

CAN WE DISCOVER  
THE QCD CRITICAL POINT  
AT RHIC?

KRISHNA RAJAGOPAL  
MIT + LBNL

BNL, March 9, 10 2006

## WHY STUDY QCD?

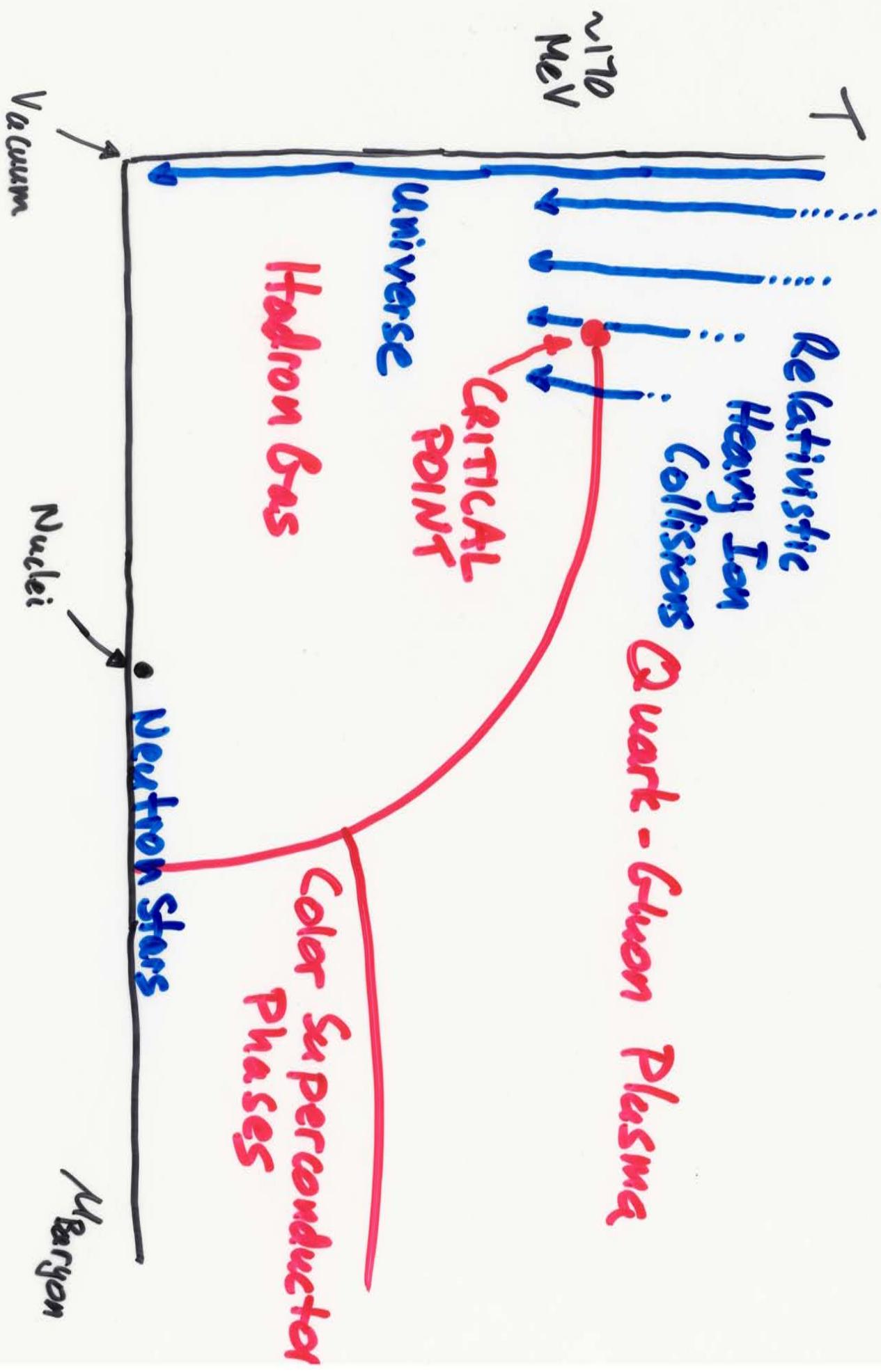
## WHY IS IT A CHALLENGE?

- The only example we know of a strongly interacting gauge theory.
- We understand the theory at short distances.
- The quasiparticles — the excitations of the vacuum — are hadrons, which do not look at all like the short distance quark + gluon degrees of freedom.

## HOW CAN WE RESPOND TO THIS CHALLENGE, EXPERIMENTALLY?

- Study the structure of the hadrons.  
(Eg , RHIC Spin)
- Get away from the vacuum. Understand other phases of QCD, and their quasiparticles. Map the QCD phase diagram.

# EXPLORING THE PHASES OF QCD

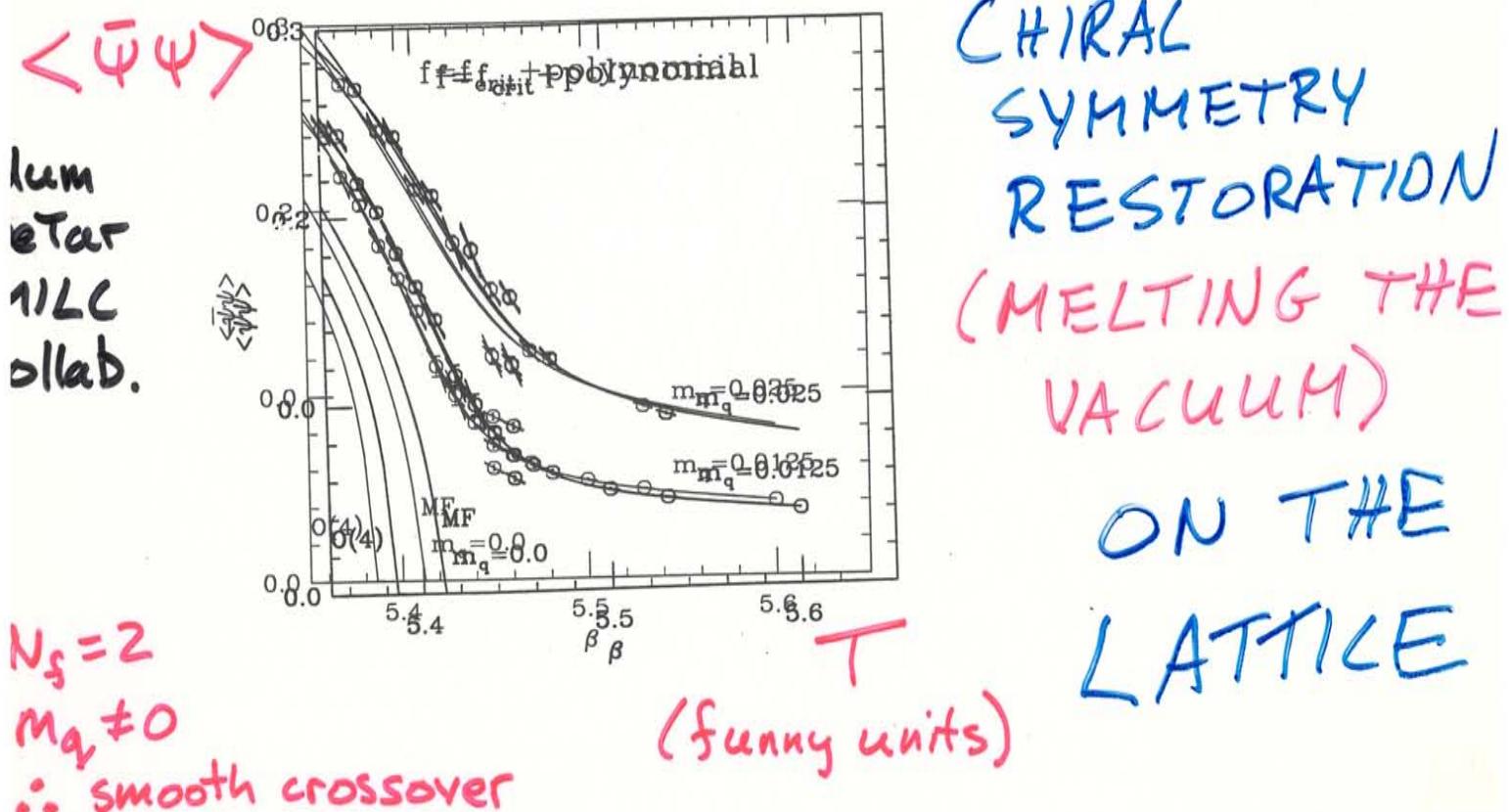
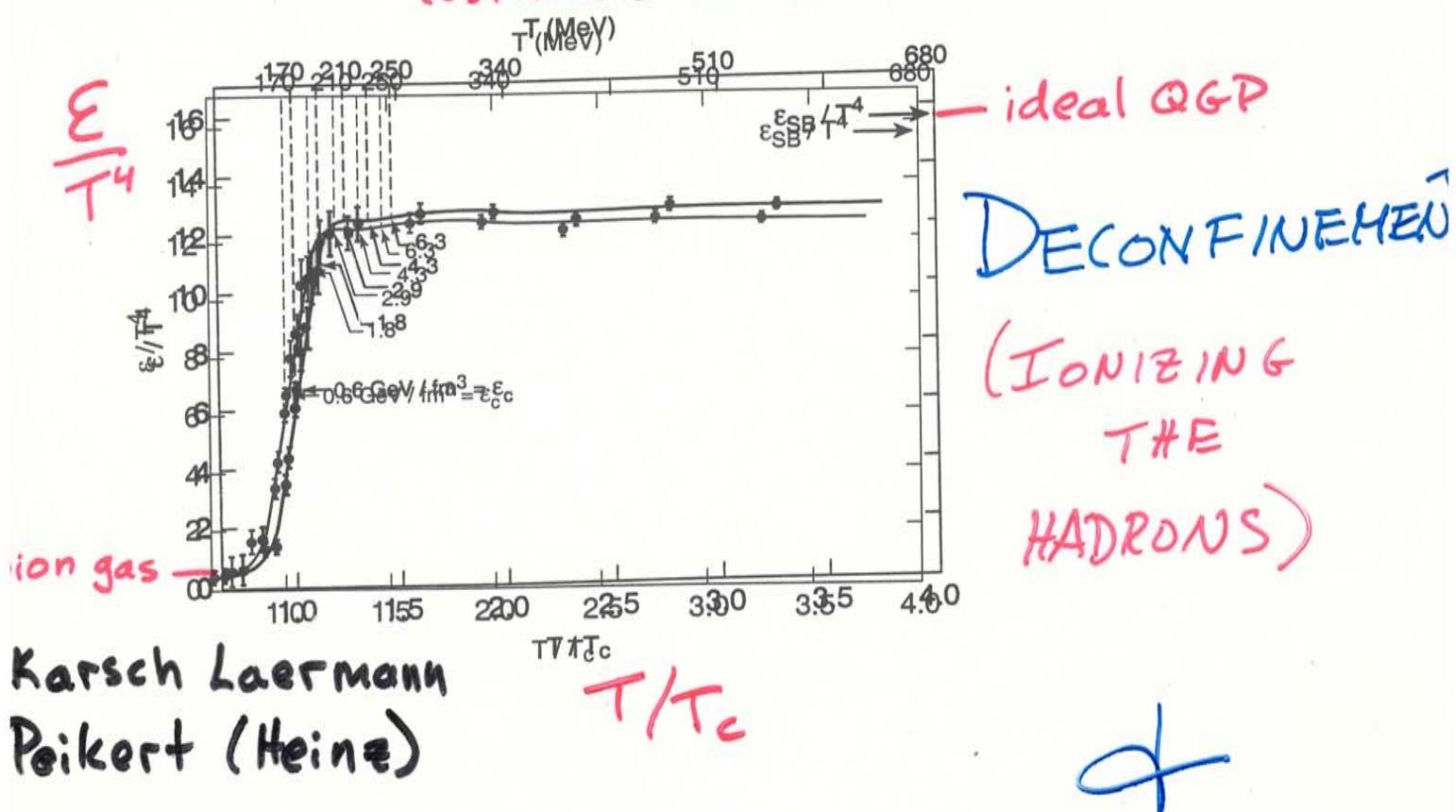


