
Update on B_K with a mixed action

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Domain Wall Fermions after ten years

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Introduction

- I. Quick review of current status of B_K
- II. Mixed action simulations and B_K
- III. Preliminary data

Current numerical status of B_K

- Benchmark calculation used in unitarity triangle fits [JLQCD]:

$$B_K^{NDR}(2\text{GeV}) = 0.628(42) \leftarrow \text{quenched!} \quad (1)$$

- Dynamical (2+1) domain-wall fermions [RBC/UKQCD]:

$$B_K^{NDR}(2\text{GeV}) = 0.557(12)(29) \leftarrow \text{impressive (still one lattice spacing)} \quad (2)$$

- Dynamical (2+1) staggered fermions [HPQCD,UKQCD]:

$$B_K^{NDR}(2\text{GeV}) = 0.618(18)(19)(130) \leftarrow \text{large mixing error} \quad (3)$$

Mixed action simulations

Our simulations use MILC lattices with asq-tad staggered quarks in the sea sector and domain wall quarks in the valence sector.

Advantages

- A large number of ensembles with different volumes, sea quark masses and lattice spacings exist and are publicly available.
- The existing ensembles have 2+1 flavors of light sea quarks ($m_{strange}/8$ for the lightest quarks)
- The good chiral properties of the valence sector make things much simpler than the staggered case. There are only two additional parameters (over pure domain wall) that appear at one loop in the mixed action ChPT for m_π , f_π , and B_K . They can both be obtained from spectrum calculations.
- NPR can be carried through in the same way as in domain wall.

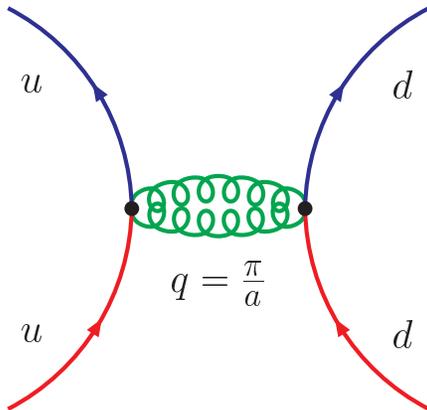
Mixed Action ChPT on a staggered sea

- Ghost quarks are introduced with $m_{ghost} = m_{valence}$, as in partially quenched ChPT, to cancel the valence contributions in the loops
- No terms linear in a because there are no dimension 5 operators compatible with all lattice symmetries
- Three types of dimension 6 operators
 1. Contain only sea quarks - “usual” staggered ChPT
 2. Contain both valence and sea - this is new
 3. Contain only valence quarks - nothing new beyond the continuum

Staggered Fermions

- Staggered quarks come in 4 tastes \Rightarrow staggered mesons come in **16 tastes**
- Labeled by the *taste matrix* in the lattice operator: $\pi_T \equiv \bar{Q}_i(\gamma_5 \otimes \xi_T)Q_j$

| | | |
|----------------------|-------------------------|---------------------------|
| 1 Singlet – ξ_I | 1 Goldstone – ξ_5 | |
| 4 Vector – ξ_μ | 4 Axial – $\xi_{\mu 5}$ | 6 Tensor – $\xi_{\mu\nu}$ |



- On the lattice, quarks of one taste can turn into another by exchanging high-momentum gluons

Good, Bad or Ugly? (a la Steve Sharpe)

- In the continuum limit, the four staggered tastes become degenerate
- In principle, taste breaking can be removed by taking the continuum limit, but in practice one must take the fourth root at finite lattice spacing. It is possible that the continuum limit of this theory is not QCD, i.e. the theory is “bad.”
- This is an open theoretical issue, which I will not discuss further here. See hep-lat/0610094 for a recent review, where it was concluded that staggered fermions are ugly.
- Assuming the validity of the 4th root trick, if the staggered quarks live only in the sea and a chiral valence quark is used, then staggered fermions are not even ugly!

The Chiral Effective Mixed Action

$$\mathcal{L}_{\text{MAXPT}} = \frac{f^2}{8} \text{Tr}(\partial_\mu \Sigma \partial_\mu \Sigma^\dagger) - \frac{1}{4} \mu f^2 \text{Tr}(\mathcal{M} \Sigma + \mathcal{M} \Sigma^\dagger) + a^2 \mathcal{U}_S + a^2 \mathcal{U}_V$$

$a^2 \mathcal{U}_S$ from mixed valence and sea 4-fermion operators

$a^2 \mathcal{U}_V$ from sea quark only 4-fermion operators

Tree level valence-valence pions proportional to quark masses plus a residual mass as usual in DWF's:

$$m_{xy}^2 = \mu(m_x + m_y + 2m_{res}) \quad (4)$$

Tree level sea-sea pions are:

$$m_{SS'}^2 = \mu(m_S + m_{S'}) + a^2 \Delta_t \quad (5)$$

Of the staggered splittings, only Δ_I appears in quantities of interest!

Mixed valence-sea pions

Mixed valence-sea pions at tree level are given by:

