collaboration] Z. Phys. C56 (1992) 63
[E802 collaboration] Phys. Rev. C52 (1995) 2663

Universally, hadron multiplicity distributions in high energy collisions conform to the NBD.

Negative binomial distribution (NBD)

$$P_n = n^{-n} / (1+\mu)^{n+1}$$

Bose-Einstein distribution
μ : average multiplicity

$$P_n^{(k)} = \frac{\Gamma(n+k)}{\Gamma(n-1)\Gamma(k)} \left(\frac{\mu / k}{1+\mu / k} \right)^n \frac{1}{(1+\mu / k)^k}$$

$$\frac{\sigma^2}{\mu^2} = \frac{1}{\mu} + \frac{1}{k(\delta)}$$

$$\sigma \equiv \sqrt{<n^2> - <n>^2}$$

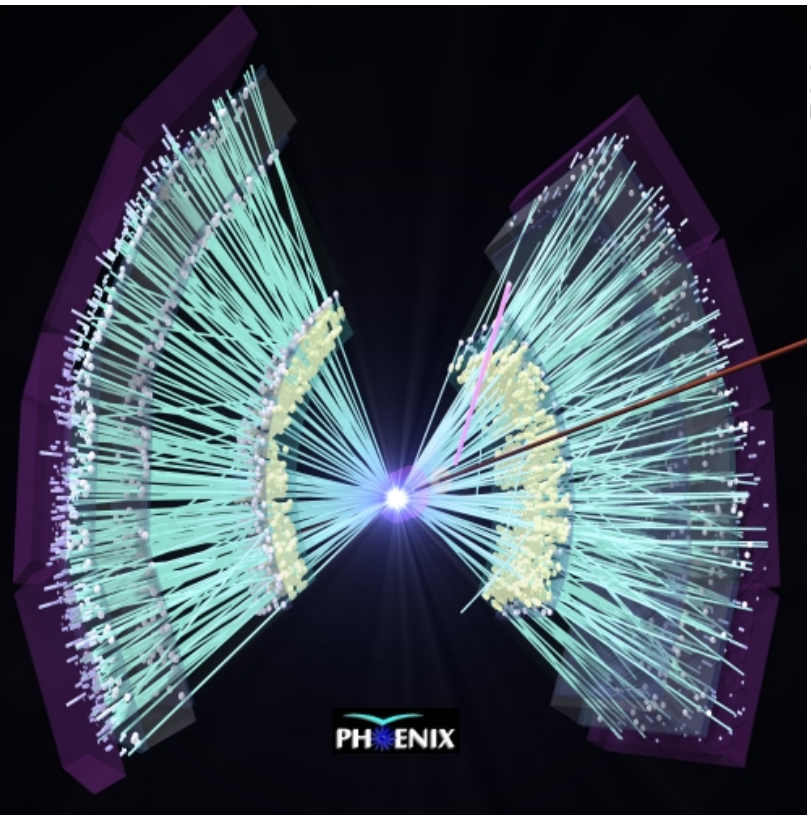
$$\frac{1}{k(\delta)} = \frac{\sigma^2}{\mu^2} - \frac{1}{\mu} = F_2(\delta) - 1$$

$$F_2(\delta) = \frac{<n>^2 - <n>}{<n>^2}$$

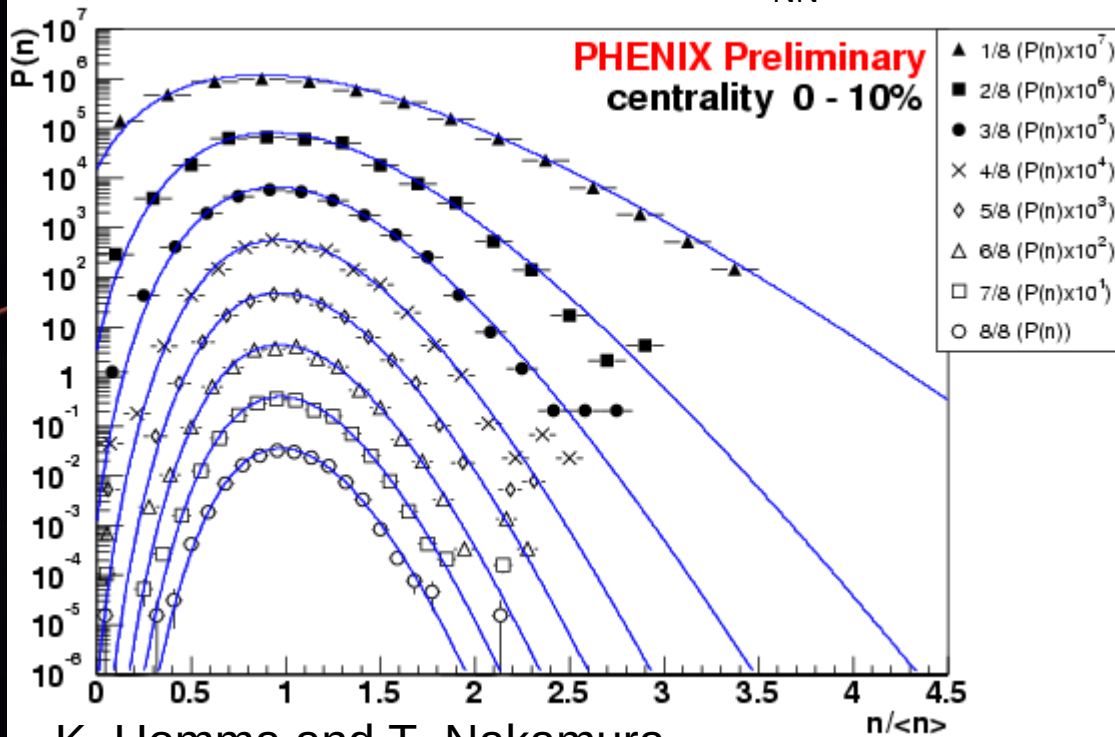
F_2 : second order normalized factorial moment

NBD correspond to multiple Bose-Einstein distribution and the parameter k indicates the multiplicity of Bose-Einstein emission sources. It is also corresponds to the Poison distribution with the infinite k value in the statistical mathematics.

Charged particle multiplicity distributions in PHENIX



PHENIX: Au+Au $\sqrt{s_{NN}}$=200GeV



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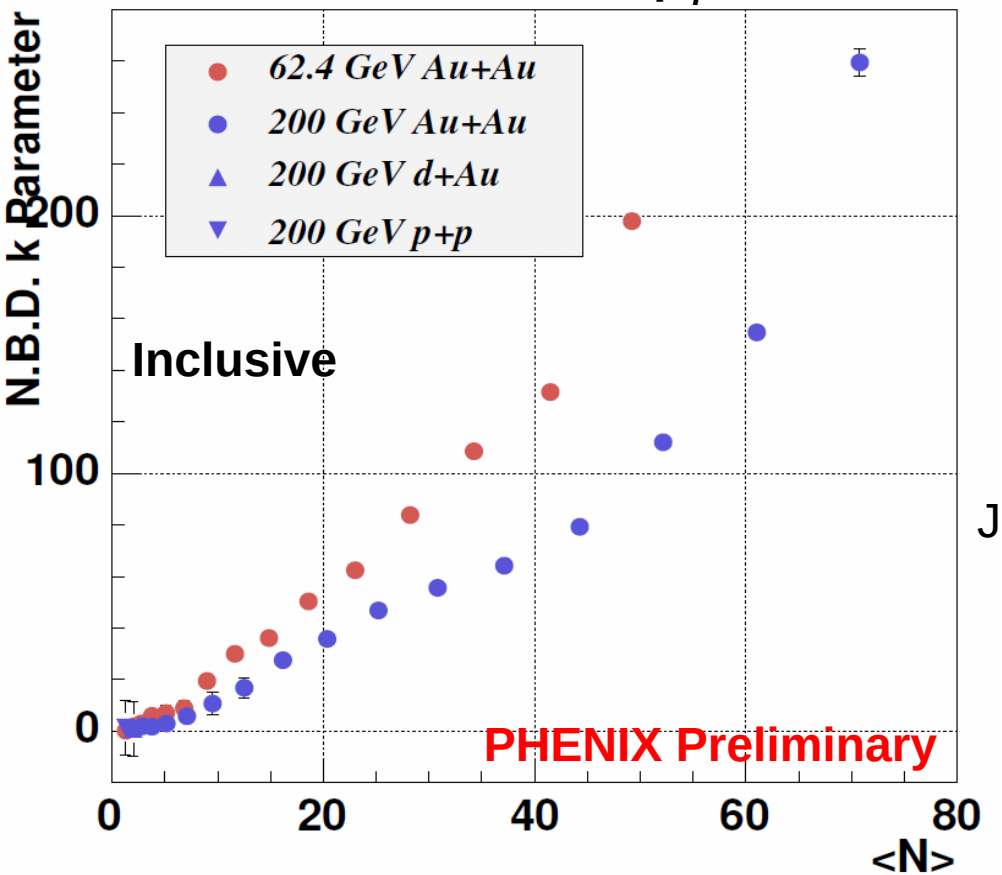
No magnetic field condition
$\Delta\eta$ <0.7, $\Delta\Phi < \pi/2$ rad

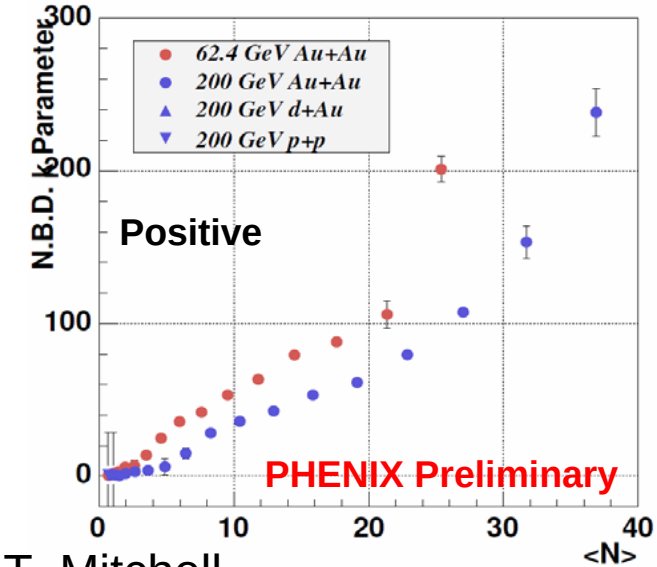
$\delta\eta = 0.09$ (1/8) : P(n) x 10^7
$\delta\eta = 0.18$ (2/8) : P(n) x 10^6
$\delta\eta = 0.35$ (3/8) : P(n) x 10^5
$\delta\eta = 0.26$ (4/8) : P(n) x 10^4
$\delta\eta = 0.44$ (5/8) : P(n)

Multiplicity distributions observed in Au+Au, d+Au and p+p collisions at PHENIX also conform to the negative binomial distribution.

