Hadron Structure with Domain Wall Fermions on a Staggered Sea

J.W. Negele

For the LHP Collaboration

Domain Wall Fermions at 10 Years

BNL March 16, 2007

Collaborators

Lattice Hadron Physics Collaboration

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Outline

- Introduction
  - Staggering towards QCD with DW fermions
- Highlights from hadron structure
  - Moments of quark distributions
  - Form factors
  - Generalized form factors
    - Transverse structure
    - Origin of nucleon spin
  - Baryon shapes
- Summary and future challenges

Goals

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- Quantitative calculation of hadron observables from first principles
  - Agreement with experiment
  - Credibility for predictions and guiding experiment

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- Quantitative calculation of hadron observables from first principles
  - Agreement with experiment
  - Credibility for predictions and guiding experiment
- Insight into how QCD works
  - Mechanisms
    - Paths that dominate action - instantons
    - Variational wave functions
    - Diquark correlations
  - Dependence on parameters
    - N_c, N_f, gauge group, m_q

Parton and generalized parton distributions

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High energy scattering: light-cone correlation function $(\lambda = p^+ x^-)$

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n)\ \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n \cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$$

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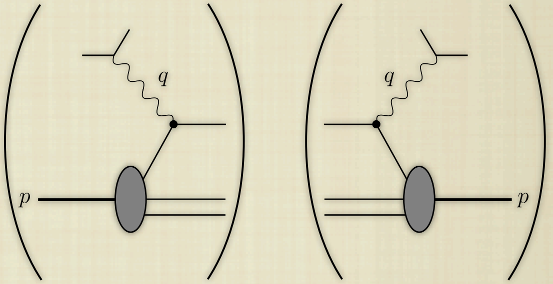
Deep inelastic scattering: diagonal matrix element

$$\langle P|\mathcal{O}(x)|P\rangle = q(x)$$



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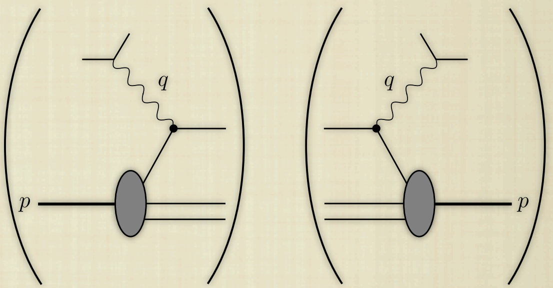
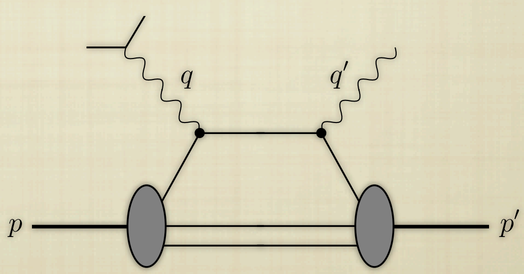


Deeply virtual Compton scattering: off-diagonal matrix element

$$\langle P'|\mathcal{O}(x)|P\rangle = \langle\gamma\rangle H(x,\xi,t) + \tfrac{i\Delta}{2m}\langle\sigma\rangle E(x,\xi,t)$$
$$\Delta = P' - P, \quad t = \Delta^2, \quad \xi = -n\cdot\Delta/2$$



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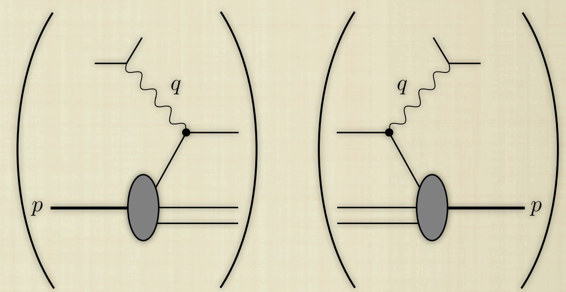
$$[\not{n} \rightarrow \not{n}\gamma_5 : \quad \Delta q(x)]$$



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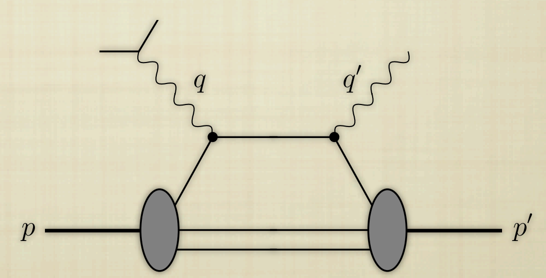

Moments of parton distributions

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Expansion of $\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n) \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha n\cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$

Generates tower of twist-2 operators

$$\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \overline{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}} \psi_q$$

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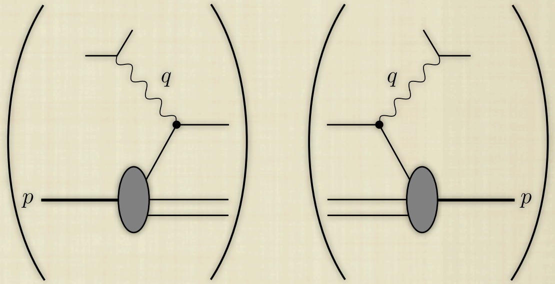
Diagonal matrix element

$$\langle P | \mathcal{O}_q^{\{\mu_1 \mu_2 \ldots \mu_n\}} | P \rangle \sim \int dx\, x^{n-1} q(x)$$



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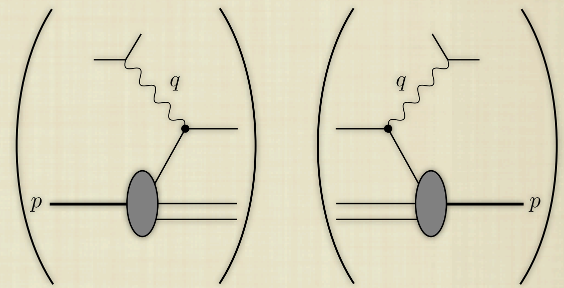
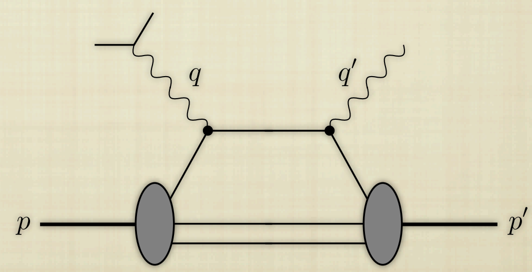


Off-diagonal matrix element

$$\langle P' | \mathcal{O}_q^{\{\mu_1 \mu_2 \ldots \mu_n\}} | P \rangle \rightarrow A_{ni}(t), B_{ni}(t), C_{n0}(t)$$

$$\int dx \, x^{n-1} H(x, \xi, t) \sim \sum \xi^i A_{ni}(t) + \xi^n C_{n0}(t)$$

$$\int dx \, x^{n-1} E(x, \xi, t) \sim \sum \xi^i B_{ni}(t) - \xi^n C_{n0}(t)$$



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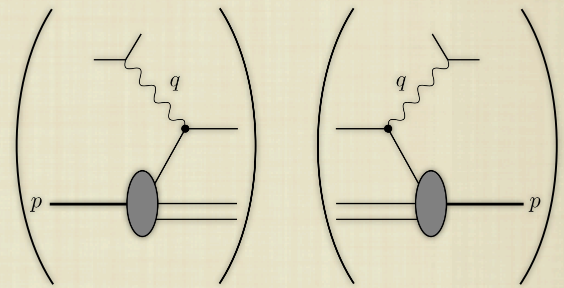

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$$\langle P'|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle \to A_{ni}(t), B_{ni}(t), C_{n0}(t)$$

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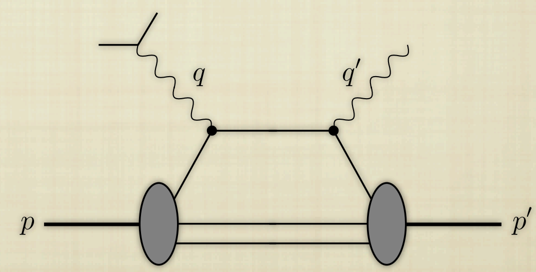

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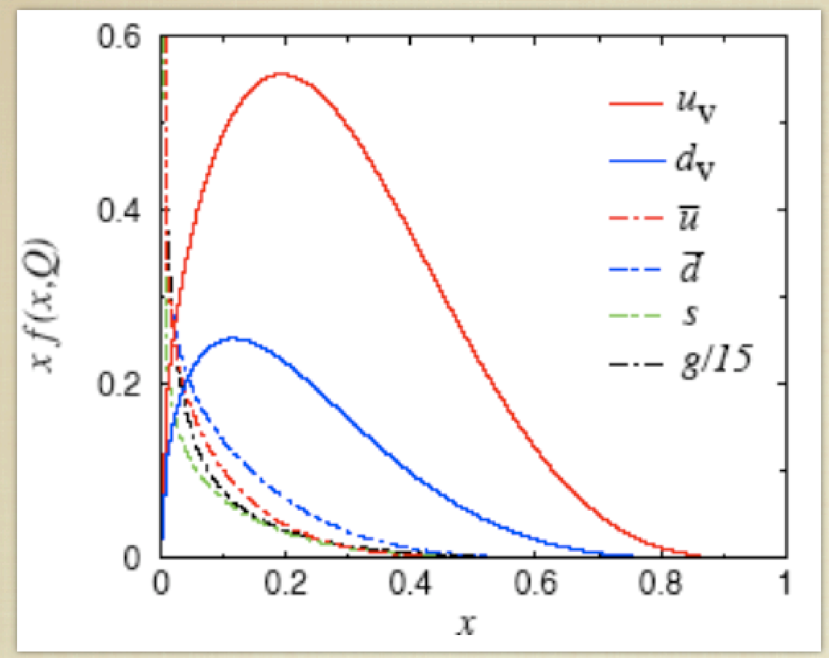
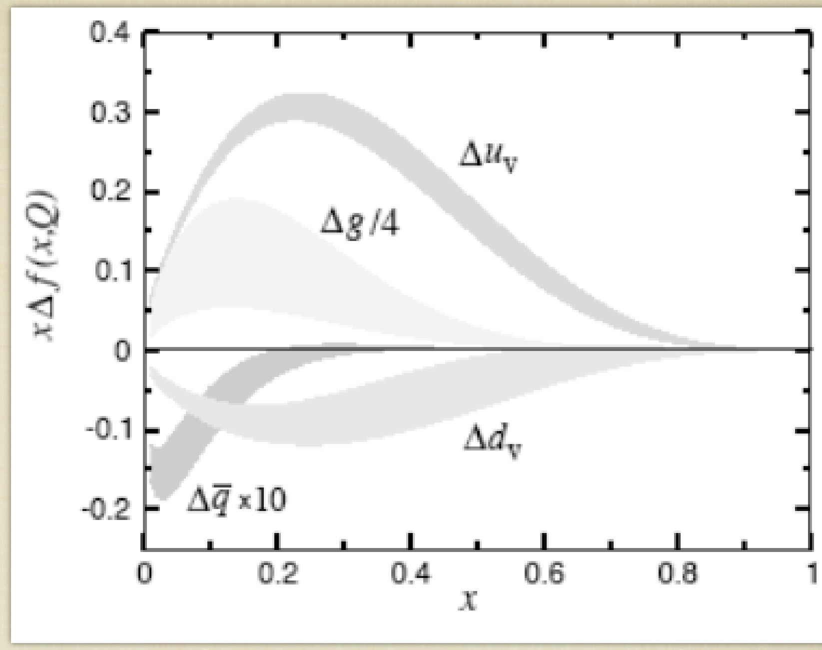




$$\langle p|\bar{\psi}\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_q \quad = \int_0^1 dx x^n[q(x)+(-1)^{(n+1)}\bar{q}(x)]$$

$$\langle p|\bar{\psi}\gamma_5\gamma_\mu D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_{\Delta q} \quad = \int_0^1 dx x^n[\Delta q(x)+(-1)^{(n)}\Delta\bar{q}(x)]$$

$$\langle p|\bar{\psi}\gamma_5\sigma_{\mu\nu} D_{\mu_1}\cdots D_{\mu_n}\psi|p\rangle \rightarrow \quad \langle x^n\rangle_{\delta q} \quad = \int_0^1 dx x^n[\delta q(x)+(-1)^{(n+1)}\delta\bar{q}(x)]$$

where $\quad q = q_\uparrow + q_\downarrow, \quad \Delta q = q_\uparrow - q_\downarrow, \quad \delta q = q_\top + q_\perp,$

Lattice operators: irreducible representations of hypercubic group with minimal operator mixing and minimal non-zero momentum components

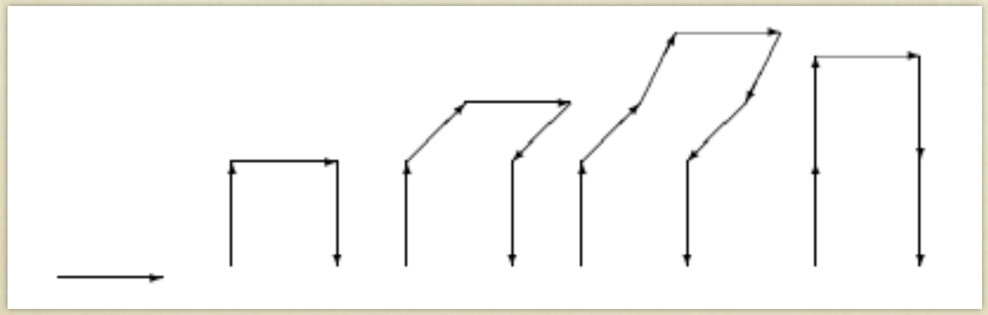
$\langle x\rangle_q^{(a)}$	6_3^+	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_{4\}}\psi$
$\langle x\rangle_q^{(b)}$	3_1^+	$\bar\psi\gamma_4\overleftrightarrow{D}_4\psi - \frac{1}{3}\sum_{i=1}^{3}\bar\psi\gamma_i\overleftrightarrow{D}_i\psi$
$\langle x^2\rangle_q$	8_1^-	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_{4\}}\psi - \frac{1}{2}\sum_{i=2}^{3}\gamma_{\{i}\overleftrightarrow{D}_i\overleftrightarrow{D}_{4\}}\psi$
$\langle x^3\rangle_q$	2_1^+	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_4\overleftrightarrow{D}_{4\}}\psi + \bar\psi\gamma_{\{2}\overleftrightarrow{D}_2\overleftrightarrow{D}_3\overleftrightarrow{D}_{3\}}\psi - \{3\leftrightarrow 4\}$
$\langle 1\rangle_{\Delta q}$	4_4^+	$\bar\psi\gamma^5\gamma_3\psi$
$\langle x\rangle_{\Delta q}^{(a)}$	6_3^-	$\bar\psi\gamma^5\gamma_{\{1}\overleftrightarrow{D}_{3\}}\psi$
$\langle x\rangle_{\Delta q}^{(b)}$	6_3^-	$\bar\psi\gamma^5\gamma_{\{3}\overleftrightarrow{D}_{4\}}\psi$
$\langle x^2\rangle_{\Delta q}$	4_2^+	$\bar\psi\gamma^5\gamma_{\{1}\overleftrightarrow{D}_3\overleftrightarrow{D}_{4\}}\psi$
$\langle 1\rangle_{\delta q}$	6_1^+	$\bar\psi\gamma^5\sigma_{34}\psi$
$\langle x\rangle_{\delta q}$	8_1^-	$\bar\psi\gamma^5\sigma_{3\{4}\overleftrightarrow{D}_{1\}}\psi$
d_1	6_1^+	$\bar\psi\gamma^5\gamma_{[3}\overleftrightarrow{D}_{4]}\psi$
d_2	8_1^-	$\bar\psi\gamma^5\gamma_{[1}\overleftrightarrow{D}_{\{3]}\overleftrightarrow{D}_{4\}}\psi$

Staggering toward QCD with DW Fermions

- Domain wall quarks on staggered sea opportunistic first step
- Improved staggered sea quarks (MILC)
  - Economical - lattices with large L, small m_π , several a
  - Fourth root appears manageable
    - RG indicates coefficient of nonlocal term → 0
    - Partially quenched staggered XPT accounts well for ugly properties
  - Order a^2 improved
- Domain wall valence quarks
  - Chiral symmetry avoids operator mixing
  - Order a^2
  - Conserved 5-d axial current facilitates renormalization
- Hybrid ChPT available
  - One-loop results have simple chiral behavior

