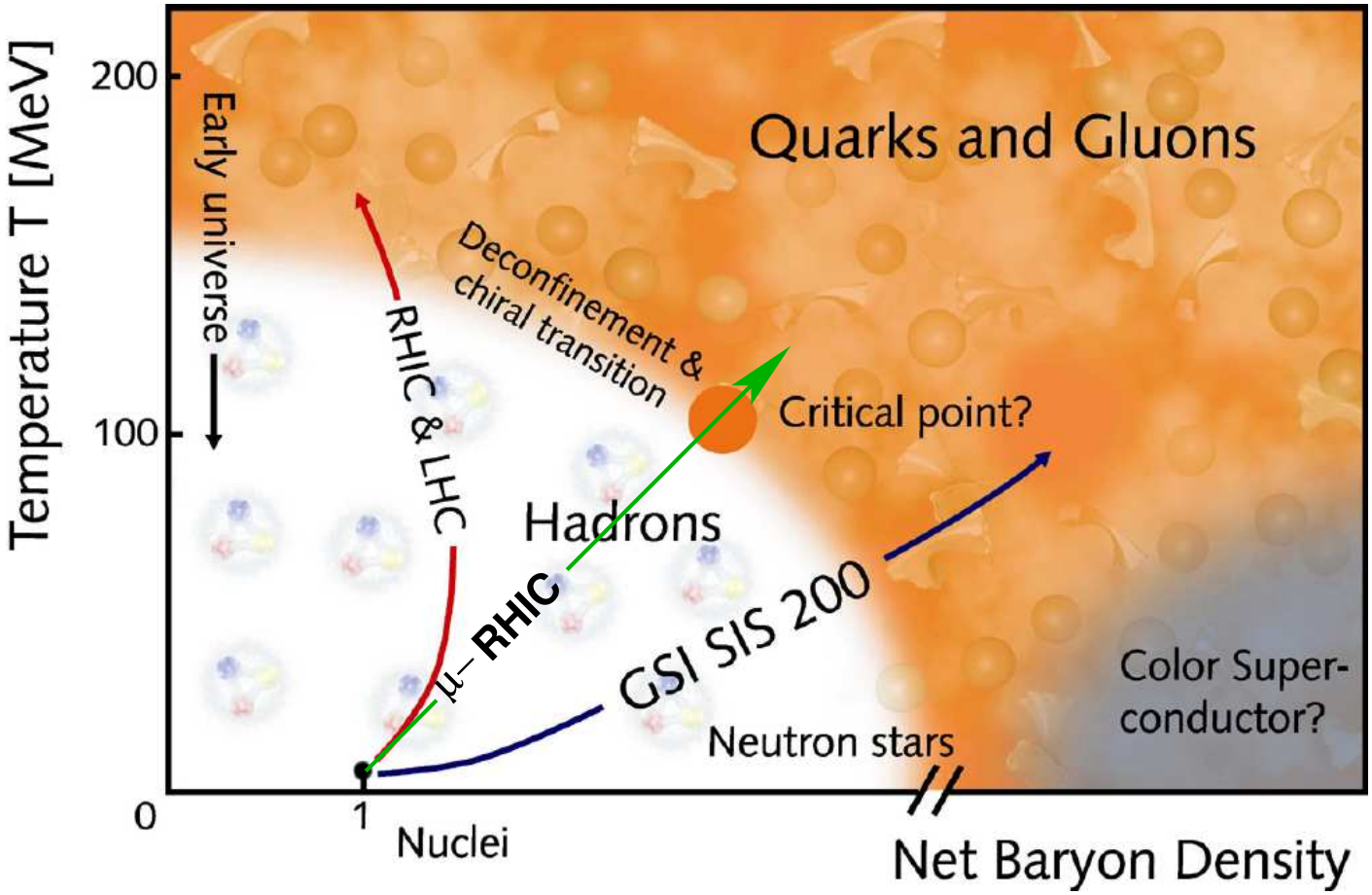
Lattice results on the QCD critical point

Frithjof Karsch, BNL



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- My charge:

  Give an answer to the question:

  - What do we know from LGT calculations about

    - the existence of a critical point and

    - the location of the critical point

    in the QCD phase diagram?

Lattice results on the QCD critical point

Frithjof Karsch, BNL

Outline

- From $\mu = 0$ to $\mu > 0$

  the crossover line in the $T - \mu$ phase diagram

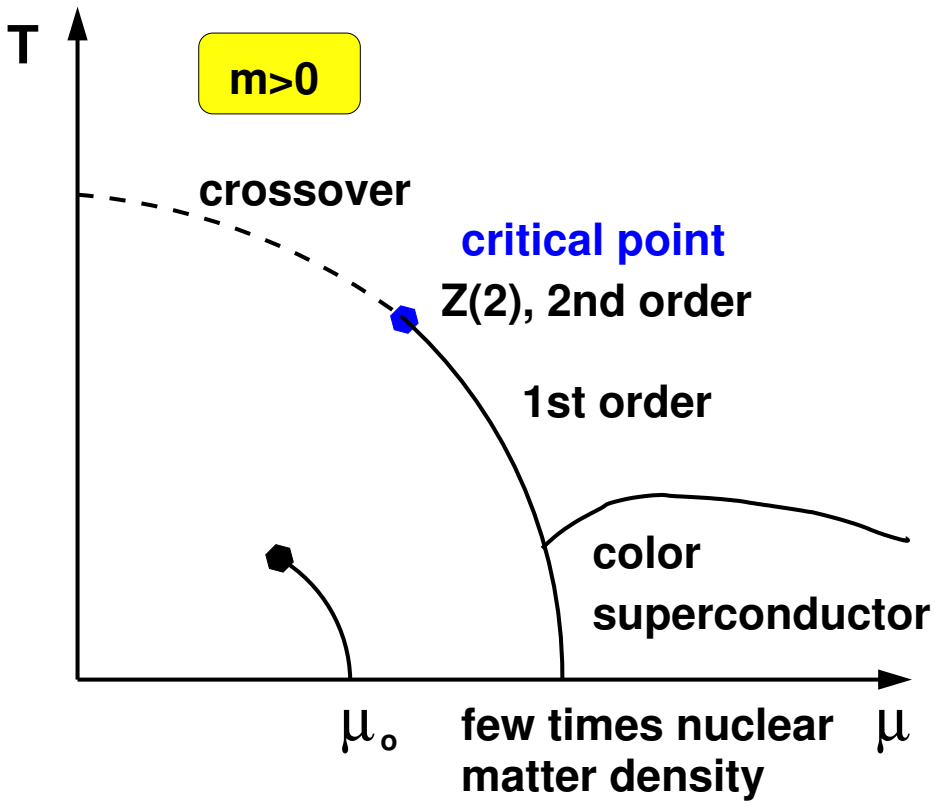
- attempts to determine the QCD critical point
  - reweighting around $\mu_q = 0$
    Z. Fodor, S. Katz, JHEP 0203 (2002) 014 and 0404 (2004) 050
  - Taylor expansion around $\mu_q = 0$
    C. R. Allton et al., Phys. Rev. D71 (2005) 054508
    R.V. Gavai, S. Gupta, Phys. Rev. D71 (2005) 114014

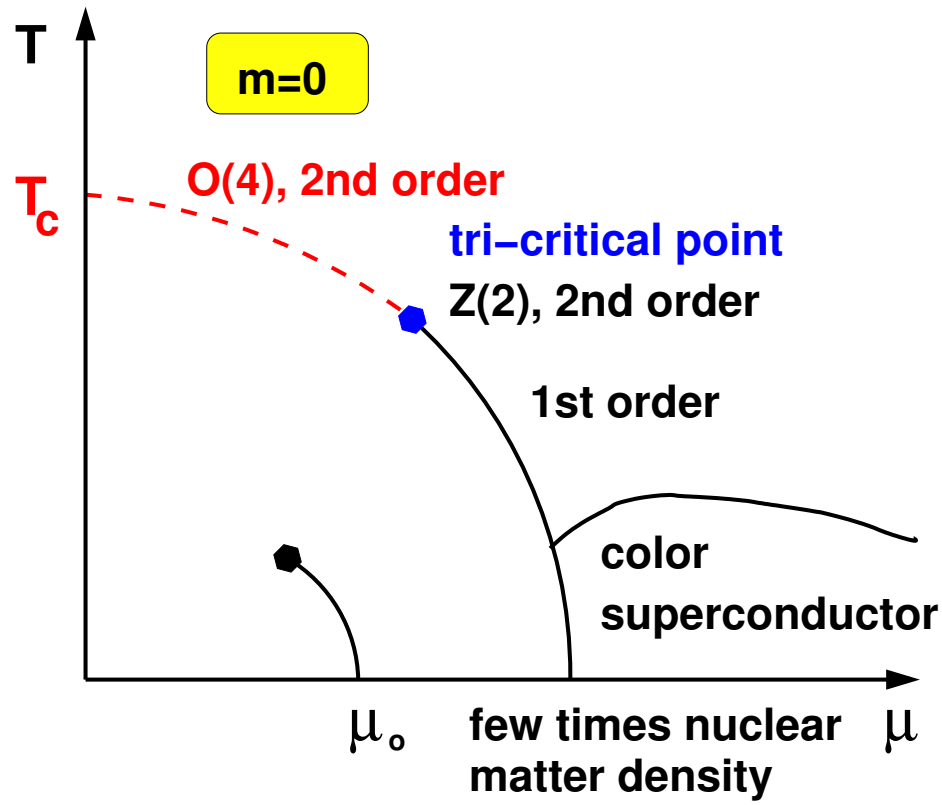
imaginary μ, finite-B simulation and further references:
O. Philipsen, hep-lat/0510077 (Lattice 2005)

The crossover line in the T-μ phase diagram

generic QCD phase diagram ($n_f = 2$)

(also holds for $n_f = 2 + 1$ for m_s 'large enough')





(Hadronic) Fluctuations at $\mu_q = 0$

- expect 2nd order transition in 3-d, O(4) symmetry class;

  scaling field: $t = \left|\frac{T - T_c}{T_c}\right| + A\left(\frac{\mu_q}{T_c}\right)^2$, $\mu_{crit} = 0$

  singular part: $f_s(T, \mu_u, \mu_d) = b^{-1} f_s(t b^{1/(2-\alpha)}) \sim t^{2-\alpha}$

$$c_2 \sim \frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_q^2} \sim t^{1-\alpha} \quad , \quad c_4 \sim \frac{\partial^4 \ln \mathcal{Z}}{\partial \mu_q^4} \sim t^{-\alpha} \qquad (\mu = 0)$$

- O(4)/O(2): $\alpha < 0$, small $\Rightarrow$

  $c_2 \sim \langle (\delta N_q)^2 \rangle$ dominated by T-dependence of regular part

  $c_4 \sim \langle (\delta N_q)^4 \rangle - 3\langle (\delta N_q)^2 \rangle^2$ develops a cusp

  Y. Hatta, T. Ikeda, PRD67 (2003) 014028

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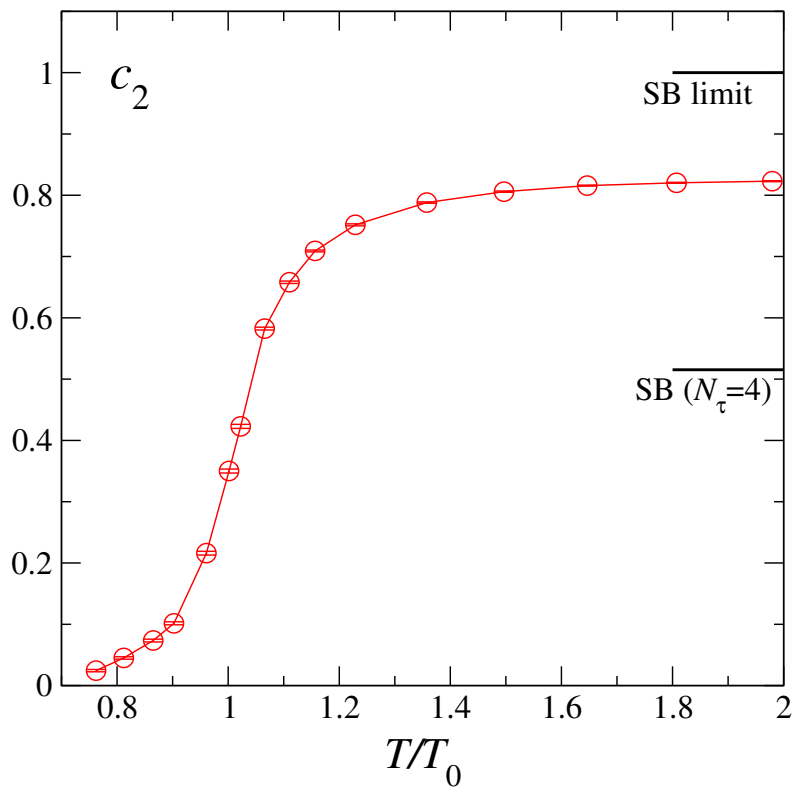
$$\epsilon \sim \frac{\partial \ln \mathcal{Z}}{\partial T} \sim t^{1-\alpha} \quad , \quad C_V \sim \frac{\partial^2 \ln \mathcal{Z}}{\partial T^2} \sim t^{-\alpha} \qquad (\mu = 0)$$

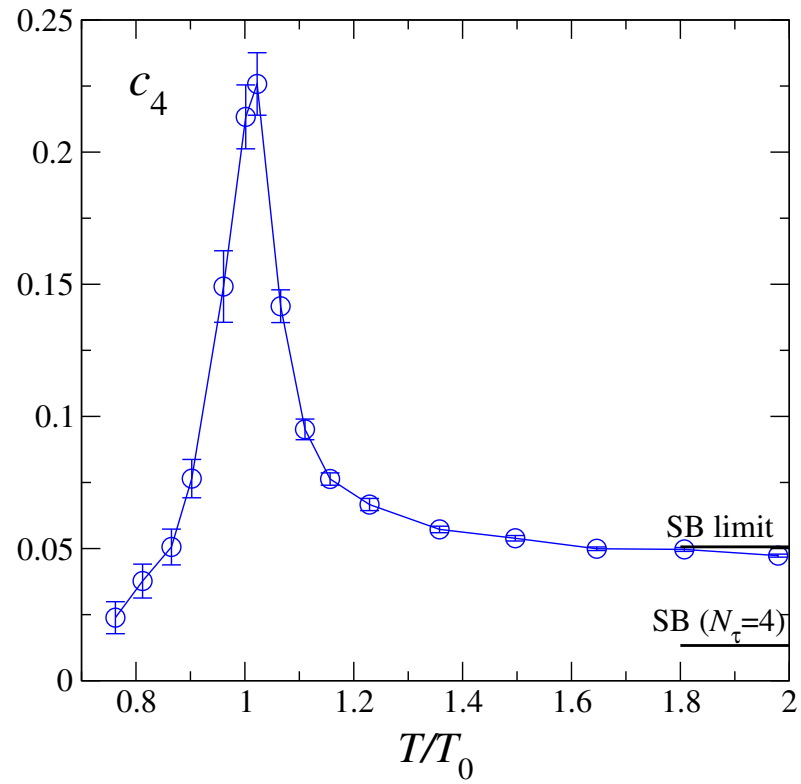
$\Rightarrow$ 2nd derivative w.r.t μ_q "looks like energy density"
$\Rightarrow$ 4th derivative w.r.t μ_q "looks like specific heat"