NBD k parameter as a function of average multiplicity

$\Delta\eta$ <0.7, $\Delta\Phi < \pi$ rad, 0.2<p_T<2.0 GeV/c





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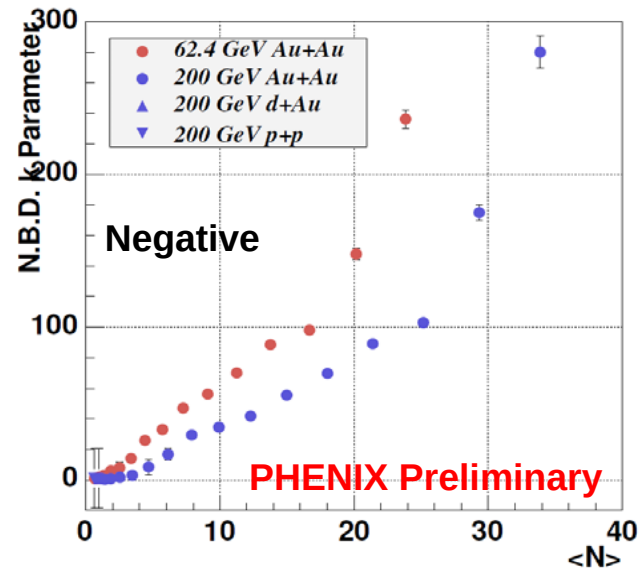
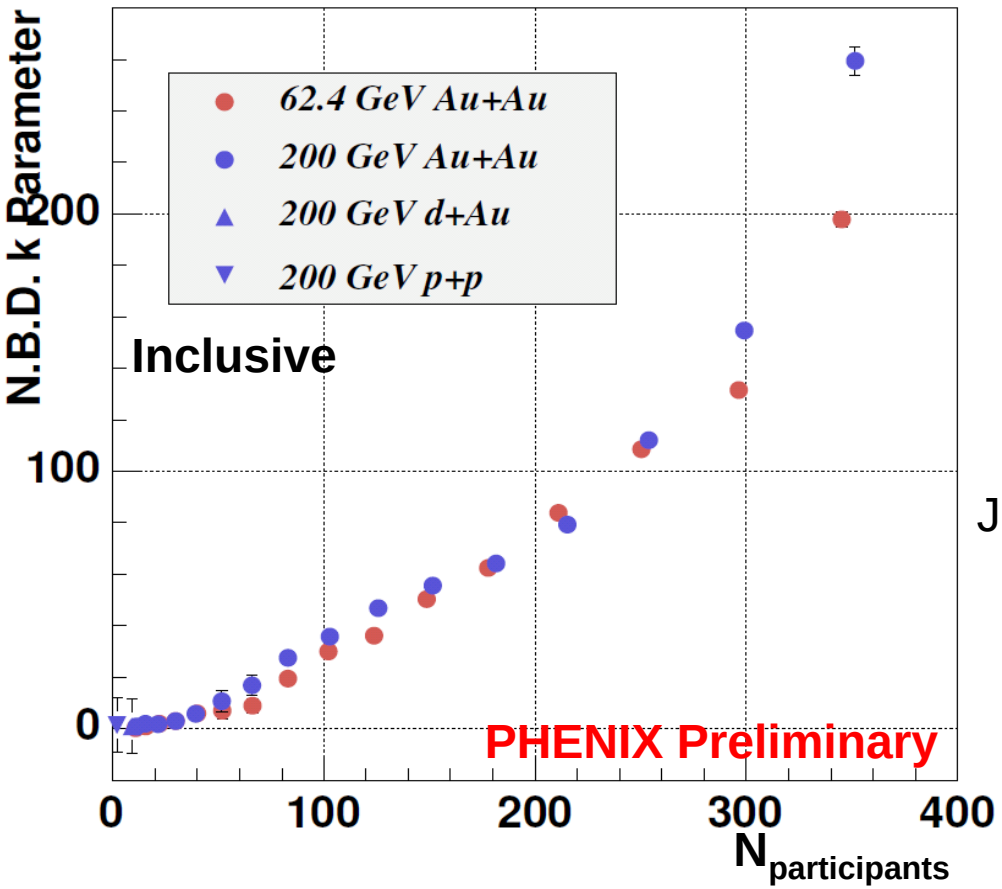
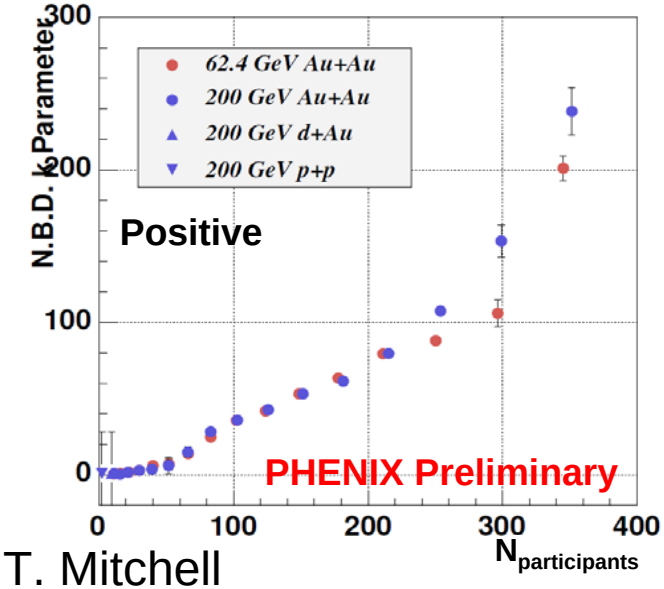


There is the difference about the average multiplicity dependence of NBD k parameters between the 200GeV and 62.4 GeV.

NBD k parameter as a function of number of participants

$\Delta\eta$ <0.7, $\Delta\Phi < \pi$ rad, 0.2<p_T<2.0 GeV/c





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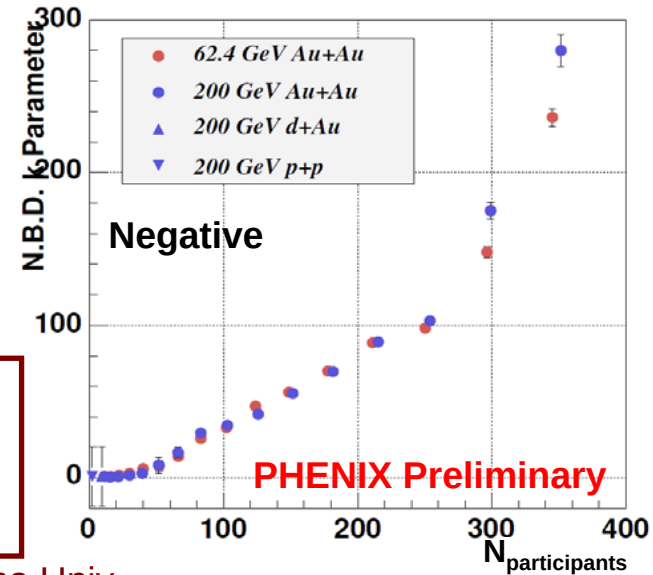


NBD k parameters as an observable of multiplicity fluctuation are not scaled by the average multiplicity but scaled by the number of participants.

Extraction of the correlation length from NBD k parameter

Normalized correlation function

$$R(y_1, y_2) = \frac{C_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_1(y_2)} - 1$$

$\rho_1(y)$: inclusive single particle density

$\rho_2(y_1, y_2)$: inclusive two-particle density

$C_2(y_1, y_2)$: two-particle correlation function

Relation with NBD k parameter

$$\frac{1}{k(\delta\eta)} = F_2 - 1 = \frac{\int^{\delta\eta} dy_1 dy_2 C_2(y_1, y_2)}{< n(\delta\eta) >^2}$$

Correlation function used in E802

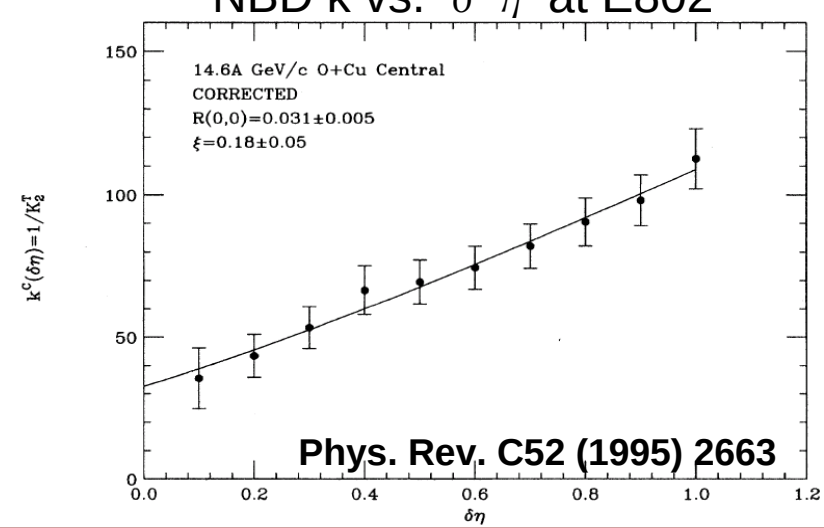
P. Carruthers and Isa Sarcevic, Phys. Rev. Lett. 63 (1989) 1562

$$C_2 = 1 + R(0,0)e^{-|y_1 - y_2|/\xi}$$

$$k(\delta\eta) = \frac{1}{R_0} \frac{\delta\eta/2\xi}{[1-(\xi/\delta\eta)(1-e^{-\delta\eta/\xi})]}$$

