Chiral Extrapolations for Domain Wall Fermions

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Outline

1. Introduction
2. Overview of current 2+1 flavor simulations
3. Taking the chiral limit for pion masses and decay constants
4. Summary and Outlook
The need for chiral extrapolations

- Quark masses in lattice simulations are far heavier than the physical up and down quark masses
  Simulations: \( m_l/m_s \approx 1/5 \) (Recent 2+1 flavor DWF simulations)
  Physical: \( m_l/m_s \approx 1/25 \)

- Extrapolations needed! Use chiral perturbation theory as a theoretical guide \( \Leftarrow \) more controlled systematics
Chiral extrapolations for DWF

- Domain wall fermions have good chiral symmetry, with only mild chiral symmetry breaking effect

\[ \Delta_\mu \langle A^a_\mu(x)O(y) \rangle = 2m_f \langle J^a_5(x)O(y) \rangle + 2\langle J^a_{5q}(x)O(y) \rangle + i\langle \delta^a O(y) \rangle. \]

Can be accounted for by the shift in quark mass:

\[ m_f \rightarrow m_f + m_{\text{res}} \]

- Easier and cleaner chiral extrapolations

- Will focus on pseudoscalar meson masses and decay constants in this talk
NLO Chiral formulae for $M_{PS}^2$ and $f_{PS}$

2+1 flavor, partially quenched, degenerate

(Sharpe and Shoresh '00)

$$M_{PS}^2 = \chi_V \left\{ 1 + \frac{48}{f^2} (2L_6 - L_4) \bar{\chi} + \frac{16}{f^2} (2L_8 - L_5) \chi_V \\
+ \frac{1}{24 f^2 \pi^2} \left[ \frac{2 \chi_V - \chi_l - \chi_s}{\chi_V - \chi_\eta} \chi_V \ln \chi_V - \frac{(\chi_V - \chi_l)(\chi_V - \chi_s)}{(\chi_V - \chi_\eta)^2} \chi_V \ln \chi_V \\
+ \frac{(\chi_V - \chi_l)(\chi_V - \chi_s)}{\chi_V - \chi_\eta} (1 + \ln \chi_V) + \frac{(\chi_\eta - \chi_l)(\chi_\eta - \chi_s)}{(\chi_V - \chi_\eta)^2} \chi_\eta \ln \chi_\eta \right] \right\}$$

$$f_{PS} = f \left\{ 1 + \frac{8}{f^2} (3L_4 \bar{\chi} + L_5 \chi_V) \\
- \frac{1}{16 \pi^2 f^2} \left[ (\chi_V + \chi_l) \ln \frac{\chi_V + \chi_l}{2} + \frac{\chi_V + \chi_s}{2} \ln \frac{\chi_V + \chi_s}{2} \right] \right\}$$

6 free parameters in combined fits.

$$\chi_i = 2B_0 (m_i + m_{res})$$
Difficulty in chiral extrapolations

- Chiral perturbation theory fails if the masses are too heavy!

Pion masses: 390 - 620 MeV
NLO fit fails! (MFL hep-lat/0600052)

- In this talk......
  lighter quark masses: down to $m_s/5$
  larger volume: $(3\text{fm})^3$
  ⇒ Can the data be described by NLO ChPT then??
Simulation Overview

Gauge Action

\[
S_G[U] = -\frac{\beta}{3} [(1 - 8c_1) \sum_{x;\mu<\nu} P[U]_{x,\mu\nu} + c_1 \sum_{x;\mu\neq\nu} R[U]_{x,\mu\nu}]
\]

For our 2+1 flavor simulations, we choose \((D.J.\ Antonio\ et.\ al.,\ hep-lat/0612005)\)

\[c_1 = -0.331(Iwasaki)\]

The Algorithm

RHMC + Quotient force + Omelyan integrator + Hasenbusch preconditioning

\(\text{(C. Allton et. al., hep-lat/0701013)}\)

\[
\det \left\{ \frac{D(m_l)}{D(1)} \right\} \det \left\{ \frac{D(m_s)}{D(1)} \right\}^{1/2} = \det \left\{ \frac{D(m_l)}{D(m_s)} \right\} \left[ \det \left\{ \frac{D(m_s)}{D(1)} \right\}^{1/2} \right]^3
\]
### Simulation Overview

**RBC and UKQCD 2+1 flavor DWF ensembles:**

<table>
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<tr>
<th>$\beta$</th>
<th>Volume</th>
<th>$L_s$</th>
<th>$m_l/m_s$</th>
<th># Time Units</th>
<th>$m_\pi$ (MeV)</th>
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<td>$16^3 \times 32$</td>
<td>16</td>
<td>0.01/0.04</td>
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<td>390</td>
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<td>620</td>
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<td>0.005/0.04</td>
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<td>0.03/0.04</td>
<td>2813</td>
<td>620</td>
</tr>
</tbody>
</table>

We are hoping to have better chiral fits with the lighter mass simulations!
The Residual Mass

- The ratio $R(t)$ is used to compute $m_{\text{res}}$ on the lattice

\[ R(t) = \frac{\langle \sum \bar{x} J_5^a(x, t) \pi^a(0) \rangle}{\langle \sum \bar{x} J_5^a(x, t) \pi^a(0) \rangle} \]

- Fit $R(t)$ where plateaux are reached to a constant to get $m_{\text{res}}$

- $O(m_f a)$ quark mass dependence

- Defined in the chiral limit:

\[ m_f \to 0 \text{ (strictly speaking, } m_f = -m_{\text{res}}, \text{ but not a big effect)} \]

\[ am_{\text{res}} = 0.00314(2) \]
Lattice Scale

From vector meson mass
Linear ansatz: \( m_\rho = A + B m_f \)

From static quark potential by Min Li

\[
a^{-1}|_\rho = 1.68(3) \text{ GeV} \\
\text{Preliminary}
\]

\[
a^{-1}|_{r_0} = 1.64(3) \text{ GeV} \\
\text{Preliminary}
\]
Simple Linear Fits

Using the linear ansatz for both $M_{PS}^2$ and $f_{PS}$:

\[ \text{am} = 0.00167(8), \text{am}_s = 0.037(1) \]
\[ f_\pi = 128(2) \text{ MeV}, f_K = 163(2) \text{ MeV} \]
\[ f_K/f_\pi = 1.27(2) \text{ Preliminary!} \]
The simultaneous NLO fit of $M_{PS}^2$ and $f_{PS}$ fails. $\chi^2$/dof = 11

Pion mass range: $\sim$ 230 - 530 MeV
Going to lighter masses

Decreasing the range of pion masses in the fit helps. $\chi^2$/dof $= 2$

Using the quark masses from linear fit of $M_{PS}^2$:

$$f_\pi = 124(3) \text{ MeV}, f_K = 153(3) \text{ MeV}$$

$$f_K/f_\pi = 1.24(2) \text{ Preliminary!}$$
Simulations with lighter quark masses allow us to do chiral extrapolations with more controlled systematics.

NLO ChPT does not work beyond $M_{PS} > 400$ MeV.

Preliminary results for light quark masses and pseudoscalar decay constants are shown.

More to do:
- finite volume corrections... expected to be small, but should check
- continuum extrapolation... weaker coupling runs in progress