Bulk thermodynamics for small μ_q/T on $16^3 \times 4$ lattices

- Taylor expansion of pressure up to $\mathcal{O}\left((\mu_q/T)^4\right)$

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu_q}{T}\right)^n \simeq c_0 + c_2 \left(\frac{\mu_q}{T}\right)^2 + c_4 \left(\frac{\mu_q}{T}\right)^4 + \mathcal{O}((\mu_q/T)^6)$$





(Hadronic) Fluctuations at $\mu_q > 0$

- expect 2nd order transition in 3-d, O(4) symmetry class;

$$\text{scaling field: } t = \left|\frac{T - T_c}{T_c}\right| + A\left(\left(\frac{\mu_q}{T_c}\right)^2 - \left(\frac{\mu_{crit}}{T_c}\right)^2\right)$$

$$\Uparrow \quad A\left(\frac{\mu_q}{T_c} + \frac{\mu_{crit}}{T_c}\right)\left(\frac{\mu_q}{T_c} - \frac{\mu_{crit}}{T_c}\right)$$

$$\frac{\partial^2 \ln \mathcal{Z}}{\partial \mu_q^2} \sim t^{-\alpha} \quad , \quad \frac{\partial^4 \ln \mathcal{Z}}{\partial \mu_q^4} \sim t^{-(2+\alpha)} \qquad (\mu > 0)$$

- already second derivative w.r.t. μ_q "looks like a specific heat"

  $\langle(\delta N_q)^2\rangle$ develops a cusp

  $\langle(\delta N_q)^4\rangle$ diverges on the O(4) critical line;

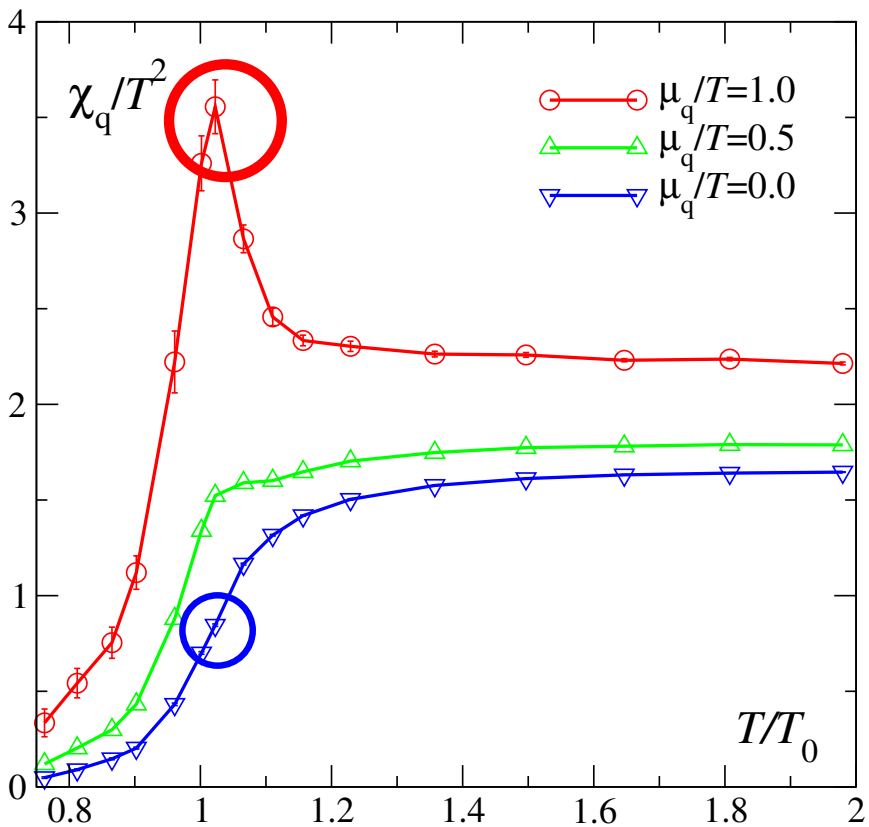
  strength $\sim \left(\frac{\mu_{crit}}{T_c}\right)^4$ ($\sim 10^{-4}$ at RHIC)

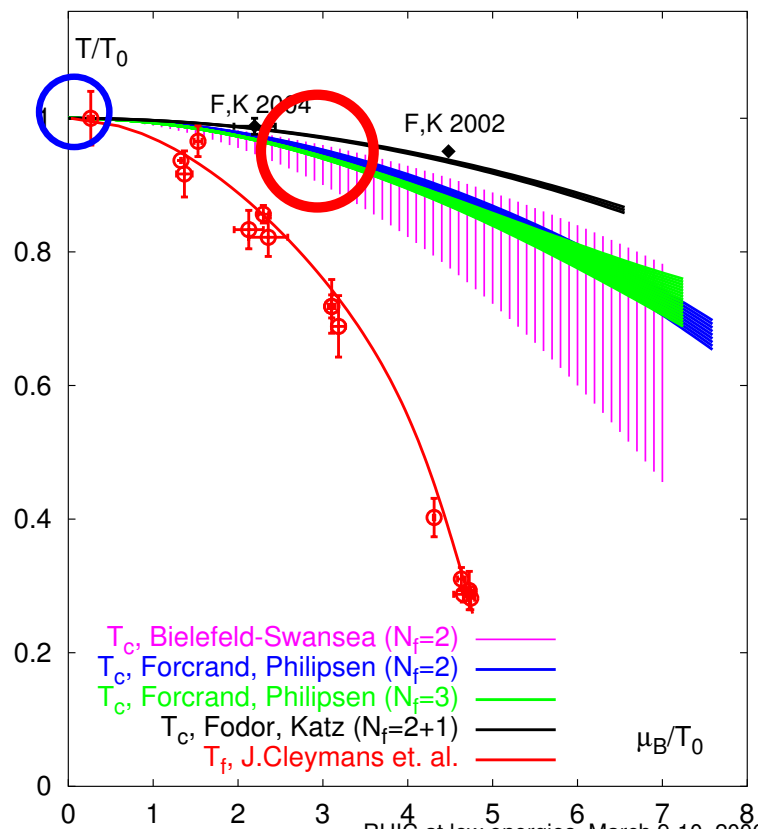
Fluctuations of the baryon number density ($\mu \geq 0$)

baryon number density fluctuations:
(Bielefeld-Swansea, PRD68 (2003) 014507)
$\mu \geq 0, n_f = 2$

$$\frac{\chi_q}{T^3} = \left(\frac{d^2}{d(\mu/T)^2} \frac{p}{T^4} \right)_{T \text{ fixed}}$$

$$= \frac{9\,T}{V} \left(\langle N_B^2 \rangle - \langle N_B \rangle^2 \right)$$





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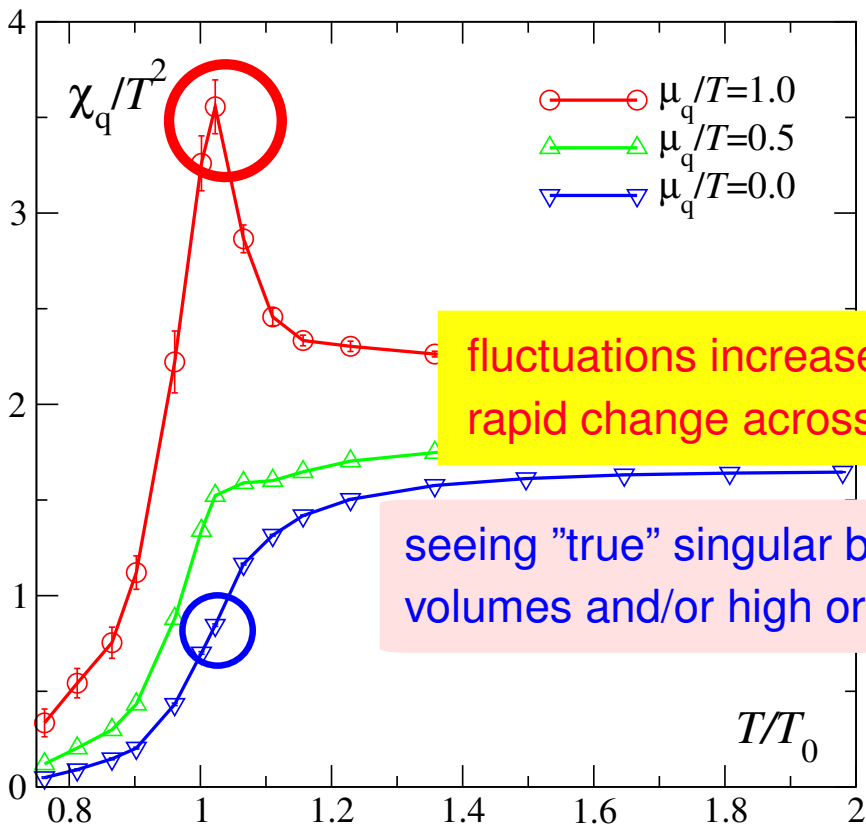
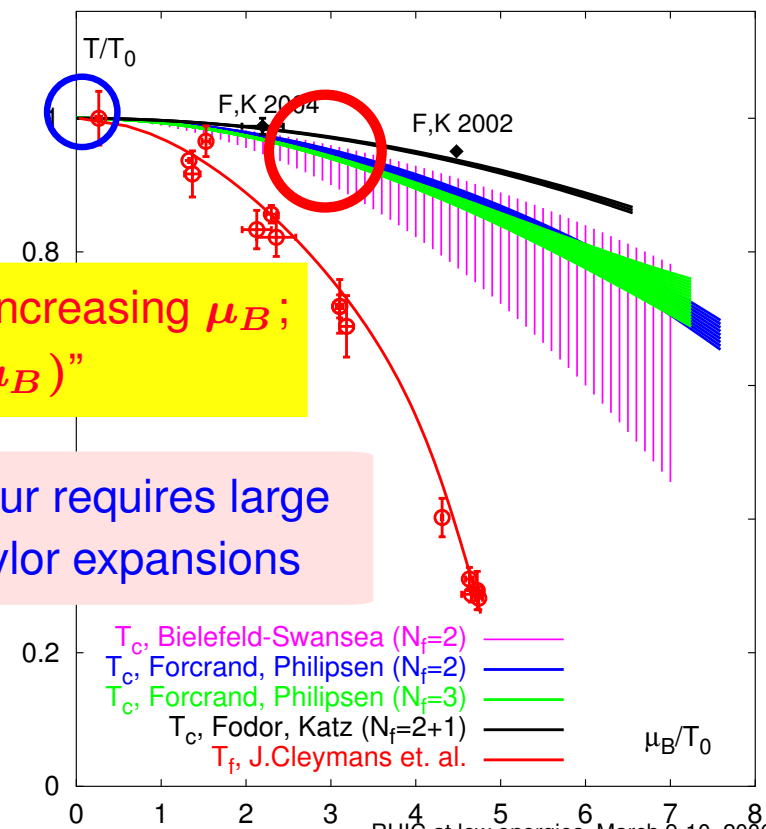


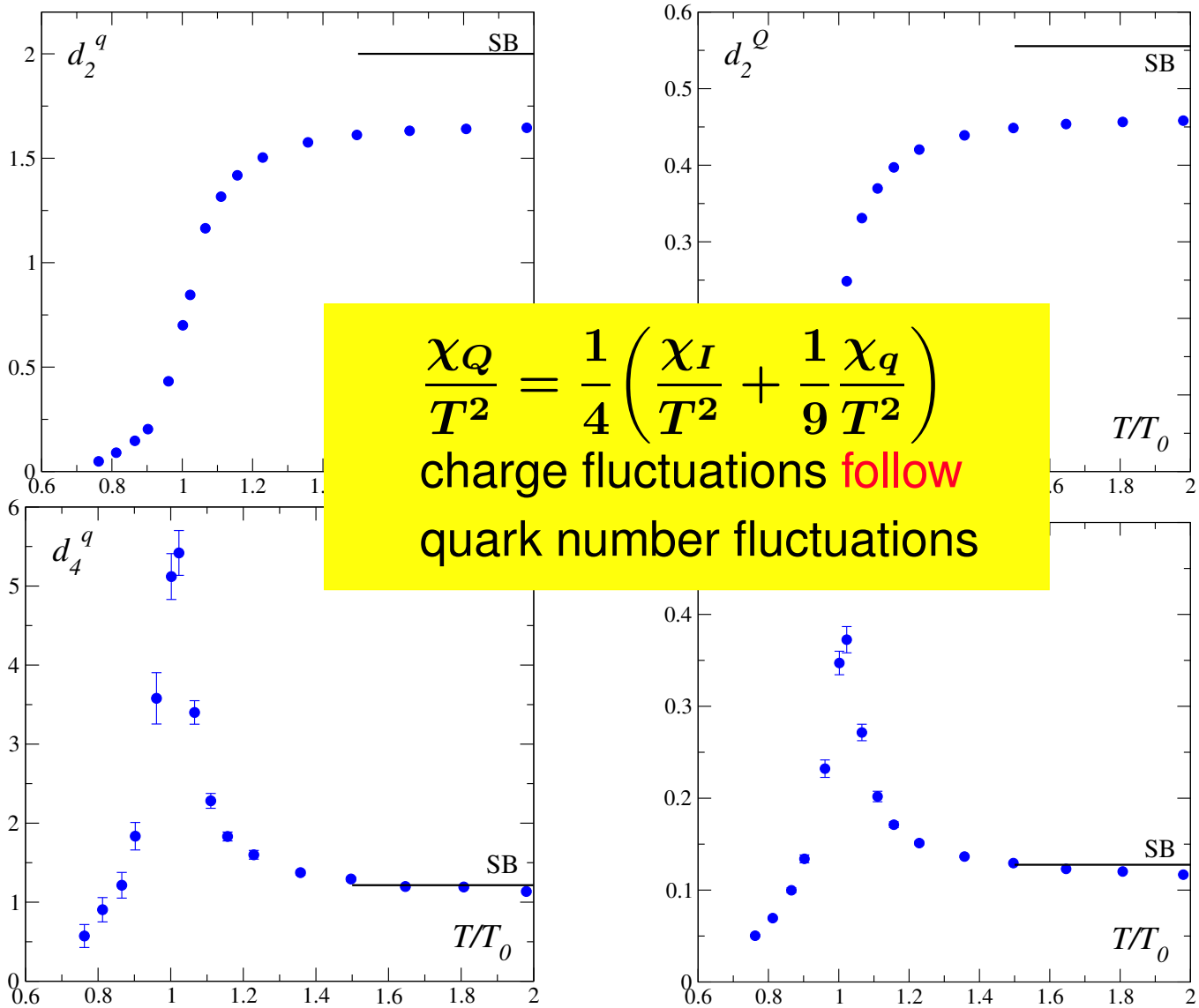


fluctuations increase with increasing μ_B; rapid change across "$T_c(\mu_B)$"

seeing "true" singular behaviour requires large volumes and/or high order Taylor expansions