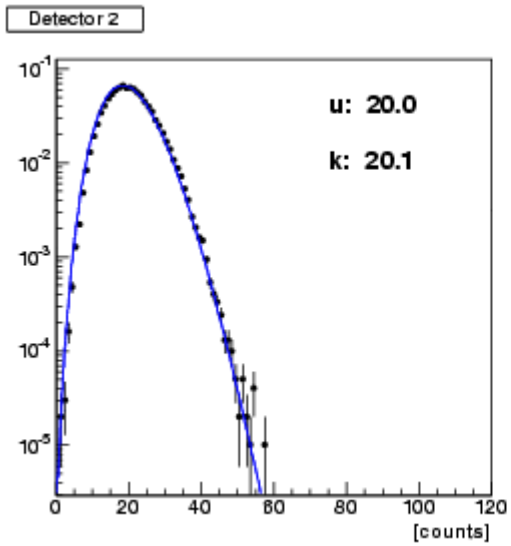
ξ : correlation length

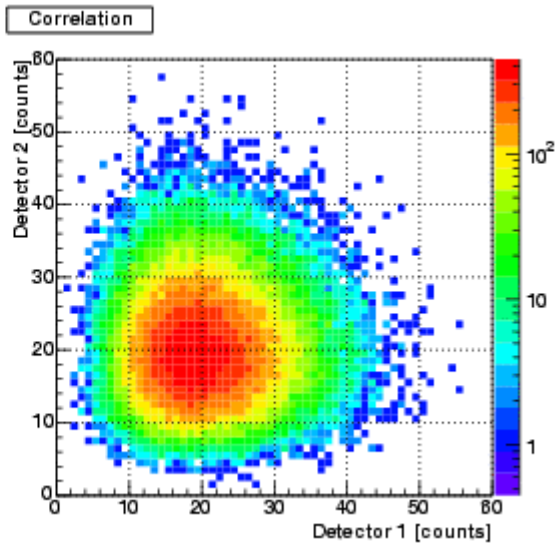
NBD k vs. $\delta\eta$ at E802



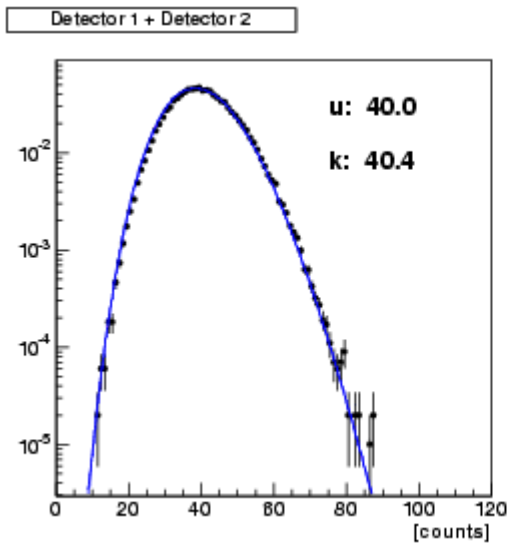
Phys. Rev. C52 (1995) 2663

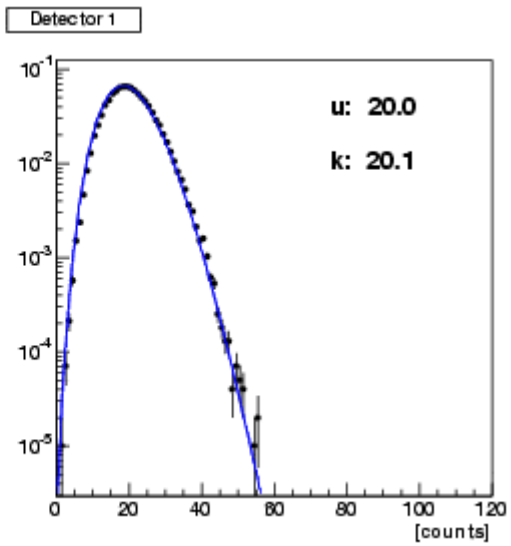
Random particle emission pattern based on NBD





Detector 1	Detector 2

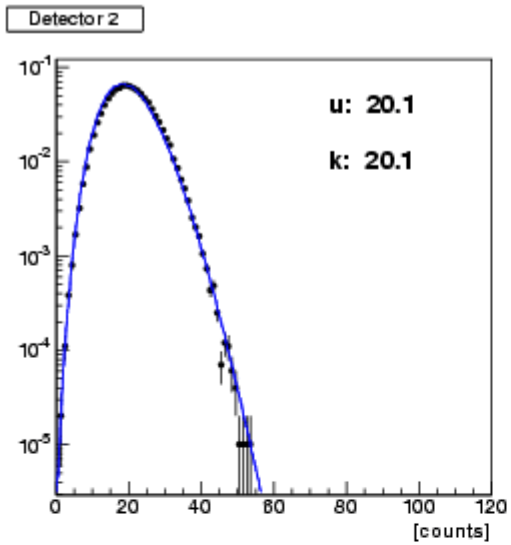


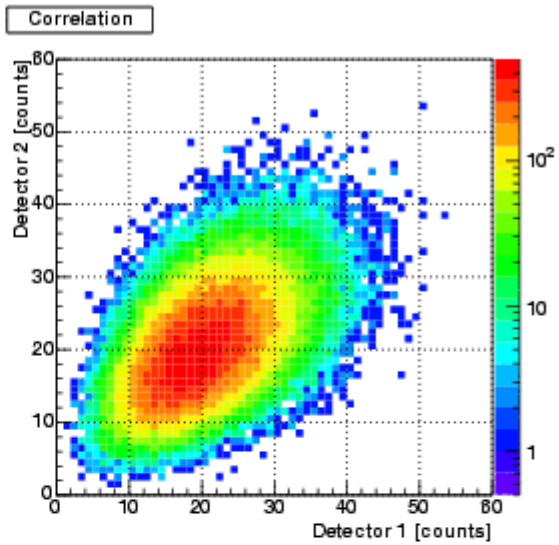


Emission source

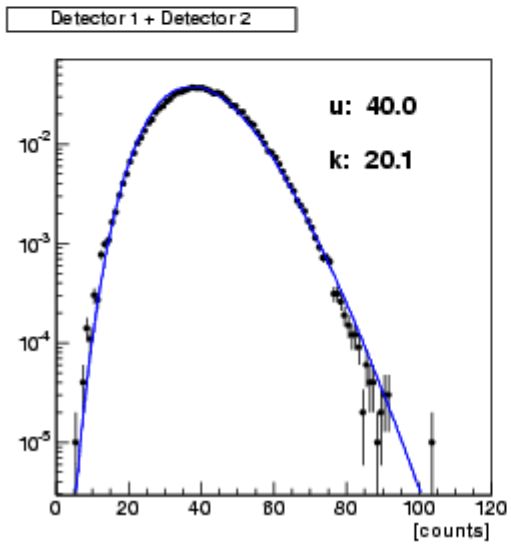
In the case of there are no correlation about the particle emission, the value of NBD k parameters are summed up.

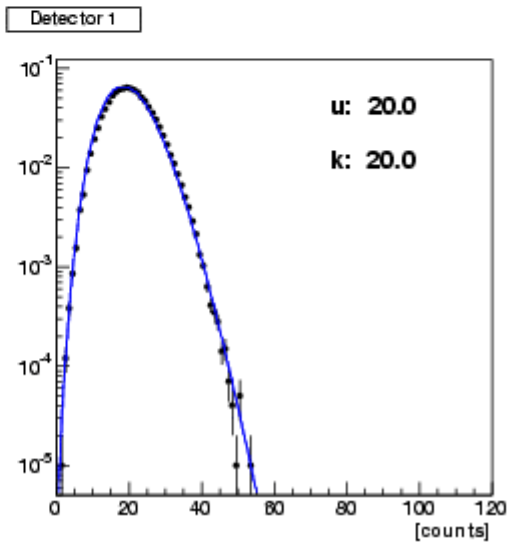
Correlated particle emission pattern into the phase-space