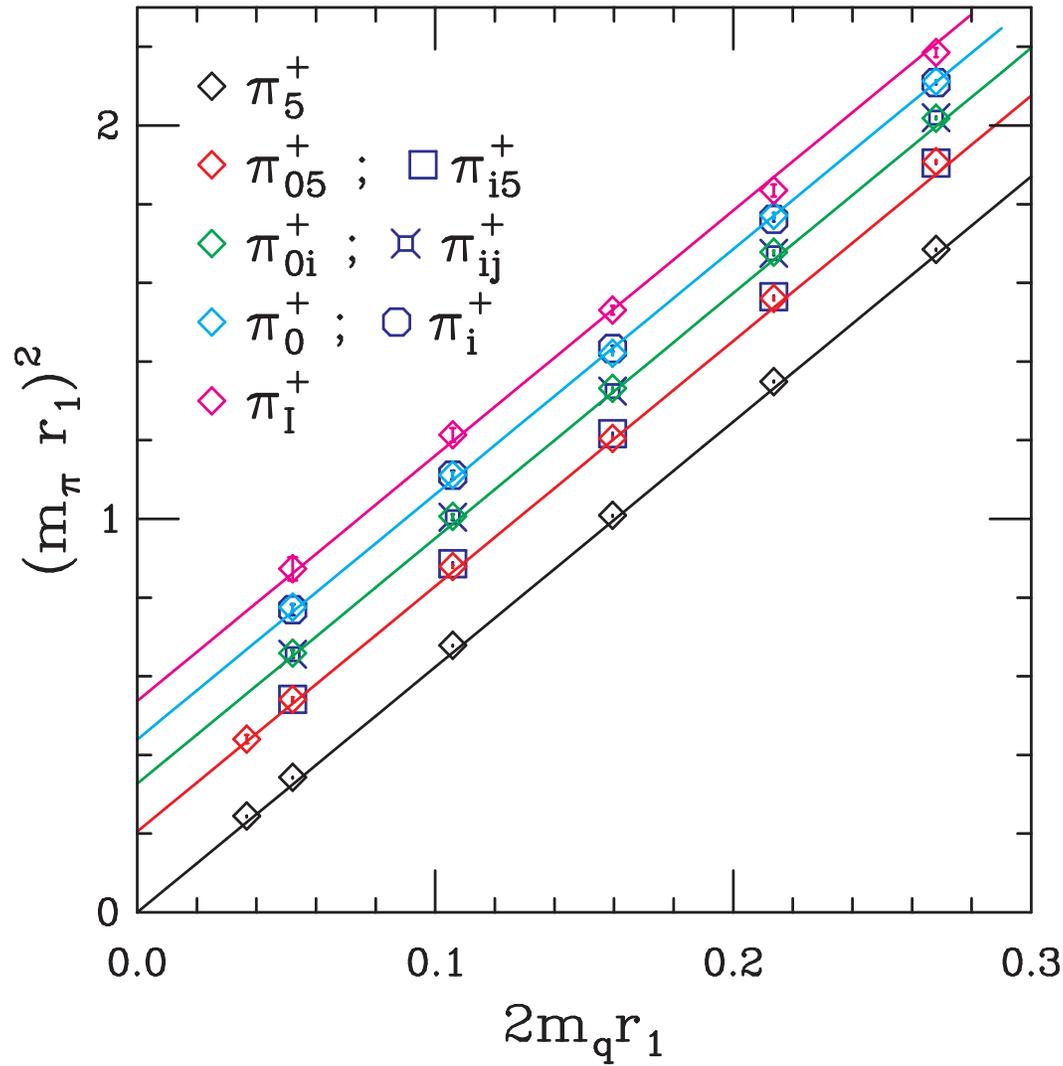
$$m_{SV}^2 = \mu(m_S + m_V) + a^2 \Delta_{Mix} \quad (6)$$

This new parameter $a^2 \Delta_{Mix}$ has not been measured yet. It can be obtained by looking at the mixed quark spectrum.

Fortunately, it does not appear in B_K at NLO! However, it does appear in f_π and f_K , which are needed as a cross-check of the B_K calculation.

Note that both $a^2 \Delta_I$ and $a^2 \Delta_{mix}$ scale as $\alpha_s a^2$ and will decrease as we go to finer lattices.

MILC Taste Splittings



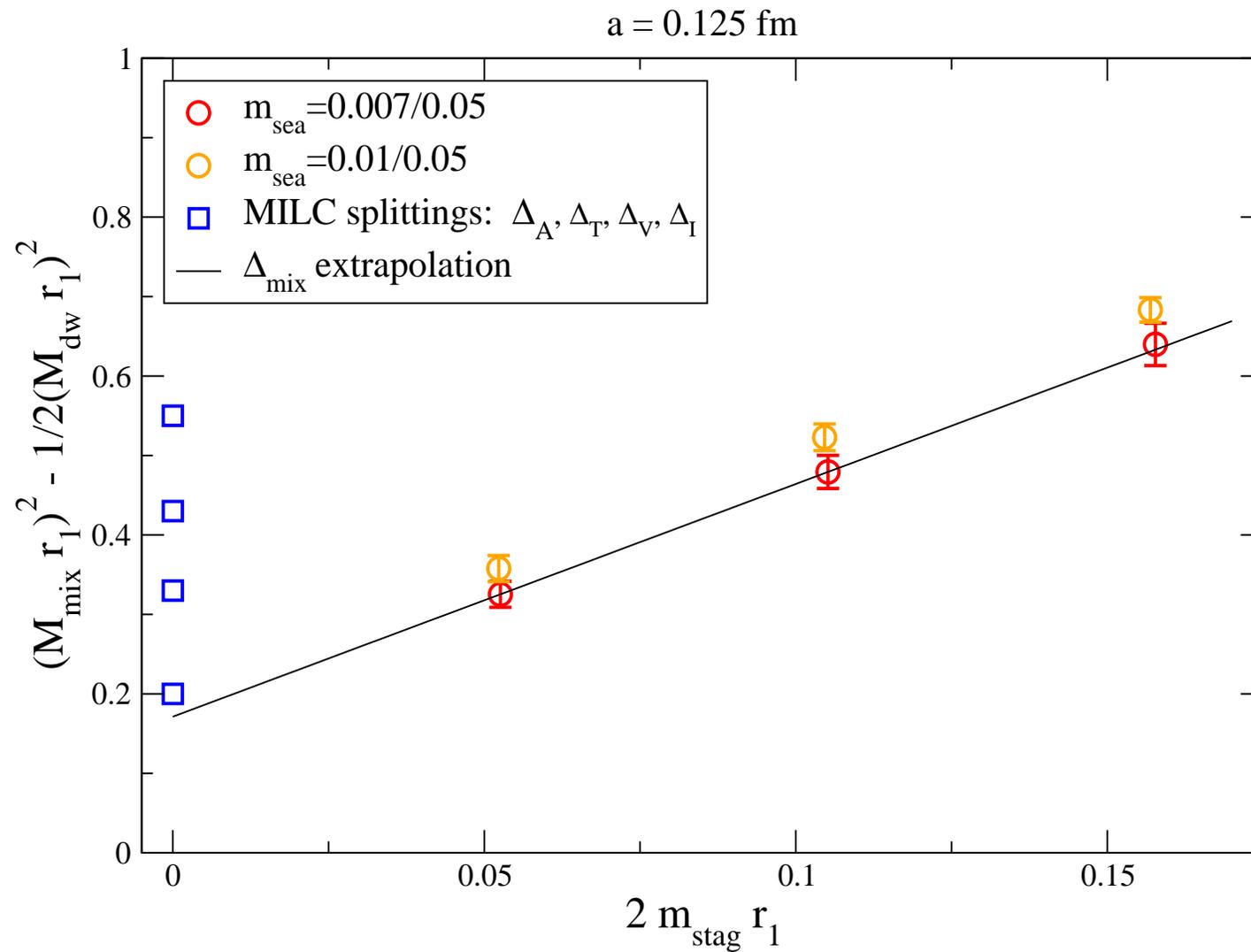
Obtaining Δ_{mix}

$$M_{\text{mix}}^2 = \mu_{\text{dw}}(m_{\text{dw}} + m_{\text{res}}) + \mu_{\text{stag}}m_{\text{stag}} + a^2\Delta_{\text{mix}}, \quad (7)$$