Quark number and charge fluctuations at $\mu_B = 0$; 2-flavor QCD ($m_\pi \simeq 770\ MeV$)

C. Allton et al. (Bielefeld-Swansea), PRD71 (2005) 054508



$$\frac{\chi_Q}{T^2} = \frac{1}{4}\left(\frac{\chi_I}{T^2} + \frac{1}{9}\frac{\chi_q}{T^2}\right)$$

charge fluctuations follow quark number fluctuations

monotonic increase; close to ideal gas value for $T \gtrsim 1.5 T_c$

develops cusp at T_c

reaches ideal gas value for $T \gtrsim 1.5 T_c$

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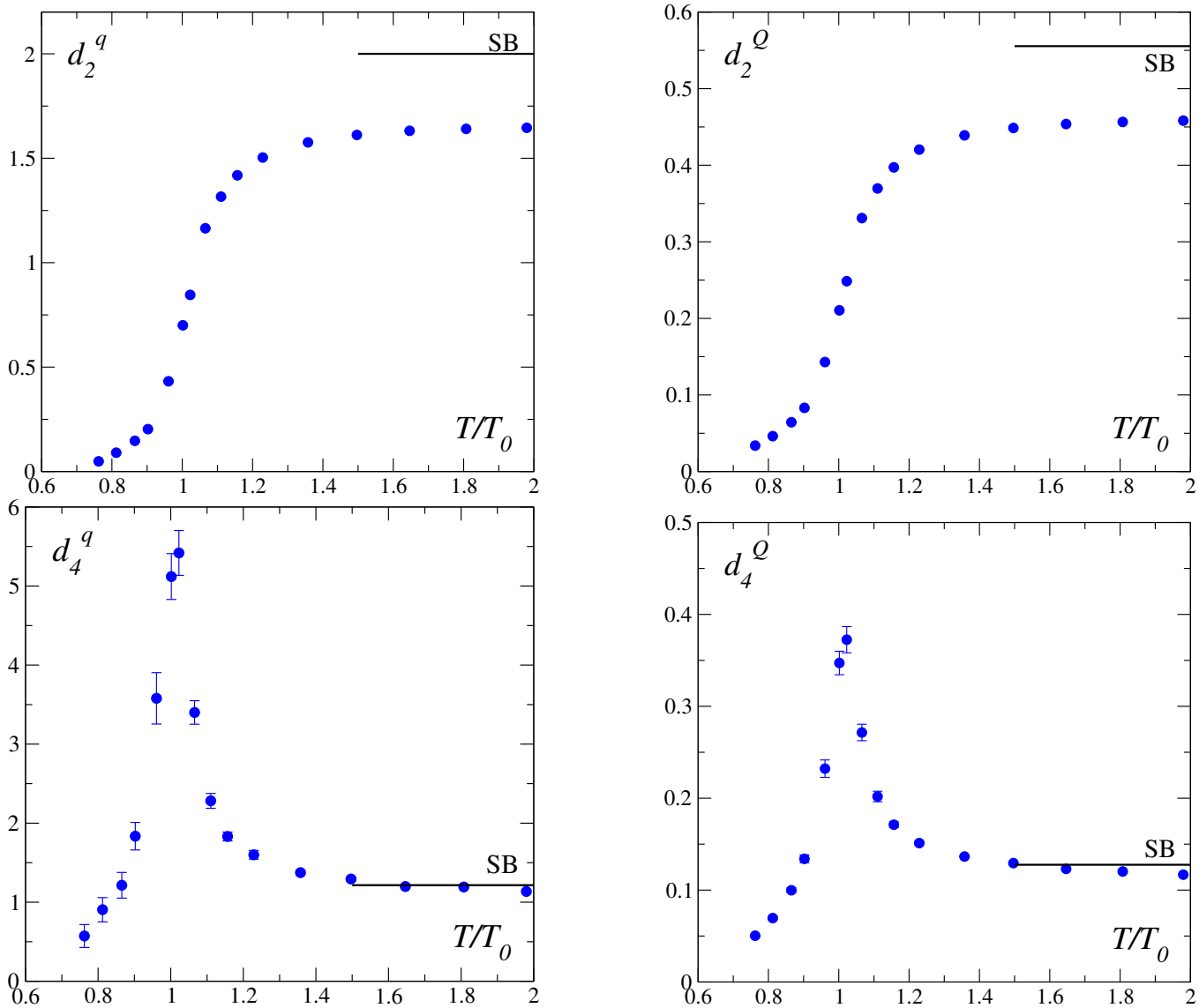

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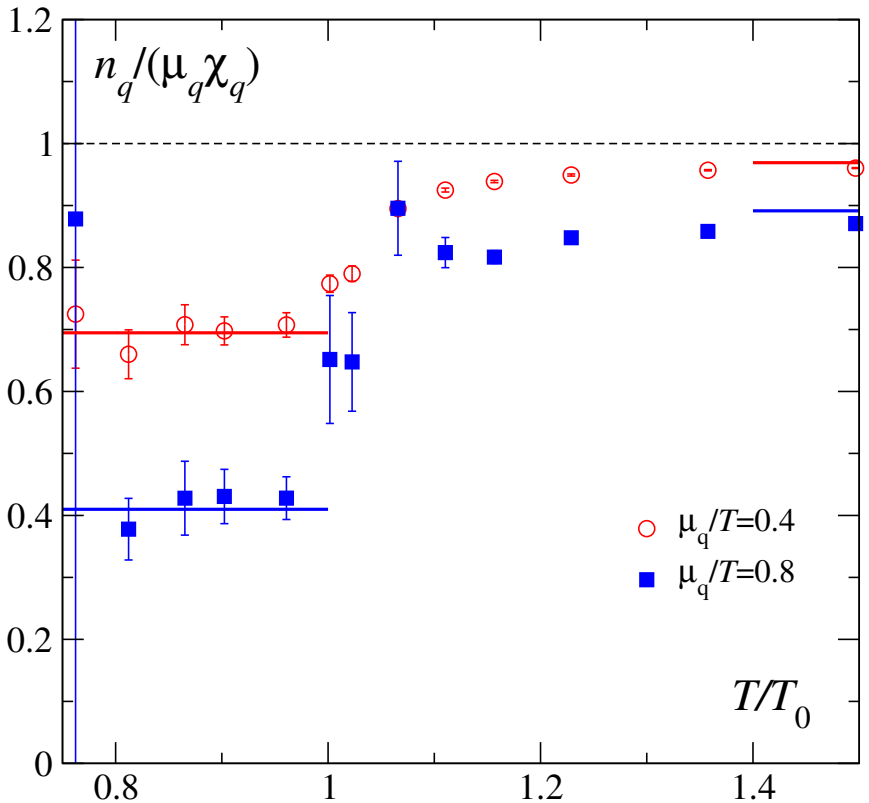
Isothermal compressibility of the quark gluon plasma

inverse compressibility:

$$\kappa_T^{-1} = \frac{n_q}{\chi_q} = \left(\frac{\partial p}{\partial n_q}\right)_{T \text{ fixed}}$$



high-T, massless limit: polynomial in (μ_q/T)

$$\frac{n_q}{\chi_q} = \mu_q + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^3\right)$$

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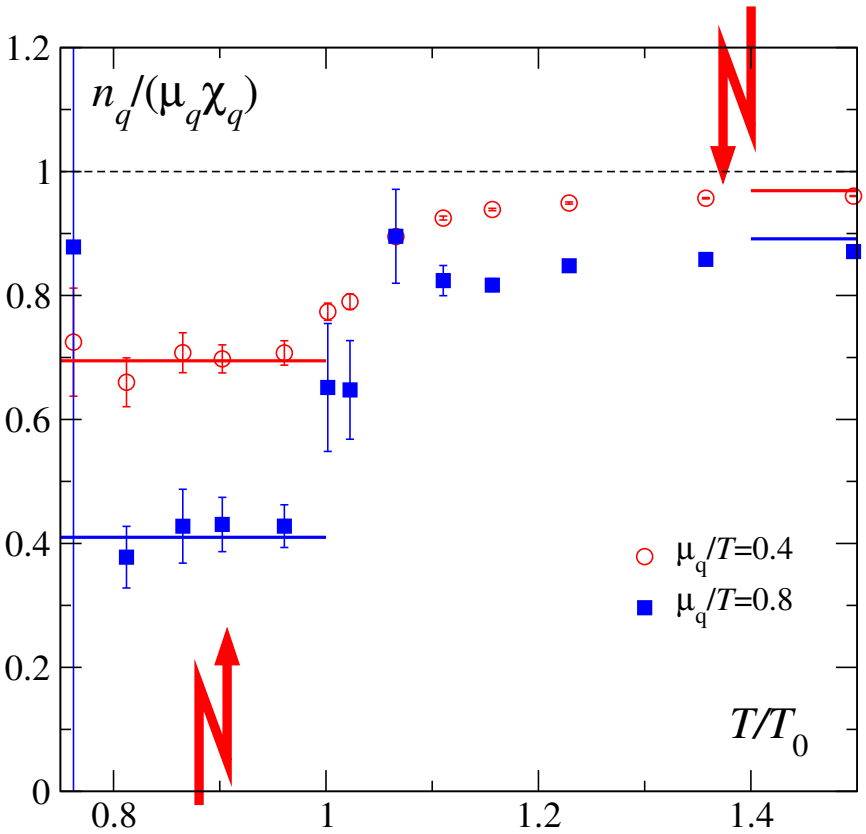
ideal $q\bar{q}$ gas



resonance gas

high-T, massless limit: polynomial in (μ_q/T)

$$\frac{n_q}{\chi_q} = \mu_q + \mathcal{O}\left(\left(\frac{\mu_q}{T}\right)^3\right)$$

large density fluctuations for $\mu_q > 0,\ T < T_c$

"saturated" by fluctuations in a

hadron resonance gas

expect: $\left(\frac{\partial p}{\partial n_q}\right)_T = \frac{n_q}{\chi_q} = 0$

at chiral critical point

Isothermal compressibility of the quark gluon plasma

inverse compressibility:

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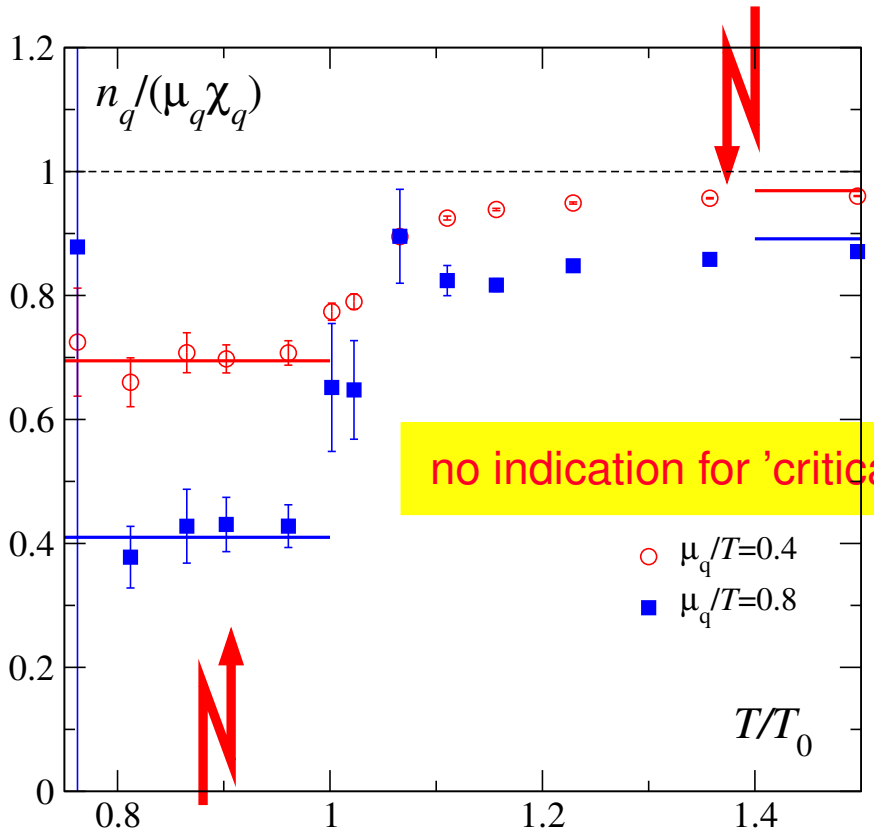

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large density fluctuations for $\mu_q > 0,\ T < T_c$

"saturated" by fluctuations in a hadron resonance gas

no indication for 'critical' fluctuations

expect: $\left(\frac{\partial p}{\partial n_q}\right)_T = \frac{n_q}{\chi_q} = 0$

at chiral critical point

$m_\pi \simeq 770$ MeV, smaller m_q or larger V needed??!!

Lattice results on the QCD critical point

- Where is the critical point?