$$\underline{T \neq 0 ; \mu = 0}$$

- vertical axis
  - we know a lot from lattice QCD. e.g. →
  - QCD describes a transition
- |                                      |                                      |
|--------------------------------------|--------------------------------------|
| FROM                                 | TO                                   |
| gas of hadrons                       | plasma of quarks<br>and gluons       |
| with chiral symmetry<br>badly broken | with chiral sym.<br>almost restored. |
- $T_c \approx 175 \pm 15$  MeV
  - The transition is a smooth crossover, like ionization of a gas, occurring in a narrow range of T.  
IF  $m_s \gtrsim \frac{1}{5} m_s^{\text{physical}}$ , and is in Nature UB: In world with  $m_u = m_d = m_s$ , crossover if  $m_s \gtrsim \frac{1}{5} m_s^{\text{physical}}$

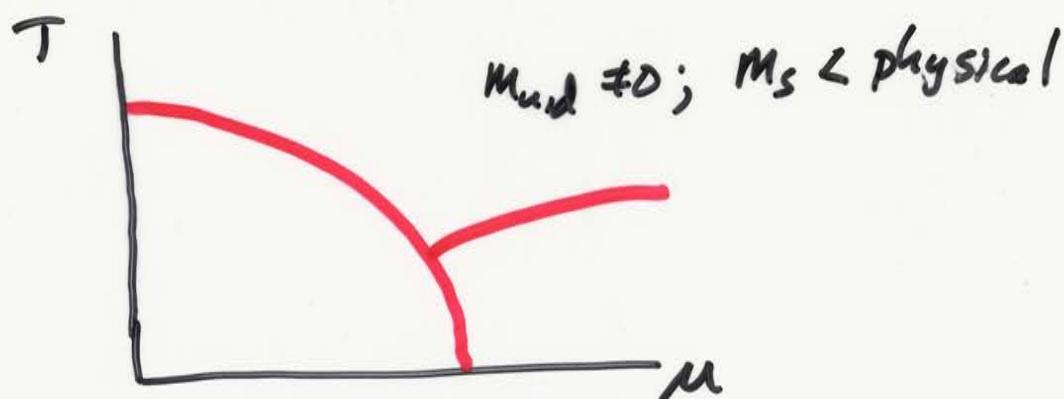
Bielefeld  
Columbia

$T$  (MeV), assuming  $T_c = 170$  MeV.  
(estimate is  $140 < T_c < 190$ )



# WHY EXPECT A CRITICAL POINT?

- Models; lattice QCD calculations at  $M \neq 0$  with varying quark masses; suggest:



- Need lattice calculations with  $T \neq 0$ ,  $\mu \neq 0$  to locate it
- Universality class known (Ising)

$T \neq 0 ; M \neq 0 ; M/T \text{ NOT LARGE}$

$I \neq 0 \rightarrow$  complex Euclidean action.

$\rightarrow$  sign problem that makes difficulty of standard Monte Carlo  $\sim e^V$ .

Nevertheless, we are learning about this regime from lattice calculations that rely on smallness of  $M/T$ .

These methods may be used to locate the...

### CRITICAL POINT

A 2<sup>nd</sup> order point in the phase diagram where a line of 1<sup>st</sup> order transitions end. (Location is sensitive to quark masses. Moves leftward as masses  $\downarrow$ .)