Detector 1	Detector 2



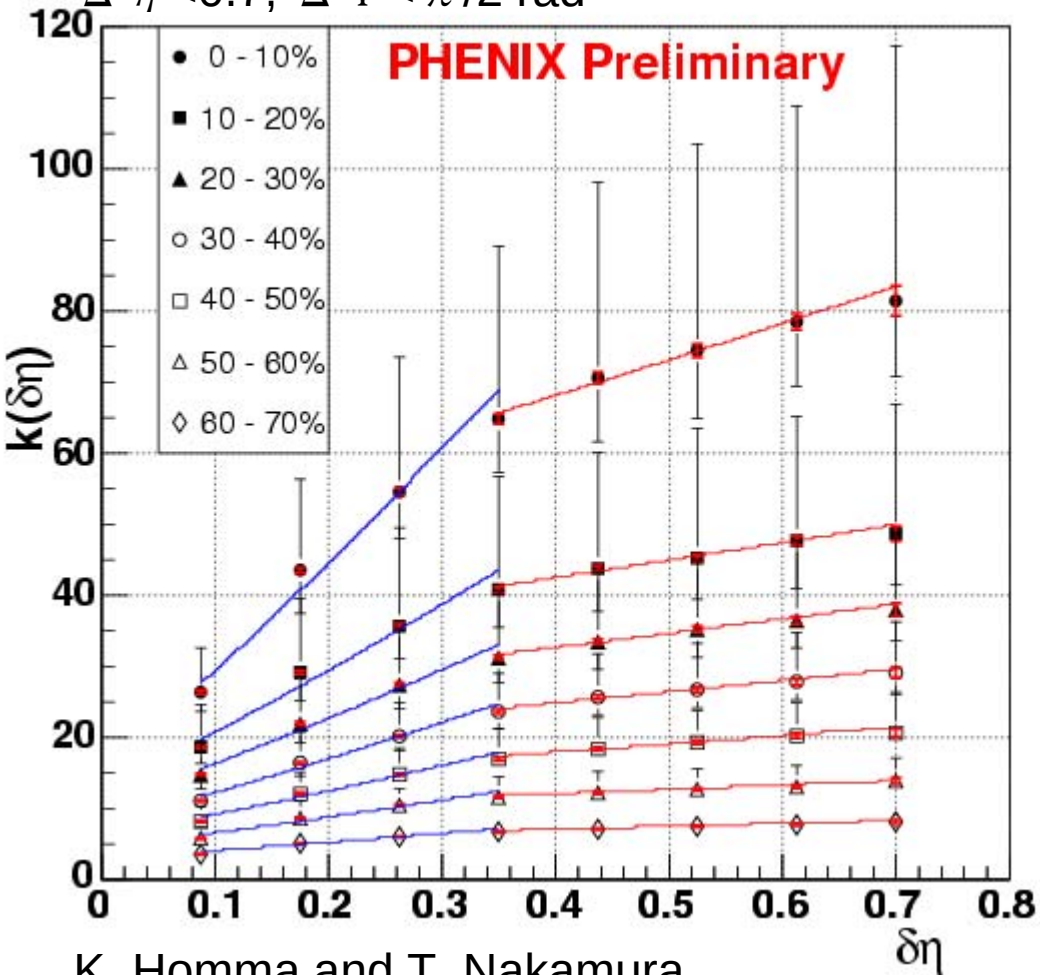


Emission source

If there are correlations, NBD k parameters do not increase according to the size of detector acceptance.

$\delta\eta$ dependence of NBD k parameter

Au+Au 200 GeV, no magnetic field
$\Delta\eta$ <0.7, $\Delta\Phi < \pi/2$ rad



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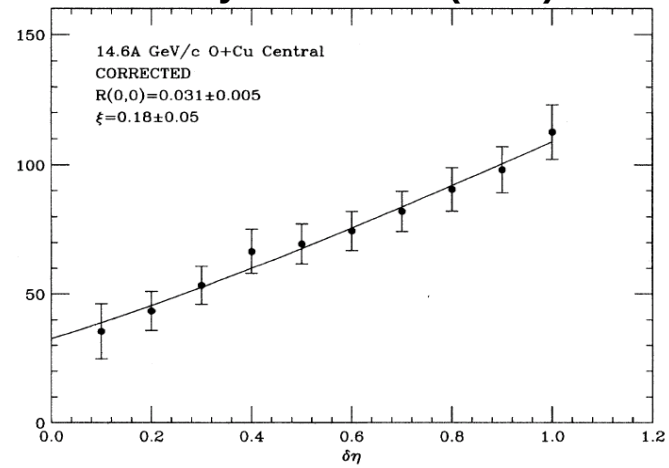
Blue: $\delta\eta \leq 0.35$
Red : $\delta\eta \geq 0.35$

E802: Phys. Rev. C52 (1995) 2663



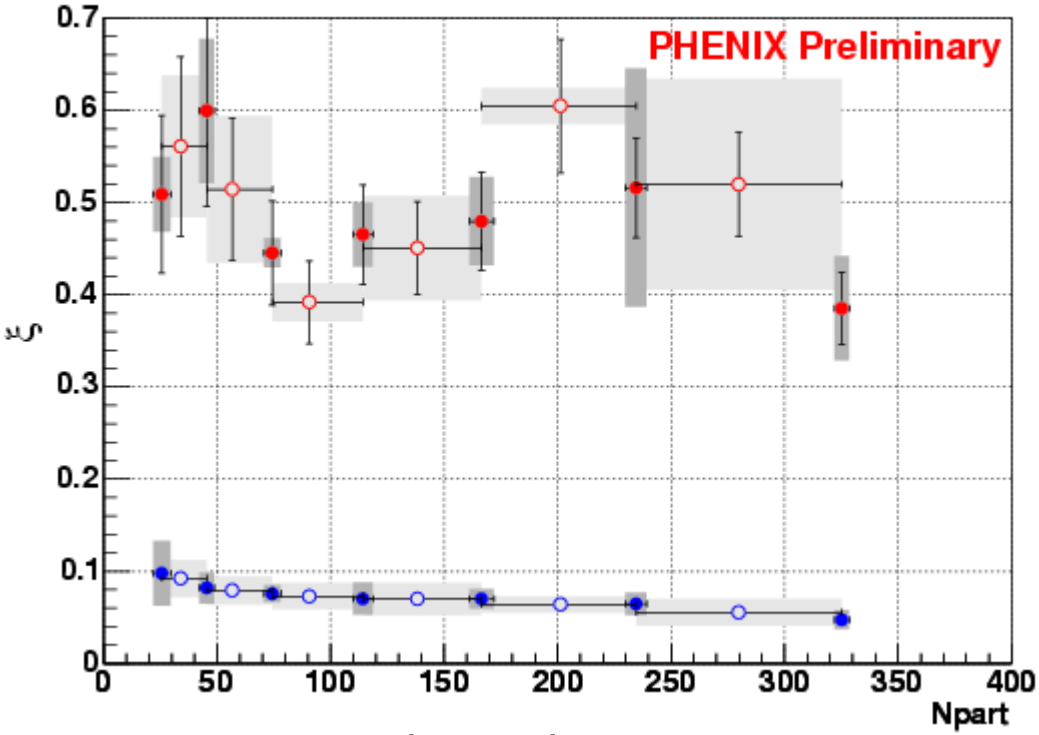
- Fitting funciton

$$k(\delta\eta) = \frac{1}{R_0}\frac{\delta\eta/2\xi}{[1-(\xi/\delta\eta)(1-e^{-\delta\eta/\xi})]}$$

NBD fit was performed for the different range of pseudo rapidity gap as shown in blue and red curve to extract the correlation length using the E802 type correlation function.

Number of participants dependence of correlation length ξ

Au+Au 200 GeV, no magnetic field
$\Delta\eta$ <0.7, $\Delta\Phi < \pi/2$ rad



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- **Fitting Range**

Blue: $\delta\eta \leq 0.35$
Red : $\delta\eta \geq 0.35$

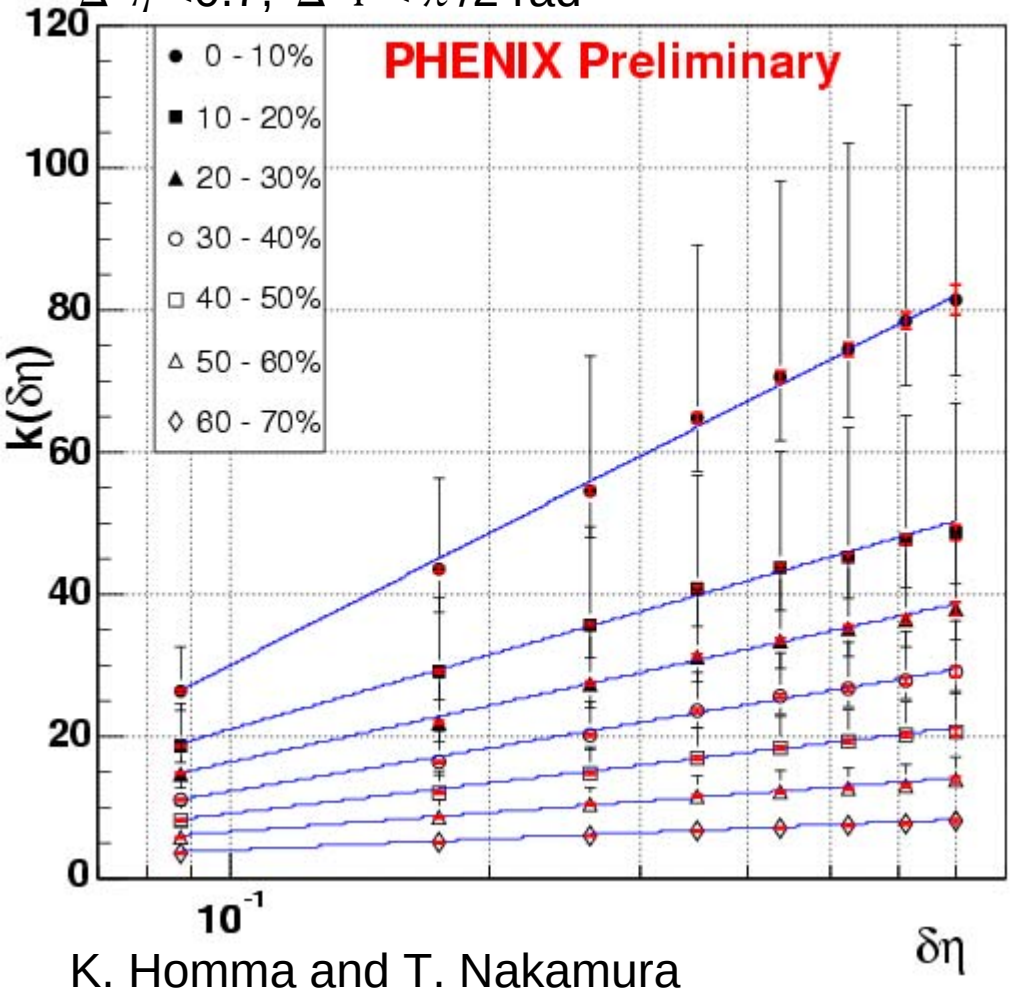
- **Centrality**

filled circle : 0-70 % (10% interval)
open circle : 5-65 % (10% interval)

Different behaviors about the extracted correlation length (ξ) as a function of number of participants are observed in the different range of the pseudo rapidity gap. The correlation length at the range of large pseudo rapidity gap has a large fluctuation.