Lattice results on the QCD critical point

- attempts to determine the QCD critical point
  - reweighting around $\mu_q = 0$
    Z. Fodor, S. Katz, JHEP 0203 (2002) 014 and 0404 (2004) 050
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    R.V. Gavai, S. Gupta, Phys. Rev. D71 (2005) 114014

- results in a nutshell
  - FK: $\mu_B \sim 360$ MeV
  - GG: $\mu_B \sim 180$ MeV
  - WE: so far, no clear-cut evidence for the chiral critical point

Reweighting for non-zero chemical potential

- map configurations from one parameter set to an other using appropriate Boltzmann weights
- include "problematic" terms in the reweighted observable rather than in the weights used to generate the original data set

consider an arbitrary observable $\mathcal{O}(m,\mu)$

$$\begin{aligned}\langle\mathcal{O}\rangle_{\mu,\beta} &= \frac{1}{\mathcal{Z}(\beta,m,\mu)}\int\mathcal{D}U\mathcal{O}(m,\mu)(\det M(m,\mu))^{n_f/4}\exp(-S_G)\\ &= \frac{\langle\mathcal{O}\exp\{\mathcal{R}-\Delta S_G\}\rangle_{\mu=0,\beta_0}}{\langle\exp\{\mathcal{R}-\Delta S_G\}\rangle_{\mu=0,\beta_0}}\end{aligned}$$

with

$$\begin{aligned}\mathcal{R} &= \frac{n_f}{4}\left(\ln\det M(m,\mu)-\ln\det M(m,0)\right)\\ \Delta S_G &= S_G(\beta)-S_G(\beta_0)\end{aligned}$$

sign problem $\Leftrightarrow$ $\mathcal{R}\sim\mathcal{O}(V);\ \mathrm{Im}\mathcal{R}\neq 0$:

Reweighting and Lee-Yang zeroes

$$\mathcal{Z}_{\mathrm{norm}}(\beta_{\mathrm{Re}}, \beta_{\mathrm{Im}}, \mu) \equiv \left| \frac{\mathcal{Z}(\beta_{\mathrm{Re}}, \beta_{\mathrm{Im}}, \mu)}{\mathcal{Z}(\beta_{\mathrm{Re}}, 0, 0)} \right|$$

$$= \left| \left\langle e^{6i\beta_{\mathrm{Im}} N_{\mathrm{site}} \Delta P} e^{i\theta} \left| e^{(N_{\mathrm{f}}/4)(\ln\det M(\mu) - \ln\det M(0))} \right| \right\rangle_{(\beta_{\mathrm{Re}},0,0)} \right|$$

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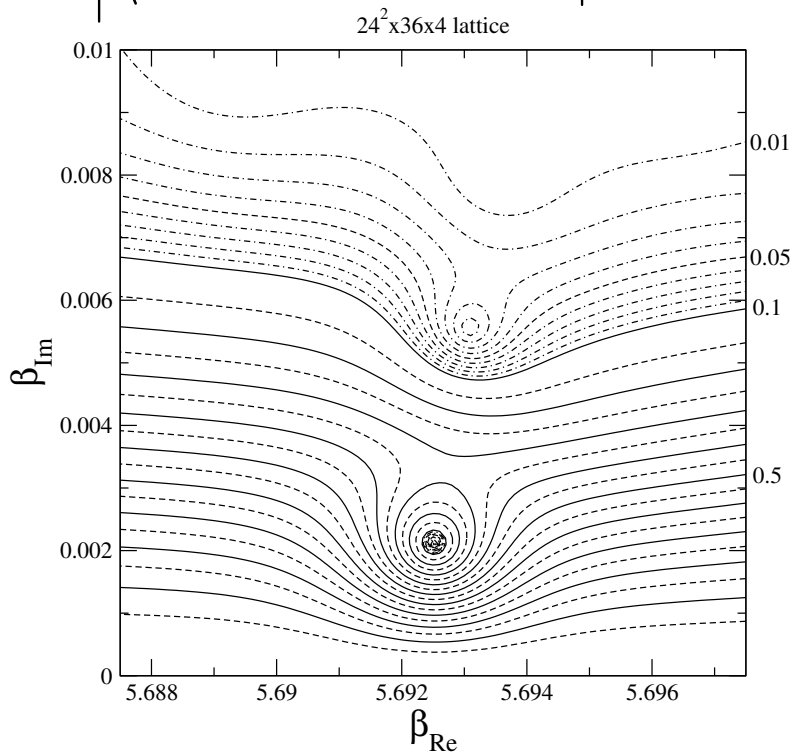

pure gauge theory $e^{i\theta} \equiv 1$: excellent on large lattices with high statistics, S. Ejiri, hep-lat/05006023

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additional complication in QCD

phase of fermion matrix contributes

$$e^{i\theta} \sim e^{i\left[c\mu V + \mathcal{O}(\mu^3 V)\right]}$$

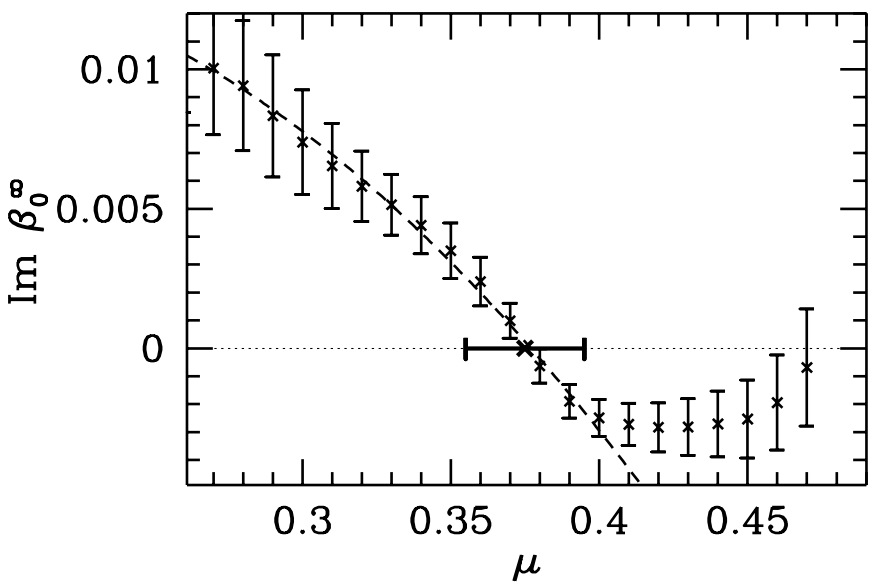
Lee-Yang zeroes from reweighting
two calculations

$V = 4^3 - 8^3$ $\qquad$ $V = 6^3 - 12^3$

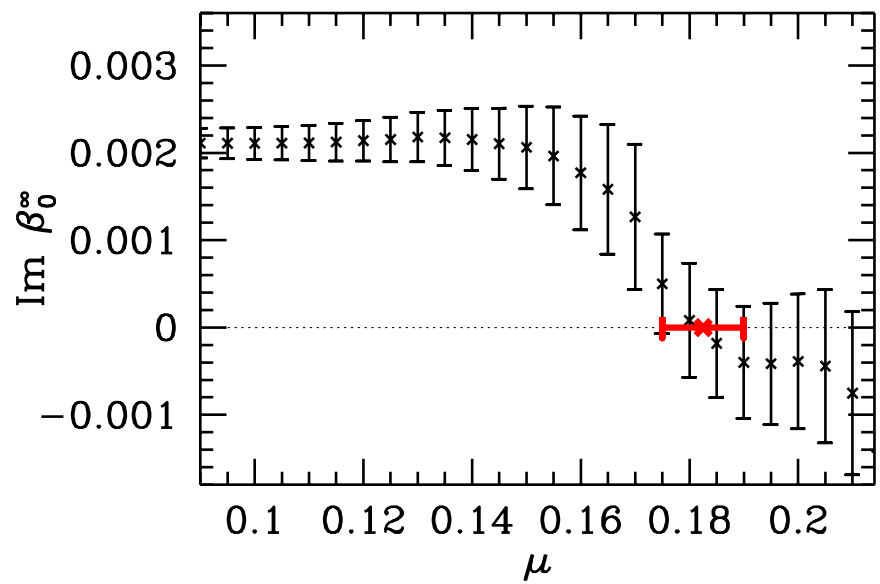
$m_q = 0.025$ $\qquad$ $m_q = 0.0092$

$\Downarrow$ $V \to \infty$ extrapolation: $Im\beta_0^V = Im\beta_0^\infty + c/V$ $\Downarrow$

$(\mu_B/T)_c = 4.50(24) \Rightarrow 725(35)$ MeV $\quad$ $(\mu_B/T)_c = 2.19(9) \Rightarrow 360(40)$ MeV

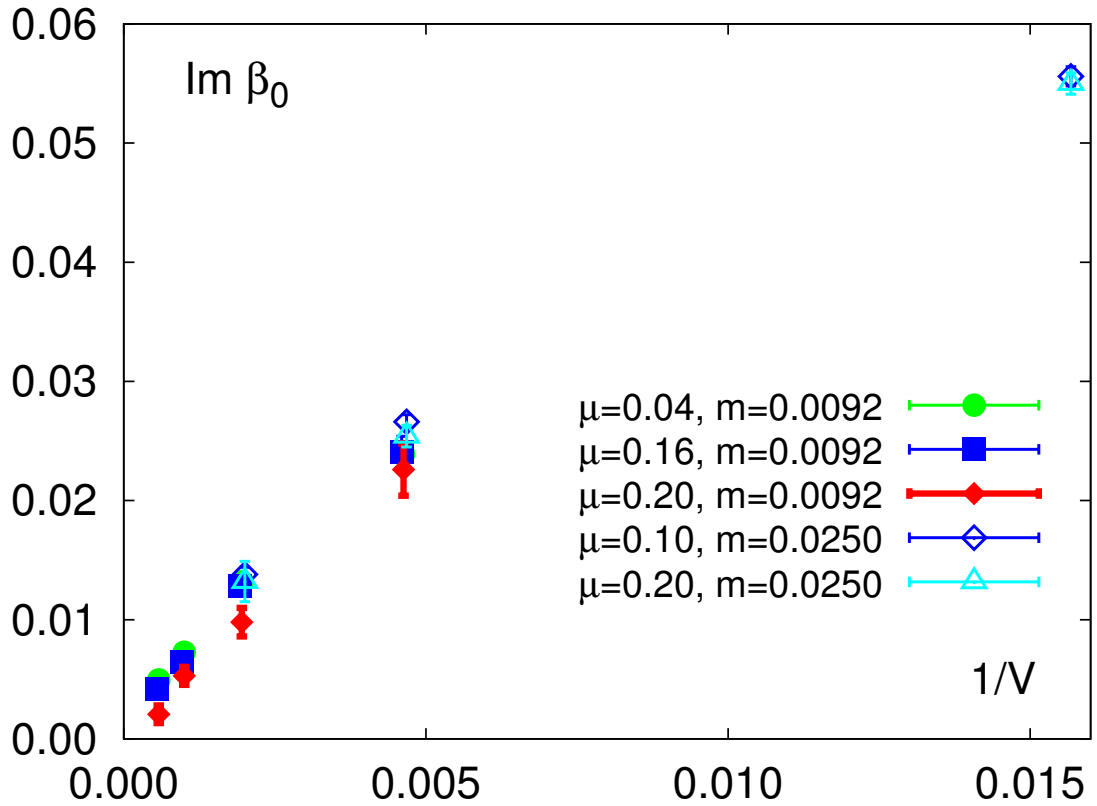


Z. Fodor and S. Katz, JHEP 0203 (2002) 014



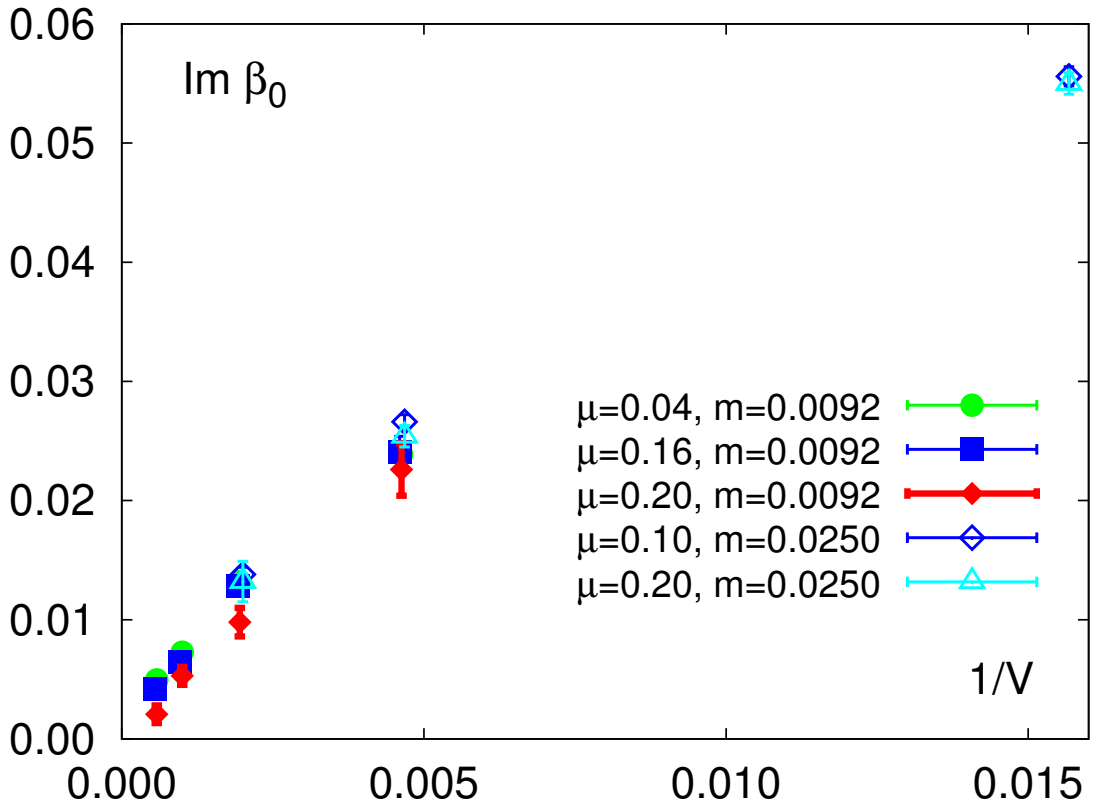
Z. Fodor and S. Katz, JHEP 04 (2004) 050