## LOCATING THE CRITICAL POINT

Range of estimates:  $\Gamma_{NB}: \mu_B = 3\mu_L$

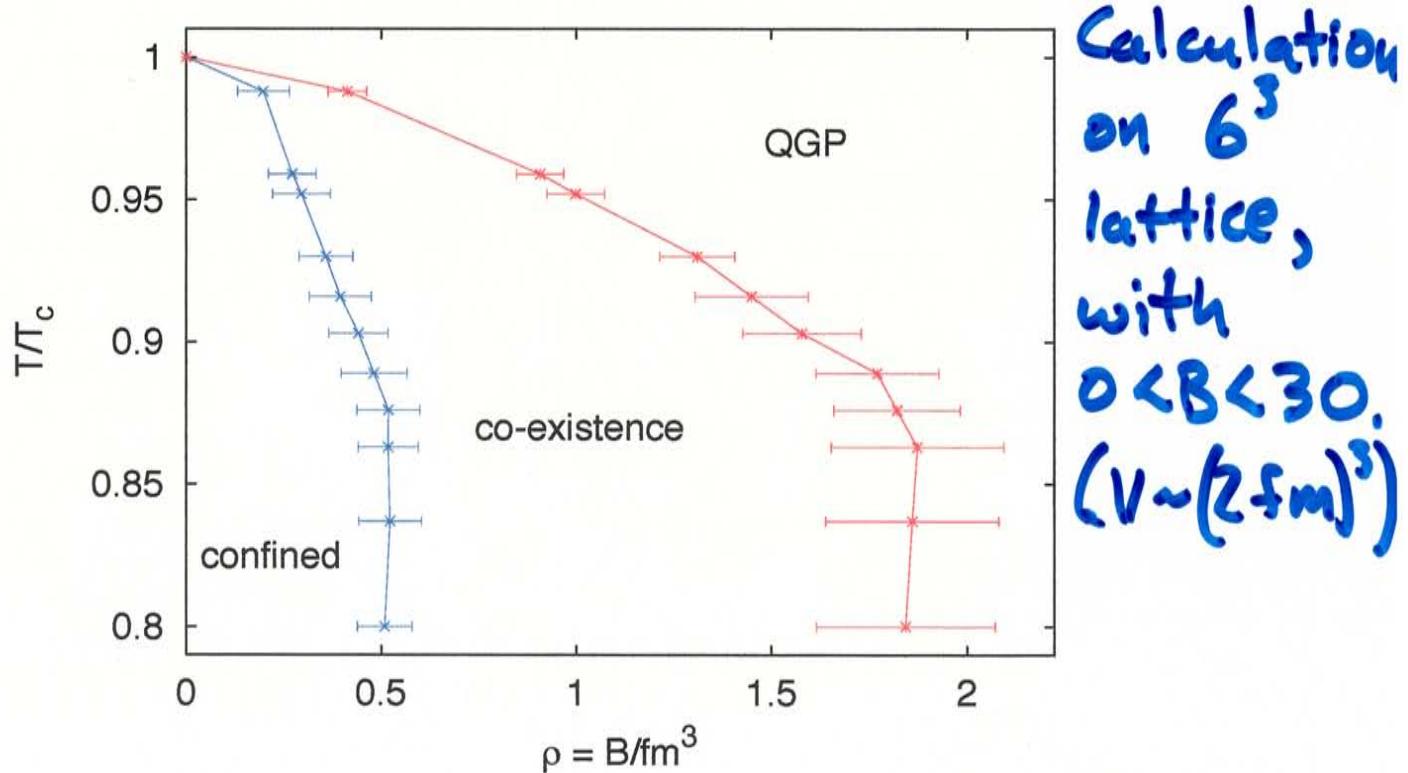
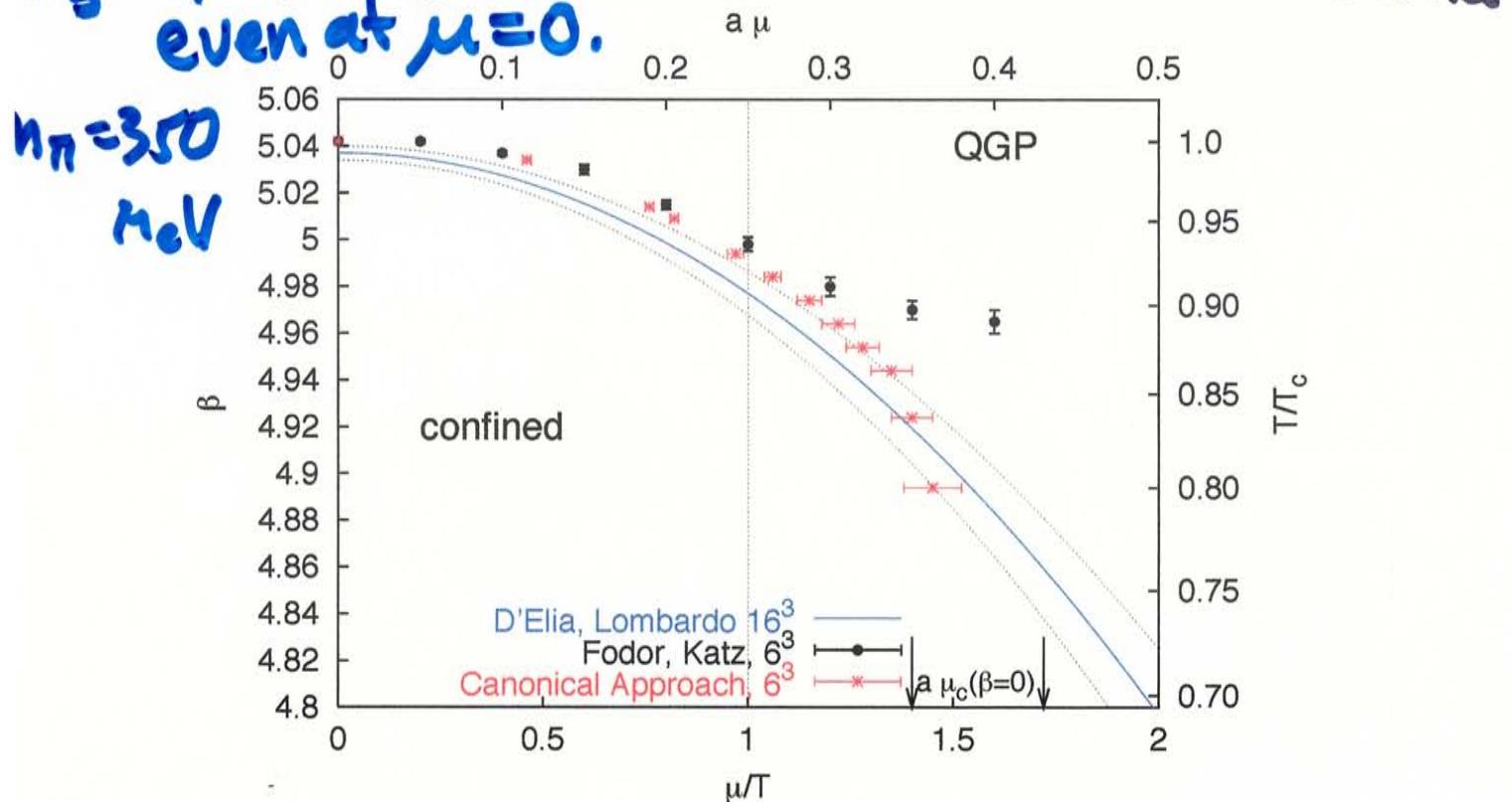
$\frac{\mu_B}{T_c(\mu=0)}$ Endpoint	$\sim 1$ , Gavai Gupta	$\sim 2$ , Fodor Kett	$\gtrsim 3$ , Ejiri et al

Error estimates uncertain and clearly still large. Not at all like calculations of  $T_c$ . Yet.

Race between lattice QCD and experiment to locate the critical point....

# LATTICE CALCULATIONS AT FIXED $n_B$

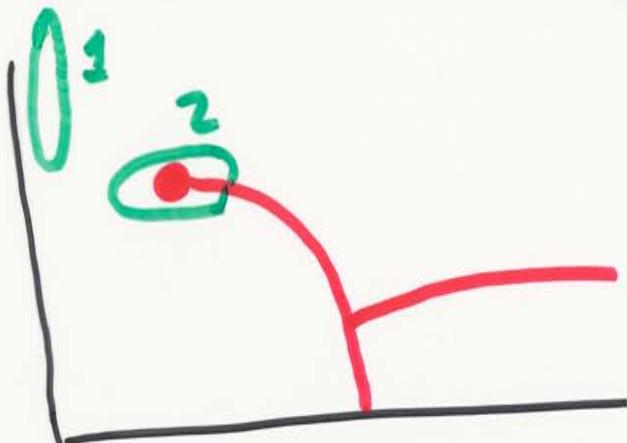
$N_f = 4 \rightarrow 1^{\text{st}} \text{ order}$   
even at  $\mu = 0$ .



- Will be very interesting to see what they find with  $N_f = 2 + 1$ .
- Determining the location of the critical point this way will have very different "systematic error" relative to calculations relying on  $\mu/T < 1$ . (ie reweighting Fodor Katz or Taylor expansion Ejiri et al, Gauri Gupta)
- In principle can be pushed to larger  $\mu/T$ , but remains to be seen how large a  $V$  can be reached at a given  $\mu$  or  $n_B$ .

# GOALS FOR EXPERIMENTS

Given a phase diagram:



what do we want to learn from heavy ion collisions?

- ① Characterize the properties of the quark-gluon plasma at  $T > T_c$ . RHIC is doing so.
- ② Equally important, discover and hence locate the critical point.  
Mapping the phase diagram. Finding the landmark that would go in any future book on QCD. Can RHIC do this?

## HOW CAN EXPERIMENTS LOCATE THE CRITICAL POINT?

- ① Need evidence that at large  $\sqrt{s}$ , i.e small  $\mu$ , collisions equilibrate well above the crossover.  $v_2 @ RHIC$ .
- ② Decrease  $\sqrt{s}$ , moving freezeout point to larger and larger  $\mu_B$ .
- ③ Look for signatures:
  - a) Of the critical point itself. Those relying on the long wavelength gaussian fluctuations occurring only near  $\bullet$ . Rise and then fall as  $\mu_B$  increases.
  - b) Onset of signatures of non-equilibrium "lumpy" final state expected after cooling through a first order transition.  
Mishustin; Dusmire; Paech Stöcker; Raudrup;  
Koch Majumder Raudrup; ...