Asqtad Action: O(a^2) perturbatively improved

- Symansik improved glue
  - $S_g(U) = C_0 W^{1\times1} + C_1 W^{1\times2} + C_2 W^{cube}$
- Smeared staggered fermions $S_f(V,U)$
  - Fat links remove taste changing gluons
  - Tadpole improved



HYP Smearing

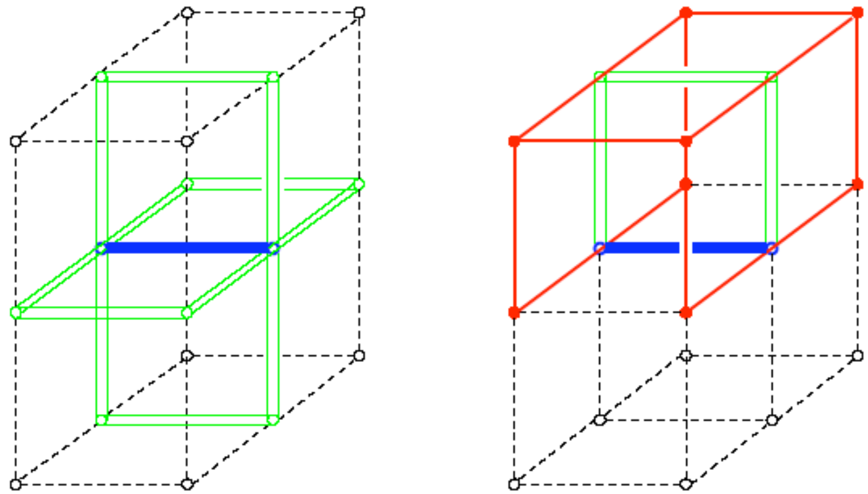
- Three levels of SU(3) projected blocking within hypercube

- Minimize dislocations - important for DW fermions

$$V_{i,\mu} = Proj_{SU(3)}\Big[(1-\alpha_1)U_{i,\mu} + \frac{\alpha_1}{6}\sum_{\pm\nu\neq\mu} \tilde{V}_{i,\nu;\mu}\tilde{V}_{i+\hat{\nu},\mu;\nu}\tilde{V}^{\dagger}_{i+\hat{\mu},\nu;\mu}\Big],$$

$$\tilde{V}_{i,\mu;\nu} = Proj_{SU(3)}\Big[(1-\alpha_2)U_{i,\mu} + \frac{\alpha_2}{4}\sum_{\pm\rho\neq\nu,\mu} \bar{V}_{i,\rho;\nu\mu}\bar{V}_{i+\hat{\rho},\mu;\rho\nu}\bar{V}^{\dagger}_{i+\hat{\mu},\rho;\nu\mu}\Big],$$

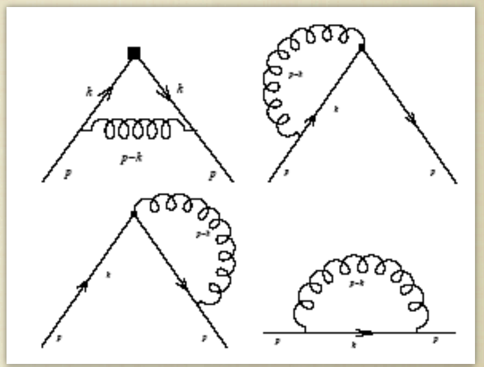
$$\bar{V}_{i,\mu;\nu\rho} = Proj_{SU(3)}\Big[(1-\alpha_3)U_{i,\mu} + \frac{\alpha_3}{2}\sum_{\pm\eta\neq\rho,\nu,\mu} U_{i,\eta}U_{i+\hat{\eta},\mu}U^{\dagger}_{i+\hat{\mu},\eta}\Big].$$

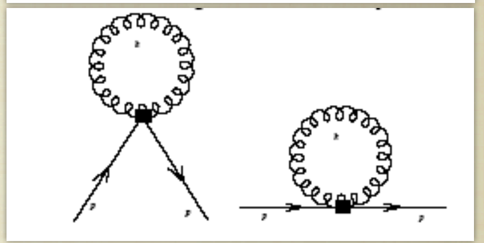


Perturbative renormalization

HYP smeared domain wall fermions - B. Bistrovic

operator	$H(4)$	NOS	HYP	APE
$\bar{q}[\gamma_5]q$	$1_1^{\pm}$	0.792	0.981	1.046
$\bar{q}[\gamma_5]\gamma_\mu q$	$4_4^{\mp}$	0.847	0.976	0.994
$\bar{q}[\gamma_5]\sigma_{\mu\nu} q$	$6_1^{\mp}$	0.883	0.992	0.993
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$6_3^{\pm}$	0.991	0.979	0.954
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_{\nu\}}q$	$3_1^{\pm}$	0.982	0.975	0.951
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$8_1^{\mp}$	1.134	0.988	0.934
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	mixing	5.71×10^{-3}	1.88×10^{-3}	8.21×10^{-4}
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_{\alpha\}}q$	$4_2^{\mp}$	1.124	0.987	0.934
$\bar{q}[\gamma_5]\gamma_{\{\mu}D_\nu D_\alpha D_{\beta\}}q$	$2_1^{\pm}$	1.244	0.993	0.919
$\bar{q}[\gamma_5]\sigma_{\mu\{\nu}D_{\alpha\}}q$	$8_1^{\pm}$	1.011	0.994	0.964
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\nu]}q$	$6_1^{\mp}$	0.979	0.982	0.989
$\bar{q}[\gamma_5]\gamma_{[\mu}D_{\{\nu]}D_{\alpha\}}q$	$8_1^{\pm}$	0.955	0.959	0.965





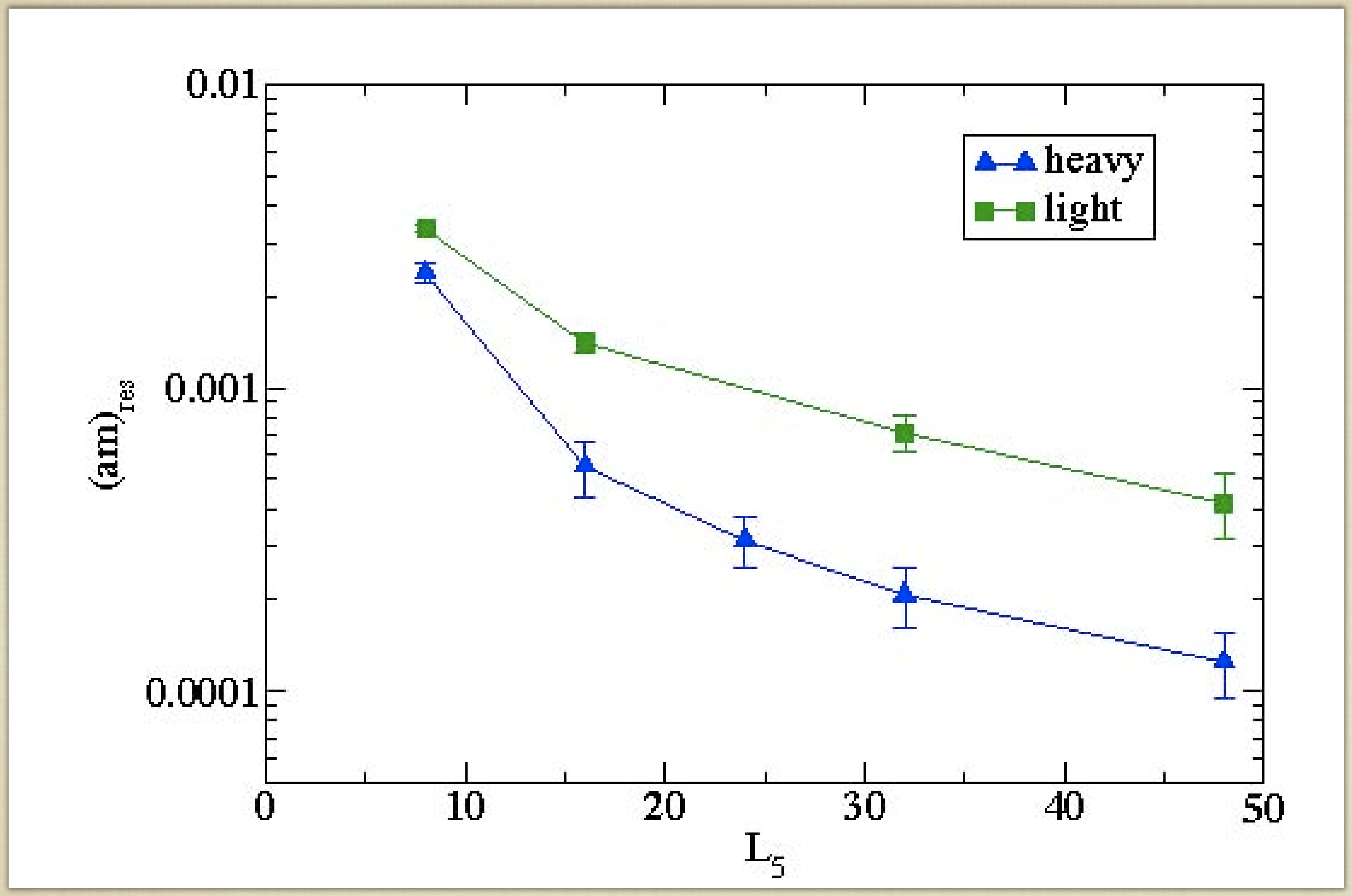
$$O_i^{\overline{MS}}(Q^2) = \sum_j \left(\delta_{ij} + \frac{g_0^2}{16\pi^2}\frac{N_c^2-1}{2N_c}\left(\gamma_{ij}^{\overline{MS}} \log(Q^2a^2) - (B_{ij}^{LATT} - B_{ij}^{\overline{MS}})\right)\right) \cdot O_j^{LATT}(a^2)$$

Numerical calculations

- Improved staggered sea quarks (MILC configurations)
  - $N_F = 3$, $a = 0.125$ fm
- Domain wall valence quarks
  - $L_S = 16$, $M = 1.7$
  - Masses and volumes:

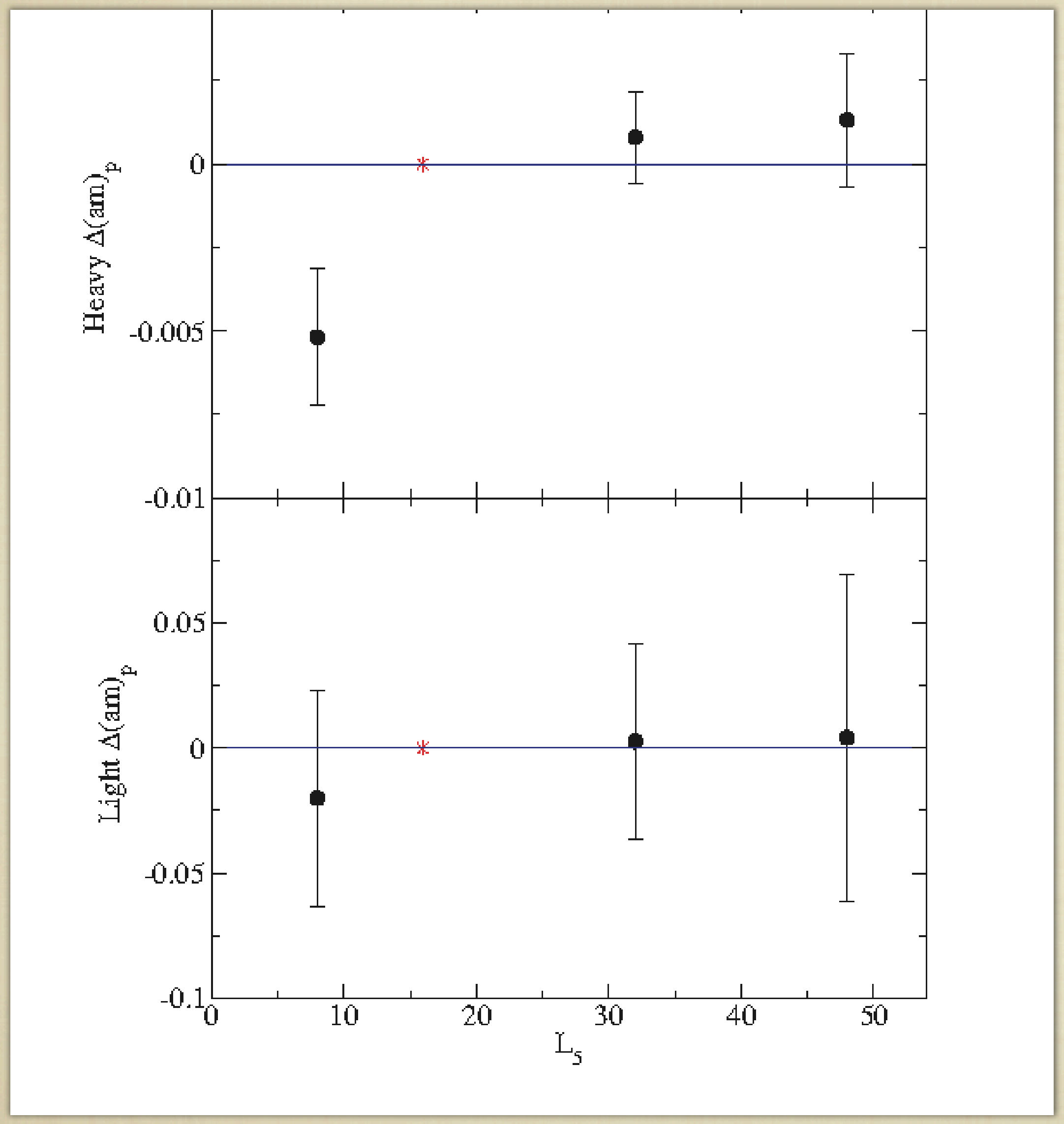
m_π	configs	Vol	L (fm)
761	425	20^3	2.5
693	350	20^3	2.5
544	564	20^3	2.5
486	498	20^3	2.5
354	655	20^3	2.5
354	270	28^3	3.5

Residual Mass



residual mass for pion masses 790 and 350 MeV

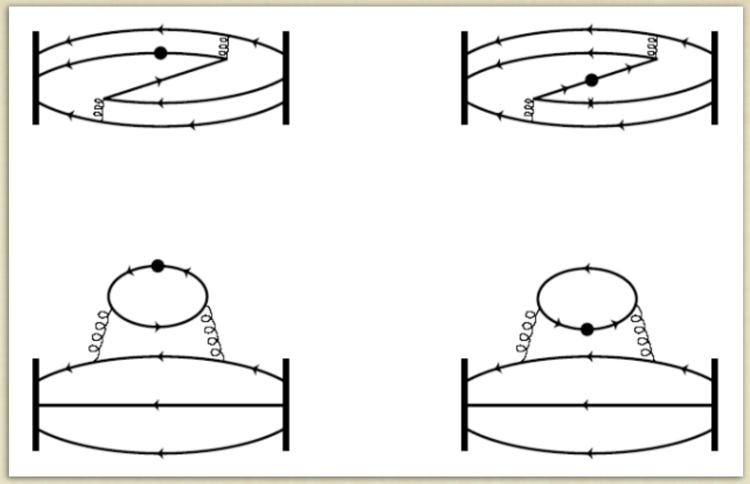
L_5 Dependence of Nucleon Mass



m_π = 790 MeV

m_π = 350 MeV

Hadron matrix elements on the lattice



- Measure $\langle \mathcal{O} \rangle$ for m_q, a, L
- Connected diagrams
- Disconnected diagrams (cancel for $\langle \mathcal{O} \rangle_u - \langle \mathcal{O} \rangle_d$)
- Extrapolate m_q, a, L

Overdetermined system for form factors

Calculate ratio

$$R_{\mathcal{O}}(\tau,P',P)=\frac{C^{3\mathrm{pt}}_{\mathcal{O}}(\tau,P',P)}{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}},P')}\left[\frac{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}}-\tau+\tau_{\mathrm{src}},P)\,C^{2\mathrm{pt}}(\tau,P')\,C^{2\mathrm{pt}}(\tau_{\mathrm{snk}},P')}{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}}-\tau+\tau_{\mathrm{src}},P')\,C^{2\mathrm{pt}}(\tau,P)\,C^{2\mathrm{pt}}(\tau_{\mathrm{snk}},P)}\right]^{1/2}$$

Perturbative renormalization

$$\mathcal{O}_i^{\overline{\mathrm{MS}}}(\mu)=\sum_j Z_{ij}(\mu,a)\mathcal{O}_j^{\mathrm{lat}}(a)$$

$$\langle P'|\,\mathcal{O}_i^{\overline{\mathrm{MS}}}\,|P\rangle=\sqrt{E(P')E(P)}\sum_j Z_{ij}\overline{R}_j$$

$$\langle P'|\,\mathcal{O}^q_{\{\mu_1\mu_2\ldots\mu_n\}}\,|P\rangle=\sum_i a_i A^q_{ni}+\sum_j b_j B^q_{nj}+cC^q_n$$

Schematic form

$$\begin{aligned}
\langle\mathcal{O}_i^{cont}\rangle &= \sum_j a_{ij}\mathcal{F}_j\\
\langle\mathcal{O}_i^{cont}\rangle &= \sqrt{E'E}\sum_j Z_{ij}\overline{R}_j\\
\overline{R}_i &= \frac{1}{\sqrt{E'E}}\sum_{jk} Z^{-1}_{ij}a_{jk}\mathcal{F}_k\\
&\equiv \sum_j a'_{ij}\mathcal{F}_j\,.
\end{aligned}$$

Nucleon axial charge in full lattice QCD

- Why g_A?