Volume and quark mass dependence of Lee-Yang zeroes



Volume and quark mass dependence of Lee-Yang zeroes

no visible quark mass dependence



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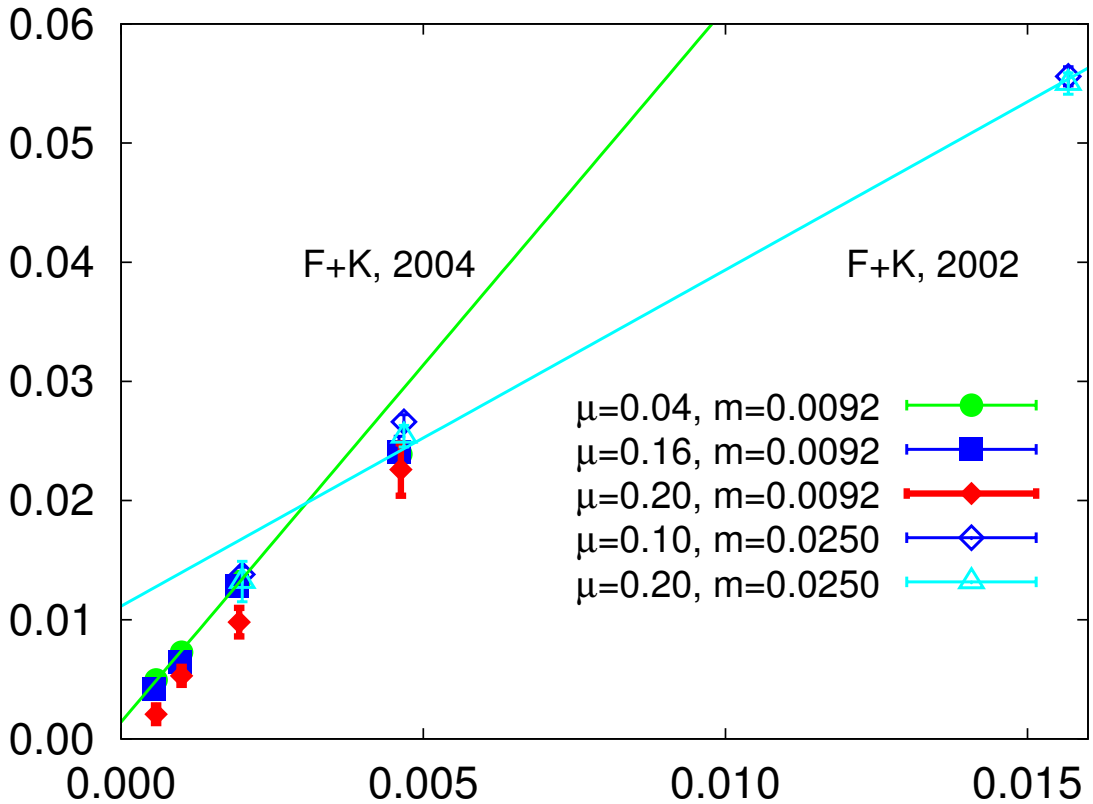
strong volume dependence ⇒ fit sensitive to fitting range



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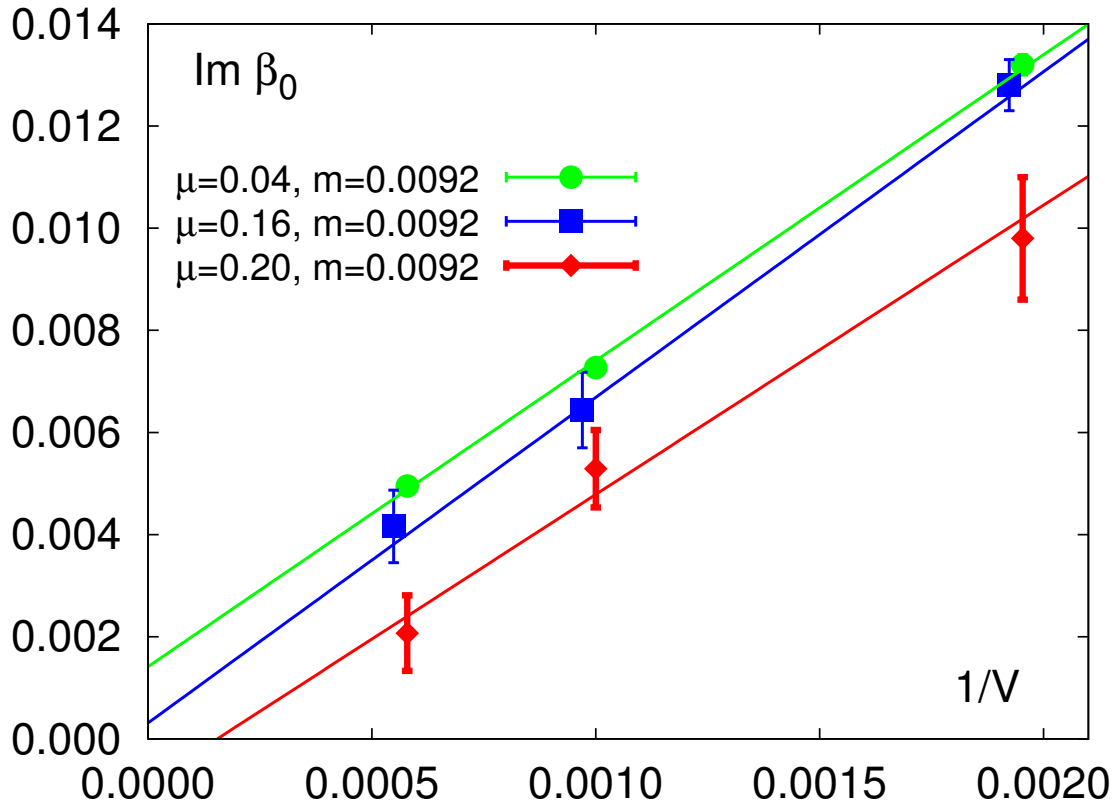

$$\mu_q^{crit} = 0.1825(75)$$

$$\Downarrow$$

$$\mu_B^{crit}/T = 2.19(9)$$

F+K 2004

$V \longrightarrow \infty$ limit difficult to control, S. Ejiri, hep-lat/05006023

the sign problem pops up again

Bulk thermodynamics with non-vanishing chemical potential

$$Z(V,T,\mu) = \int \mathcal{D}\mathcal{A}\mathcal{D}\psi\mathcal{D}\bar{\psi}\ \mathrm{e}^{-S_E(V,T,\mu)}$$

$$= \int \mathcal{D}\mathcal{A}\ \left[det\ M(\mu)\right]^f\ \mathrm{e}^{-S_G(V,T)}$$

$\Uparrow$complex fermion determinant;

$\Downarrow$Taylor expansion;

$$\frac{p}{T^4} = \frac{1}{VT^3}\ln Z(V,T,\mu)$$

$$\equiv \sum_{n=0}^{\infty} c_n(T)\left(\frac{\mu}{T}\right)^n$$

$$= c_0 + c_2\left(\frac{\mu}{T}\right)^2 + c_4\left(\frac{\mu}{T}\right)^4 + \mathcal{O}((\mu/T)^6)$$

$$\mu = 0 \quad \Rightarrow \quad \frac{p}{T^4} \equiv c_0(T)$$

Bulk thermodynamics for small μ_q/T on $16^3 \times 4$ lattices

- Taylor expansion of pressure up to $\mathcal{O}\left((\mu_q/T)^6\right)$

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu_q}{T}\right)^n \simeq c_0 + c_2 \left(\frac{\mu_q}{T}\right)^2 + c_4 \left(\frac{\mu_q}{T}\right)^4 + c_6 \left(\frac{\mu_q}{T}\right)^6$$

quark number density $\quad \frac{n_q}{T^3} = 2c_2 \frac{\mu_q}{T} + 4c_4 \left(\frac{\mu_q}{T}\right)^3 + 6c_6 \left(\frac{\mu_q}{T}\right)^5$

quark number susceptibility $\quad \frac{\chi_q}{T^2} = 2c_2 + 12c_4 \left(\frac{\mu_q}{T}\right)^2 + 30c_6 \left(\frac{\mu_q}{T}\right)^4$

an estimator for the radius of convergence

$$\left(\frac{\mu_q}{T}\right)_{crit} = \lim_{n \to \infty} \left| \frac{c_{2n}}{c_{2n+2}} \right|^{1/2}$$

$c_n > 0$ for all n;
singularity for real μ

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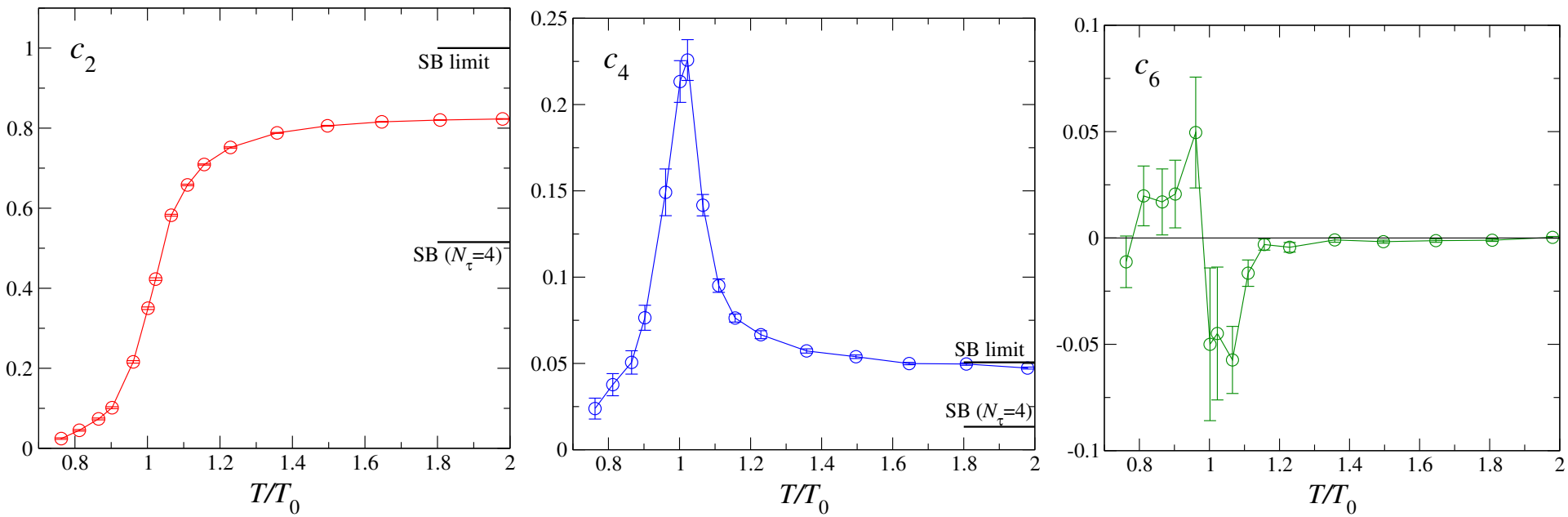

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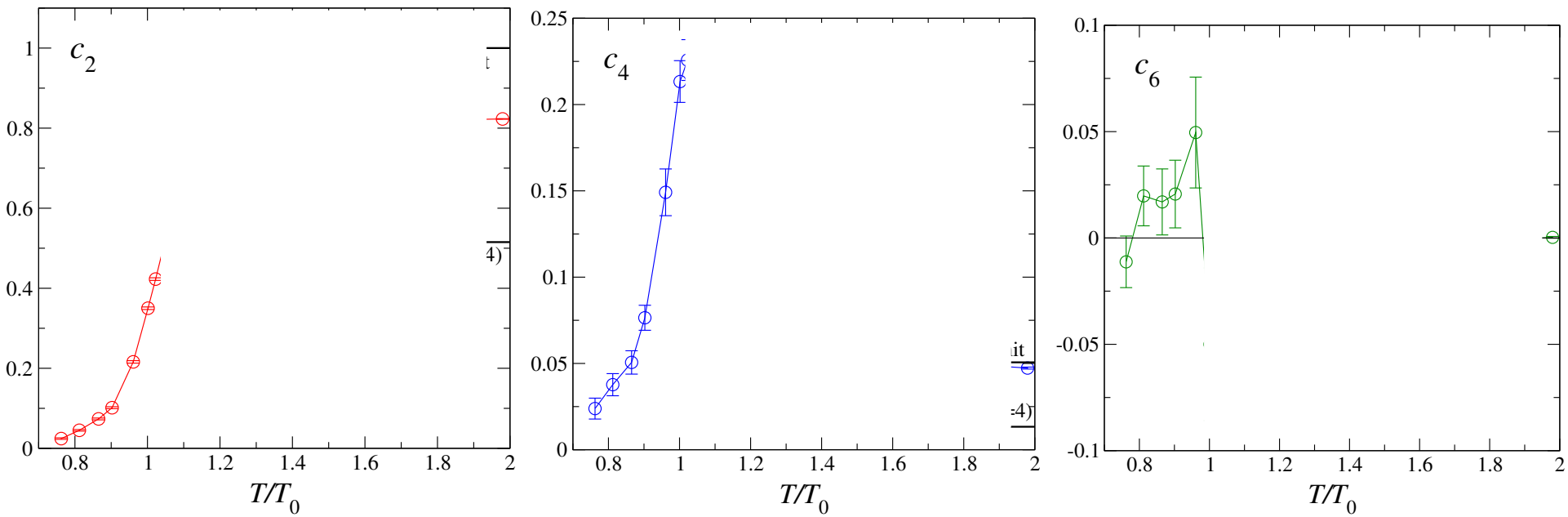
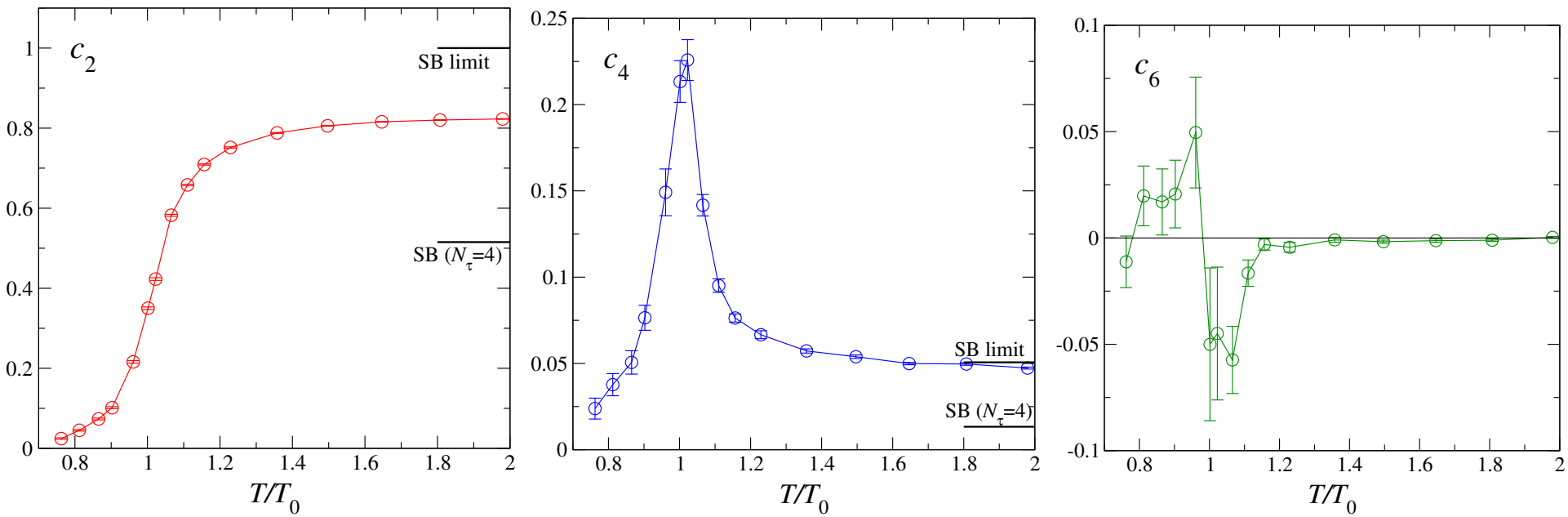