## SIGNATURES OF THE CRITICAL POINT

In those collisions that pass near the critical point as they cool, find long wavelength oscillations of a mode that is a linear combination of  $\sigma$  (means fluctuations couple to  $\pi\pi$ ) and baryon number. The more effectively equilibrium is maintained, the longer the correlation length  $\xi$  gets, the bigger the signatures:

- Gaussian event-by-event fluctuations of specific observables, calculable in magnitude in terms of  $\xi$ .  
Stephanov  
KR Shuryak
- Vary  $\mu$  by varying  $\sqrt{s}$ , search for enhancement of these fluctuations in a window in  $\sqrt{s}$ , ie  $\mu$ .
- Examples .... But first :

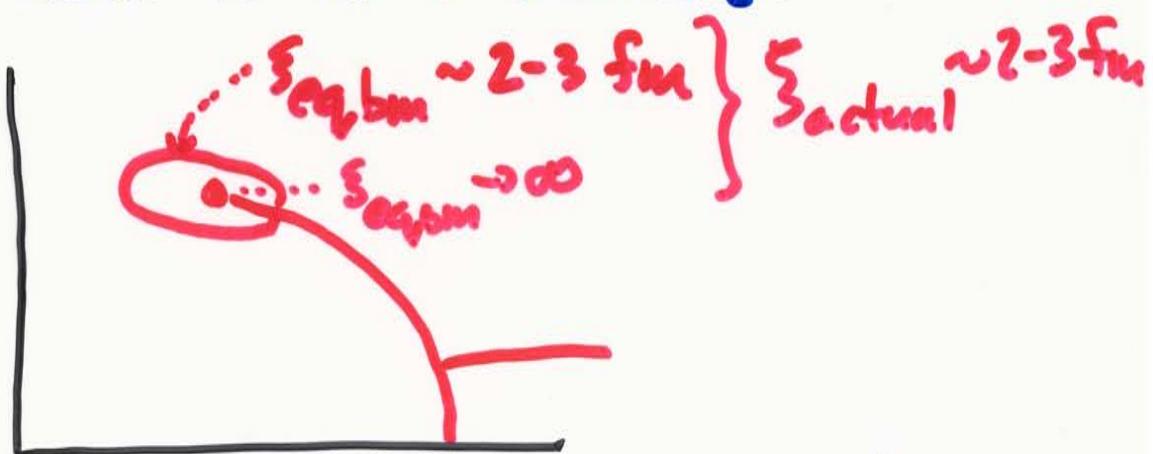
## HOW LARGE CAN $\xi$ GET?

## HOW CLOSE TO $\bullet$ NEED WE BE?

Obviously  $\xi$  limited by finite size of system. But, turns out that finite time is a more severe limitation.

Berdnikov KR; Asakawa Nonaka

- Finite time Spent in critical region means that even if equilibrium value of  $\xi$  is much larger, actual  $\xi$  won't grow bigger than 2-3 fm.
- Means no need to hit  $\bullet$  precisely.

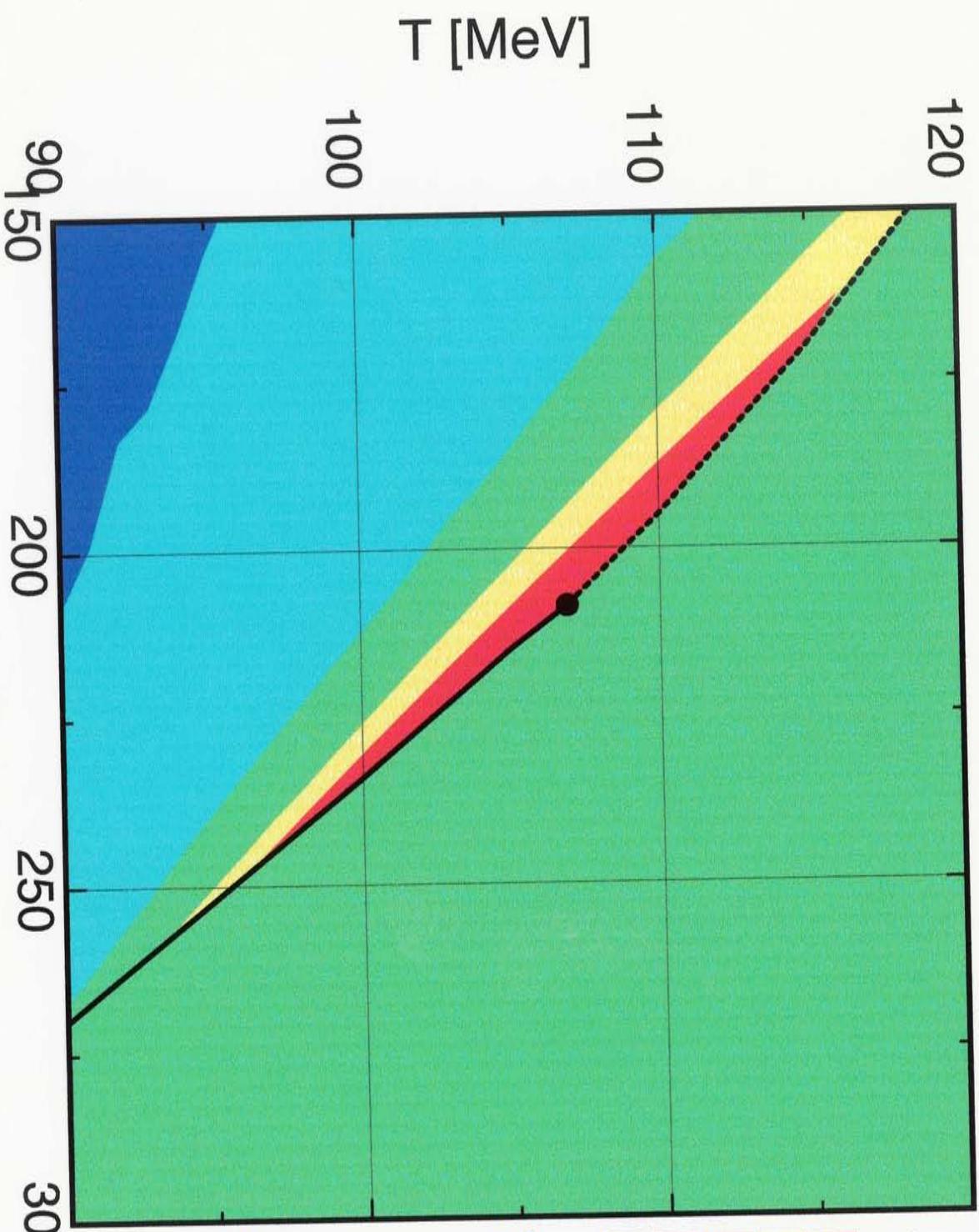


Signatures will be just as big if you pass anywhere in  $\circ$ . No bigger, even if you hit  $\bullet$ .

- Hatta + Ikeda calculated "O's" in a model, but did so with contours of  $\chi_B$  rather than  $\beta$ .  $\rightarrow$  Figs.  
 The robust point is that the extent of these O's in  $\mu_B$  is not small. Width in  $\mu_B$  is  $\sim 100$  MeV, an estimate that is both crude and uncertain.  
 Can this be obtained on lattice ??
- NB also: since  $\beta$  cannot be  $> 2\text{-}3\text{ fm}$ , heavy ion collision experiments can never be used to measure the critical exponents of the 2nd order critical point. That's OK: we know it is Ising. What we don't know, and need experiments for, is where it is located.