- Matrix element of axial current $A_\mu = \bar{q}\gamma_\mu\gamma_5 \frac{\vec{\tau}}{2} q$

  $$\langle N(p+q)|A_\mu|N(p)\rangle = \bar{u}(p+q)\frac{\vec{\tau}}{2}\left[g_A(q^2)\gamma_\mu\gamma_5 + g_P(q^2)q_\mu\gamma_5\right]u(p)$$

  $$g_A(0) = 1.2695 \pm 0.0029$$

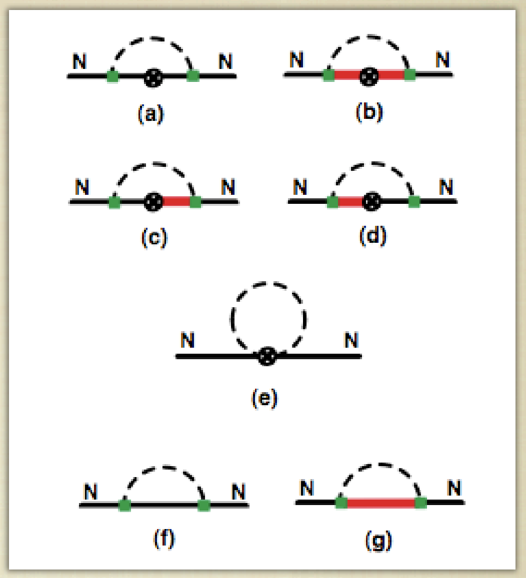
- Adler Weisberger $g_A^2 - 1 \sim \int(\sigma_{\pi^+p} - \sigma_{\pi^-p})$

- Goldberger Treiman $g_A \to f_\pi g_{\pi NN}/M_N$

- Spin content $\langle 1\rangle_{\Delta q} = \int_0^1 dx[\Delta q(x) + \Delta\bar{q}(x)]$

  $$g_A = \langle 1\rangle_{\Delta u} - \langle 1\rangle_{\Delta d} \qquad \Sigma = \langle 1\rangle_{\Delta u} + \langle 1\rangle_{\Delta d} + \langle 1\rangle_{\Delta s}$$

Nucleon Axial Charge



- Chiral perturbation theory $g_A(m_\pi^2, V)$
  - Beane and Savage hep-ph/0404131
  - Detmold and Lin hep-lat/0501007
- 1-loop theory has 6 parameters
  - Fix $f_\pi,\, m_\Delta - m_N,\, g_{\Delta N}$ (0.3% error)
  - Fit $g_A,\, g_{\Delta\Delta},\, C$
  - Result $g_A(m_\pi = 140) = 1.212 \pm 0.084$

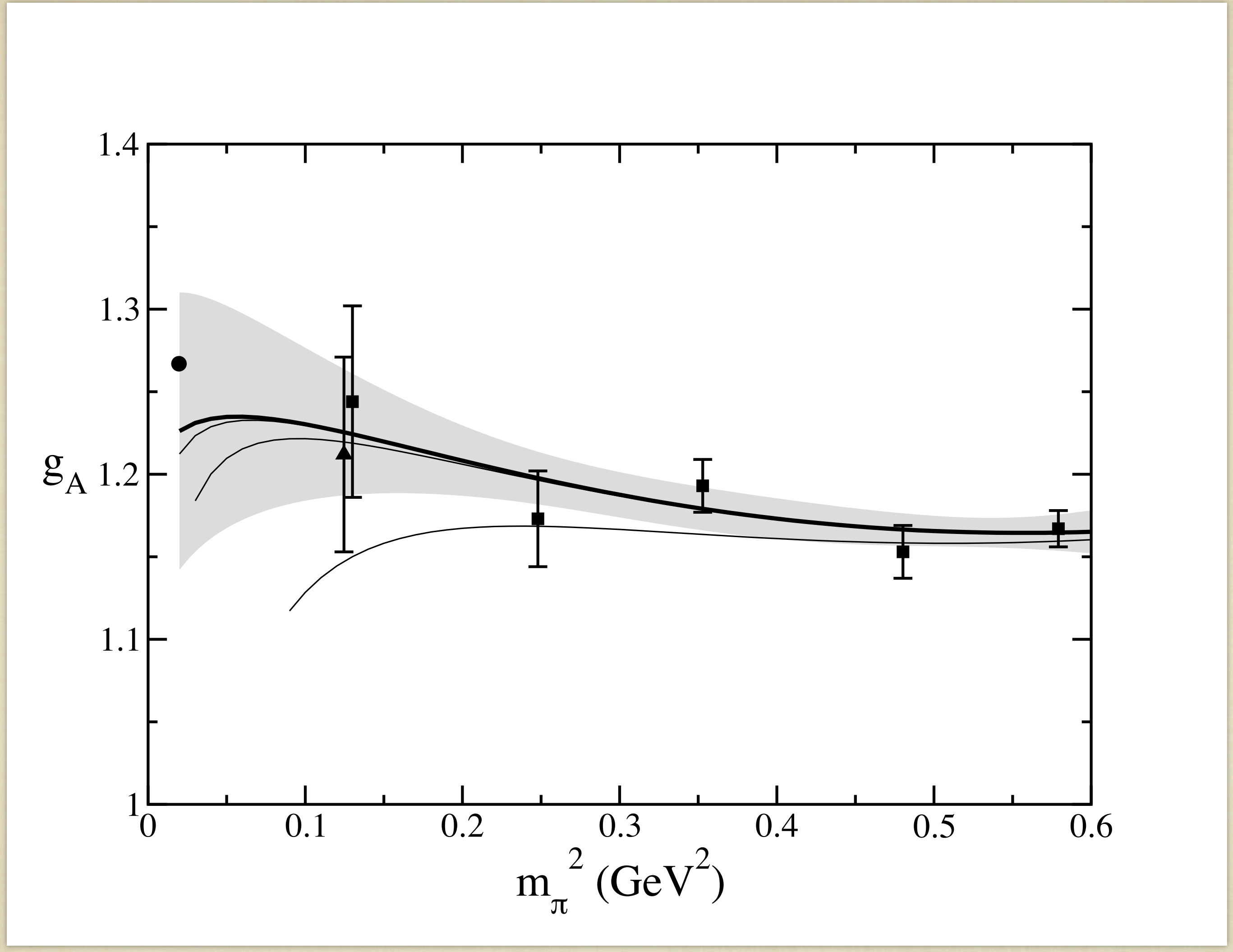
Chiral expansion of axial charge

Beane and Savage hep-ph/0404131

$$
\begin{aligned}
\Gamma_{NN} = g_A \quad & - \quad i\frac{4}{3f^2}[4g_A^3 J_1(m_\pi, 0, \mu) \\
& + \quad 4(g_{\Delta N}^2 g_A + \frac{25}{81} g_{\Delta N}^2 g_{\Delta\Delta}) J_1(m_\pi, \Delta, \mu) \\
& + \quad \frac{3}{2} g_A R_1(m_\pi, \mu) \\
& - \quad \frac{32}{9} g_{\Delta N} g_A N_1(m_\pi, \Delta, \mu)] \\
& + \quad C m_\pi^2
\end{aligned}
$$

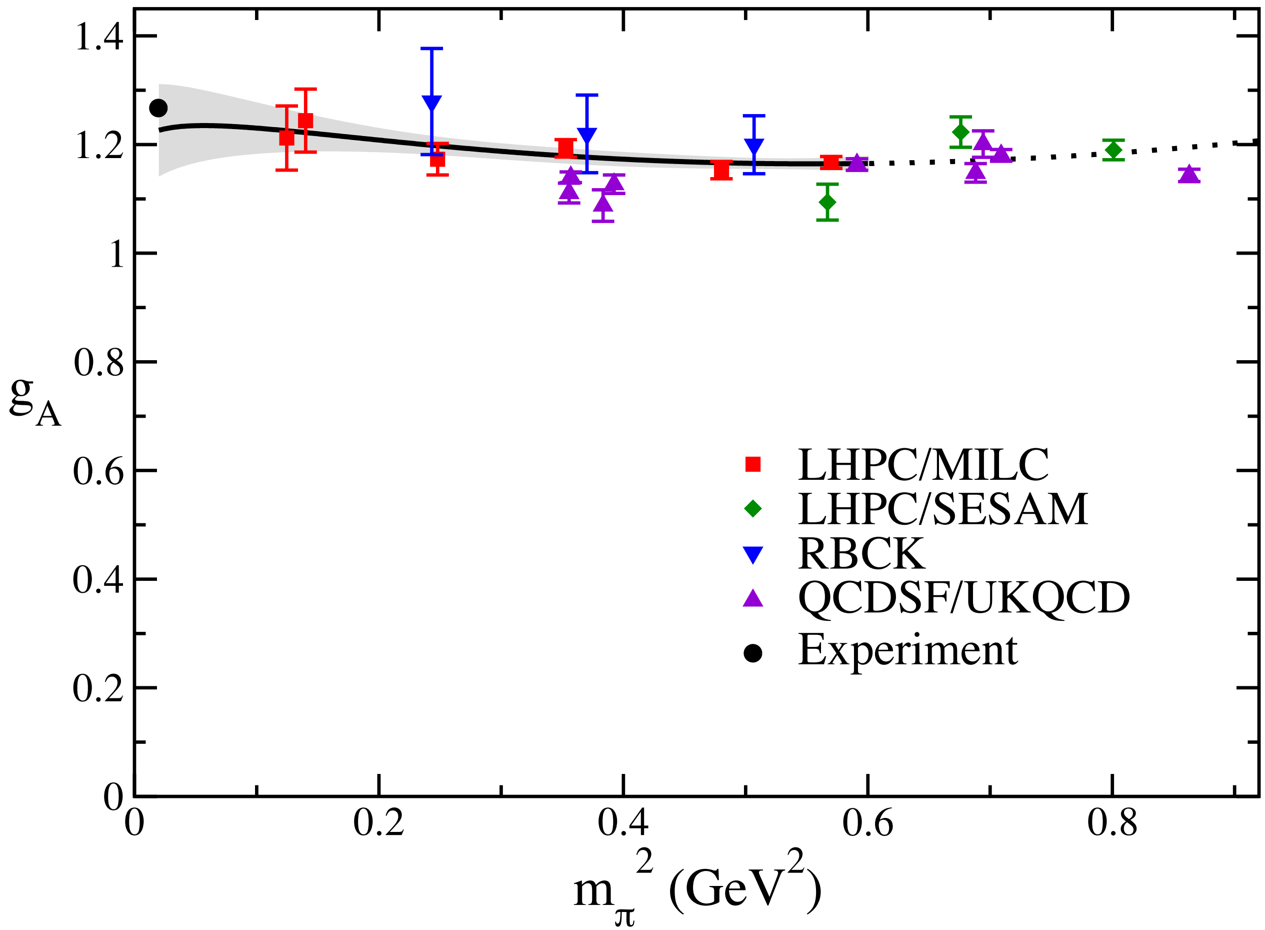
$$
\begin{aligned}
J_1(m, \Delta, \mu) &= -\frac{3}{4}\frac{i}{16\pi^2}\left[(m^2 - 2\Delta^2)\log\frac{m^2}{\mu^2} + 2\Delta F(m, \Delta)\right] \\
R_1(m, \mu) &= \frac{i}{16\pi^2} m^2 \left[\Gamma(\epsilon) + 1 - \log\frac{m^2}{\mu^2}\right] \\
N_1(m, \Delta, \mu) &= -\frac{3}{4}\frac{i}{16\pi^2}\left[(m^2 - \frac{2}{3}\Delta^2)\log\frac{m^2}{\mu^2} + \frac{2}{3}\Delta F(m, \Delta) + \frac{2}{3}\frac{m^2}{\Delta}[\pi m - F(m, \Delta)]\right] \\
f(m, \Delta) &= \sqrt{\Delta^2 - m^2 - i\epsilon}\,\log\left(\frac{\Delta - \sqrt{\Delta^2 - m^2 - i\epsilon}}{\Delta + \sqrt{\Delta^2 - m^2 - i\epsilon}}\right)
\end{aligned}
$$

Nucleon axial charge g_A $\langle 1 \rangle_{\Delta q}^{u-d}$



LHPC hep-lat/0510062

Nucleon axial charge g_A $\langle 1 \rangle_{\Delta q}^{u-d}$



Chiral Extrapolation of Moments

- for example, unpolarized moments

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_{A,0}^2 + 1)}{(4\pi f_{\pi,0})^2} m_\pi^2 \ln\left(\frac{m_\pi^2}{\mu^2}\right)\right) + b_n'(\mu) m_\pi^2$$

- choose $\mu = f_{\pi,0}$, and at one loop $g_{A,0} \to g_{A,m_\pi}$ and $f_{\pi,0} \to f_{\pi,m_\pi}$

$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_{A,m_\pi}^2 + 1)}{(4\pi)^2} \frac{m_\pi^2}{f_{\pi,m_\pi}^2} \ln\left(\frac{m_\pi^2}{f_{\pi,m_\pi}^2}\right)\right) + b_n \frac{m_\pi^2}{f_{\pi,m_\pi}^2}$$

- self consistently $g_A \to g_{A,\text{lat}}$, $f_\pi \to f_{\pi,\text{lat}}$, $m_\pi \to m_{\pi,\text{lat}}$

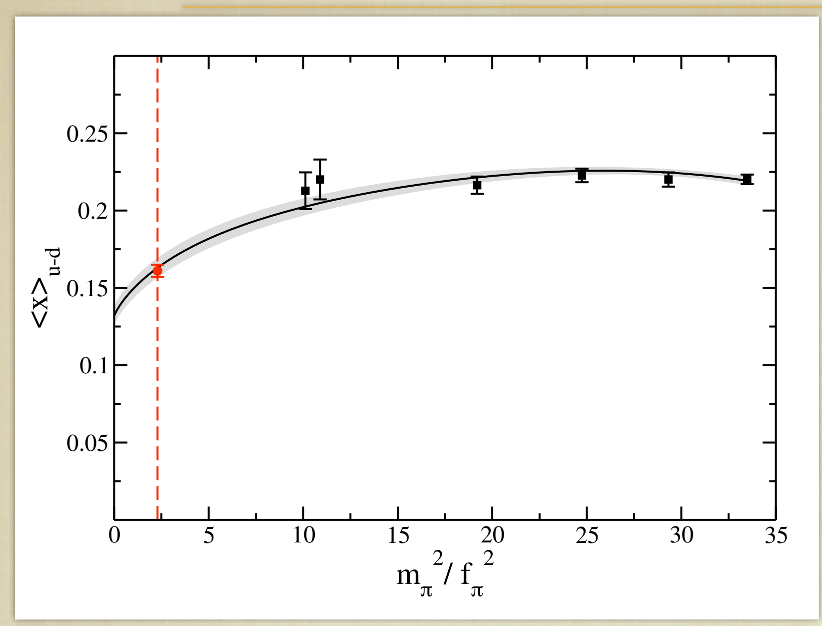
$$\langle x^n \rangle_{u-d} = a_n \left(1 - \frac{(3g_{A,\text{lat}}^2 + 1)}{(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln\left(\frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}\right)\right) + b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