where M_{mix} is the mass of a meson which is composed of a staggered quark and a domain-wall quark, m_{dw} is the bare domain wall mass, m_{stag} is the bare staggered mass, and $a^2\Delta_{\text{mix}}$ is the splitting we wish to determine. Note that this formula is in terms of the bare lattice masses, and since domain wall and staggered quarks get renormalized differently, we absorb the renormalization coefficients into different values of μ .

$$M_{\text{mix}}^2 - \frac{1}{2}M_{\text{dw}}^2 = \mu_{\text{stag}}m_{\text{stag}} + a^2\Delta_{\text{mix}}. \quad (8)$$

Values for Δ_{mix}



B_K in Mixed Action ChPT

We make use of the partially quenched ChPT expression. Thus tuning the domain wall masses is not necessary. We are computing many different valence masses to aid in the chiral extrapolation.

$$\begin{aligned} \left(\frac{B_K}{B_0} \right)^{\text{PQ},2+1} &= 1 + \frac{1}{16\pi^2 f^2 m_{xy}^2} \left[I_{\text{conn}} + I_{\text{disc}}^{2+1} \right] + c_1 a^2 + c_2 m_{xy}^2 \\ &\quad + c_3 \frac{(m_X^2 - m_Y^2)^2}{m_{xy}^2} + c_4 (2m_D^2 + m_S^2), \end{aligned} \quad (9)$$

$$I_{\text{conn}} = 2m_{xy}^4 \tilde{\ell}(m_{xy}^2) - \ell(m_X^2)(m_X^2 + m_{xy}^2) - \ell(m_Y^2)(m_Y^2 + m_{xy}^2) \quad (10)$$

$$I_{\text{disc}}^{2+1} = \frac{1}{3} (m_X^2 - m_Y^2)^2 \frac{\partial}{\partial m_X^2} \frac{\partial}{\partial m_Y^2} \left\{ \sum_j \ell(m_j^2) (m_{xy}^2 + m_j^2) R_j^{[3,2]}(\{M_{XY,I}^{[3]}\}; \{\mu_I^{[2]}\}) \right\} \quad (11)$$

where ℓ and $\tilde{\ell}$ are logarithms. Δ_I appears within the logs.

In the case of pure G-W valence quarks, the B_K operator cannot mix with operators of the wrong chirality.

With domain-wall valence quarks the desired lattice B_K operator with spin structure $VV + AA$ mixes with four other operators which do not have the same $VV + AA$ spin structure (TT , $VV - AA$, $SS + PP$, and $SS - PP$).

This contamination is suppressed by two factors of the residual mass, and the effect is small if the residual mass is small.

It could be non-negligible, but then it can be removed nonperturbatively using the standard method of Rome-Southampton and pioneered for this quantity by RBC.

There is no mixing with taste breaking operators. The mixing is the same as in the pure domain wall case!

Parameters of the simulation

- Done on MILC lattices with improved staggered (asqtad) sea quarks
- Many MILC lattice ensembles exist. This work only makes use of the MILC coarse lattices ($a \approx 0.12$ fm). We will add the fine ($a \approx 0.09$) soon.
- The lightest quark masses have $\approx m_{strange}/8$ and $m_\pi L \geq 4$ for all data points.
- Following LHPC, we are using HYP smearing with the usual Hasenfratz parameters to reduce residual chiral symmetry breaking.
- We are using periodic+antiperiodic boundary conditions and periodic-antiperiodic boundary conditions to create forward and backward propagators, effectively doubling the time extent of the lattice.

Determining the lattice spacing

The scale on the coarse MILC lattices was determined by HPQCD using the Υ 2S-1S or 1P-1S splittings.

This determination is independent of the valence light quarks, and is more accurate than calculations of other physical quantities.

This spectrum calculation makes use of effective field theory, which is a systematic expansion of QCD when in the appropriate regime (in this case Λ_{QCD}/m_b is the expansion parameter).

Note that calculations of r_1 (or the related r_0) are based on the quark potential model, and cannot be compared directly with experiment, introducing a larger systematic error.