$c_n > 0$ for all n and $T \lesssim 0.95\, T_c$ $\Leftrightarrow$ singularity for real μ (positive μ^2)

Bulk thermodynamics for small μ_q/T on $16^3 \times 4$ lattices

- Taylor expansion of pressure up to $\mathcal{O}\left((\mu_q/T)^6\right)$

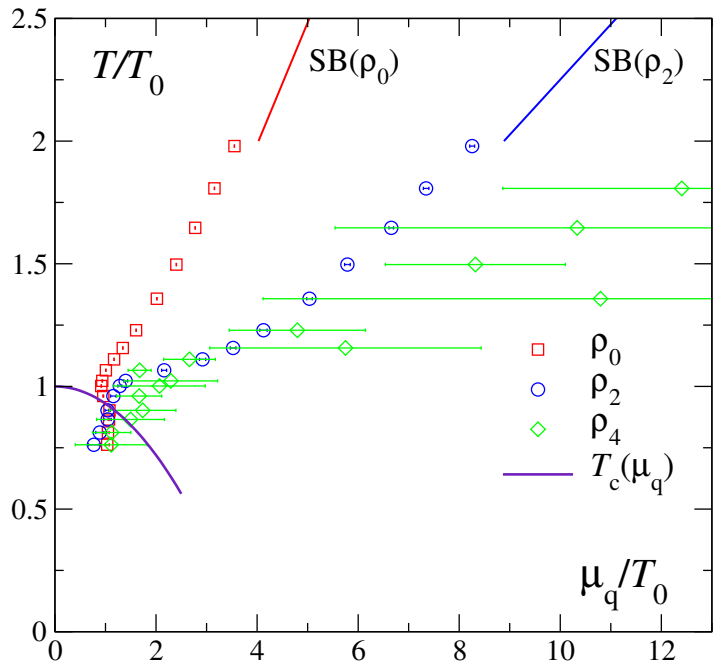
$$\frac{p}{T^4} = \frac{1}{VT^3} \ln \mathcal{Z} = \sum_{n=0}^{\infty} c_n(T) \left(\frac{\mu_q}{T}\right)^n \simeq c_0 + c_2 \left(\frac{\mu_q}{T}\right)^2 + c_4 \left(\frac{\mu_q}{T}\right)^4 + c_6 \left(\frac{\mu_q}{T}\right)^6$$

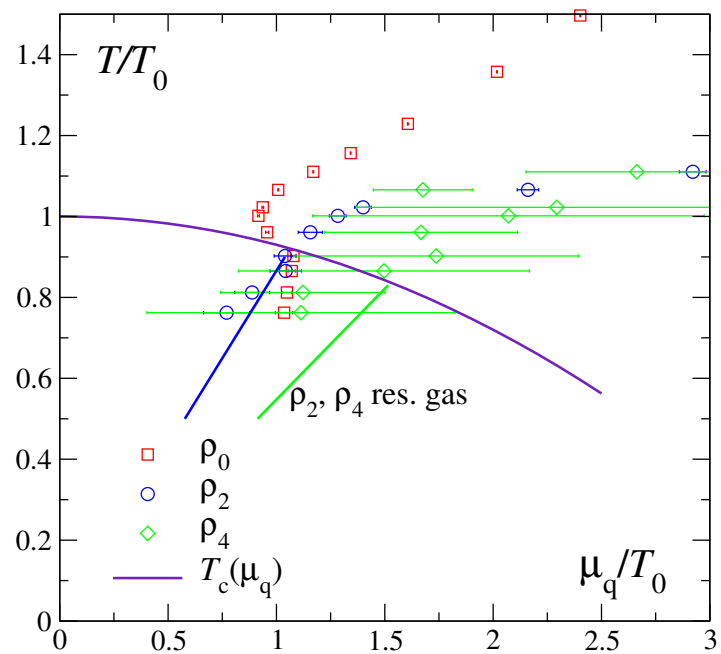


irregular sign of c_n for $T \gtrsim T_c$ $\Leftrightarrow$ singularity in complex plane

Radius of convergence: lattice estimates vs. resonance gas

- Taylor expansion $\Rightarrow$ estimates for radius of convergence $\rho_{2n} = \sqrt{\left|\frac{c_{2n}}{c_{2n+2}}\right|}$





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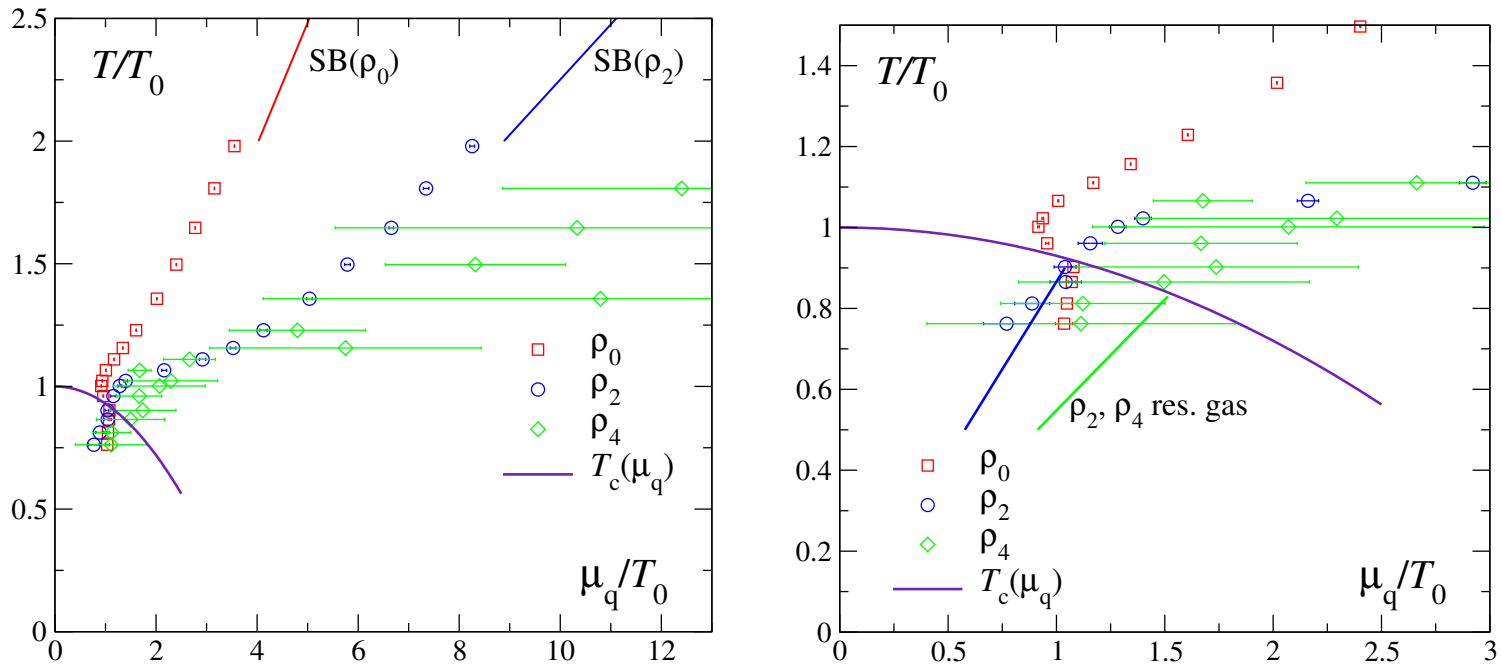

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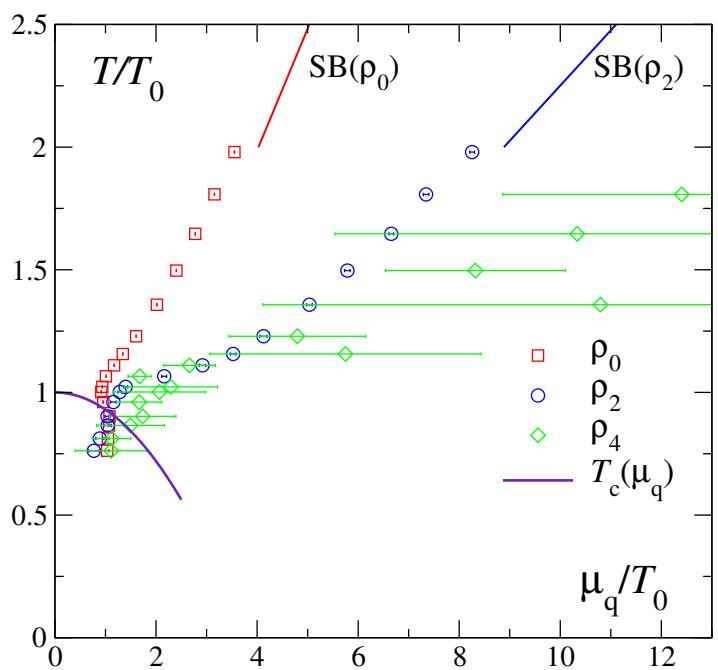
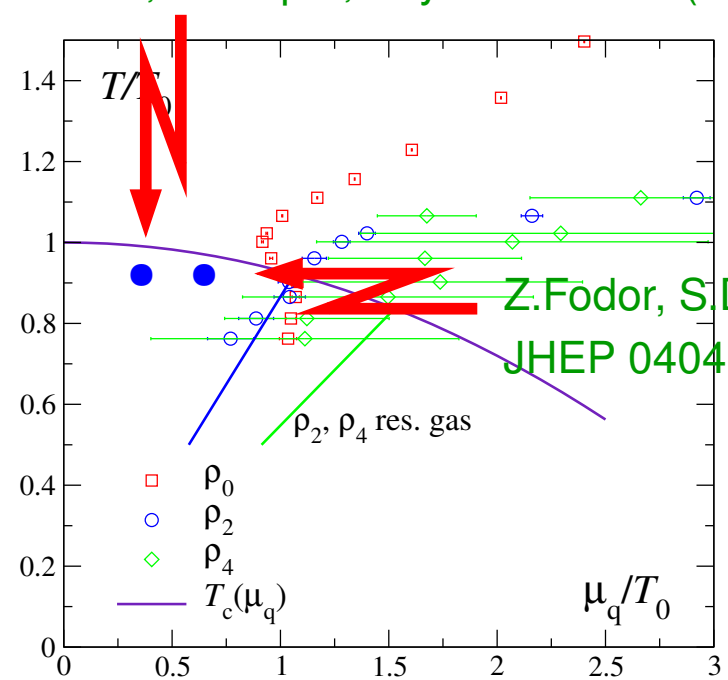
R.V. Gavai, S. Gupta, Phys. Rev. D71 (2005) 114014





Z.Fodor, S.D.Katz
JHEP 0404 (2004) 050

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The estimate of the QCD critical point by Gavai and Gupta

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  What is different?

  - standard staggered fermions vs. improved fermions
  - $m_q/T = 0.1$ vs. $m_q/T = 0.4$
  - $24^3 \times 4$ vs. $16^3 \times 4$
  - Taylor series for "flavor diagonal quark number susceptibility" up to μ_q^6 vs. Taylor series for pressure up to μ_q^6
  - statistics, statistics, statistics....
    ($\sim$ (1.000 - 5.000) vs. $\sim$ 50.000)

Taylor expansions in μ_q/T

Allton et al. (WE) vs. Gavai+Gupta (GG)

- Taylor expansion of pressure up to $\mathcal{O}\left((\mu_q/T)^6\right)$

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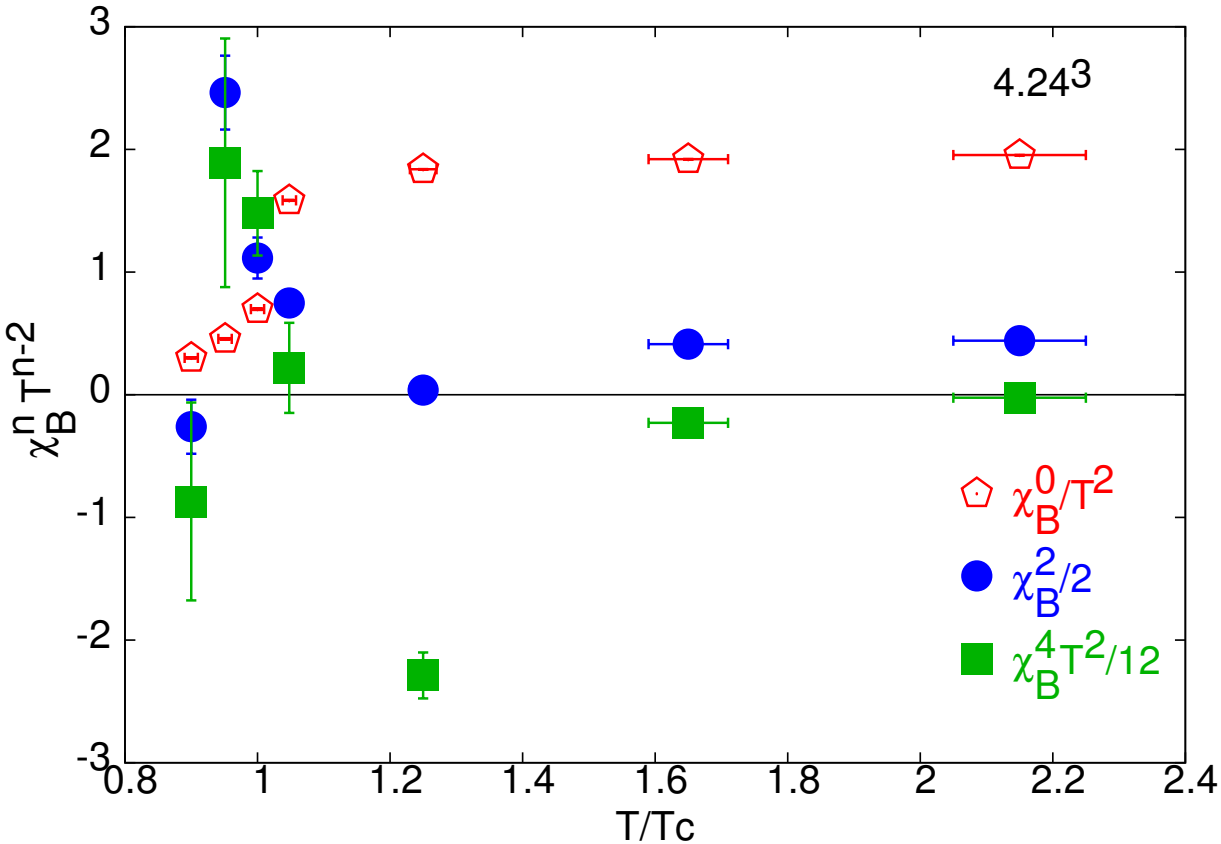
quark number susceptibility $\frac{\chi_q}{T^2} = 2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2 + 30c_6\left(\frac{\mu_q}{T}\right)^4$

flavor diagonal susceptibility $\frac{\chi_q^{uu}}{T^2} = 2c_2^{uu} + 12c_4^{uu}\left(\frac{\mu_q}{T}\right)^2 + 30c_6^{uu}\left(\frac{\mu_q}{T}\right)^4$

$$c_n^{uu} = (c_n + c_n^I)/4$$

in notation of Gavai and Gupta $\chi_{20} = \chi_B^0 + \chi_B^2\mu_q^2 + \chi_B^4\mu_q^4 + \ldots$