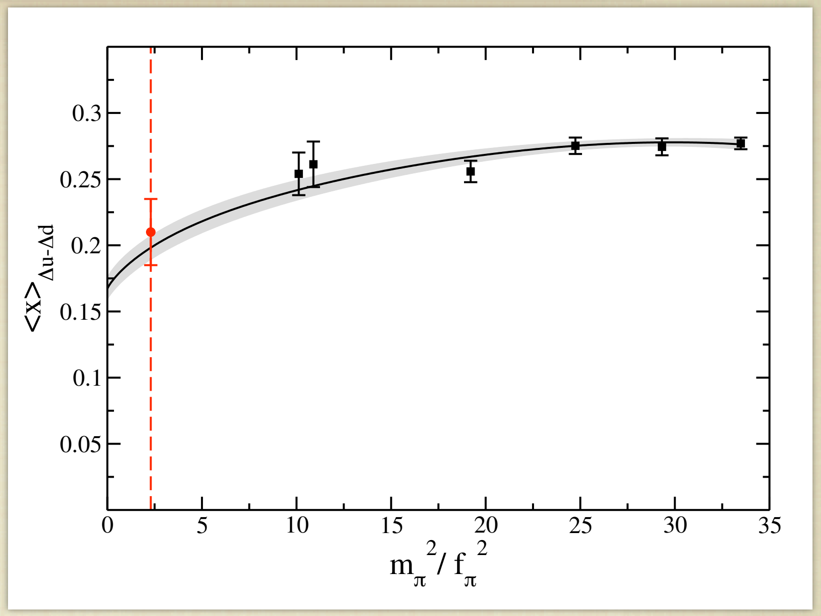
- similarly for the helicity and transversity moments

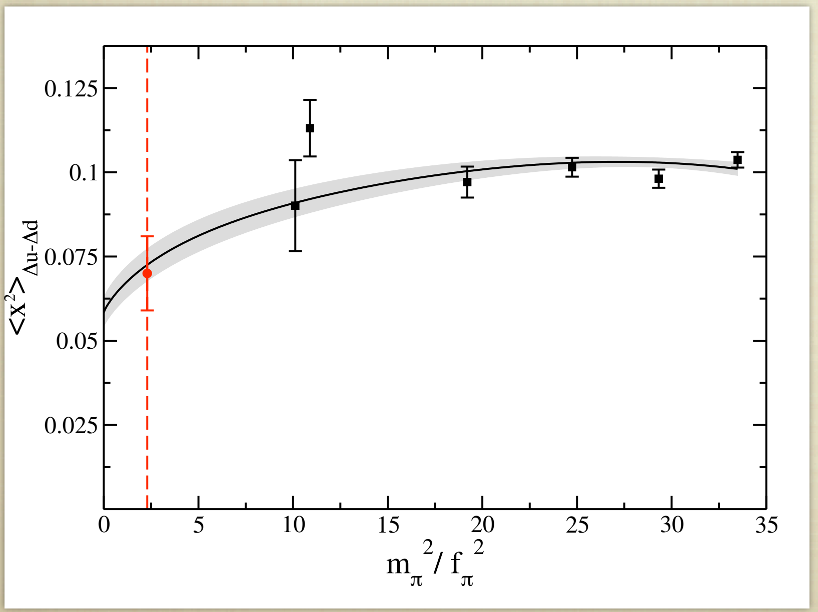
$$\langle x^n \rangle_{\Delta u-\Delta d} = \Delta a_n \left(1 - \frac{(2g_{A,\text{lat}}^2 + 1)}{(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln\left(\frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}\right)\right) + \Delta b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

$$\langle x^n \rangle_{\delta u-\delta d} = \delta a_n \left(1 - \frac{(4g_{A,\text{lat}}^2 + 1)}{2(4\pi)^2} \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2} \ln\left(\frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}\right)\right) + \delta b_n \frac{m_{\pi,\text{lat}}^2}{f_{\pi,\text{lat}}^2}$$

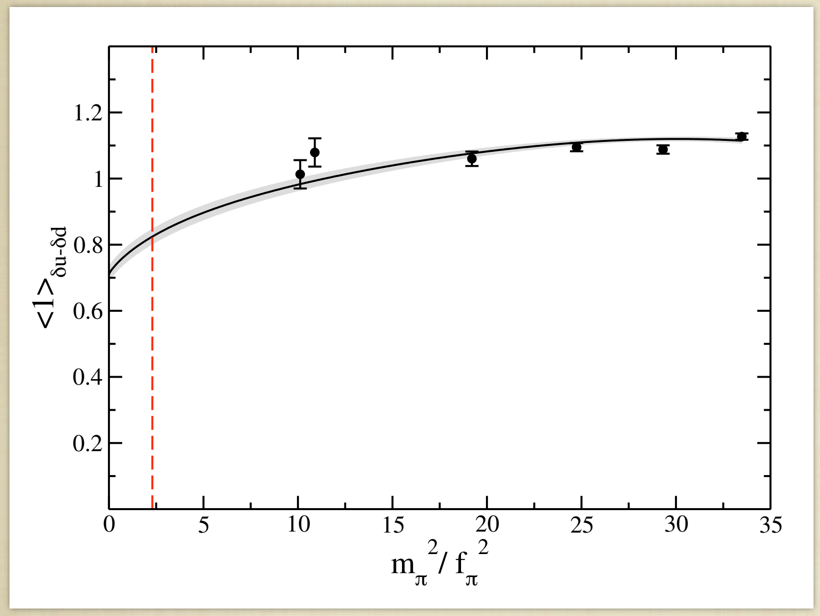
Chiral Extrapolation of Moments

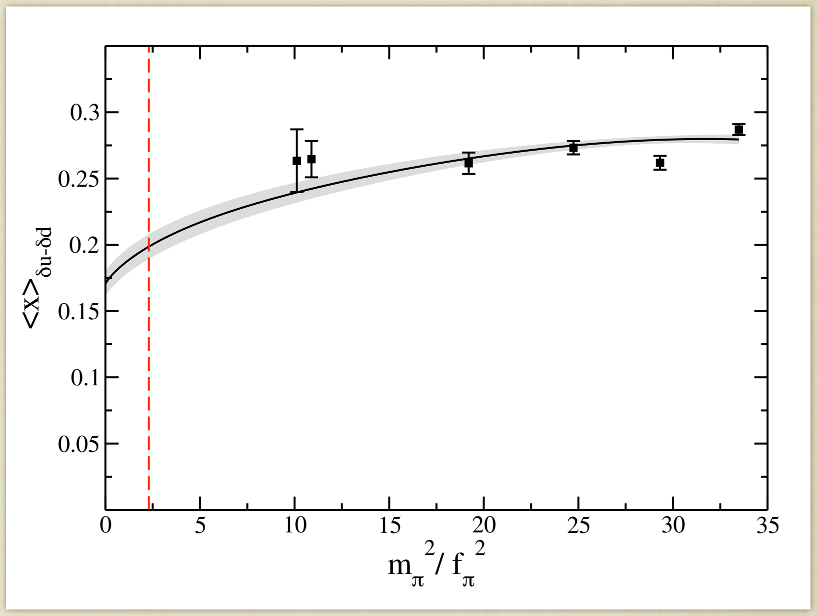




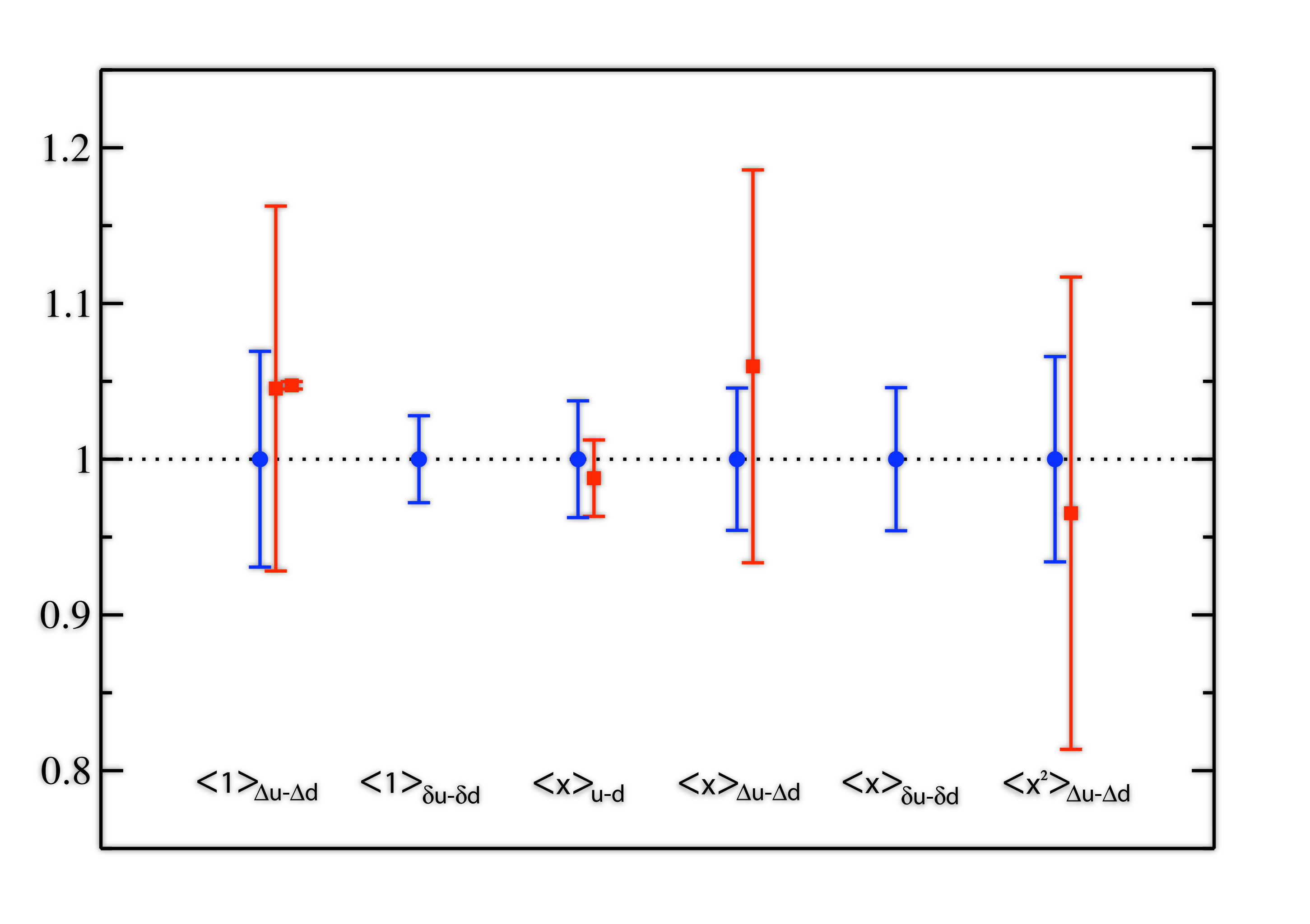


Chiral Extrapolation of Moments





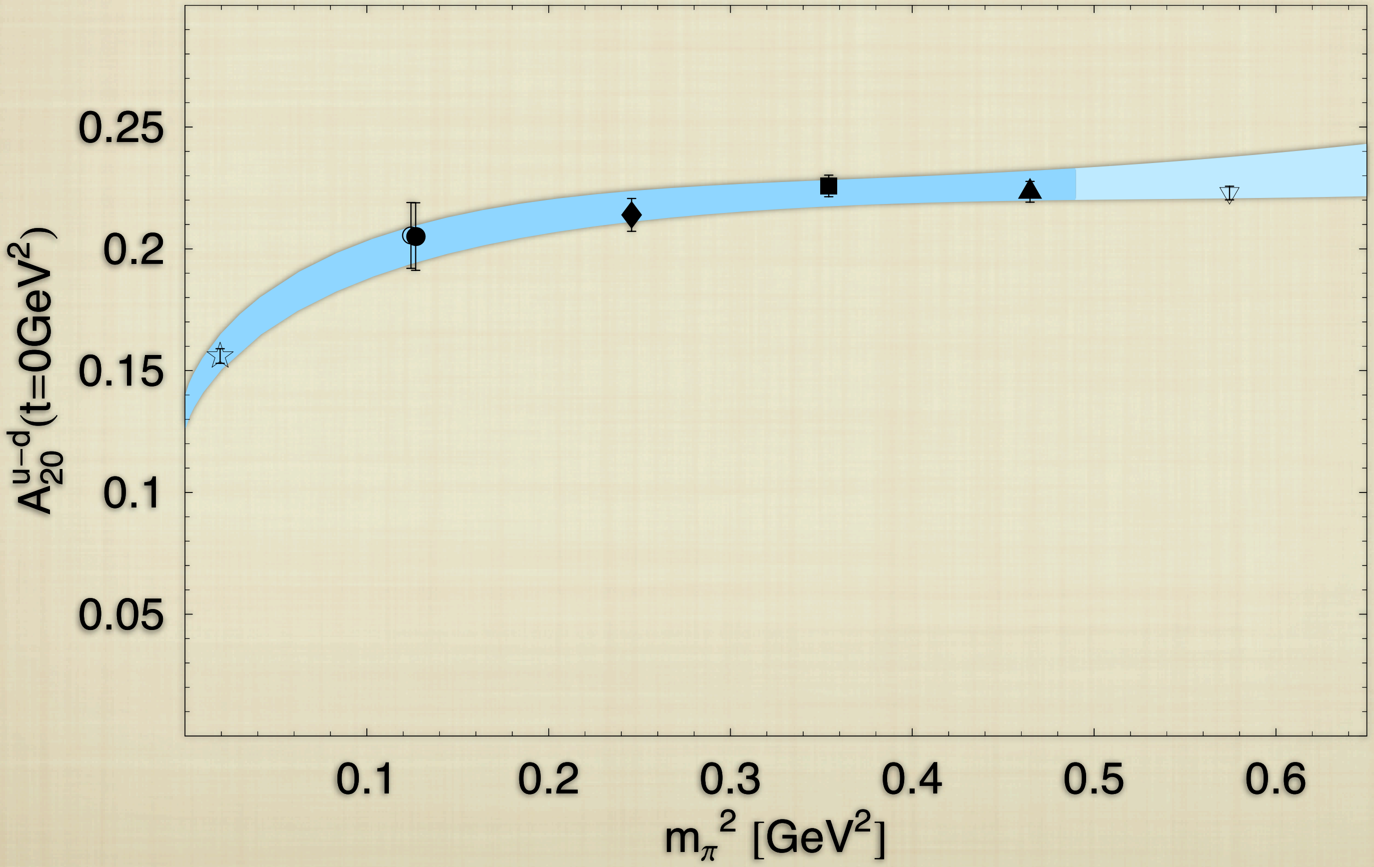
Chiral Extrapolation of Moments



Chiral extrapolation of $\langle x \rangle_q^{u-d} = A_{20}^{u-d}(t=0)$

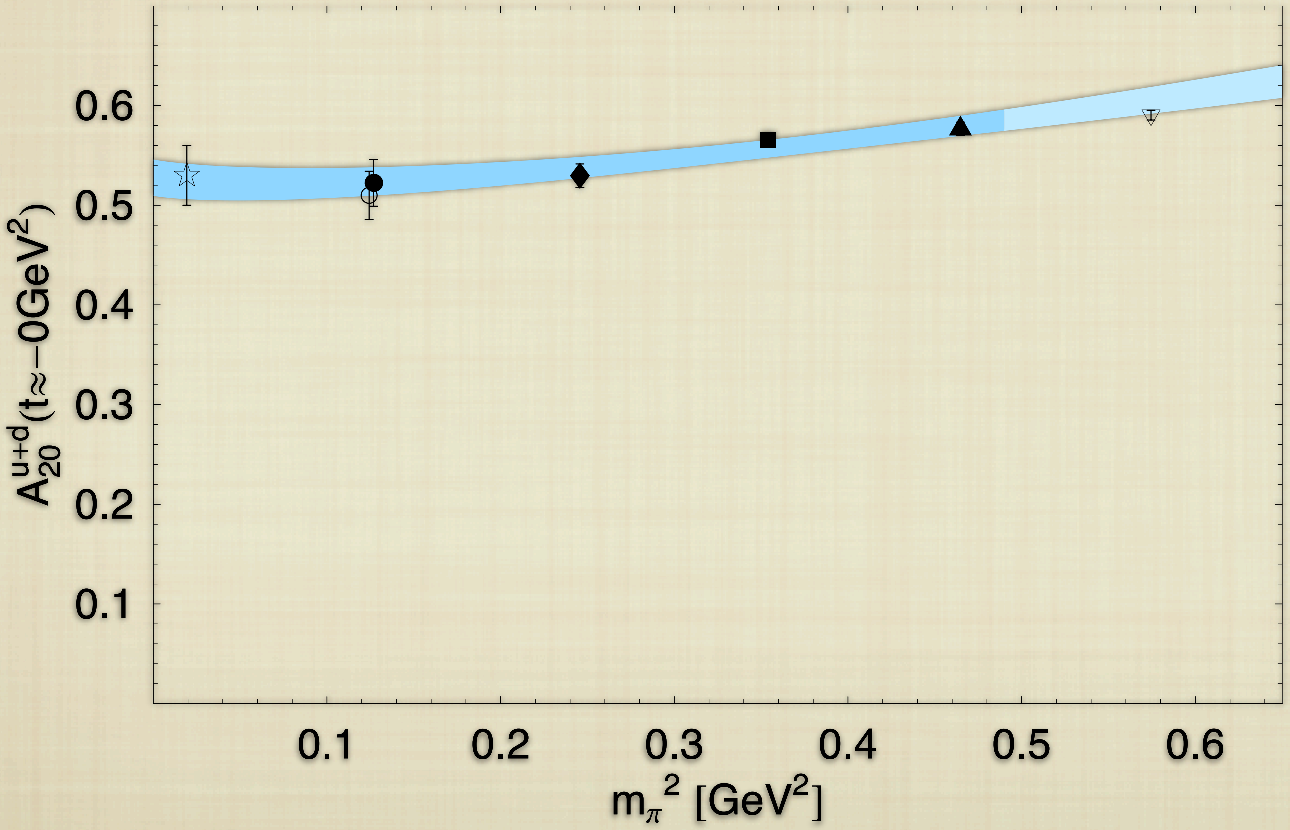
Chiral extrapolation O(p^4) relativistic ChPT (Dorati, Hemmert, et. al.)