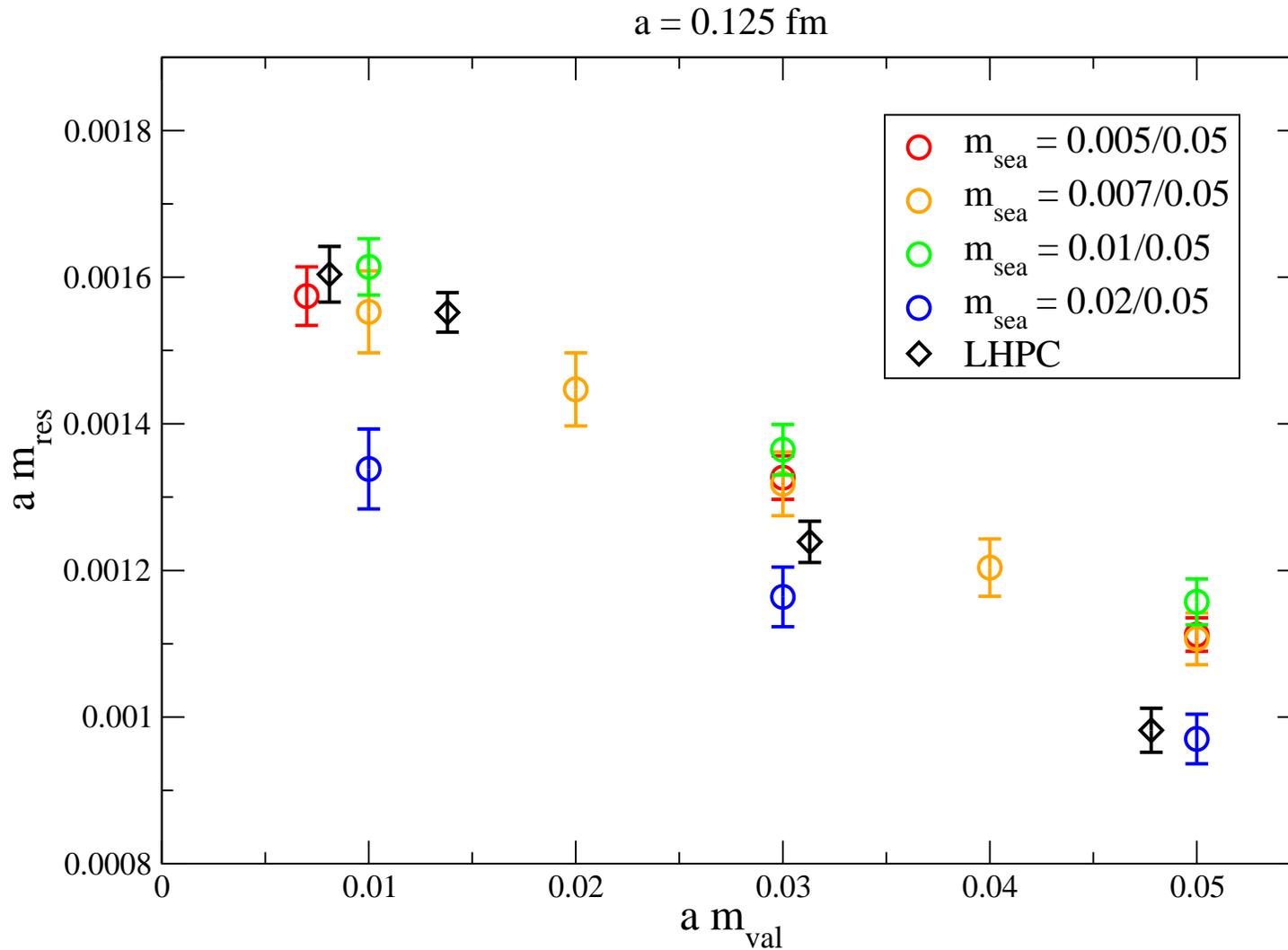
r_1 smoothing

MILC has calculated r_1/a from a fit to the static quark potential for various sea quark masses at a given lattice spacing.

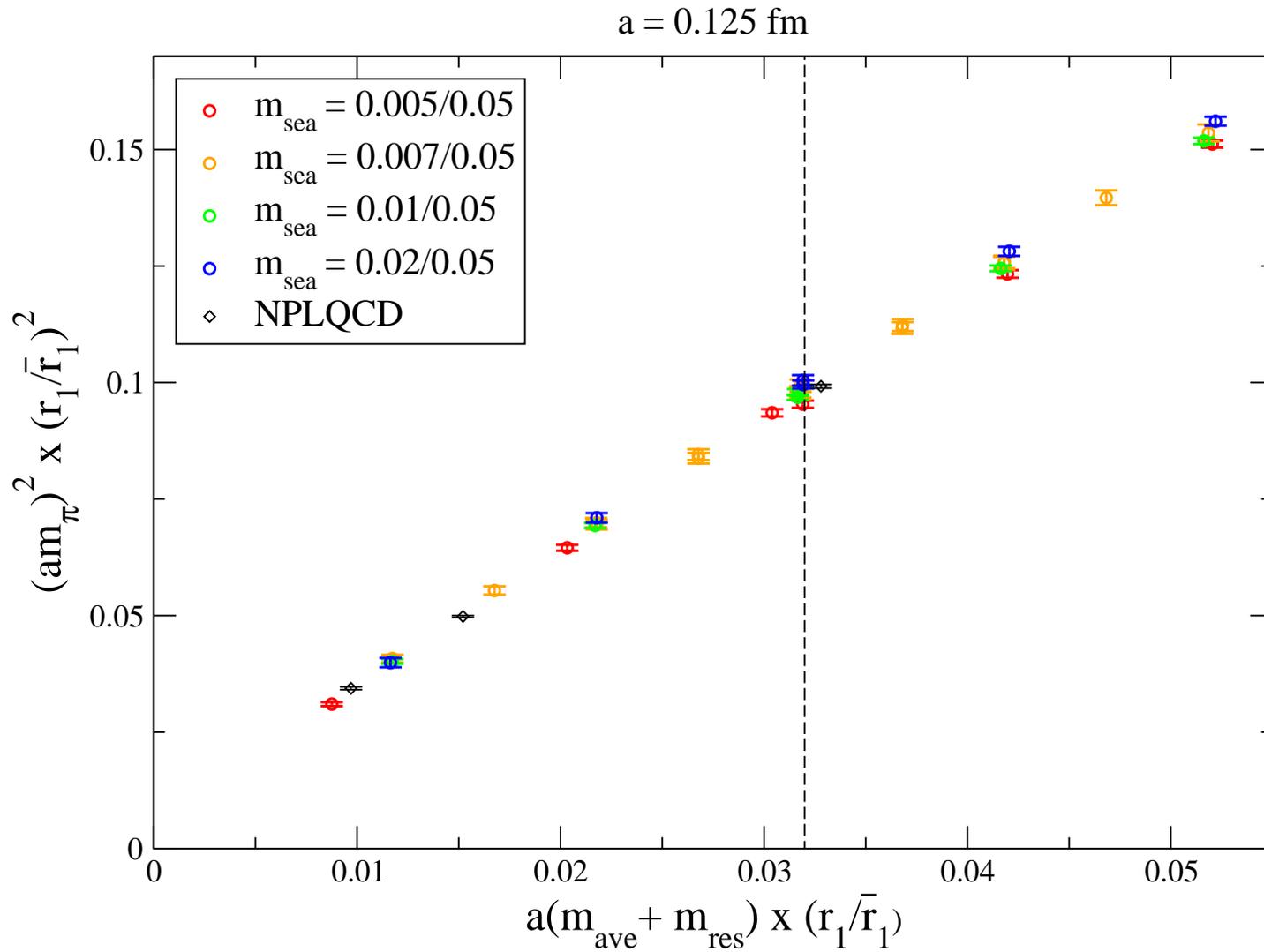
In order to smooth statistical fluctuations due to varying the sea quark mass, they have computed a “smoothed r_1 ” value by fitting r_1/a to a smooth function.