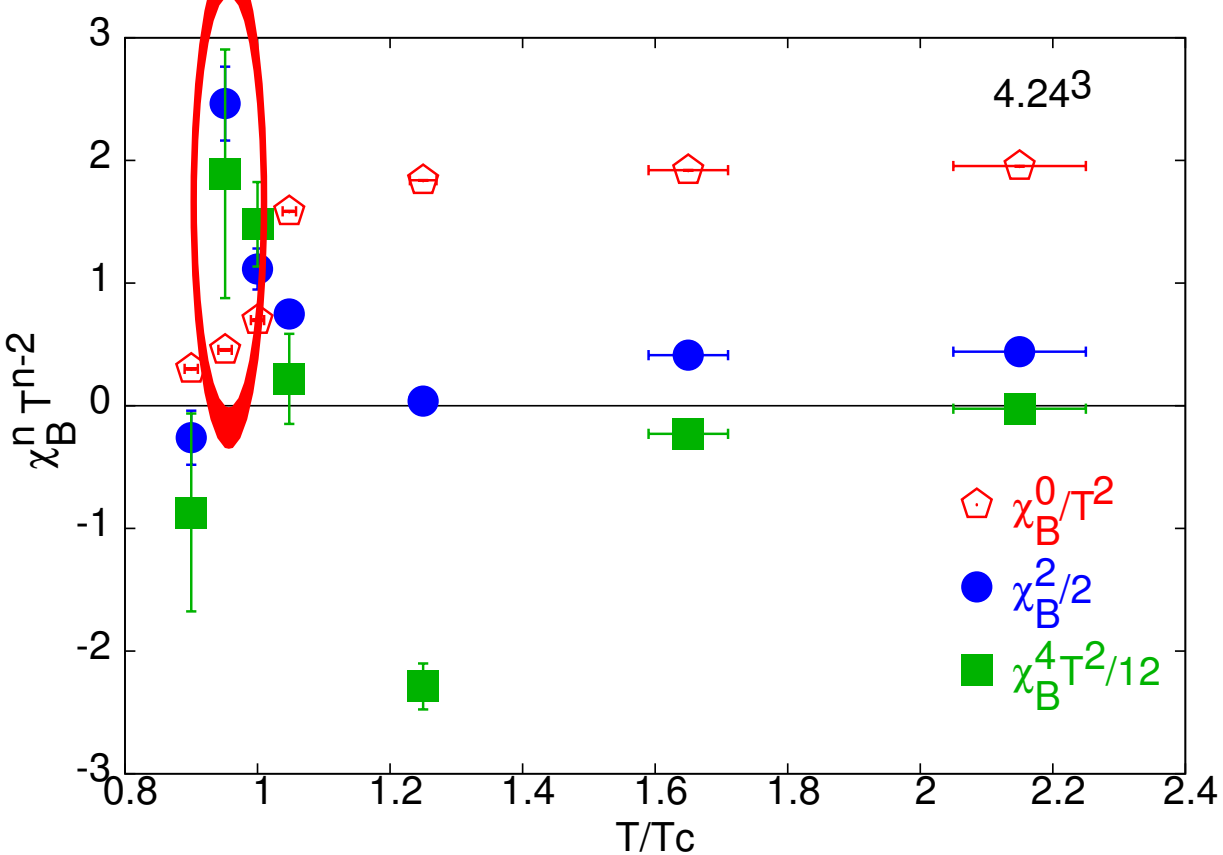
Taylor expansion coefficients of quark number susceptibilities



R.V. Gavai and S.Gupta, PRD71 (2005) 114014

Taylor expansion coefficients of quark number susceptibilities

data at $T = 0.95$ have been used to estimate μ_{crit}



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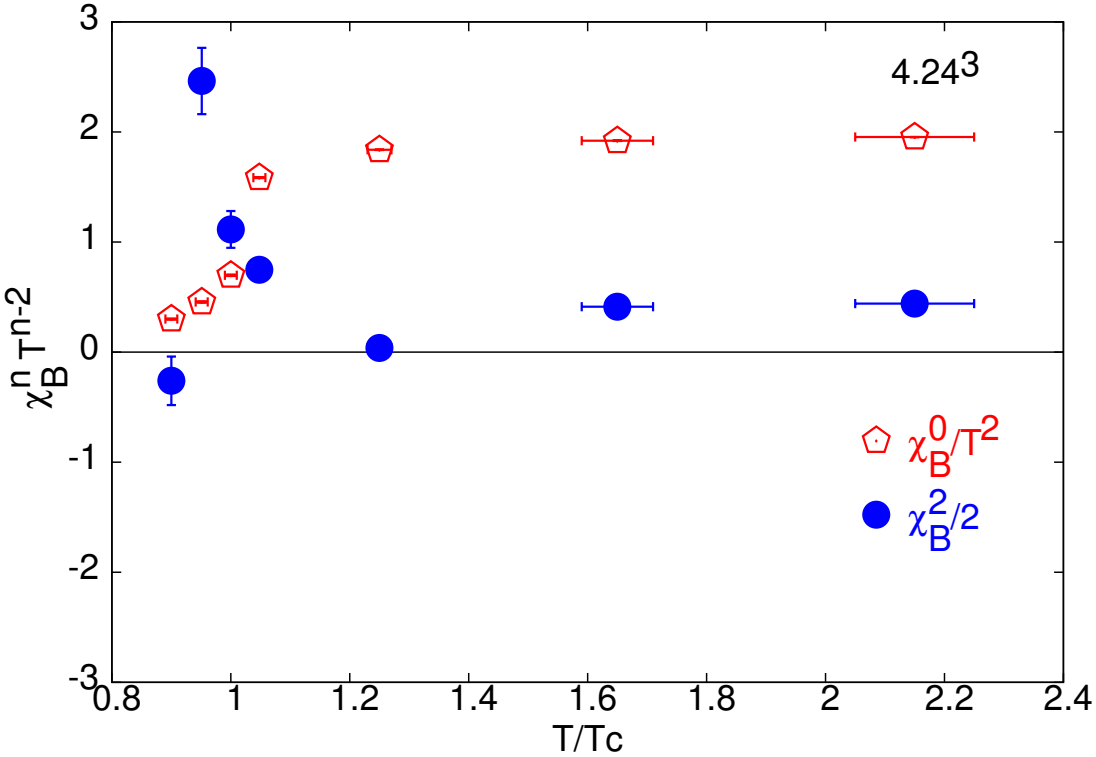

GG

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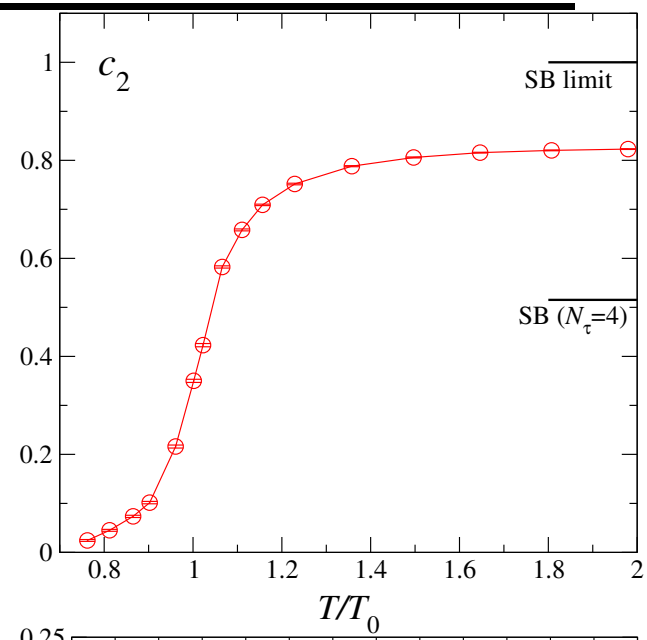
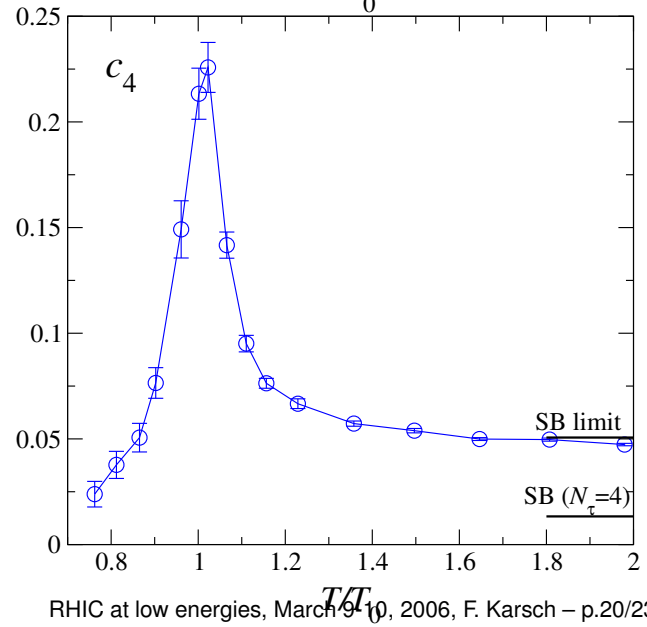
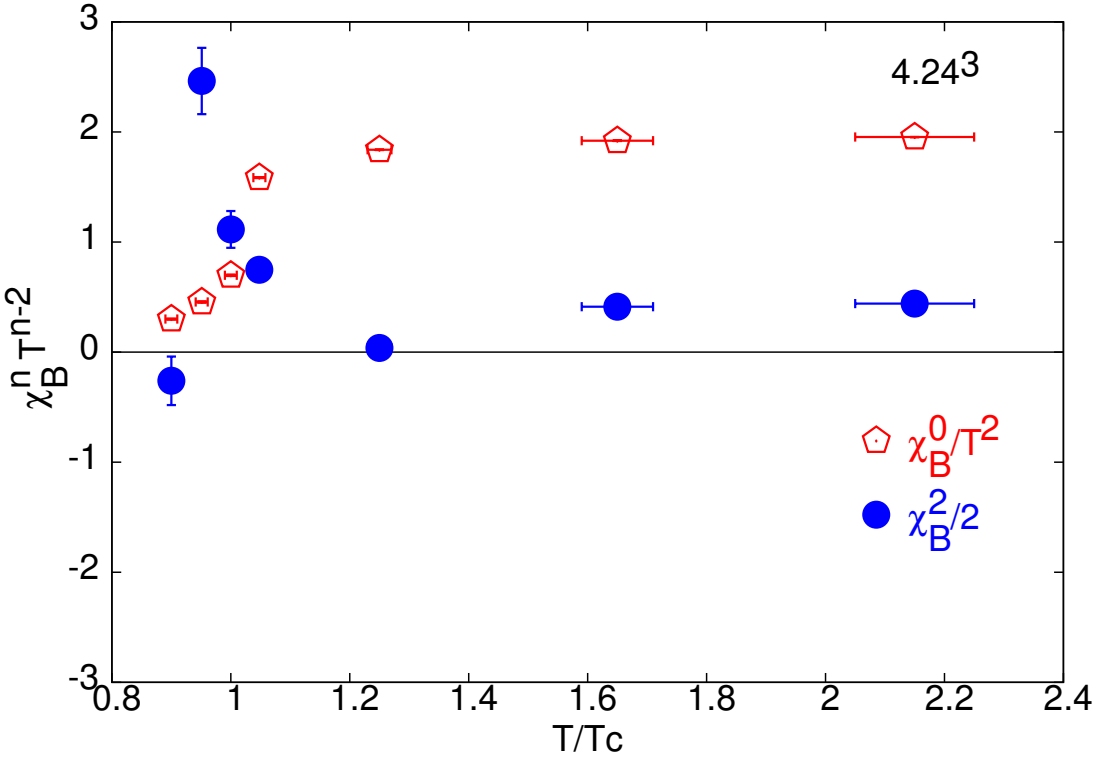