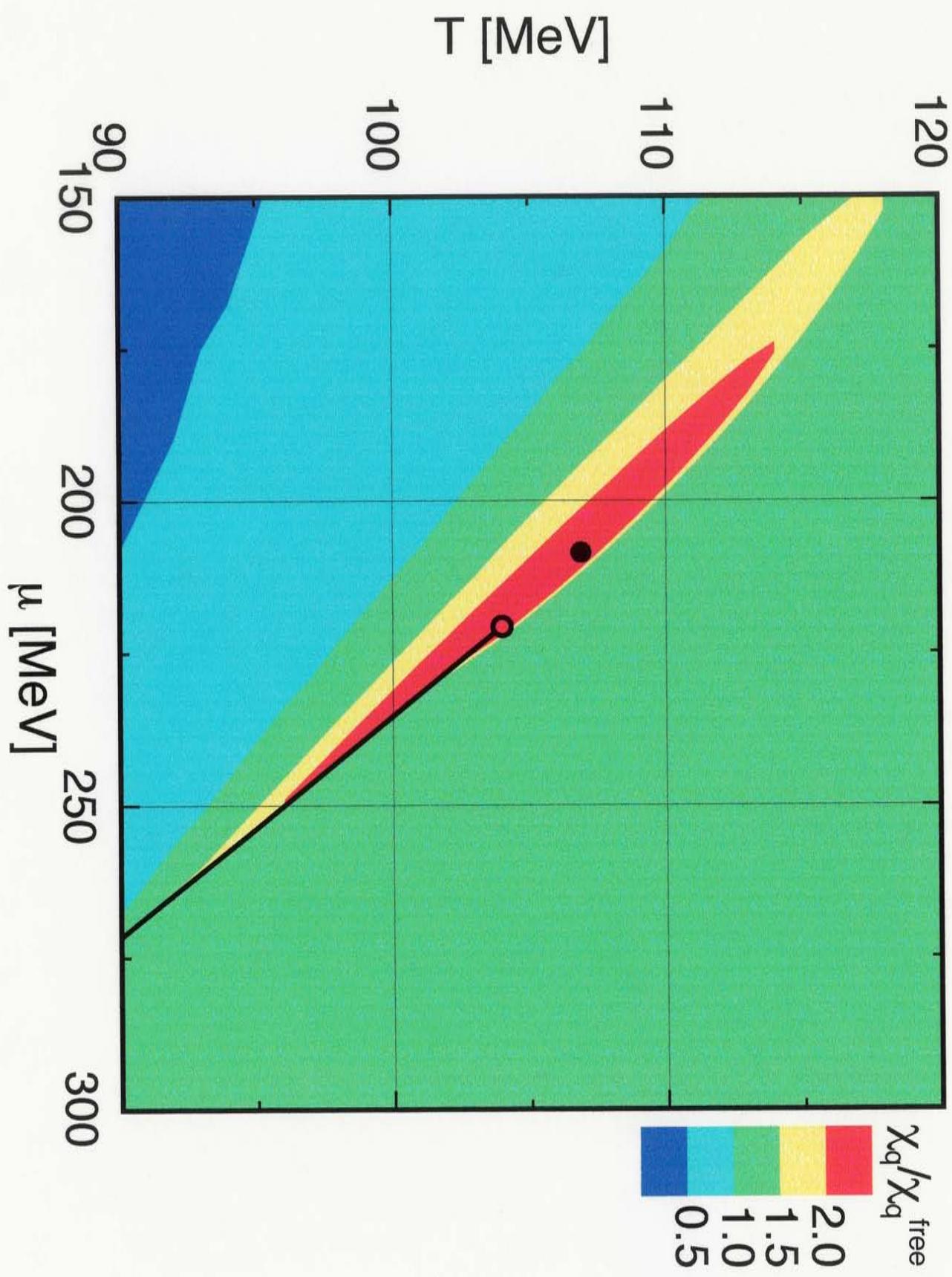
$\mu_u = \mu_d = 0$

MODEL ANALYSIS OF  
EXTENT OF CRITICAL REGION

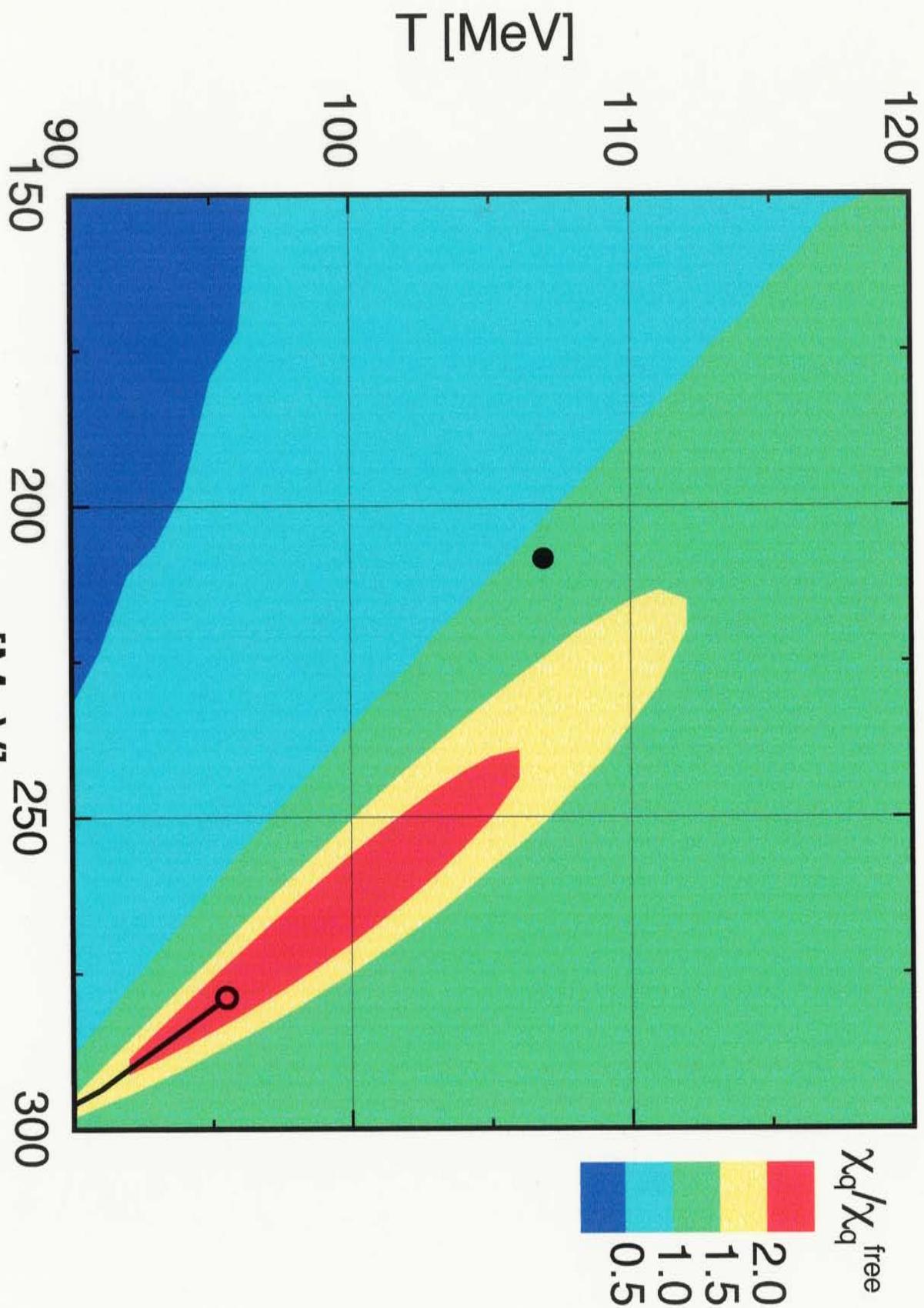


$$\chi_B \sim \xi^2 + \frac{\partial \mathcal{S}}{\partial \mu_B^2}$$

$\mu$  the Ikeda



$$m_u = m_d = 5 \text{ MeV}$$



ueda T keda

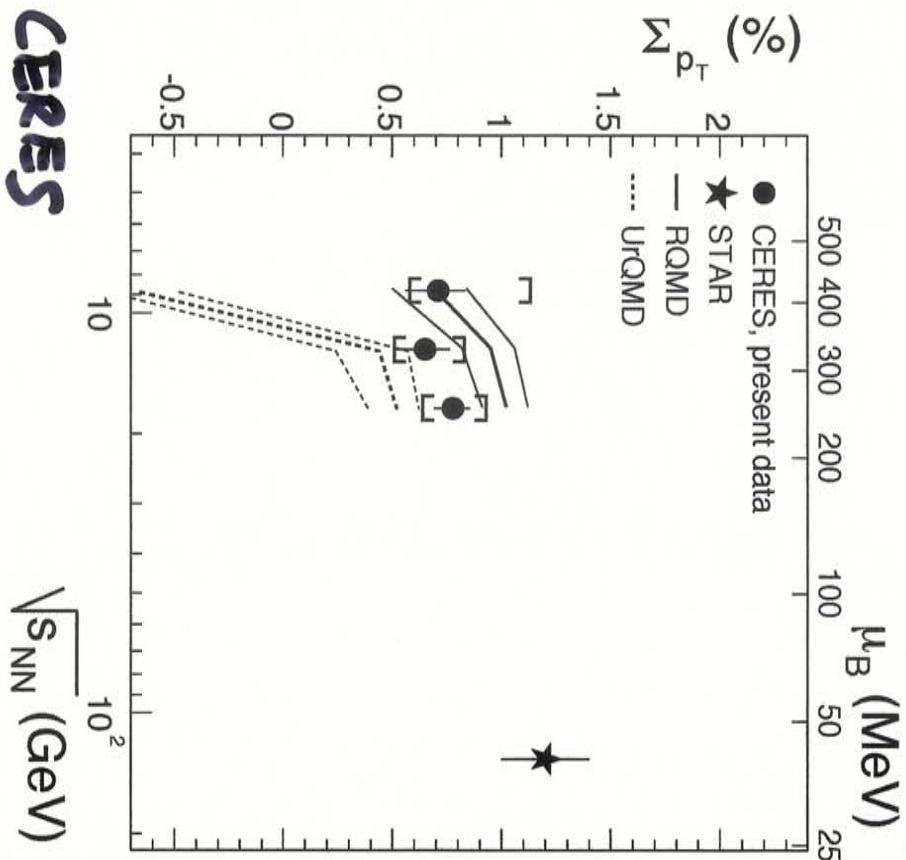
## EXAMPLES

Event-by-event fluctuations of:

- Mean  $P_T$  of low  $P_T$  pions      Stephanov KR  
Shuryak
  - fluctuations should be dominated by fluctuations of pions with  $P_T \lesssim 500 \text{ MeV}$
  - (NOT the case for fluctuations of  $\langle P_T \rangle$  at  $\sqrt{s} = 200 \text{ GeV}$ . PHENIX)
- Baryon number, and proton number.  
Hatta Ikeda; Hatta Stephanov
  - seen on lattice. Fig. Ejiri et al
- Particle ratios involving  $\pi$ 's and/or  $p$ 's.  
eg fluctuation of  $\langle k \rangle / \langle \pi \rangle$ ,  $\langle p \rangle / \langle \pi \rangle$ .
  - will better survive late time hadron gas than  $P_T$ -fluctuations, NA49

Also, light  $\sigma \rightarrow$  increased p-p attraction;  
and,  $\pi$ - $\pi$  repulsion.  $\rightarrow$  reduction in flow of protons relative to pions. Shuryak