Linear behavior of NBD k as a function of logarithmic $\delta\eta$

Au+Au 200 GeV, no magnetic field
$\Delta\eta$ <0.7, $\Delta\Phi < \pi/2$ rad



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- **Fitting function**
  k($\delta\eta$) = c1 + c2 $\times$ ln($\delta\eta$)
  c1, c2 : constant
- **Fitting Range**
  0.09 $\leq \delta\eta \leq$ 0.7

Relations of fluctuation and normalized factorial moments and fractal structure.

$$\frac{1}{k(\delta\eta)} + 1 = F_q(\delta\eta) \propto (\delta\eta)^{-\phi_q}$$

This power low relation of fluctuations and pseudo rapidity gap might suggest self similarity of correlation length!!...?

Correlation functions and correlation length

Used in E802

$$C_2 = 1 + R(0,0)e^{-|y_1 - y_2|/\xi}$$

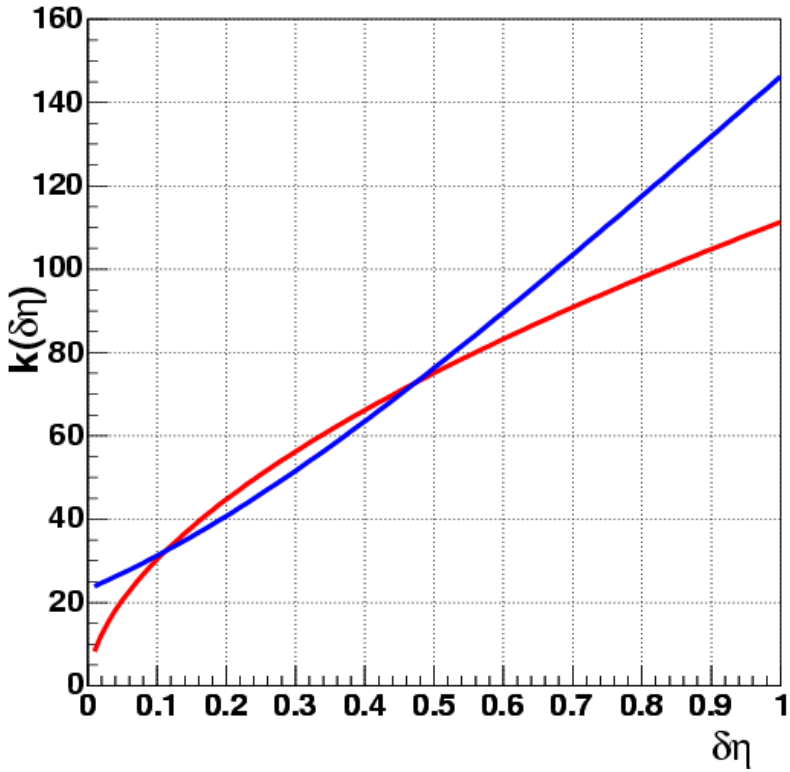
$$k(\delta\eta) = \frac{1}{R_0}\frac{\delta\eta/2\xi}{[1-(\xi/\delta\eta)(1-e^{-\delta\eta/\xi})]}$$

General correlation function

$$C_2 = 1 + \frac{R_0}{|y_1 - y_2|^{\alpha}} e^{-|y_1 - y_2|/\xi}$$

$$k(\delta\eta) = \frac{\delta\eta}{\int^{\delta\eta} \frac{R_0}{y^{\alpha}} e^{-y/\xi} dy}$$

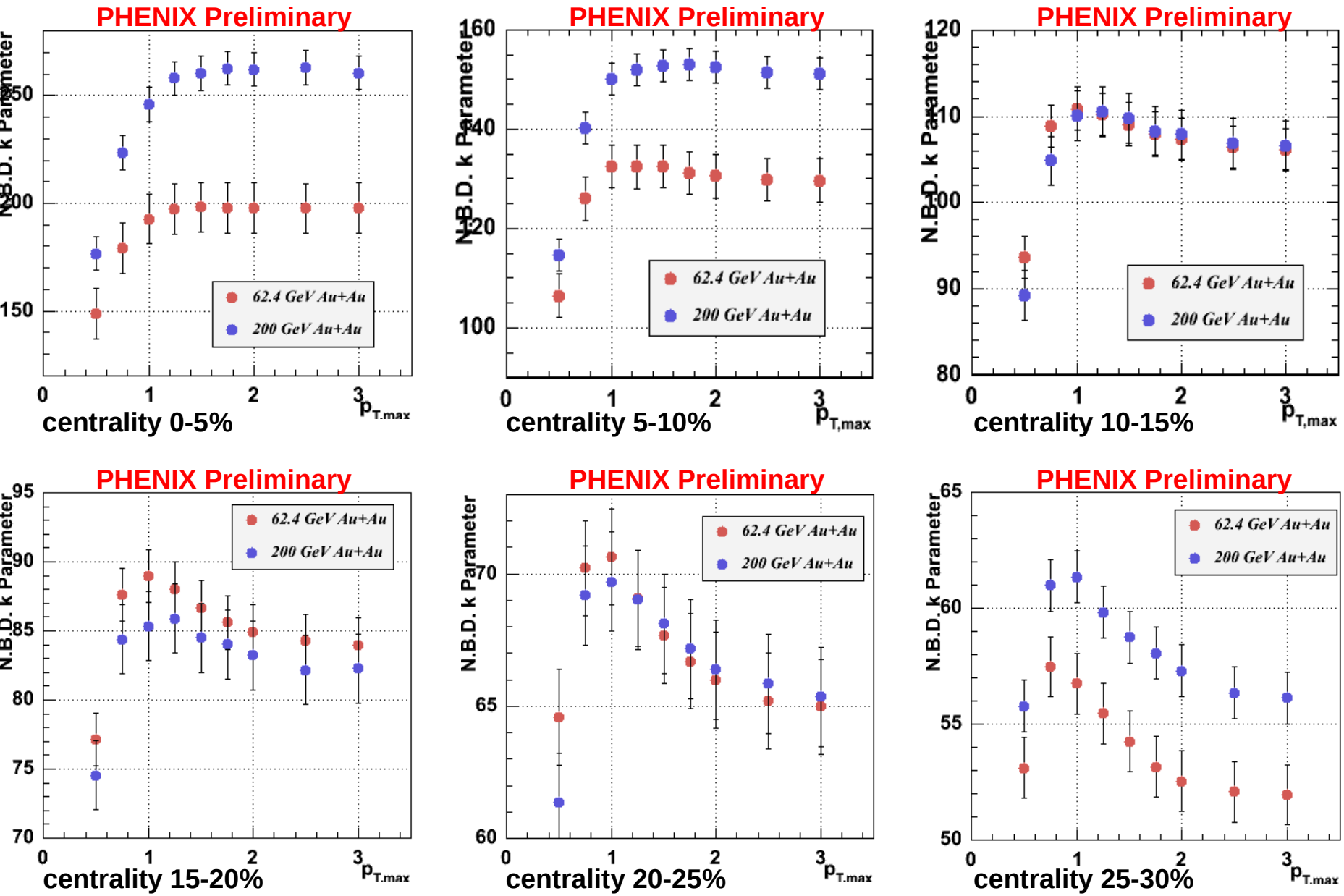
Using arbitrary R_0, ξ and α.



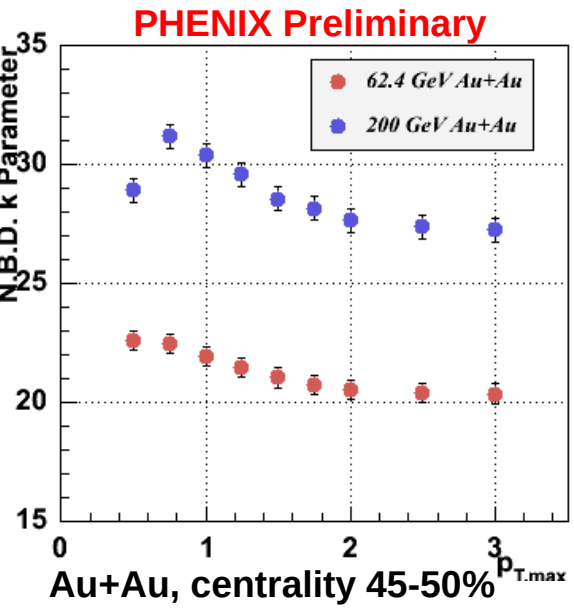
One may discuss an effective potential form of a deconfined field when assume the correlation functions.

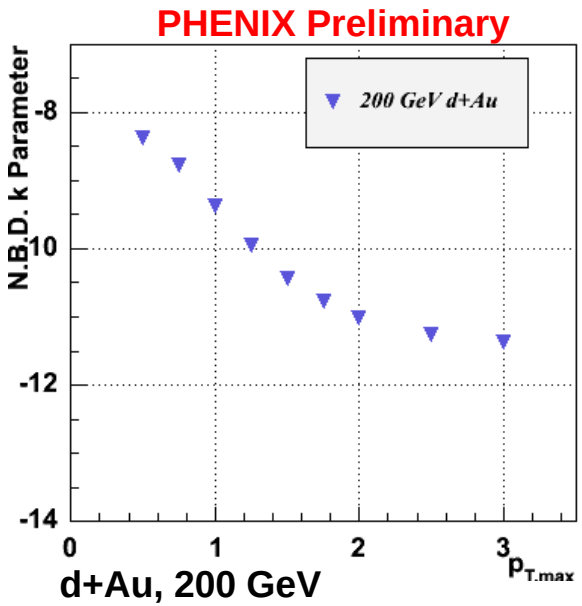
ξ : correlation length, α : critical exponent