Taylor expansion coefficients of quark number susceptibilities

estimate of CP based on data at $0.95T_c$

large value for χ_B^2 leads to low estimate for μ_{crit}



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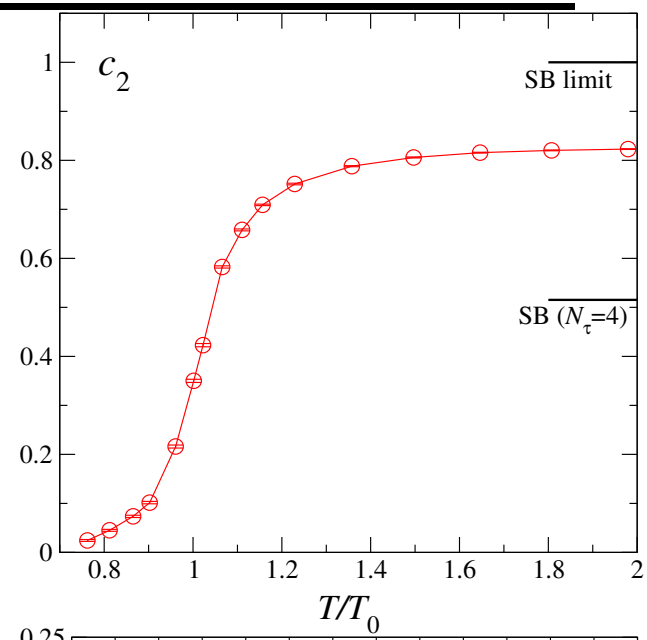
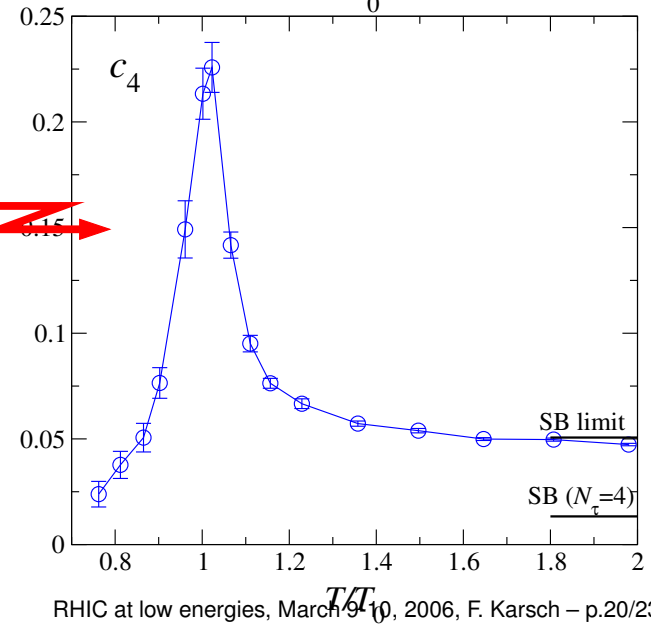
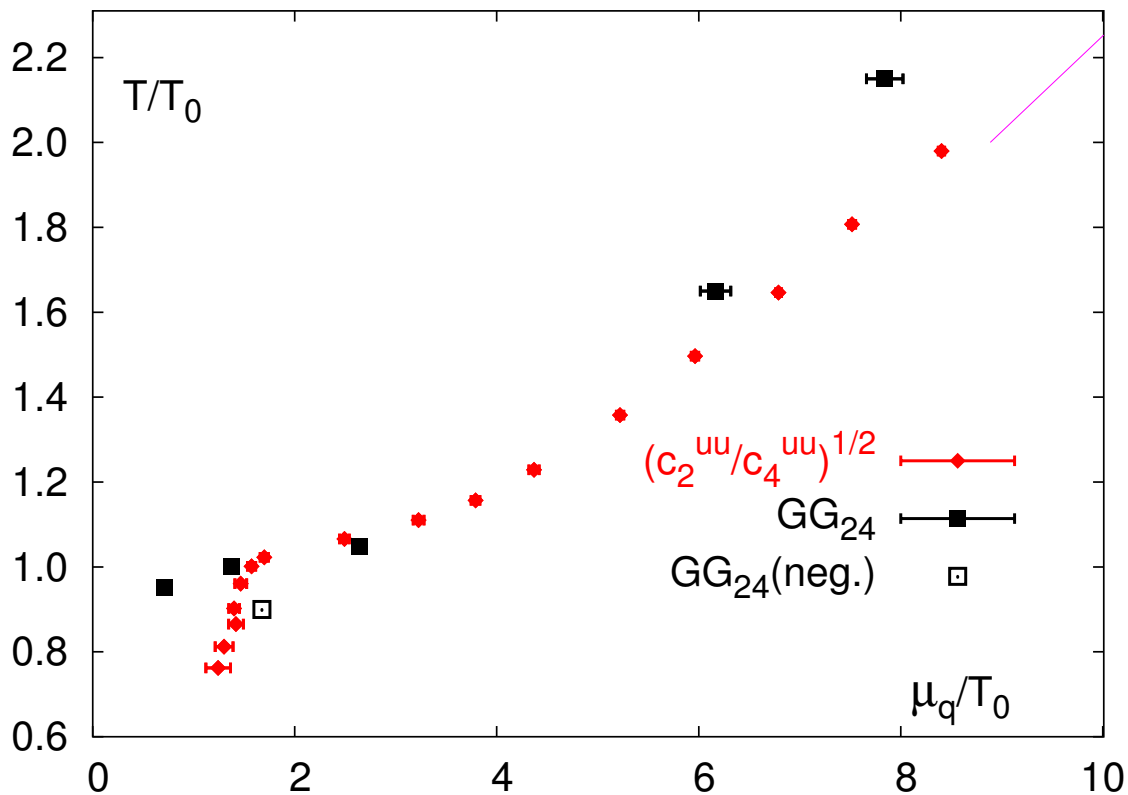




Estimator for radius of convergence

$$\left(\frac{\mu_q}{T}\right)_{crit} \simeq \sqrt{\left|\frac{c_2^{uu}}{c_4^{uu}}\right|}$$



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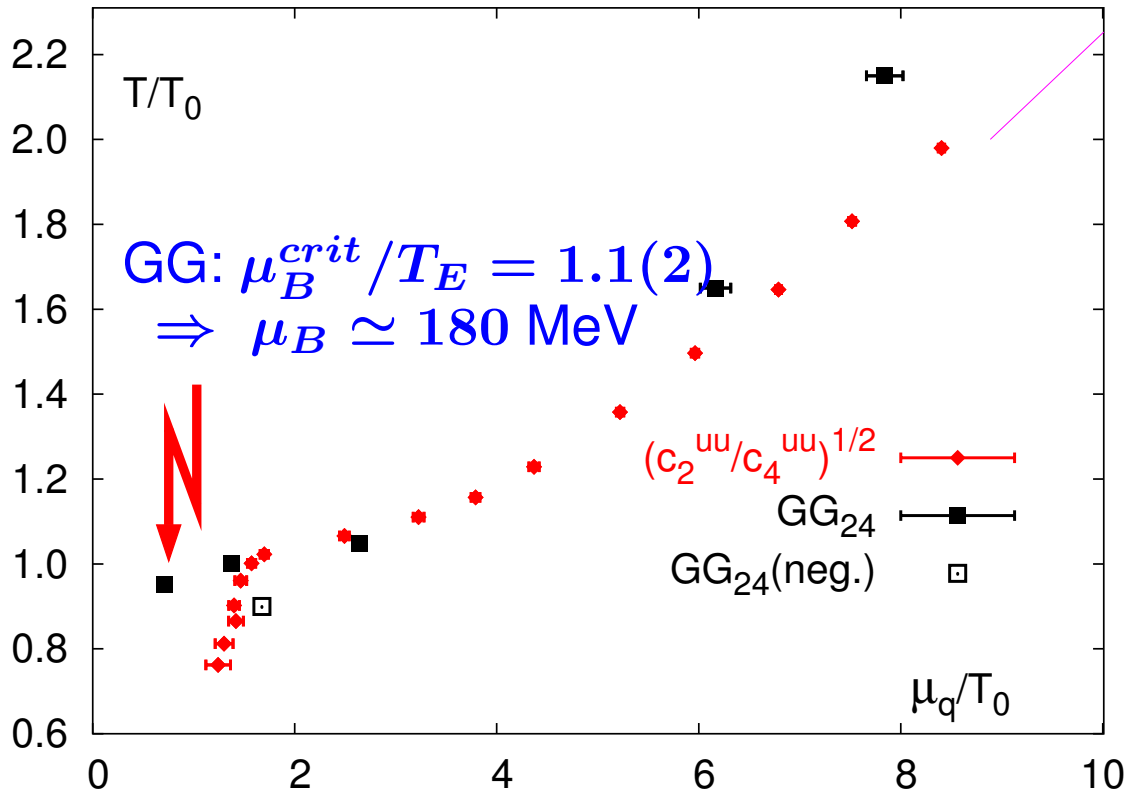

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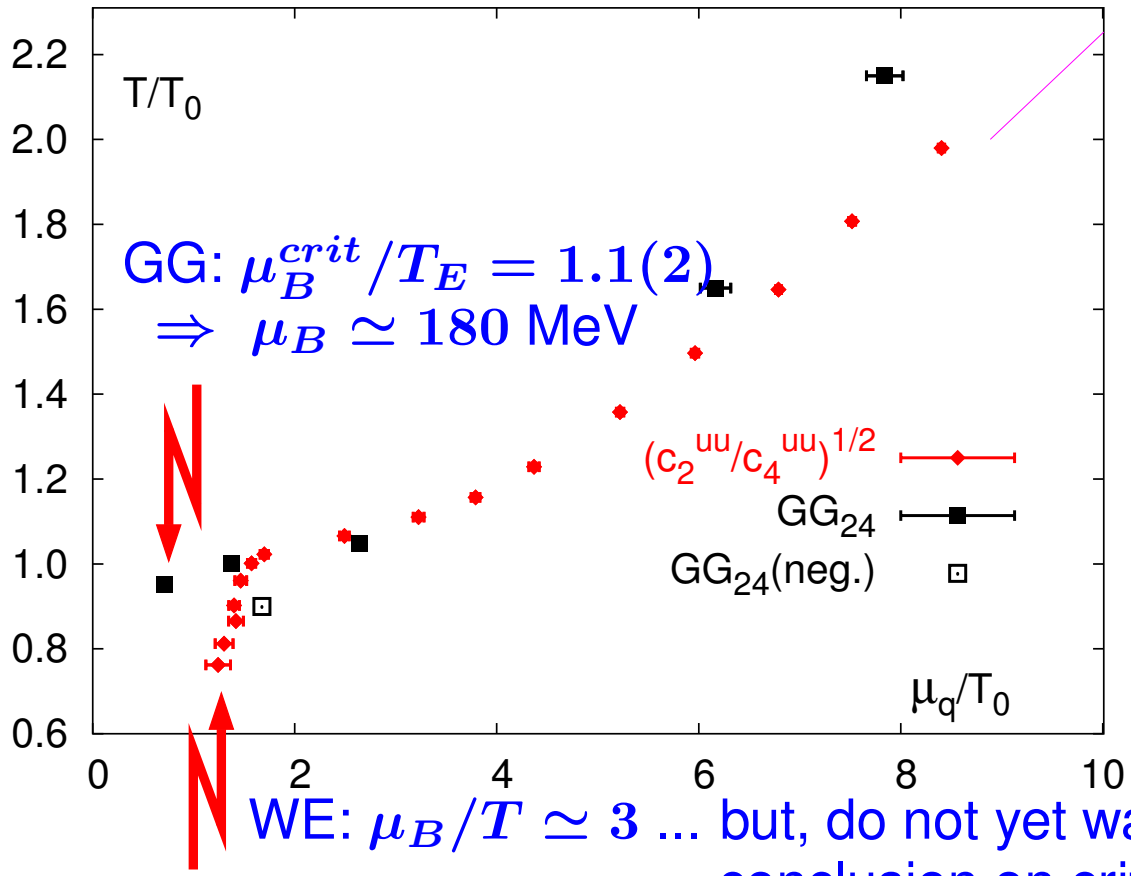


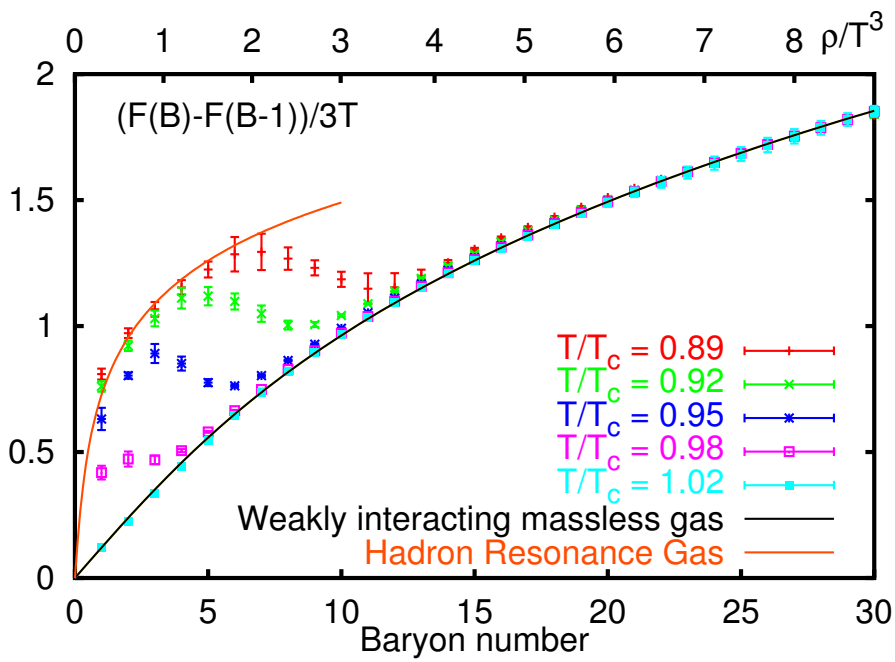
GG: $\mu_B^{crit}/T_E = 1.1(2)$
$\Rightarrow$ $\mu_B \simeq 180$ MeV

WE: $\mu_B/T \simeq 3$ ... but, do not yet want to draw a conclusion on critical behavior

some results from 4-flavor QCD

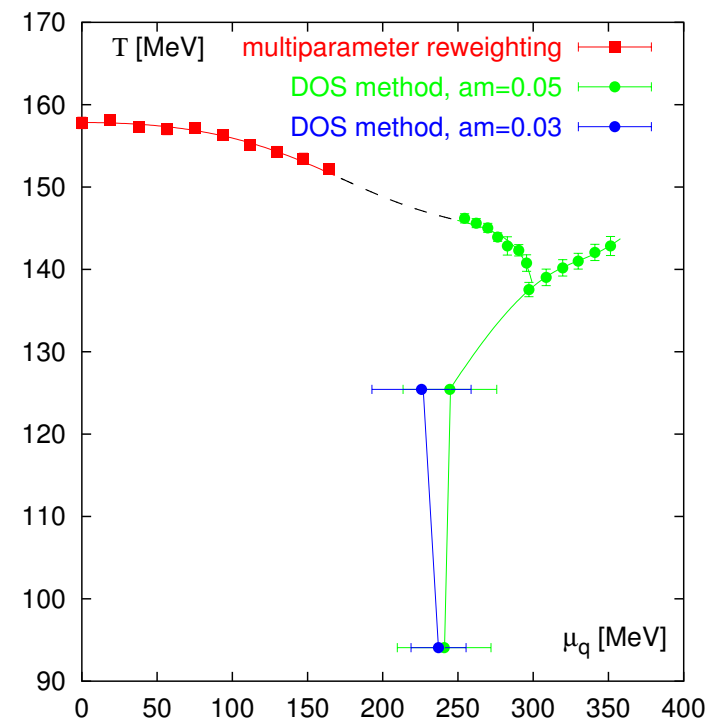
- 4-flavor QCD at $\mu = 0$ has a 1st order phase transition
- expect entire $T - \mu_q$ transition line to be 1st order

non-zero baryon number rather than non-zero μ_q



S. Kratochvilla, Ph. de Forcrand, hep-lat/0509143

density of state method $\simeq$ constraint MC at $\mu_q = 0$



Ch. Schmidt et al., hep-lat/0510087

Conclusions

- calculations for non-vanishing chemical potential ($\mu_q > 0$) show a rapid transition from a HRG to a QGP; signaled by sudden changes in EoS and susceptibilities
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