$$A_{20}^{u-d}(t, m_\pi) = A_{20}^{0,u-d}\big(f_A(m_\pi) + g_A(t, m_\pi)\big) + \widetilde{A}_{20}^{0,u-d} h_A(m_\pi) + A_{20}^{m_\pi} m_\pi^2 + A_{20}^t t$$



Chiral extrapolation of $\langle x \rangle_q^{u+d} = A_{20}^{u+d}(t=0)$

Chiral extrapolation O(p^4) relativistic ChPT (Dorati, Hemmert, et. al.)
Note: connected diagrams only



Electromagnetic form factors

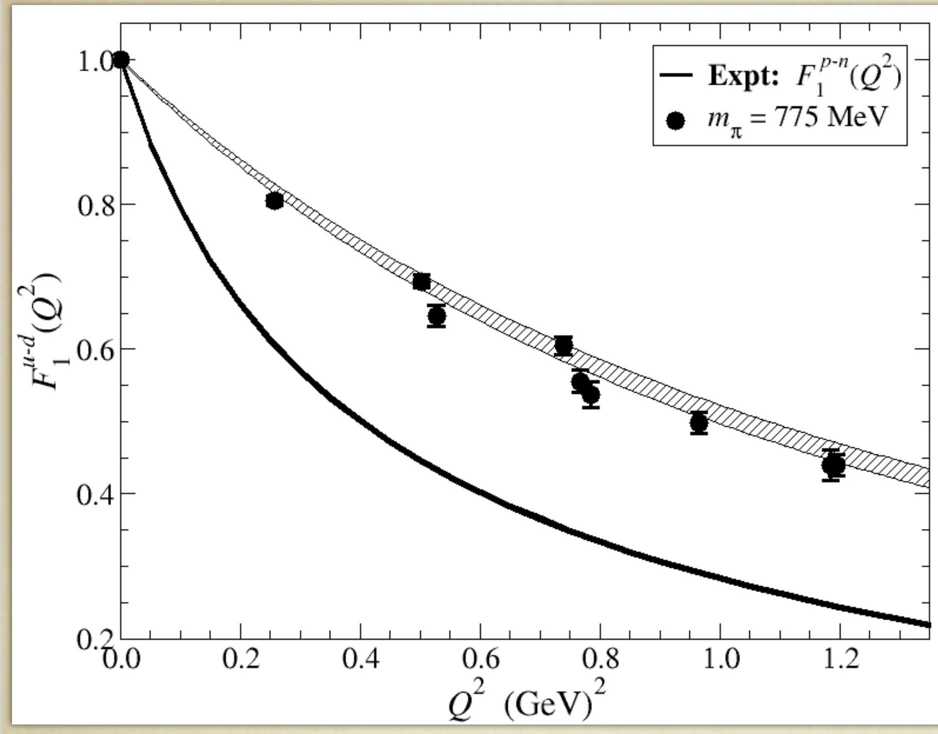
- Simplest off-diagonal matrix element

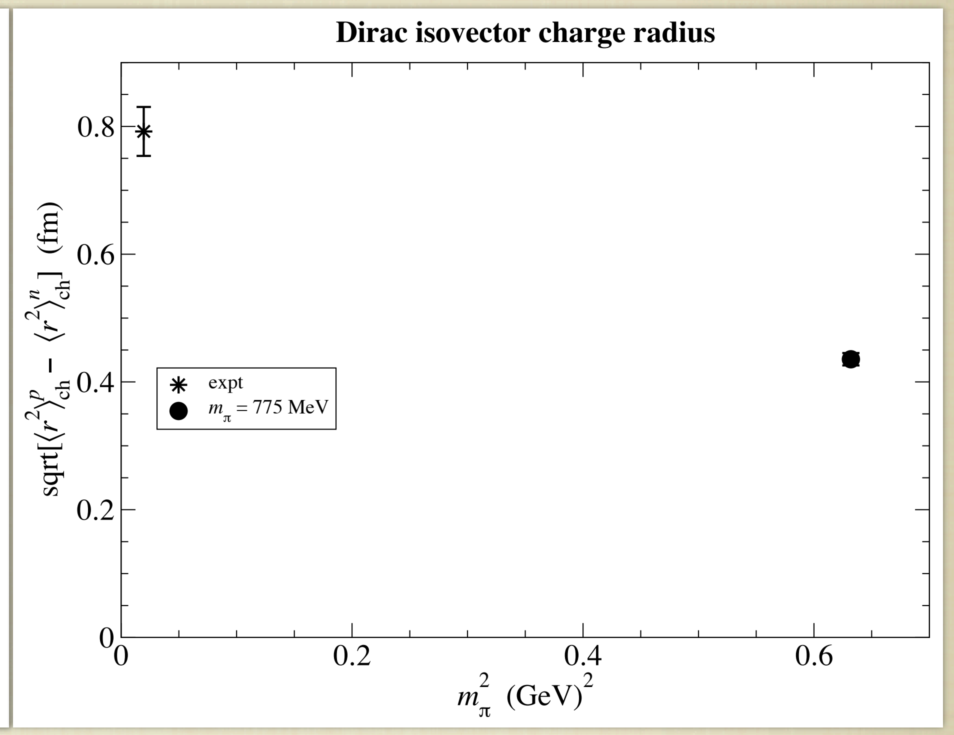
$$\langle p|\bar{\psi}\gamma^\mu\psi|p'\rangle = \bar{u}(p)[F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}]u(p')$$

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2}F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

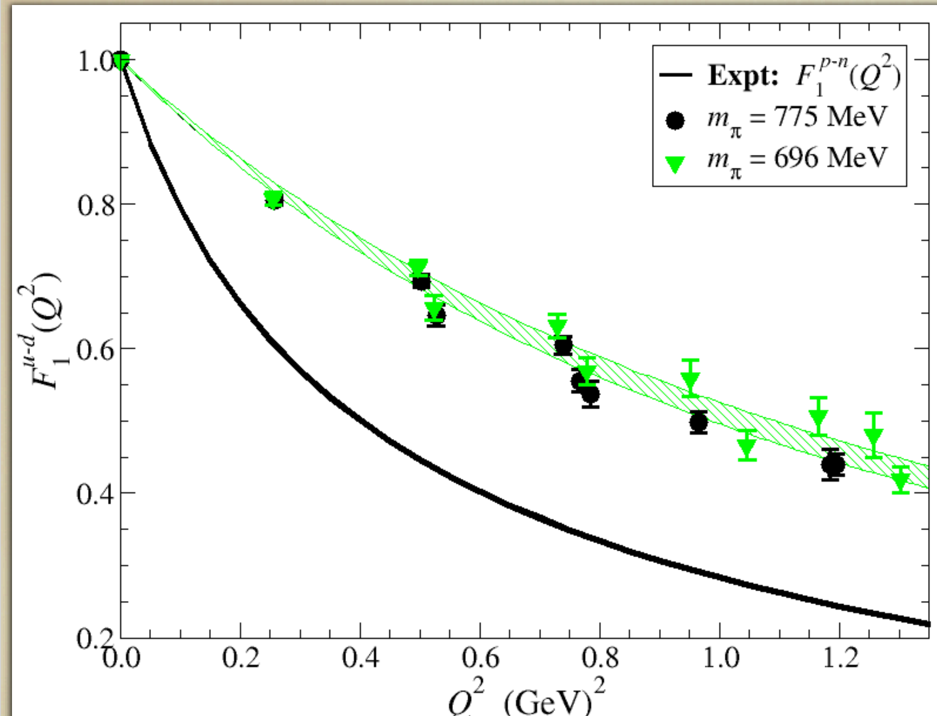
- Fourier transform of charge density if $\mathrm{L}_{\mathrm{system}} \gg \mathrm{L}_{\mathrm{wavepacket}} \gg \frac{1}{m}$
  - Pb: 5 fm >> 10^{-5} fm, Proton: 0.8 fm ~ 0.2 fm: marginal
  - For transverse Fourier transform of light cone w. f., m → p_+ ~ ∞
- Large q^2: ability of one quark to share q^2 with other constituents to remain in ground state - q^2 counting rules