# Data (example): $p_T$ fluctuations (CERES)



Near the critical point (for CERES acceptance) one expects:

$$\sim 2\% \times \left( \frac{G}{300 \text{ MeV}} \right)^2 \left( \frac{\xi_\sigma}{3 \text{ fm}} \right)^2$$

$$(\xi_\sigma = 1/m_\sigma)$$

Signal one is looking for:  
non-monotonic dependence on  $\sqrt{s}$ .

**CERES**

Also: low transverse momenta (thermal, soft)),  $p_T < 3T \sim 500 \text{ MeV}$  – where critical point correlations are (cf. CERES data –  $p_T < 1.5 \text{ GeV}$ ).

$\frac{\partial^2 \Omega}{\partial \mu_B^2} \rightarrow$  B-fluctuations  
 $\sim \xi^2 + \text{nonsingular}$

$\frac{\partial^2 \Omega}{\partial \mu_T^2} \rightarrow$  no enhanced ( $u-d$ )  
 fluctuations  
 $\sim \text{nonsingular}$

suggests

$$\frac{\mu_q}{T} \sim 1$$

getting  
close  
to  $\bullet$ .

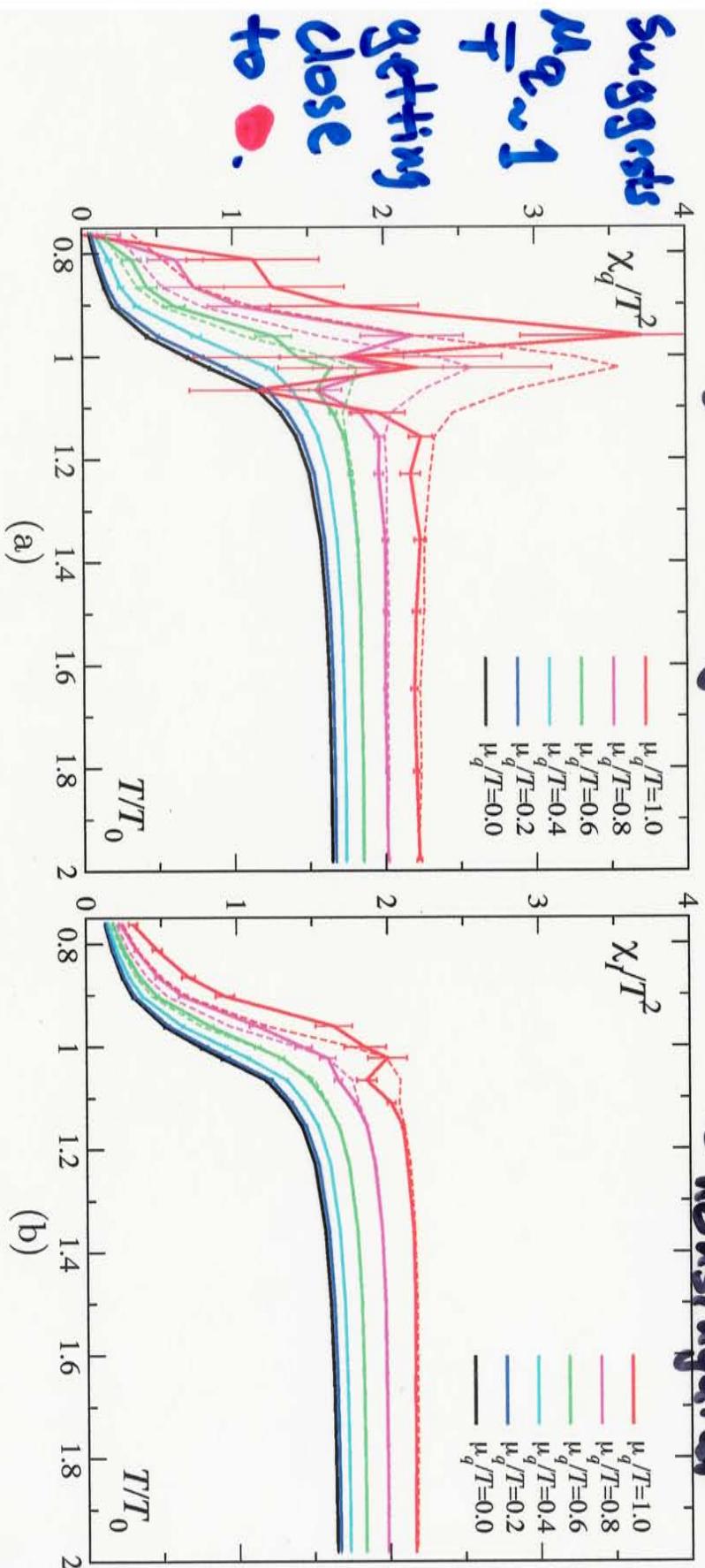


Figure 3.3: The quark number susceptibility  $\chi_q/T^2$  (left) and isovector susceptibility  $\chi_I/T^2$  (right) as functions of  $T/T_0$  for various  $\mu_q/T$  ranging from  $\mu_q/T = 0$  (lowest curve) rising in steps of 0.2 to  $\mu_q/T = 1$ , calculated from a Taylor series in 6<sup>th</sup> order. Also shown as dashed lines are results from a 4<sup>th</sup> order expansion in  $\mu_q/T$ .