$\delta\ p_T$ (p_T>0.2 GeV/c) dependence of NBD k



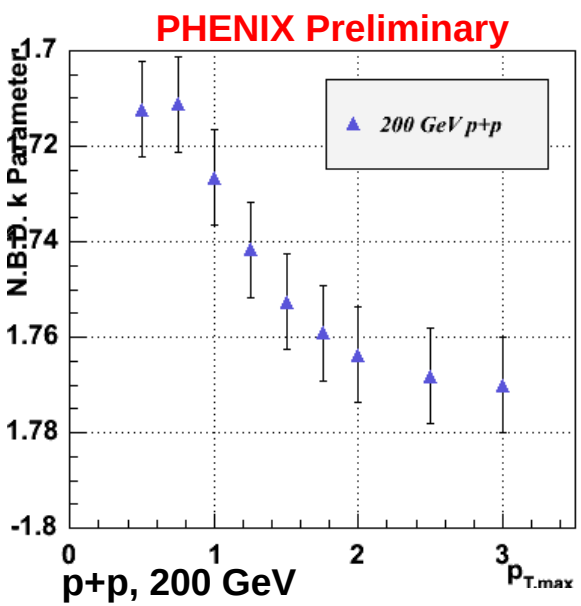
Comparison for Au+Au, d+Au and p+p

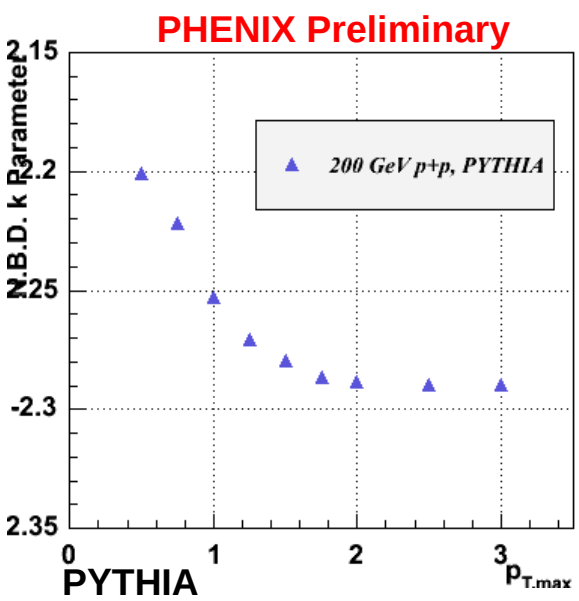




$\delta\, p_T$ dependence of NBD k parameters in Au+Au peripheral collisions are approaching toward the similar shape of d+Au and p+p with decreasing the centrality.

A behavior of NBD k parameters as a function of $\delta\, p_T$ at p+p collisions measured by PHENIX is agree with PYHTIA qualitatively.





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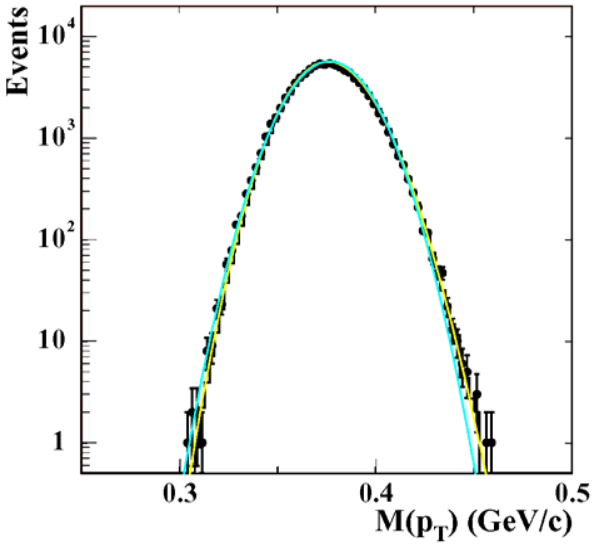
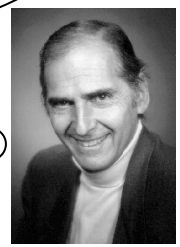
Average p_T fluctuation
Published by Phys. Rev. Lett. 93, 092301 (2004)

NA49: Phys. Lett. B 459 (1999) 679

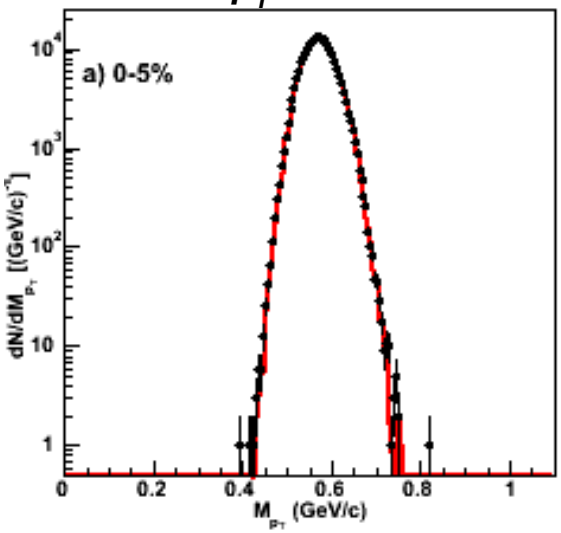


Already, famous!
It's not a Gaussian…
it's a Gamma distribution!



M.J. Tannenbaum,
Phys. Lett. B 498 (2001) 29

PHENIX: Au+Au 200GeV
$0.2 < p_T < 2.0$ GeV/c

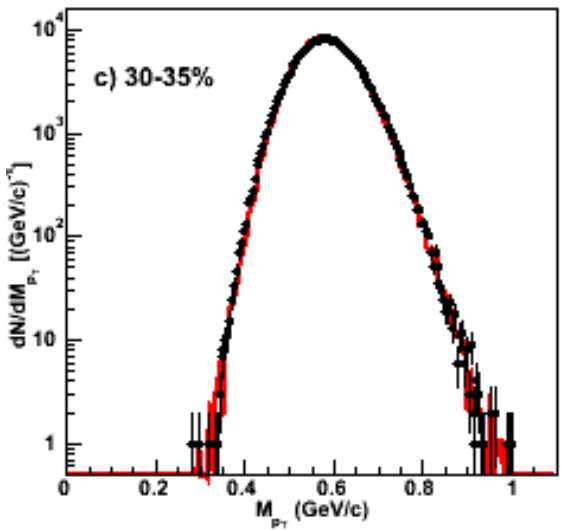


Magnitude of average p_T fluctuation

$$\omega_{p_T} = \frac{(<M_{p_T}^2> - <M_{p_T}>^2)^{1/2}}{<M_{p_T}>} = \frac{\sigma_{M_{pT}}}{<M_{p_T}>}$$