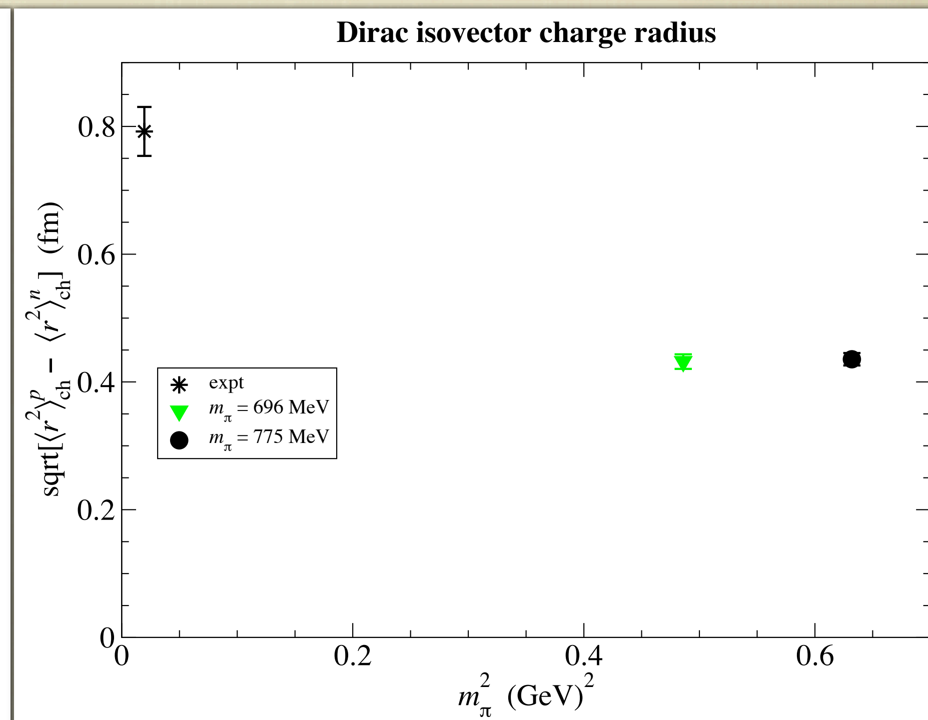
F_1 Isovector Form Factor



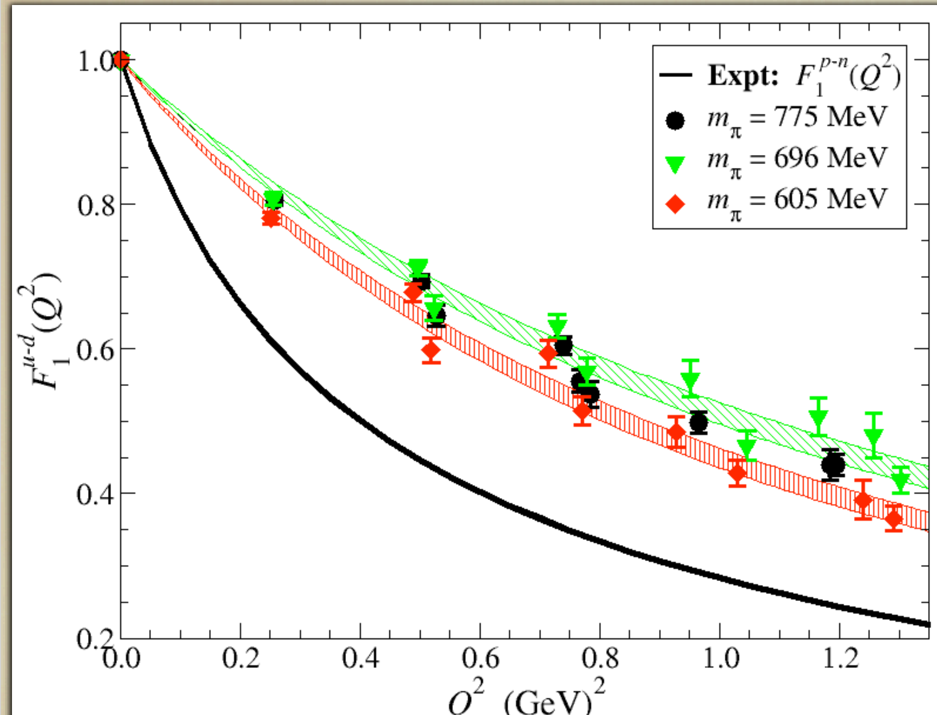


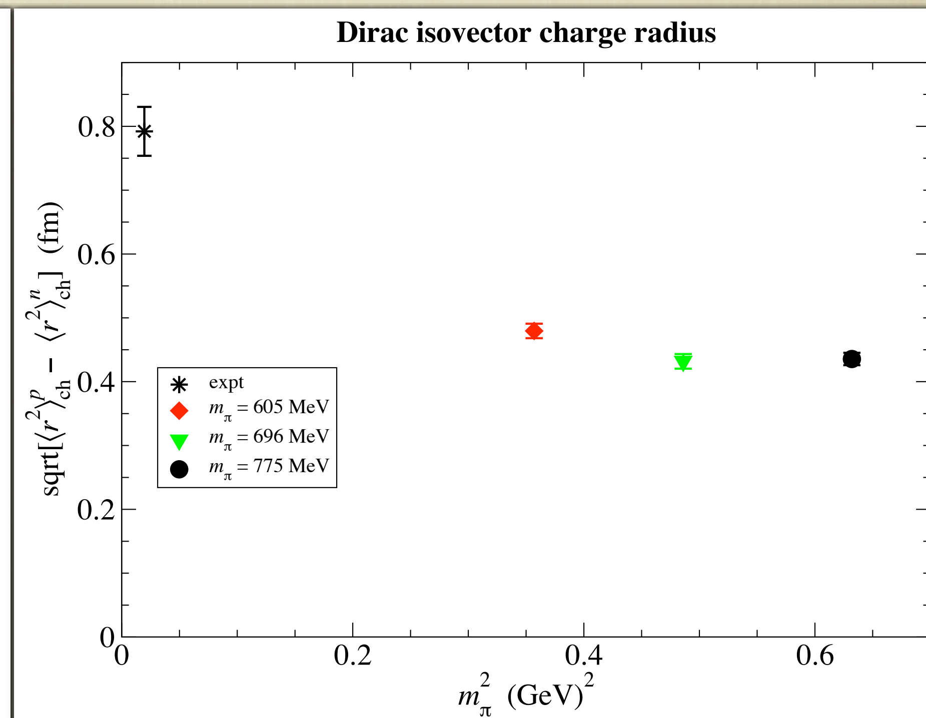
F_1 Isovector Form Factor



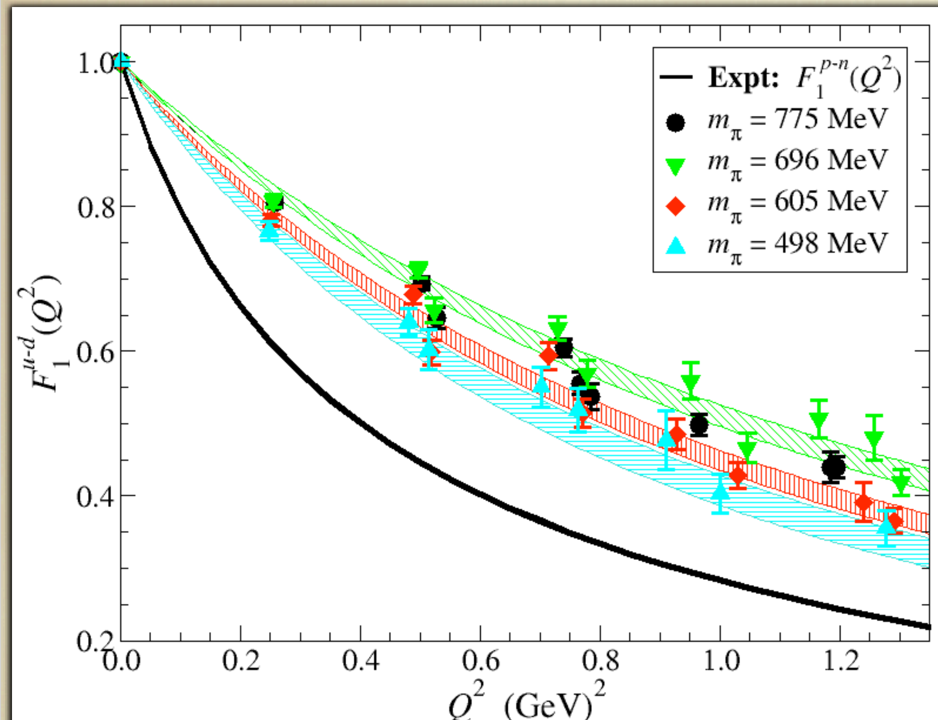


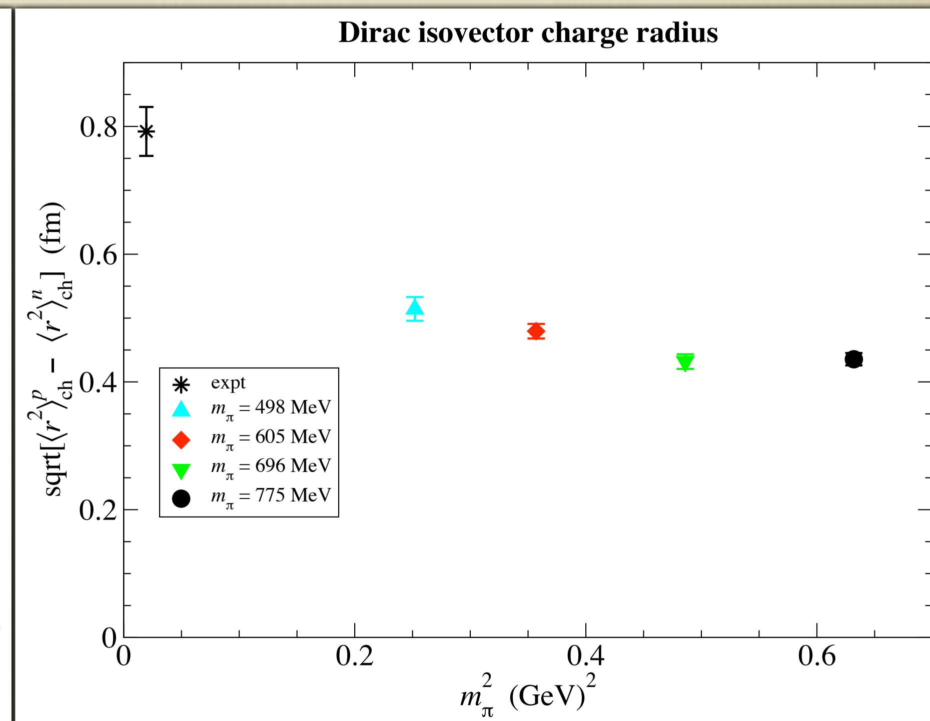
F1 Isovector Form Factor



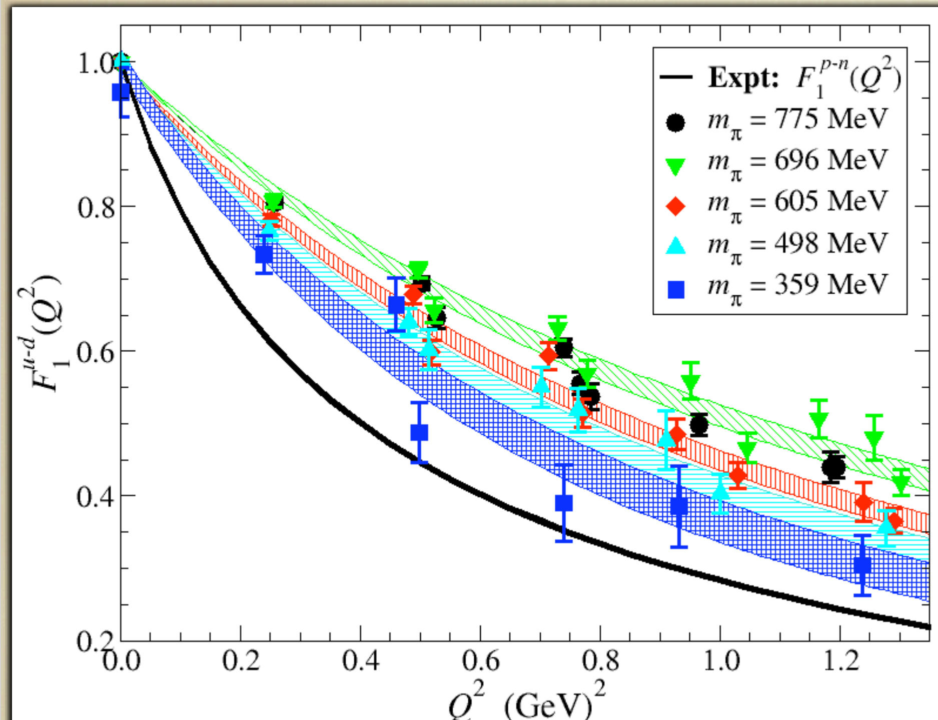


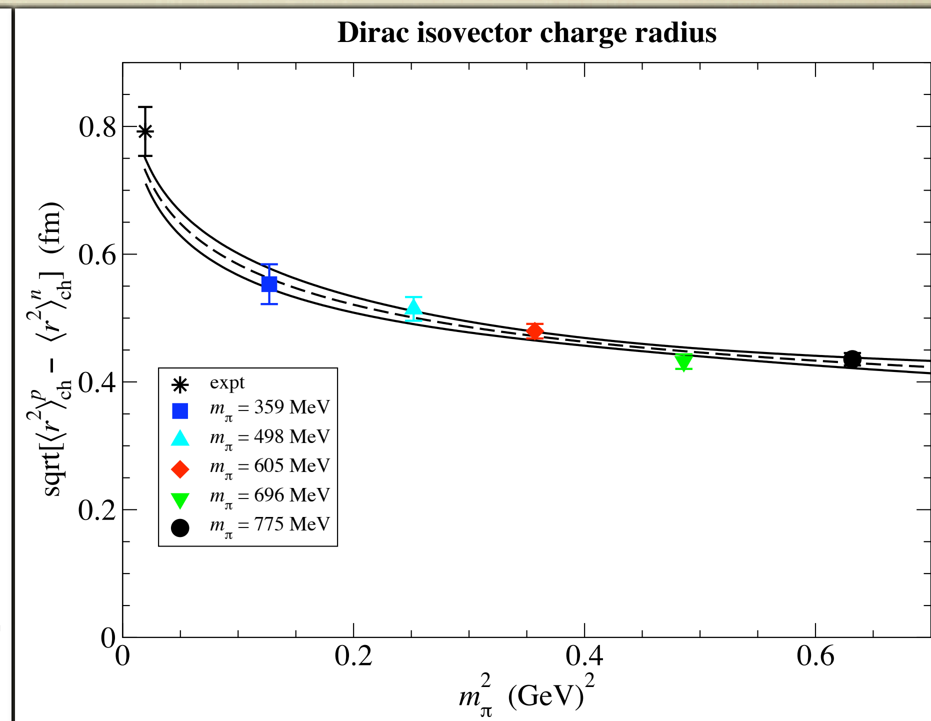
F_1 Isovector Form Factor





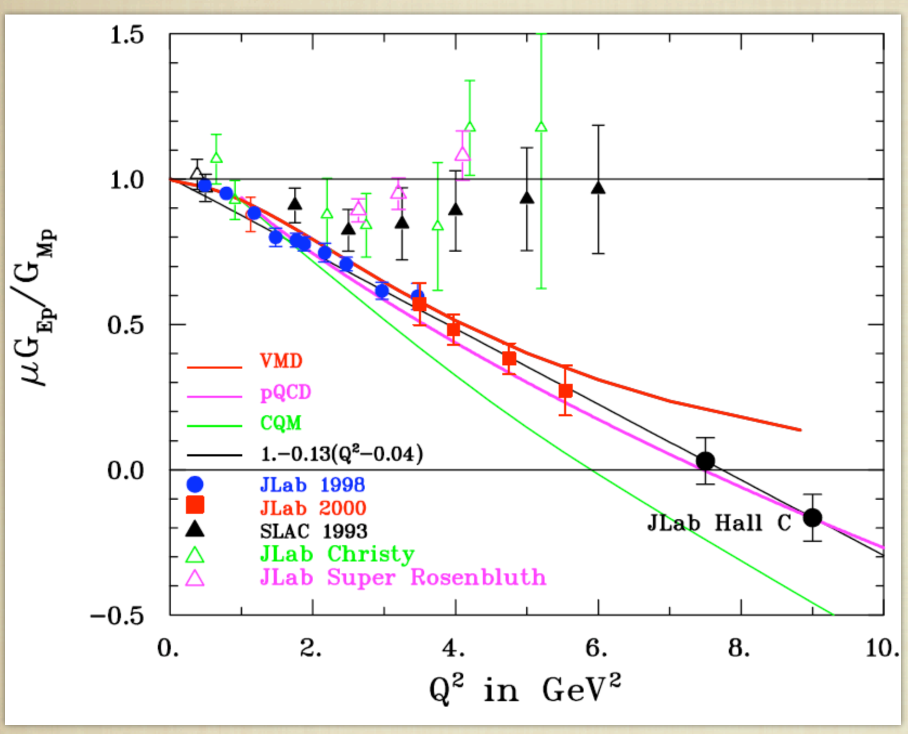
F_1 Isovector Form Factor

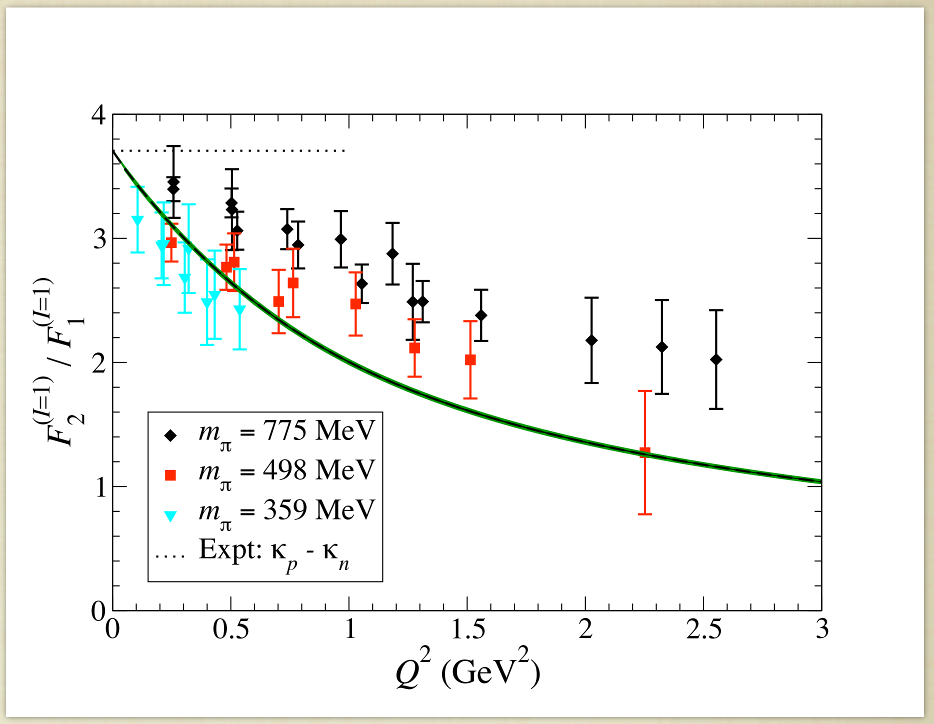




$$\langle r^2 \rangle^{u-d} = a_0 - \frac{(1+5g_A^2)}{(4\pi f_\pi)^2} \log\left(\frac{m_\pi^2}{m_\pi^2+\Lambda^2}\right)$$

Form factor ratio: F_2 / F_1



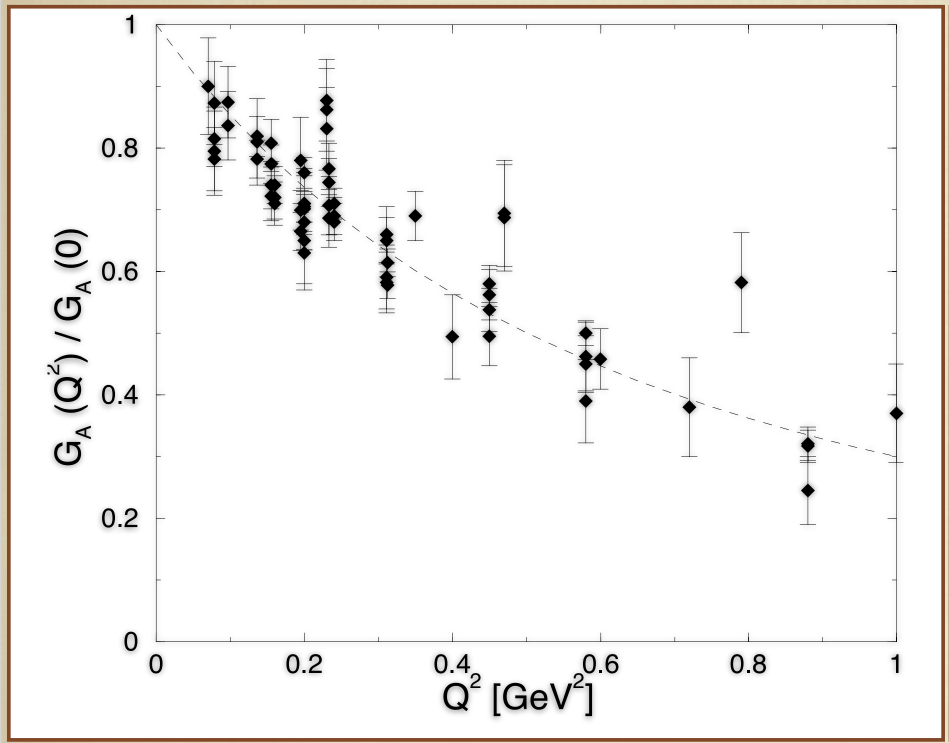


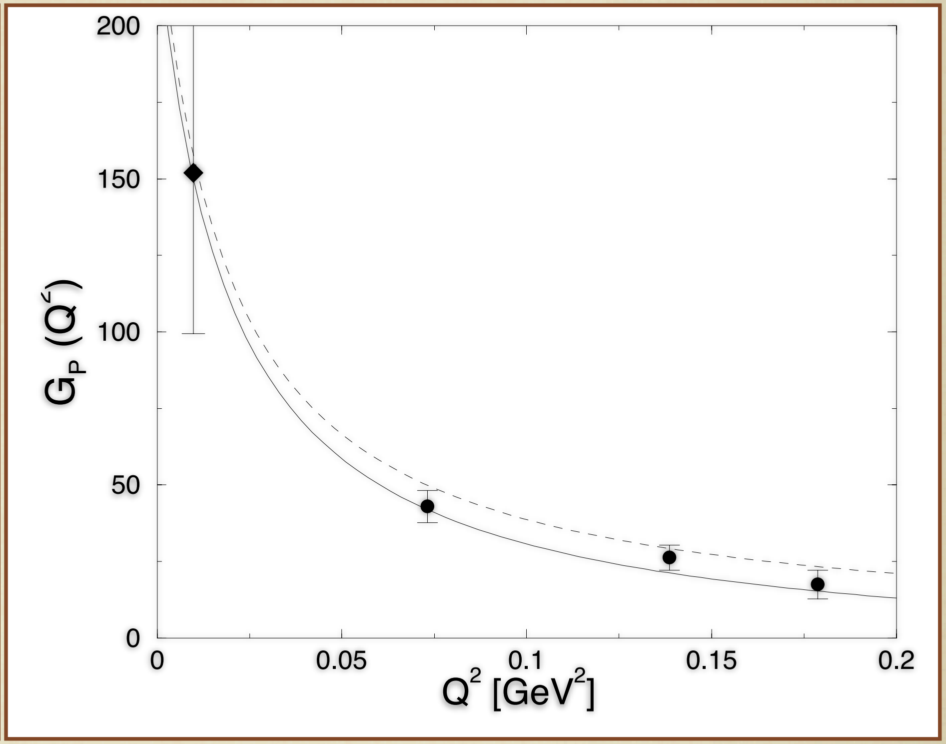
Polarization transfer at JLab

Lattice results

Polarized Nucleon Form Factors G_A and G_P

$$\langle p|\bar{\psi}\gamma^\mu\gamma_5\psi|p'\rangle = \bar{u}(p)[G_A(q^2)\gamma^\mu\gamma_5 + q^\mu\gamma_5 G_P(q^2) + \sigma^{\mu\nu}\gamma_5 q_\nu G_M(q^2)]u(p')$$