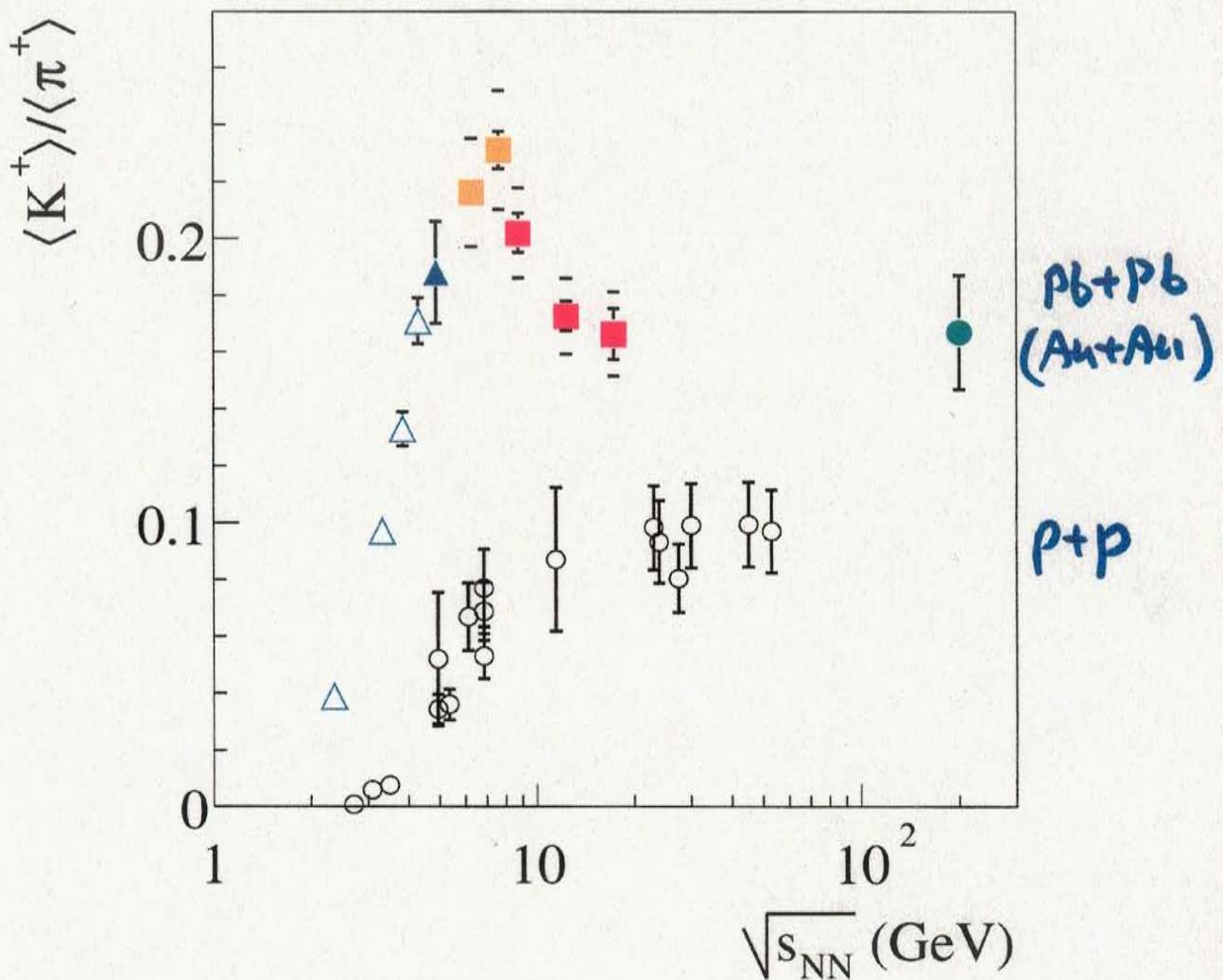
(Because B fluctuations while isospin does not, proton fluctuations  $\sim$  B fluctuations)

Elli et al

Here is another quantity — not an e-by-e fluctuation — that varies nonmonotonically with  $\sqrt{s}$ ...

Hadron Multiplicities

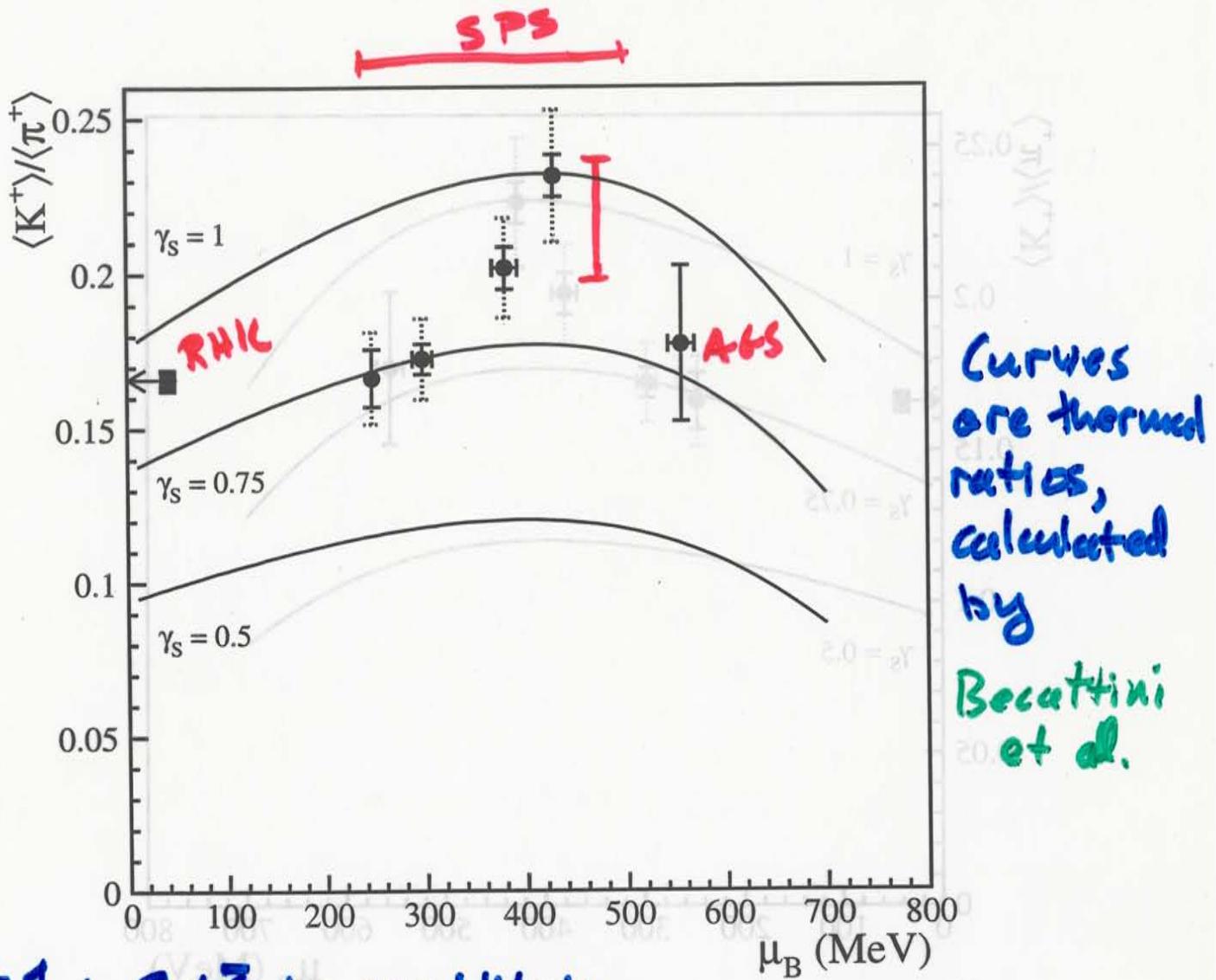
( $\bar{s}$ -QUARK CARRIER)



THE HORN

Talk by M. Gąsielicki, QM04, reporting  
NA49 results.  
(Horn also seen in  $\frac{K^- + \Lambda}{\pi^-}$ .)

FIG. 13: Measured  $\langle K^+ \rangle / \langle \pi^+ \rangle$  ratio as a function of the fitted baryon-chemical potential. The full square dot is a preliminary full phase space measurement in Au-Au collisions at  $\sqrt{s_{NN}} = 200$  GeV [37] and the error is only statistical; the arrow on the left signifies that its associated baryon chemical potential is lower than that estimated at  $\sqrt{s_{NN}} = 130$  GeV [11] used here. For the SPS energy points the statistical errors are indicated with solid lines, while the contribution of the common systematic error is shown as a dotted line. Also shown the theoretical values for a hadron gas along the fitted chemical freeze-out curve shown in fig. 11, for different values of  $\gamma_S$ .