We have made use of these smoothed r_1 values in our plots of dimensionful quantities, though the effects of this are small, $O(1\%)$.

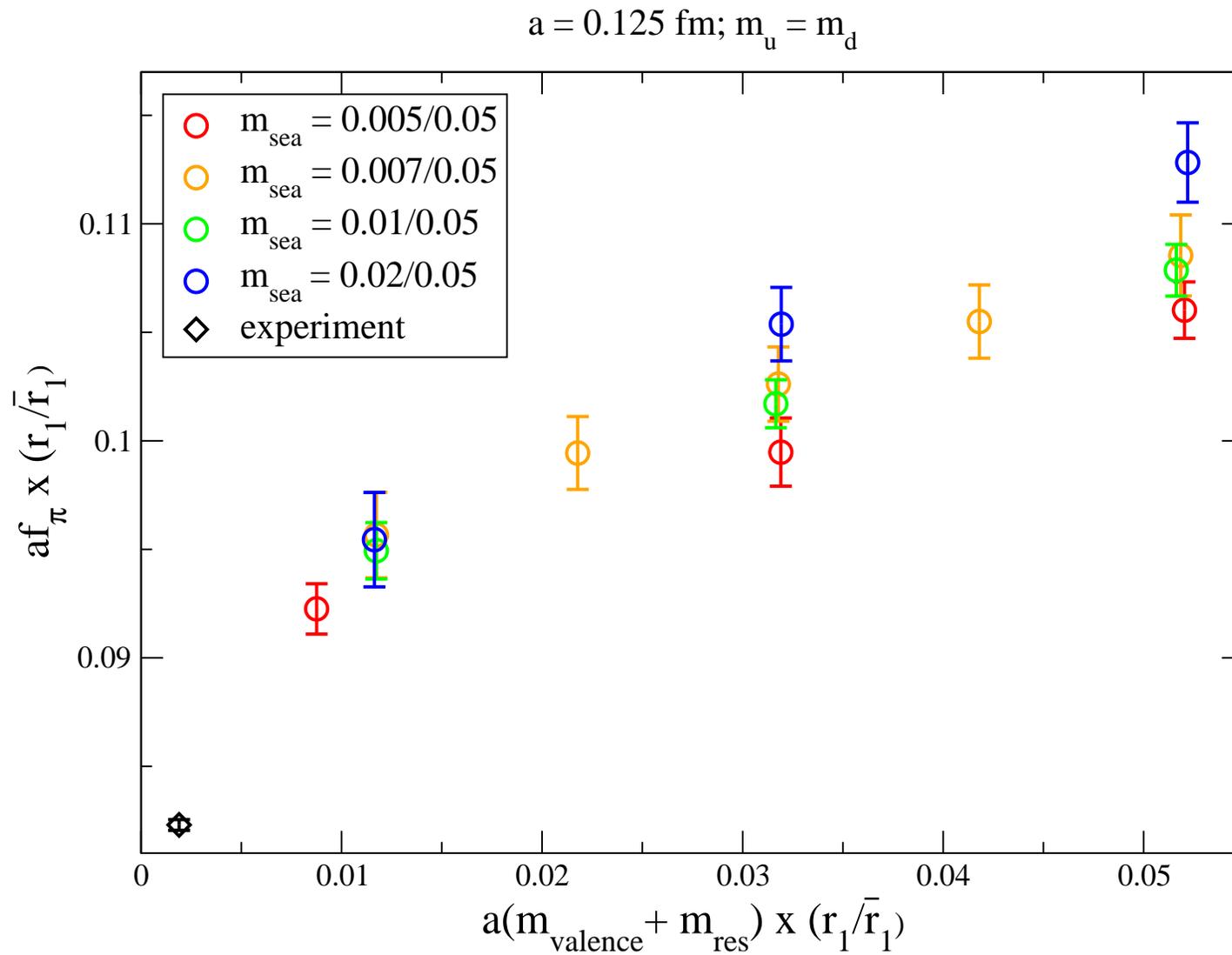
Value of m_{res}



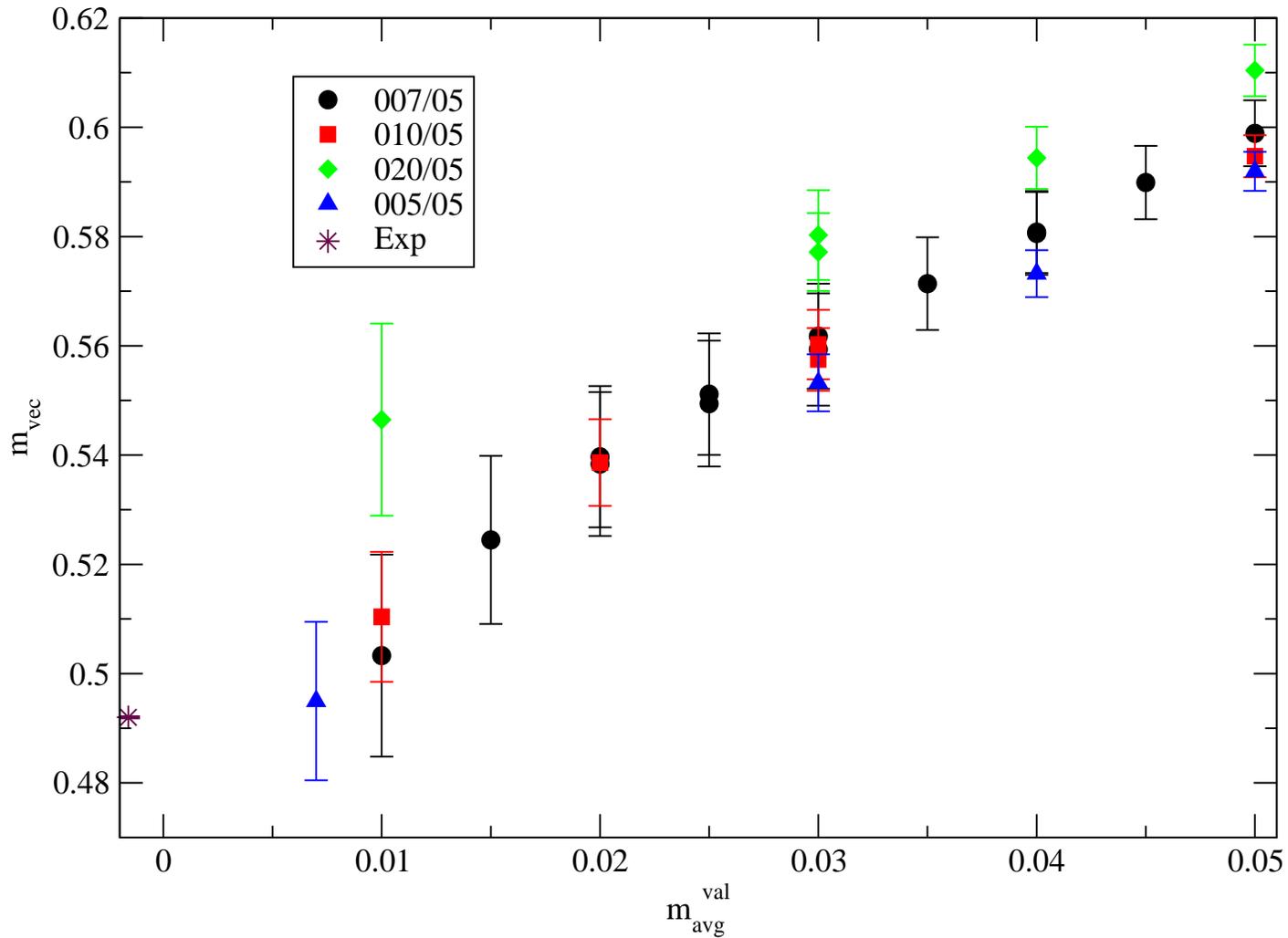
m_π^2 values



f_π data

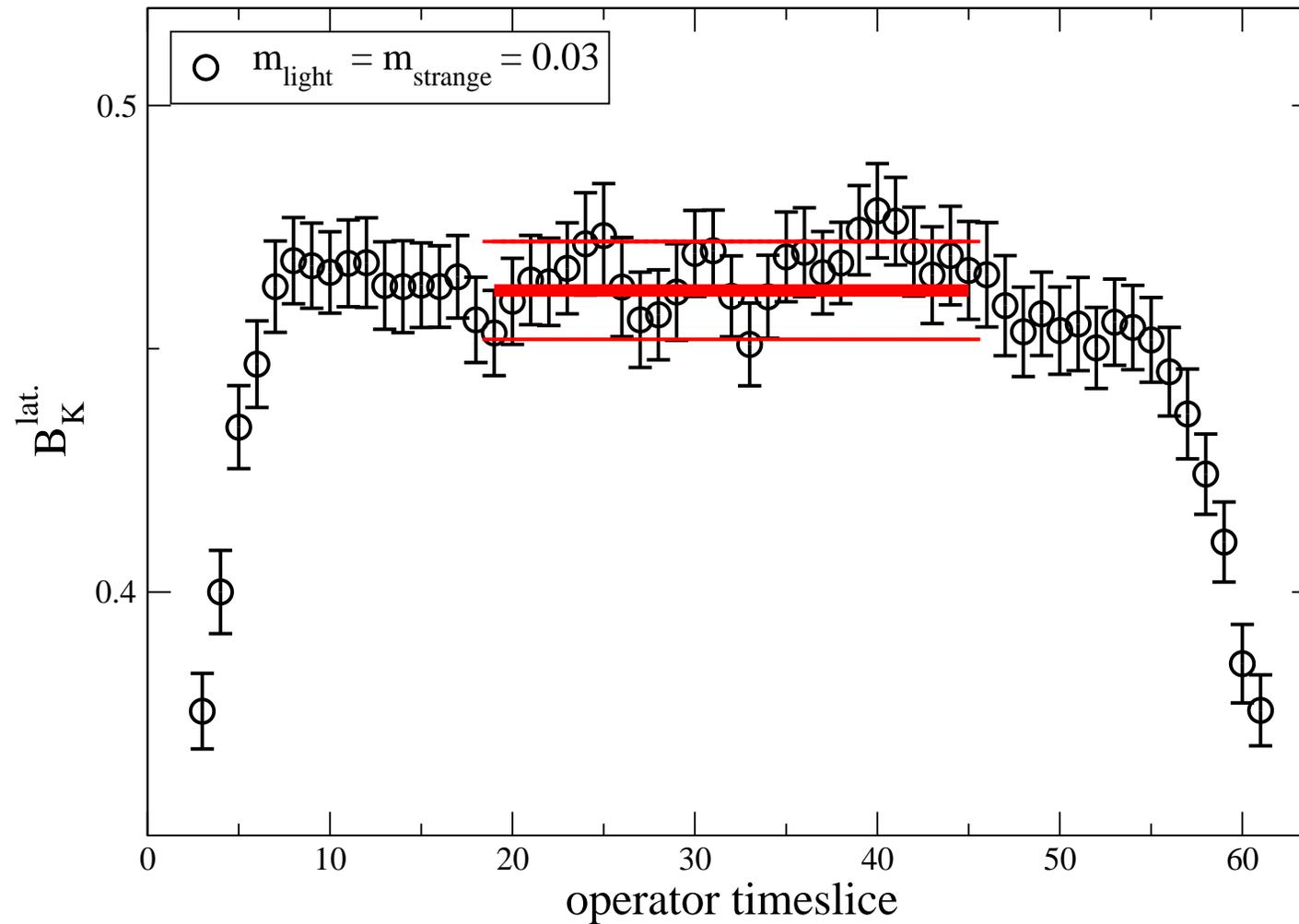


The vector



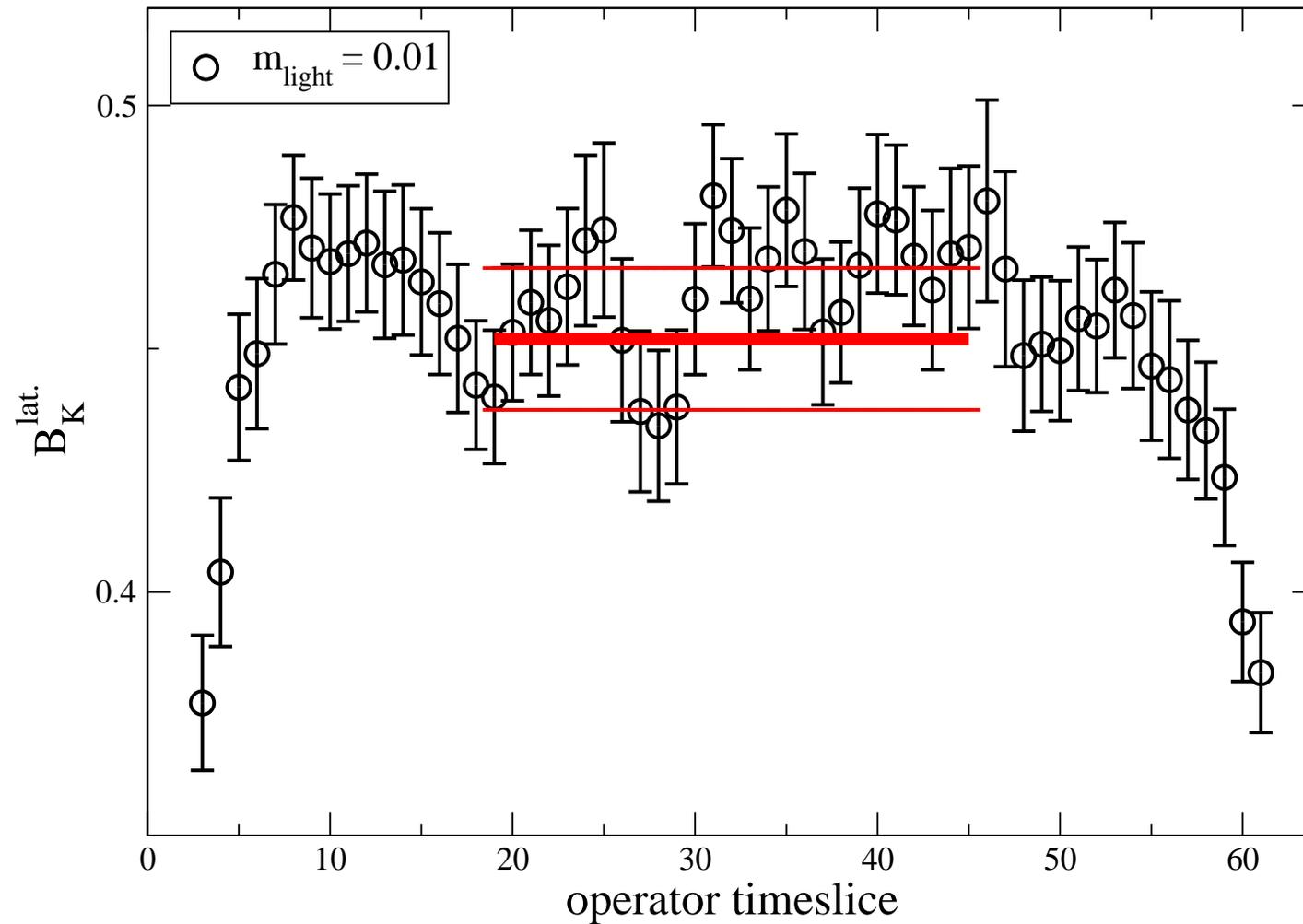
B_K plateau