Bernard, Elouadrhiri, Meissner, J. Phys. G Nucl. Part. Phys. 2002, R1

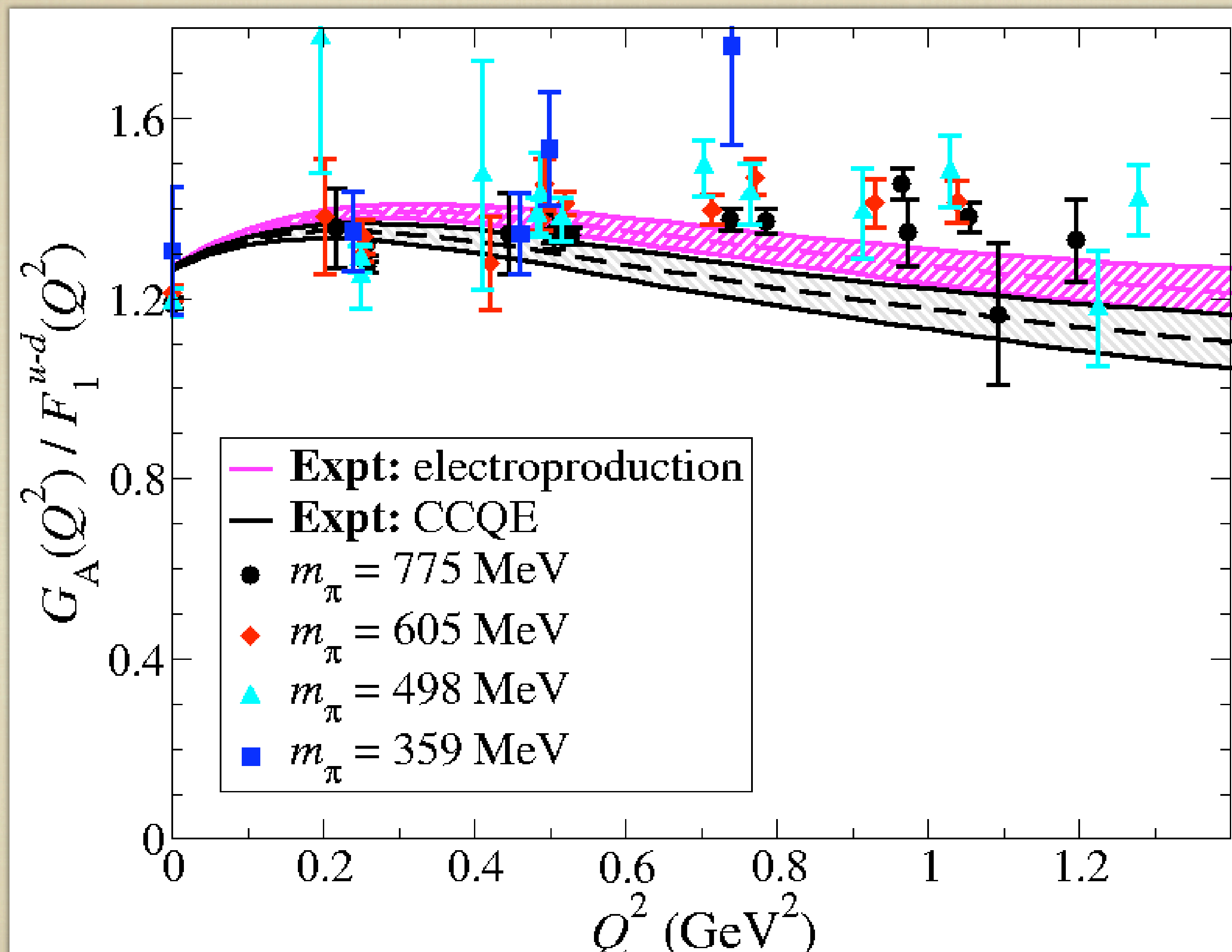
pion electroproduction ◆

$\nu_\mu\, n \rightarrow \mu^-\, p$

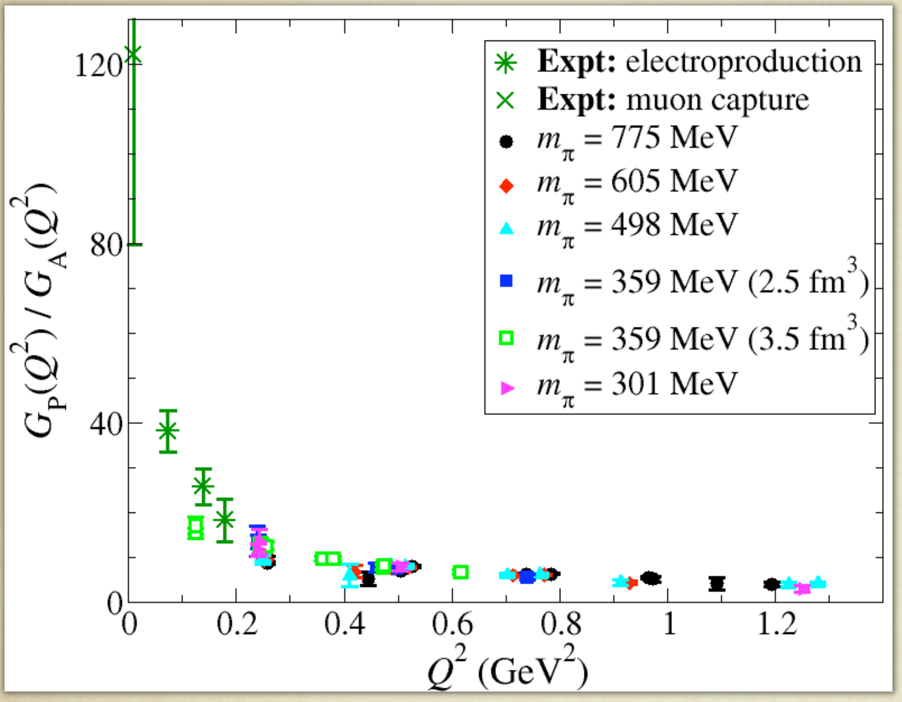
pion electroproduction ●

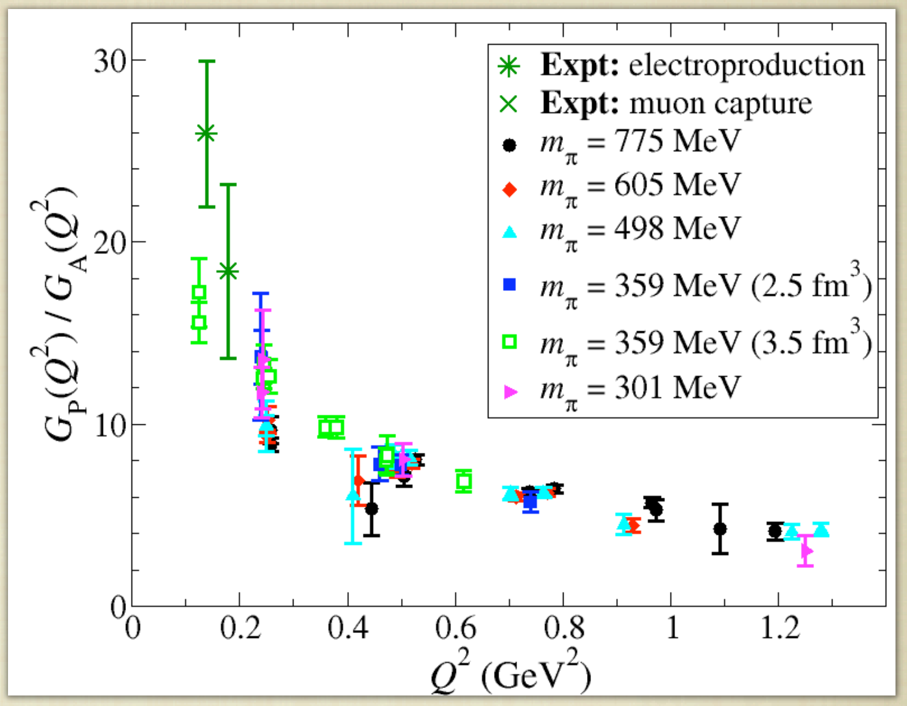
$\mu^-\, p \rightarrow \nu_\mu\, n$ ◆

Form factor ratio: G_A/F_1



Form factor ratio: G_P/G_A





soft pion pole: $G_P(q^2) \sim \dfrac{4M^2 G_A(q^2)}{q^2 - m_\pi^2}$

Generalized form factors

$$\mathcal{O}_q^{\{\mu_1\mu_2\dots\mu_n\}} = \overline{\psi}_q\gamma^{\{\mu_1}iD^{\mu_2}\dots iD^{\mu_n\}}\psi_q$$

$$\bar{P} = \tfrac{1}{2}(P' + P)$$

$$\Delta = P' - P$$

$$t = \Delta^2$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\mu_1}|P\rangle &= \langle\!\langle\gamma^{\mu_1}\rangle\!\rangle A_{10}(t) \\
&+ \frac{i}{2m}\langle\!\langle\sigma^{\mu_1\alpha}\rangle\!\rangle\Delta_\alpha B_{10}(t)\,,
\end{aligned}$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\}}|P\rangle &= \bar{P}^{\{\mu_1}\langle\!\langle\gamma^{\mu_2\}}\rangle\!\rangle A_{20}(t) \\
&+ \frac{i}{2m}\bar{P}^{\{\mu_1}\langle\!\langle\sigma^{\mu_2\}\alpha}\rangle\!\rangle\Delta_\alpha B_{20}(t) \\
&+ \frac{1}{m}\Delta^{\{\mu_1}\Delta^{\mu_2\}}C_2(t)\,,
\end{aligned}$$

$$\begin{aligned}
\langle P'|\mathcal{O}^{\{\mu_1\mu_2\mu_3\}}|P\rangle &= \bar{P}^{\{\mu_1}\bar{P}^{\mu_2}\langle\!\langle\gamma^{\mu_3\}}\rangle\!\rangle A_{30}(t) \\
&+ \frac{i}{2m}\bar{P}^{\{\mu_1}\bar{P}^{\mu_2}\langle\!\langle\sigma^{\mu_3\}\alpha}\rangle\!\rangle\Delta_\alpha B_{30}(t) \\
&+ \Delta^{\{\mu_1}\Delta^{\mu_2}\langle\!\langle\gamma^{\mu_3\}}\rangle\!\rangle A_{32}(t) \\
&+ \frac{i}{2m}\Delta^{\{\mu_1}\Delta^{\mu_2}\langle\!\langle\sigma^{\mu_3\}\alpha}\rangle\!\rangle\Delta_\alpha B_{32}(t),
\end{aligned}$$

Limits of generalized form factors

- Moments of parton distributions t →0

$$A_{n0} = \int dx x^{n-1} q(x)$$

- Electromagnetic form factors

$$A_{10} = F_1(t), \quad B_{10} = F_2(t)$$

- Total quark angular momentum

$$J_q = \frac{1}{2}[A(0)_{20} + B(0)_{20}]$$

Sum Rules

- Momentum sum rule

$$1 = A_{20,q}(0) + A_{20,g}(0) = \langle x \rangle_q + \langle x \rangle_g$$

- Nucleon spin sum rule

$$\begin{aligned} \frac{1}{2} &= \frac{1}{2}\big(A_{20,q}(0) + A_{20,g}(0) + B_{20,q}(0) + B_{20,g}(0)\big) \\ &= \frac{1}{2}\Delta\Sigma_q + L_q + J_g \end{aligned}$$

- Vanishing of anomalous gravitomagnetic moment

$$0 = B_{20,q}(0) + B_{20,g}(0)$$

Transverse structure of nucleon

Transverse structure of nucleon

$H(x, 0, -\Delta^2_\perp)$ is transverse Fourier transform of light cone quark distribution $q(x, r_\perp)$ at momentum fraction x

Transverse structure of nucleon

H(x, 0, - $\Delta^2_\perp$) is transverse Fourier transform of light cone quark distribution q(x,$r_\perp$) at momentum fraction x

$$q(x, r_\perp) \quad = \quad \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta^2_\perp)\, e^{-ir_\perp\Delta_\perp}$$

$$\int dx x^{n-1} q(x, r_\perp) \quad = \quad \int \frac{d^2\Delta_\perp}{(2\pi)^2} A(-\Delta^2_\perp)\, e^{-ir_\perp\Delta_\perp}$$

Transverse structure of nucleon

H(x, 0, - $\Delta^2_\perp$) is transverse Fourier transform of light cone quark distribution q(x,$r_\perp$) at momentum fraction x

$$
\begin{aligned}
q(x, r_\perp) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta_\perp^2)\, e^{-ir_\perp \Delta_\perp} \\
\int dx x^{n-1} q(x, r_\perp) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} A(-\Delta_\perp^2)\, e^{-ir_\perp \Delta_\perp}
\end{aligned}
$$

- x → 1: Single Fock space component ⇒ slope → 0

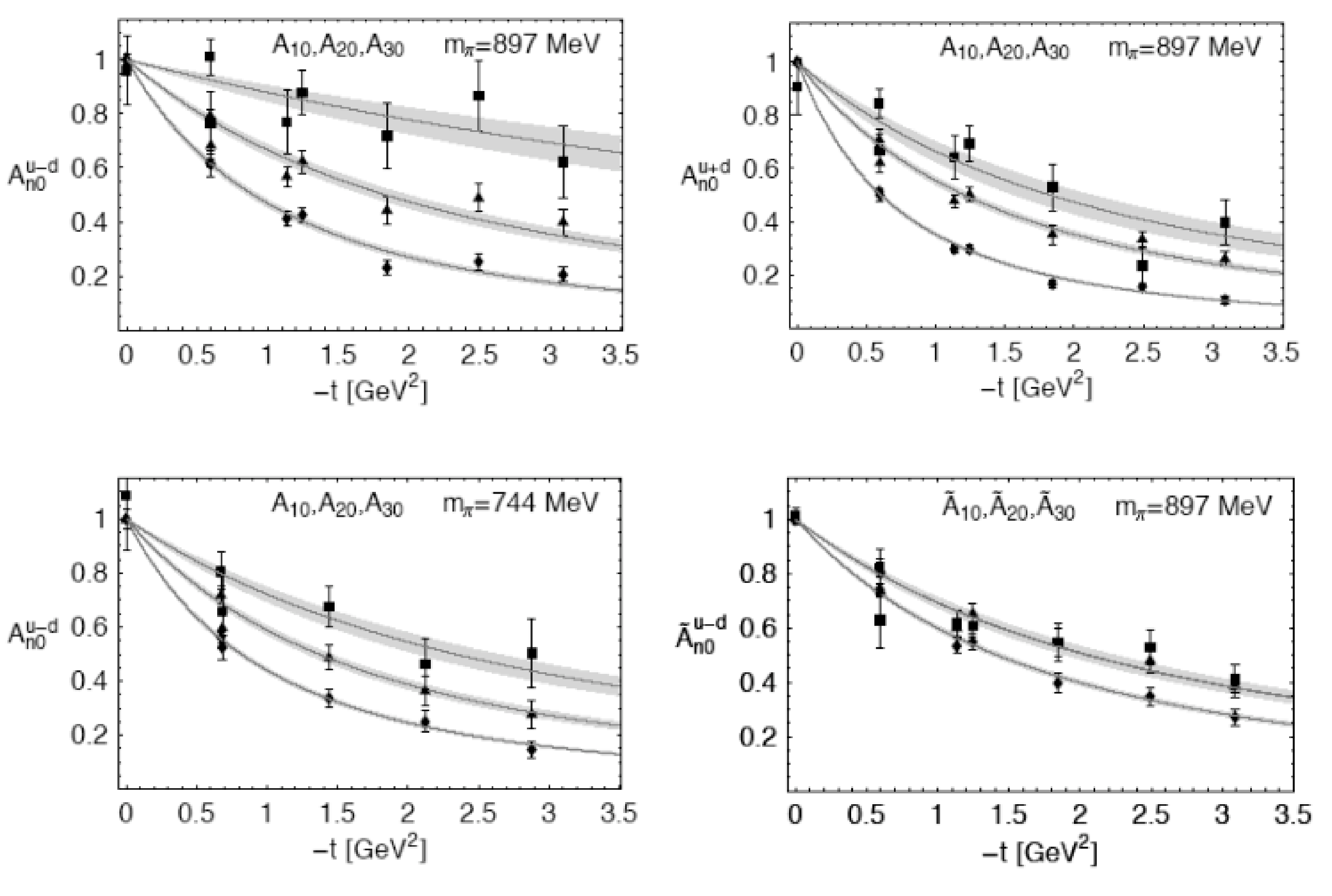
Transverse structure of nucleon

$H(x, 0, -\Delta^2_\perp)$ is transverse Fourier transform of light cone quark distribution $q(x,r_\perp)$ at momentum fraction x