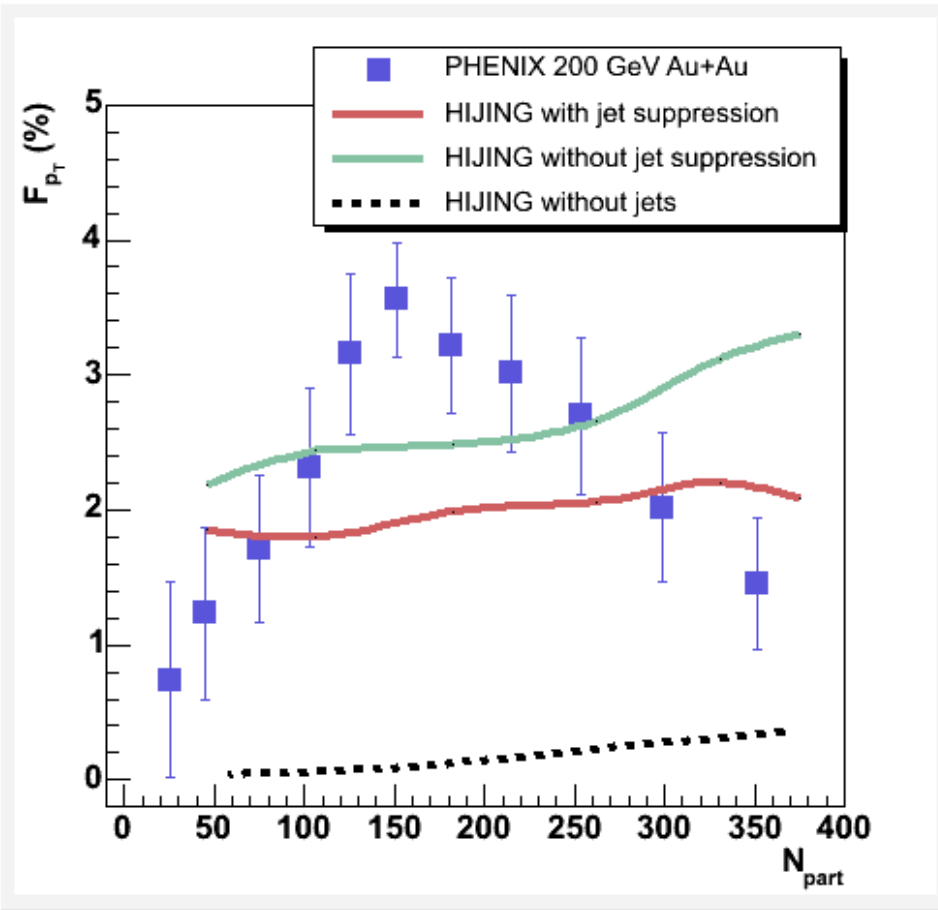
Fractional deviation from mixed events

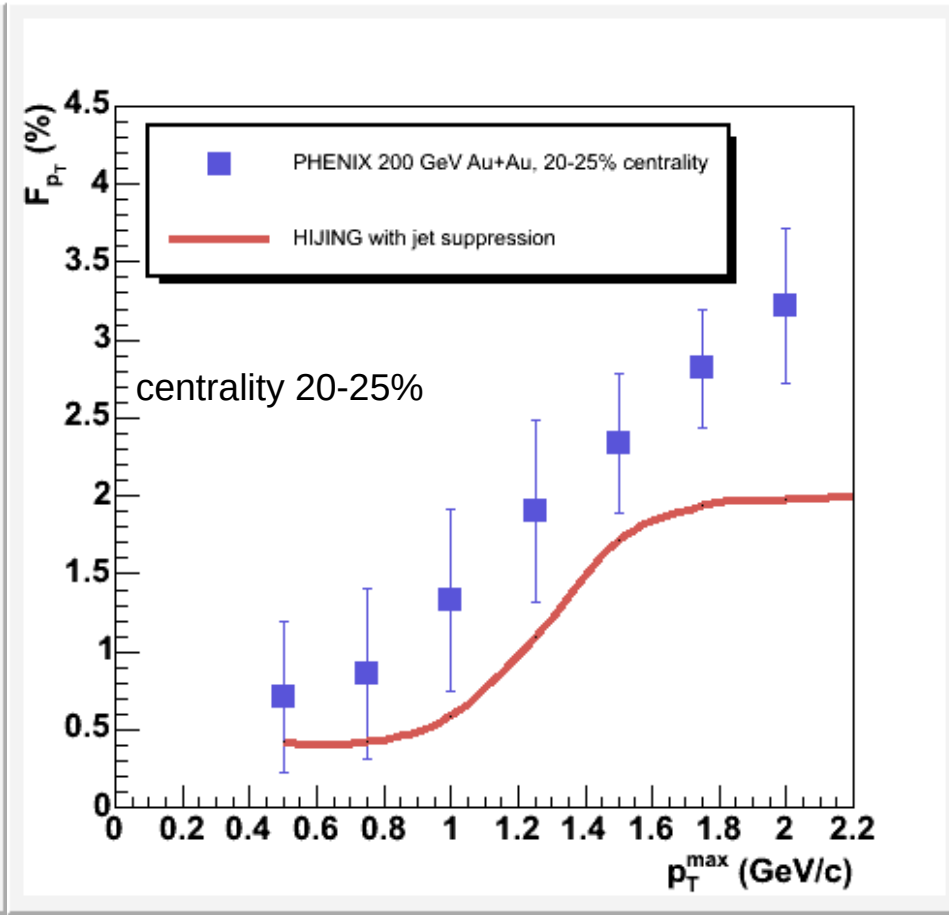
$$F_{p_T} = \frac{[\omega_{(p_T,data)} - \omega_{(p_T,mixed)}]}{\omega_{(p_T,mixed)}}$$



Result of average p_T fluctuations in Au+Au 200 GeV and comparison with HIJING

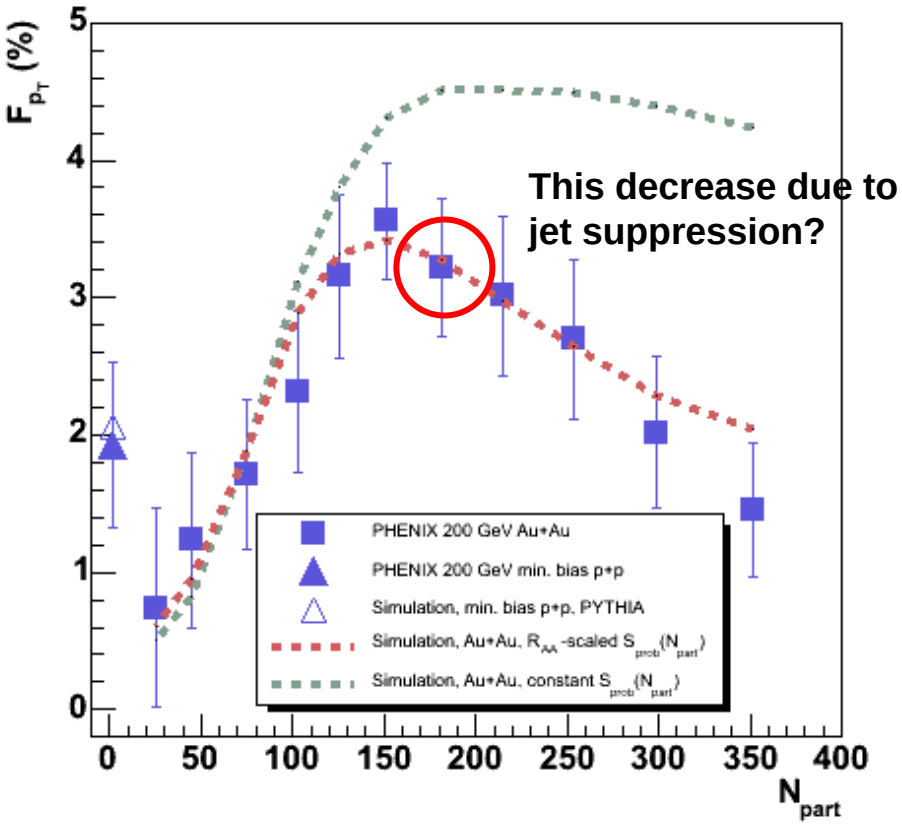
HIJING cannot reproduce the centrality dependence of the fluctuations. One problem is that <N> changes depending on the HIJING settings – it is not matched to the observed dataset.

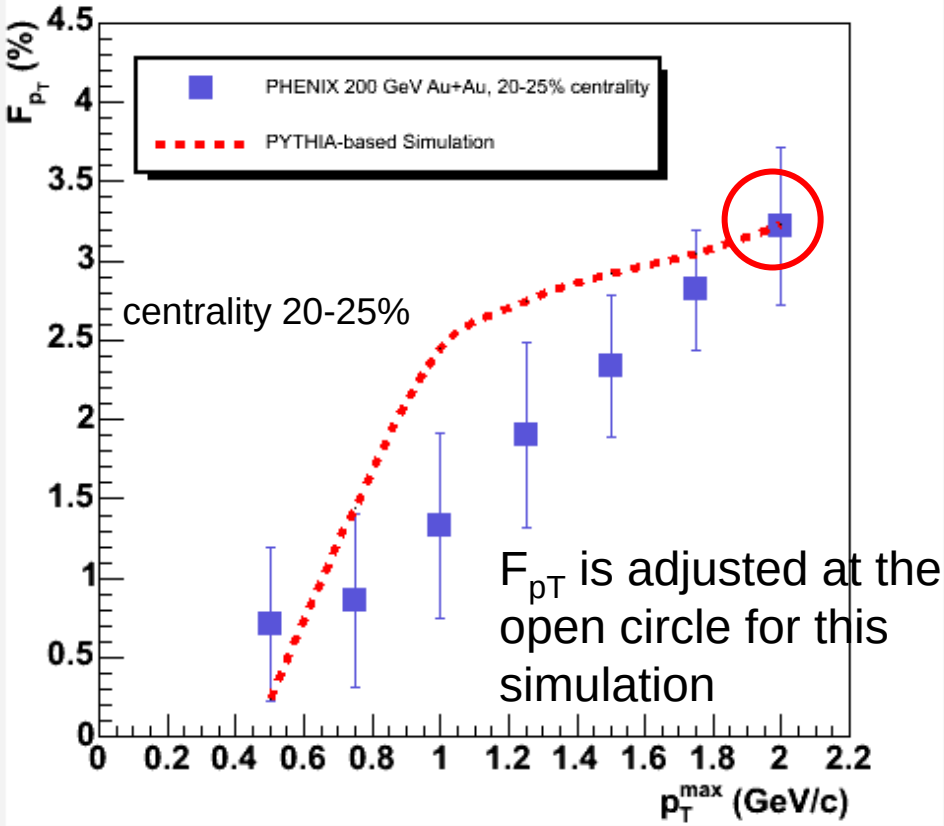




Contribution of Jet/Jet suppression to the average p_T fluctuation

PYTHIA based simulation, which contains scaled hard-scattering probability factor (S_{prob}) by the nuclear modification factor (R_{AA}), well agree with the measured F_{pT}. It might be indicate that jet suppression might contribute to the average p_T fluctuation.





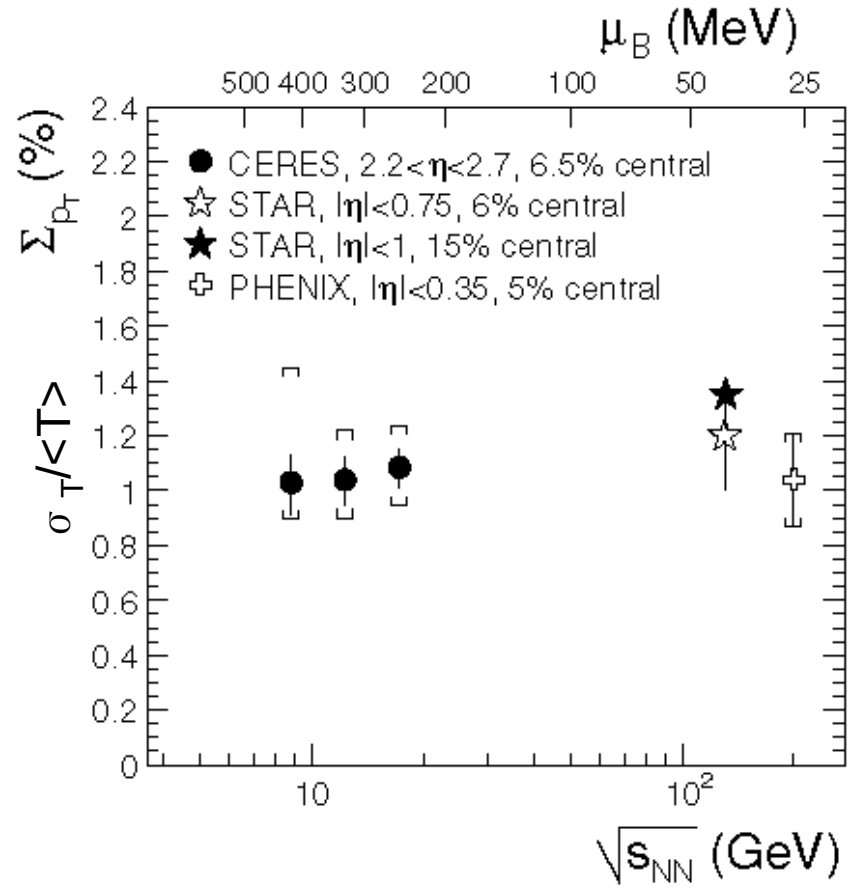
Estimation of the magnitude of residual temperature fluctuations

$$\frac{\sigma_T}{<T>} = \sqrt{\frac{2F_{p_T}}{p(<N>-1)}}$$

$p \rightarrow$ inclusive p_T

R. Korus and S. Mrowczynski, Phys. Rev. C64 (2001) 054908.

Experiment	$\sqrt{s_{NN}}$	$\sigma_T/<T>$ (most central)
PHENIX	200	1.8%
STAR	130	1.7%
CERES	17	1.3%
NA49	17	0.7%



H.Sako, et al, JPG 30, S (2004) 1371

A signal of phase transition dose not emerge in the temperature fluctuation?