$a = 0.125$ fm; $m_{\text{sea}} = 0.007/0.05$; 112 configurations

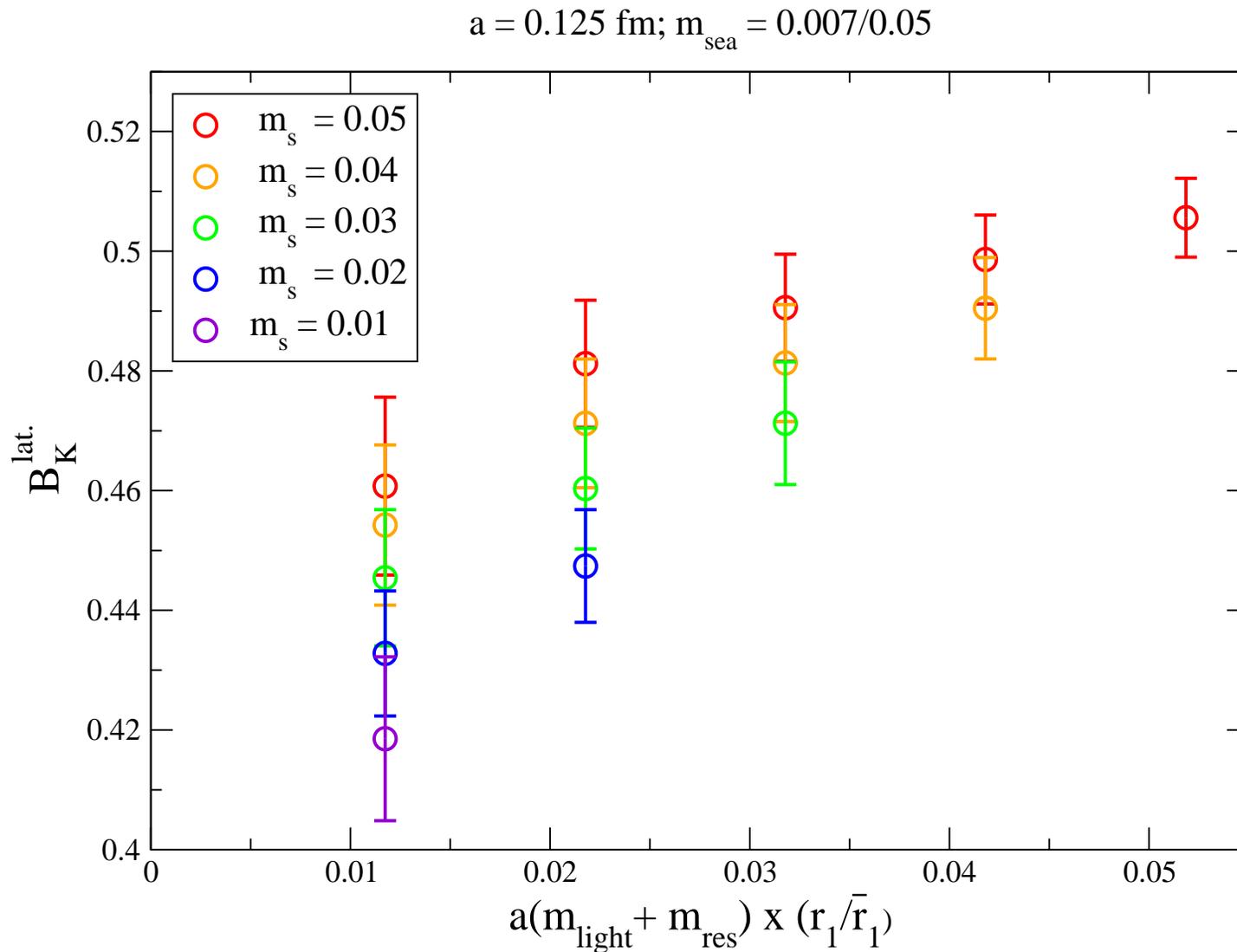


B_K plateau

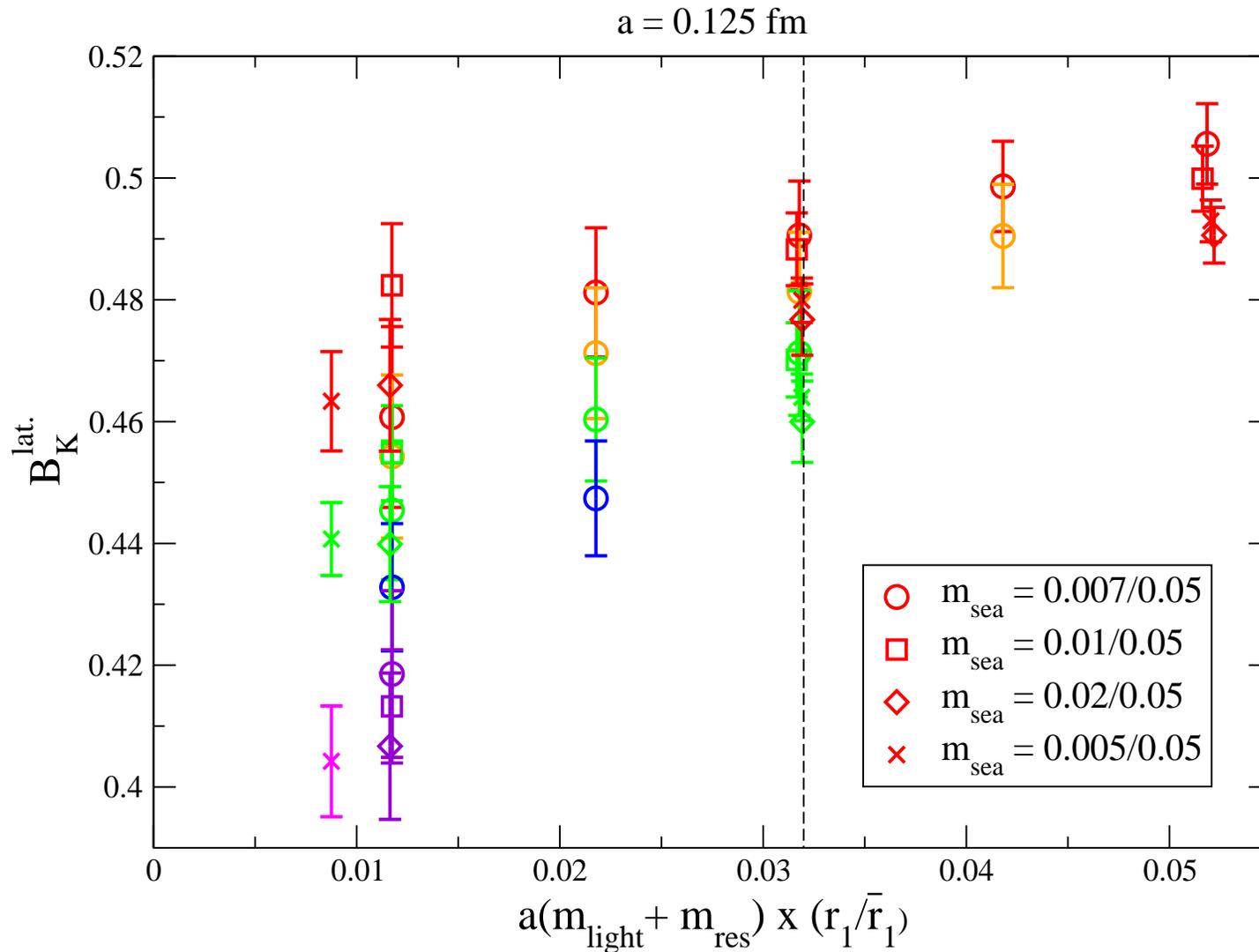
$a = 0.125$ fm; $m_{\text{sea}} = 0.007/0.05$; 112 configurations



B_K on a single ensemble



All B_K data



Estimate of future error budget

- 3% chiral extrapolation error (similar to MILC's f_K number)
- Finite Volume Effects: 1.5%
- NPR: Similar to RBC $\approx 3\%$
- lattice spacing dependence: again, similar to RBC $\approx 4\%$. This will improve with another lattice spacing.
- Statistical: 2 – 4% now. This will improve also.

Added in quadrature, we estimate around 7% now, and 5% after another year of running.

Conclusion

- There are only two new parameters in chiral fits to m_π , f_π and B_K over that of domain wall
- The NPR can be carried out in the same way. There is no mixing with taste breaking operators.
- There are a large number of MILC ensembles that already exist, and we are taking advantage of them. This will provide a valuable check of the recently announced RBC/UKQCD result.

Acknowledgements

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