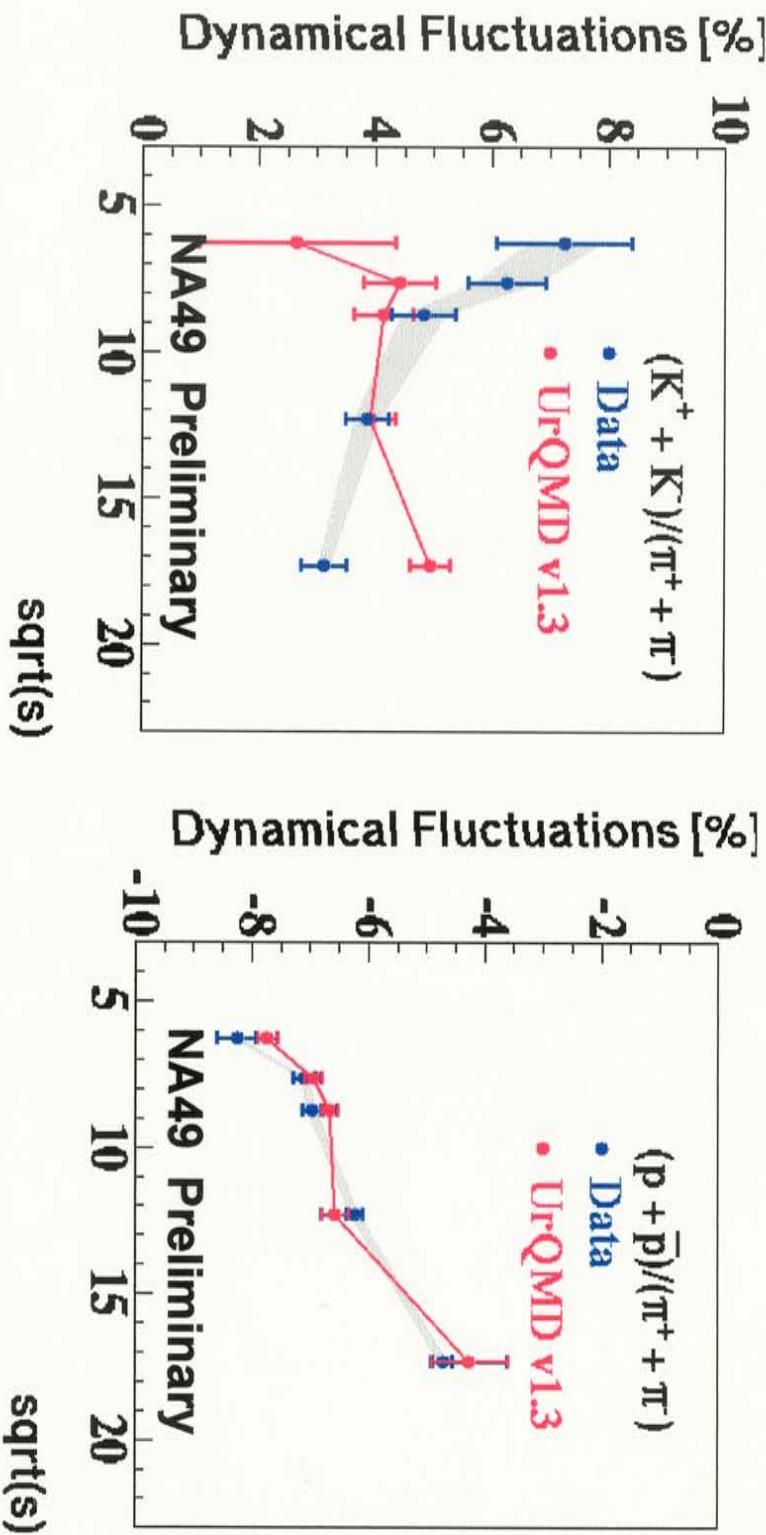
$\gamma_S = 1$  :  $S + \bar{S}$  in equilibrium

$\gamma_S = 0.75$  :  $S + \bar{S}$  is 75% of eqbm value

To explain "horn", need  $\gamma_S = 1$  at horn and  
 $\gamma_S \sim .7 - .8$  on either side of horn.

This spoils other aspects of thermal  
model fit, however. (Becattini; Cleymans)

# Summary



- $K/\pi$  fluctuations increase towards lower beam energy
- **Significant enhancement over hadronic cascade model**
- $p/\pi$  fluctuations are negative
- **indicates a strong contribution from resonance decays**

## Intriguing...

- Large event-by-event fluctuations in  $K/\pi$  at  $\mu_B \sim 400 - 450$  MeV
- Are there  $P_T$  fluctuations? Not known
- Are the  $K/\pi$  fluctuations dominated by low  $P_T \pi$ ? Apparently not!
- Why no  $P/\pi$  fluctuations??
- Koch Majumder Randrup suggest the  $K/\pi$  fluctuations due to hadronizing of "blobs" left by a first order transition.
- If so, expect greater  $K/\pi$  fluctuation, and other non-eq,bm fluctuations vs. rapidity, at higher  $\mu_B$ , lower  $\sqrt{s}$ .
- And, expect critical point at lower  $\mu_B$ !
- Cf: Fodor Katz  $\rightarrow \mu_B^{\circ} \sim 360$  MeV  $\rightarrow \sqrt{s} \sim 8$  GeV  
Gavai Gupta  $\rightarrow \mu_B^{\circ} \sim 190$  MeV  $\rightarrow \sqrt{s} \sim 25$  GeV

We have motivation-a-plenty. So....

## CAN RHIC FIND THE CRITICAL POINT?

- Gunther Roland will describe the advantages of using a collider vs. fixed target for study of event-by-event fluctuations at varying  $\sqrt{s}$ .
- I will focus on "what needs to be done". Later speakers (experimental and accelerator physicists) will address what can be done. The goal for the workshop is to understand how best to maximize the intersection.

## WHAT NEED BE MEASURED AT EACH ENERGY

- Enough  $\langle \text{particle ratios} \rangle$  to first evaluate  $M_B$  — ie you have to know where you are freezing out — and then see whether  $\langle K \rangle / \langle \pi \rangle$  enhanced — ie as a bonus confirm(?) the "horn".
- Event-by-event fluctuations in:
  - $\langle p_T \rangle$ , with equal or smaller error bars as in CERES data
  - in  $\langle K \rangle / \langle \pi \rangle$  and  $\langle p \rangle / \langle \pi \rangle$ , with smaller error bars than in NA49 data
  - All fluctuation analyses done for  $p_T < p_T^{\text{cut}}$  for several choices of  $p_T^{\text{cut}}$  down to  $p_T^{\text{cut}} \sim 500 \text{ MeV}$
- $V_2$  of  $\pi + p$ .

## WHAT RANGE OF $\sqrt{s}$ , ie $\mu_B$ ?

- I suggest RHIC explore  $\mu_B \lesssim 500$  MeV.
- Want to test NA49 observation of  $K/\pi$  fluctuations at  $\mu_B \sim 400 - 450$ .
- Why stop at  $\mu_B \sim 500$ ?
  - If  $\mu_B^{\circ} < 3T_c \sim 500$  MeV, plausibly the different lattice calculations will converge as each improves in the next few years. If  $\mu_B^{\circ} > 500$  MeV, a quantitative comparison with theory is harder to envision.
  - If  $\mu_B^{\circ} > 500$  MeV, it will also be tough to find experimentally. (Low  $T_{freezeout}$ ) equilibration ???)
- NB: if  $\bullet$  is found with  $\mu_B^{\circ} < 500$  MeV, this will give strong motivation for study of non-equilibrium signatures of first order transition at larger  $\mu_B$ , eq at FAIR.

## WHAT SPACING BETWEEN $\mu_B$ 'S ?

In the vicinity of a discovery, will want  $\mu_B$ 's separated by  $\sim 50 \text{ MeV}$ .

- A scan with steps  $\lesssim 100 \text{ MeV}$  should allow to make discoveries.
- Other than calculating  $\mu_B^*$ , second most important quantity we would like to learn from lattice QCD is width in  $\mu_B$  of region where  $\xi > 2 \text{ fm}$ . Or, say, where  $\frac{\chi_B}{\chi_B(T_c, \mu=0)} > 4$ .

## "STRAW MAN" CHOICE OF ENERGIES

$\sqrt{s} \leftrightarrow M_B$  taken from Cleymans et al's compilation of data where available and empirical fit where there is none. [hep-ph/0511094]

$\sqrt{s}$ (AGeV)	$M_B$ (MeV)
5	550
6.27	470
7.62	410
8.77	380
12.3	300
19	220
28	150
done at RHIC	60
	40
	30

peak  $\rightarrow$  f "horn"

largest fluctuations in  $\langle K \rangle / \langle \pi \rangle$

# CAN WE DISCOVER THE QCD CRITICAL POINT AT RHIC?

YES, IF:

- Accelerator + detector capabilities permit measurement of the event-by-event fluctuations of the hadronic observables I described, at a sequence of energies like that I described
- Nature is kind, and puts  $\mu_B^{\circ} < 500 \text{ MeV}$

IF YES:

- The landmark discovered. Our map of the QCD phase diagram then anchored by experiment.
- Assuming reasonable progress in lattice QCD, quantitative comparison between theory + experiment for  $\mu_B^{\circ}$
- FAIR (and RHIC) can study the first order phase transition.