Conclusions

- Charged particle multiplicity fluctuation
  - Different behavior about the normalized variance of multiplicity distribution as a fluctuation of number of participants are observed at PHENIX as compared to SPS energy.
  - NBD k parameters as an observable of the multiplicity fluctuation are not scaled by the mean multiplicity but scaled by the number of participants.
  - There are no difference on the multiplicity fluctuation with respect to the sign of electric charge for each particles.
  - Correlation length could be extracted from the multiplicity fluctuation.
  - Different pseudo rapidity gap dependence of NBD k parameter has been observed as compared to the past experiment.
- Average p_T fluctuation
  - PYTHIA based simulation with scaling the hard-scattering probability factor by the nuclear modification factor well reproduce the Au+Au 200 GeV data.
  - It is suggested that jet suppressions contribute to the average p_T fluctuations
  - No anomaly was found about the residual temperature fluctuations from the SPS energy to RHIC energy.

Outlooks for QM2005

- Further new results will be presented at QM2005
  - Systematic study of multiplicity fluctuations with respect to the collision system using Cu+Cu 200 GeV.
  - Detailed study about extraction of the correlation length by the different correlation functions and the behavior of the correlation length as a function of number of participants.
  - Jet suppression effects and extraction of specific heat in the average p_T fluctuation with Au+Au 62GeV and Cu+Cu 200GeV.
  - The other observables of event-by-event fluctuations and correlations, e.g. p_T-p_T correlation.

- We are willing to analyze the data on the event-by-event fluctuations for the QM. See you again at the Budapest.

Backup Slide

Normalized factorial moment F_q

$$F_q(\delta) = \frac{< n(n-1)\cdots(n-q+1) >}{< n >^q}$$

$$F_2(\delta) = \frac{< n(n-1) >}{< n >^2} = \frac{< n >^2 - < n >}{< n >^2} = \frac{\sigma^2 + < n >^2 - < n >}{< n >^2}$$

$$= 1 + \frac{\sigma^2}{\mu^2} - \frac{1}{\mu}$$

$\mu \equiv < n >$ average multiplicity

$\sigma \equiv \sqrt{< n^2 > - < n >^2}$ standard deviation

$F_q(\delta) \propto (\delta)^{-\phi_q}$ relation between fractal (self-similarity) structures and normalized factorial moment

NBD k and factorial moment F_q

$$P_n^{(k)} = \frac{\Gamma(n+k)}{\Gamma(n-1)\Gamma(k)} \left(\frac{\mu / k}{1+\mu / k} \right)^n \frac{1}{(1+\mu / k)^k}$$

$$\sigma = \sqrt{\mu(1+\mu / k)}$$

$$\frac{\sigma^2}{\mu^2} = \frac{1}{\mu} + \frac{1}{k} \qquad F_2 - 1 = \frac{1}{k} \qquad F_2 = 1 + \frac{1}{k}$$

$$F_3 = \left(1+\frac{1}{k}\right)\left(1+\frac{2}{k}\right)$$

$$F_4 = \left(1+\frac{1}{k}\right)\left(1+\frac{2}{k}\right)\left(1+\frac{3}{k}\right)$$

$$F_q = F_{(q-1)}\left(1+\frac{q-1}{k}\right)$$

Integral of correlation function and normalized factorial moments

$$\int^{\delta\eta} dy_1 \rho_1(y_1) = < n >$$

$$\int^{\delta\eta} dy_1 dy_2 \rho_2(y_1, y_2) = < n(n-1) > = < n >^2 F_2$$

$$\int^{\delta\eta} dy_1 \cdots dy_q \rho_q(y_1, \cdots y_q) = < n(n-1)\cdots(n-q+1) > = < n >^q F_q$$

$\rho_q(y_1, \cdots y_q)$ inclusive *q* particle density

when there are no correlation in rapidity

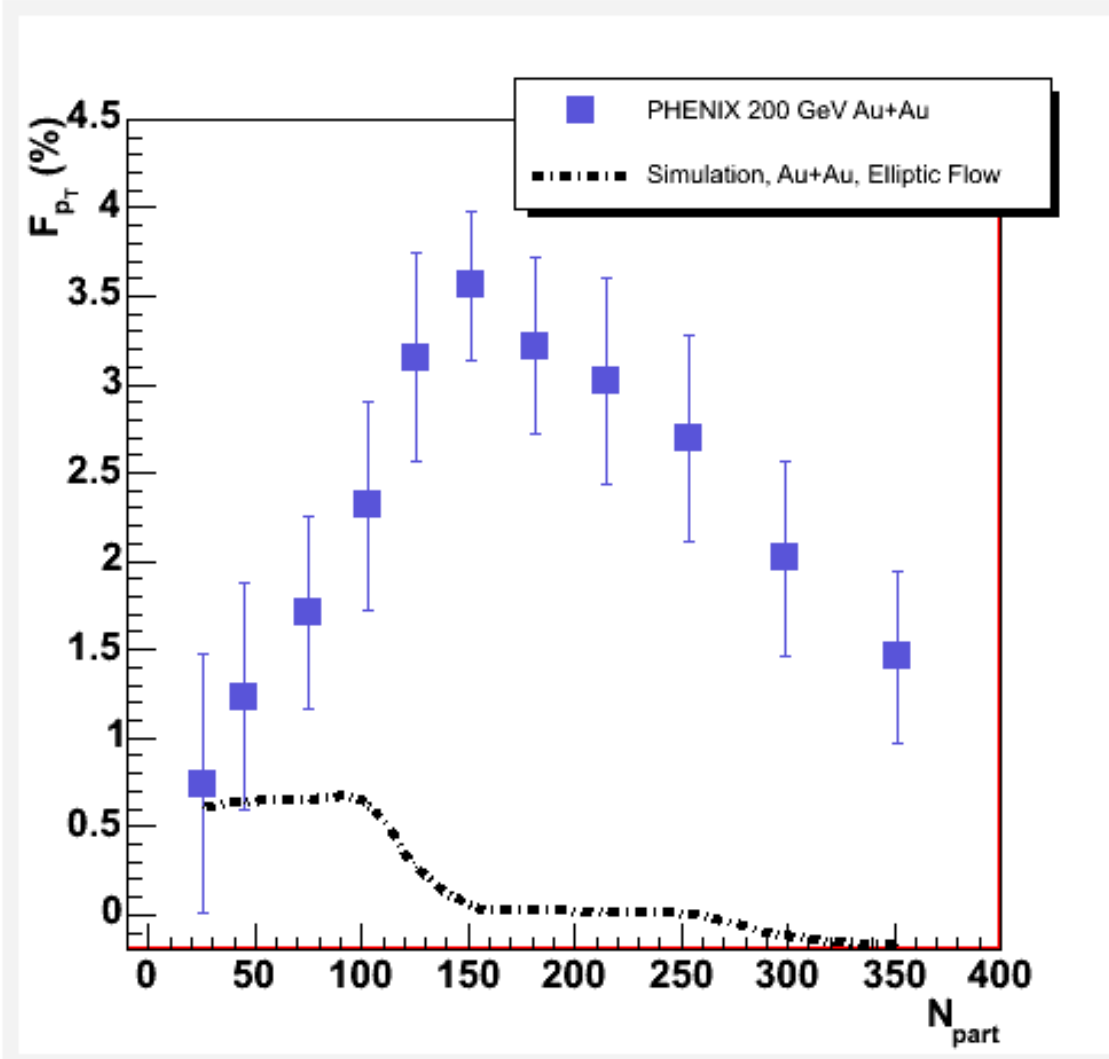
$$\rho_q(y_1, \cdots y_q) = \rho_1(y_1)\rho_2(y_2)\cdots\rho_q(y_q)$$

Elliptic flow contribution by simulation

Jeffery T. Mitchell

Particles are assigned an azimuthal coordinate based upon the PHENIX measurement of v_2 (w.r.t. the reaction plane) as a function of centrality and p_T. Only particles within the PHENIX acceptance are included in the calculation of M_{pT}.

With the exception of peripheral collisions, the elliptic flow contribution is a small fraction of the observed fluctuation.



Specific heat from average p_T fluctuation

$$F_{p_T} = \left(\frac{\sigma_{M_{p_T}}}{\mu} - \frac{1}{\sqrt{n}}\frac{\sigma_{p_T}}{\mu}\right) / \frac{1}{\sqrt{n}}\frac{\sigma_{p_T}}{\mu}$$

$$\frac{\Delta\sigma^2}{\sigma^2} = 2\frac{\Delta\sigma}{\sigma} = 2F$$

$$\frac{\sigma^2_{M_{p_T}}}{\mu^2} - \frac{1}{n}\frac{\sigma^2_{p_T}}{\mu^2} = \left(1 - \frac{1}{n}\right)\frac{\sigma^2_T}{\langle T\rangle^2}$$

Korus, et al, PRC 64, (2001) 054908

$$1/C_V = \sigma^2_T / \langle T\rangle^2$$

$$c_v = C_V / \langle N_{tot}\rangle$$

$$c_v = \frac{\langle n\rangle}{\langle N_{tot}\rangle}\frac{1}{F_{p_T}}$$

M. J. Tannenbaum,
2nd International Workshop on the Critical Point and Onset of Deconfinement

n represents the measured particles while N_{tot} is all the particles, so n/N_{tot} is a simple geometrical factor for all experiments