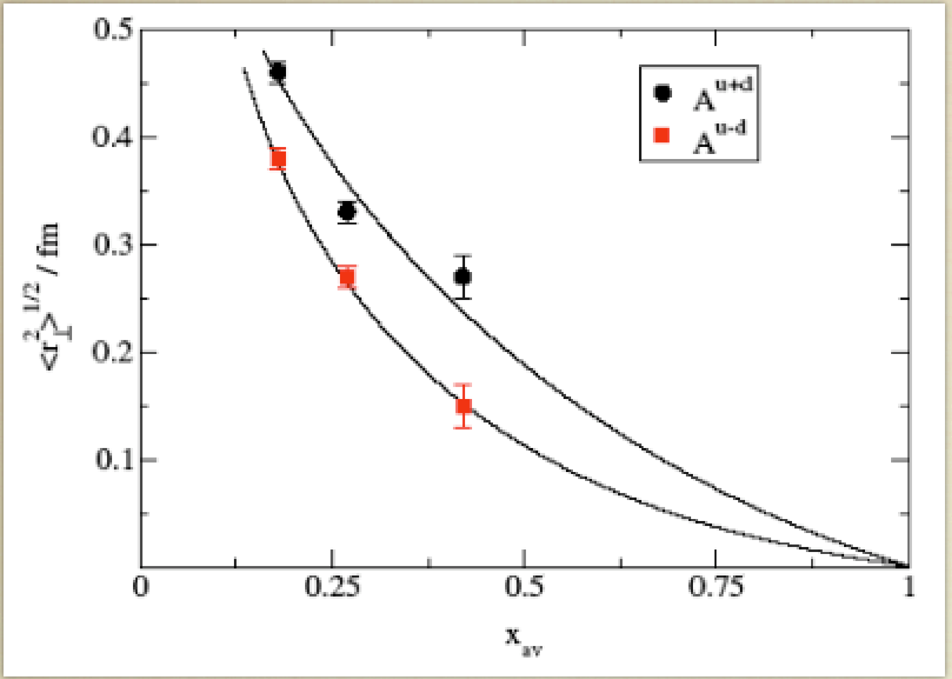
$$\begin{aligned} q(x,r_\perp) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x,0,-\Delta^2_\perp)\, e^{-ir_\perp\Delta_\perp} \\ \int dx x^{n-1} q(x,r_\perp) &= \int \frac{d^2\Delta_\perp}{(2\pi)^2} A(-\Delta^2_\perp)\, e^{-ir_\perp\Delta_\perp} \end{aligned}$$

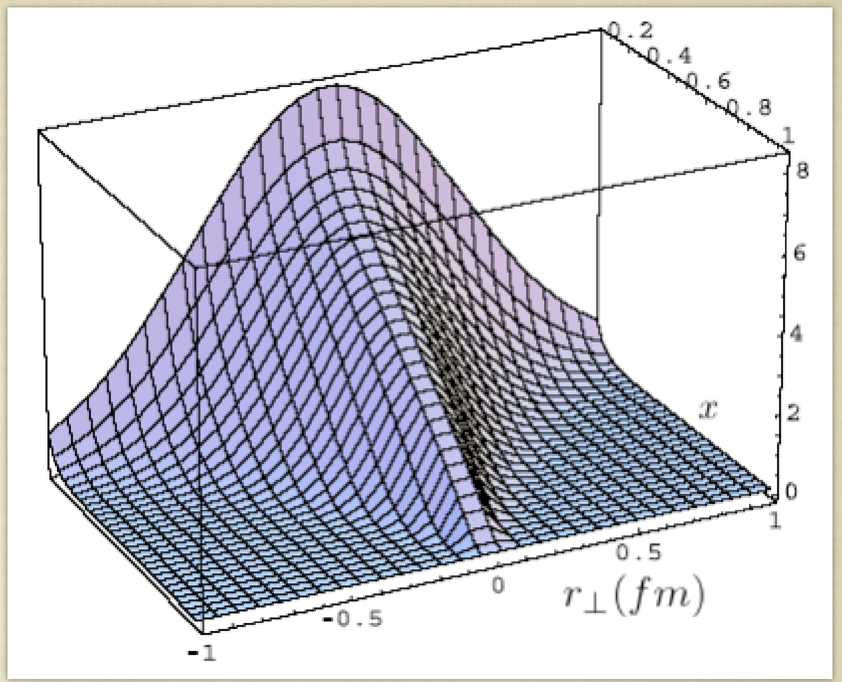
- $x \to 1$: Single Fock space component $\Rightarrow$ slope $\to 0$
- $x \neq 1$: Transverse structure $\Rightarrow$ slope steeper

Generalized form factors from lattice



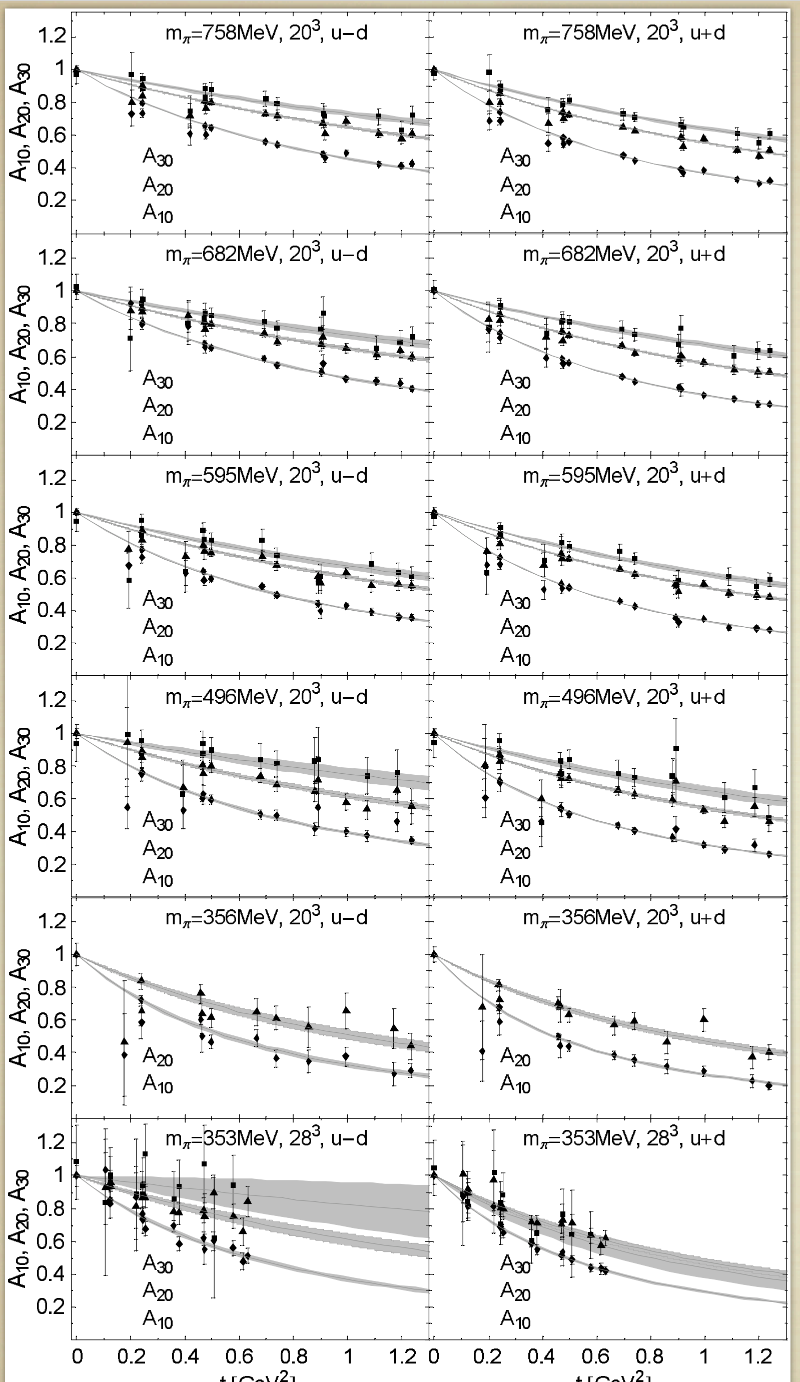
Transverse size of light-cone wave function





$$x_{\rm av}^n = \frac{\int d^2 r_\perp \int dx\, x \cdot x^{n-1} q(x, \vec{r}_\perp)}{\int d^2 r_\perp \int dx x^{n-1} q(x, \vec{r}_\perp)}$$

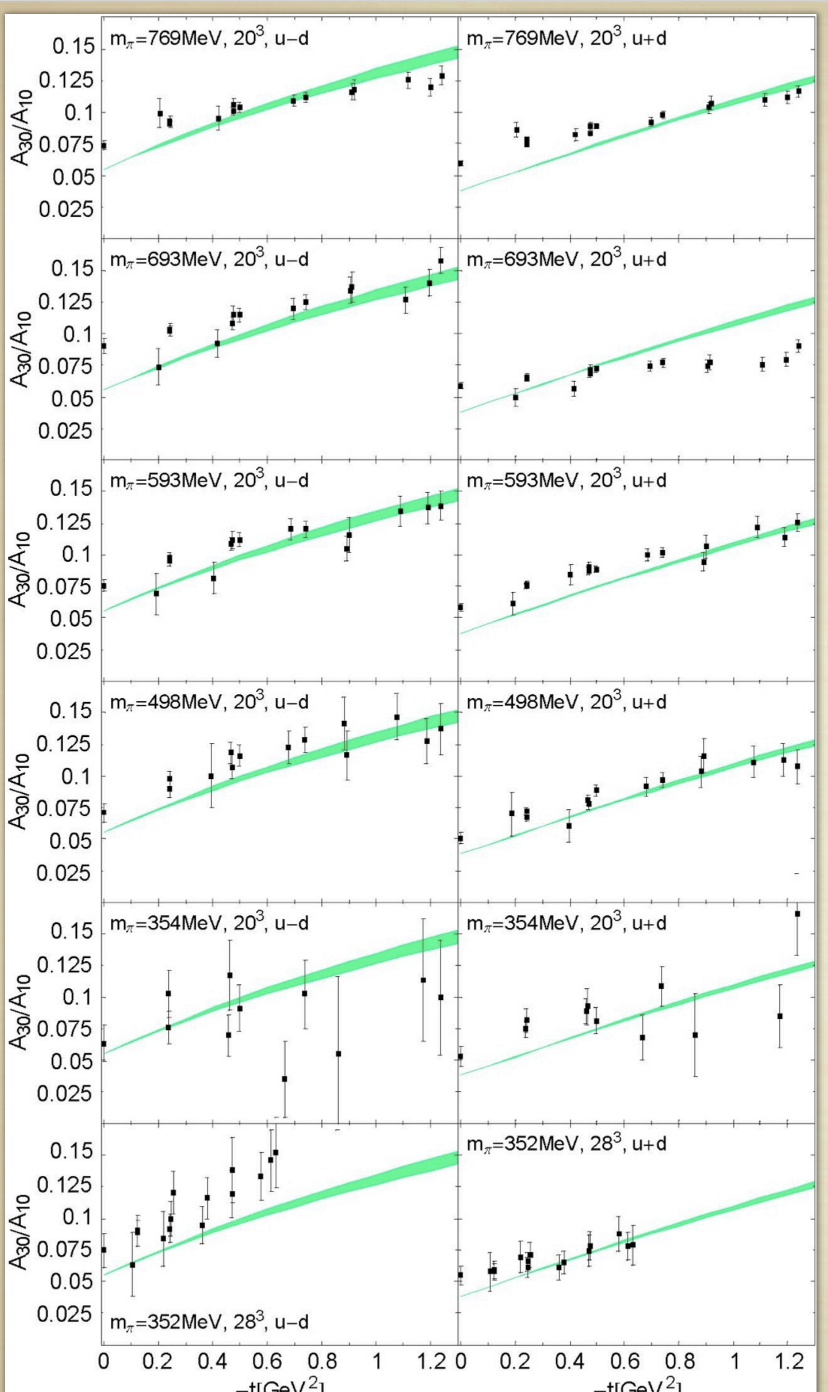
$q(x, \vec{r}_\perp)$ **model** (Burkardt hep-ph/0207047)



Generalized form factors

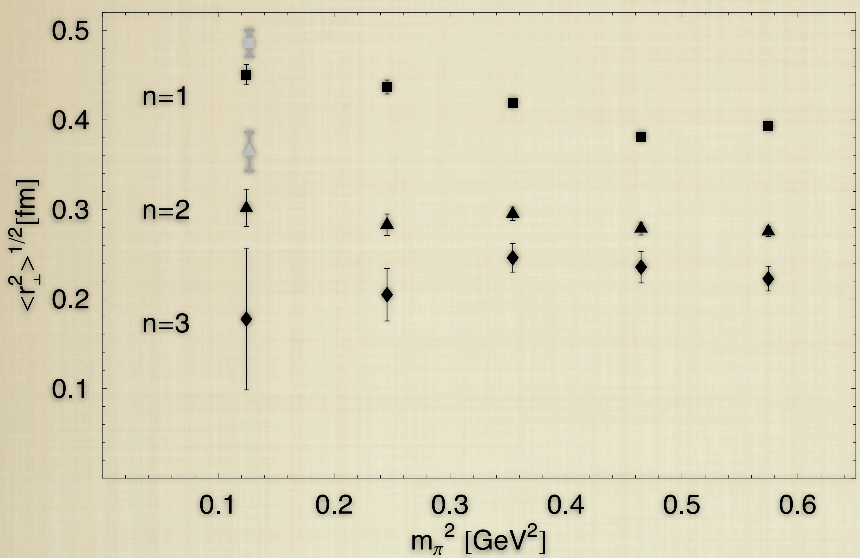
A_{10}, A_{20}, A_{30}

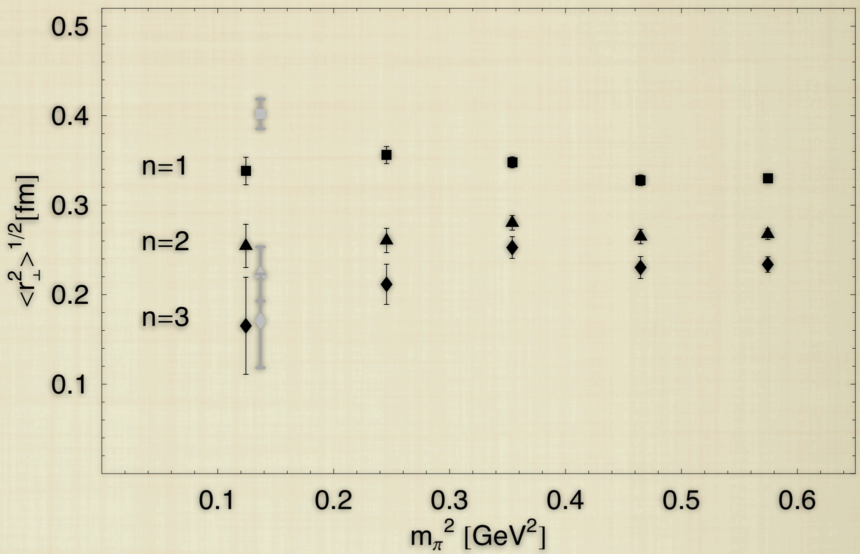
Generalized form factor ratios A_{30} / A_{10}



GPD parameterization:
Nucleon form factors,
CTEQ parton distributions,
Regge behavior,
Ansatz
Diehl, Feldmann, Jakob, Kroll EPJC 2005

2-d rms Radii for $A_{n0}, \tilde{A}_{n0}$

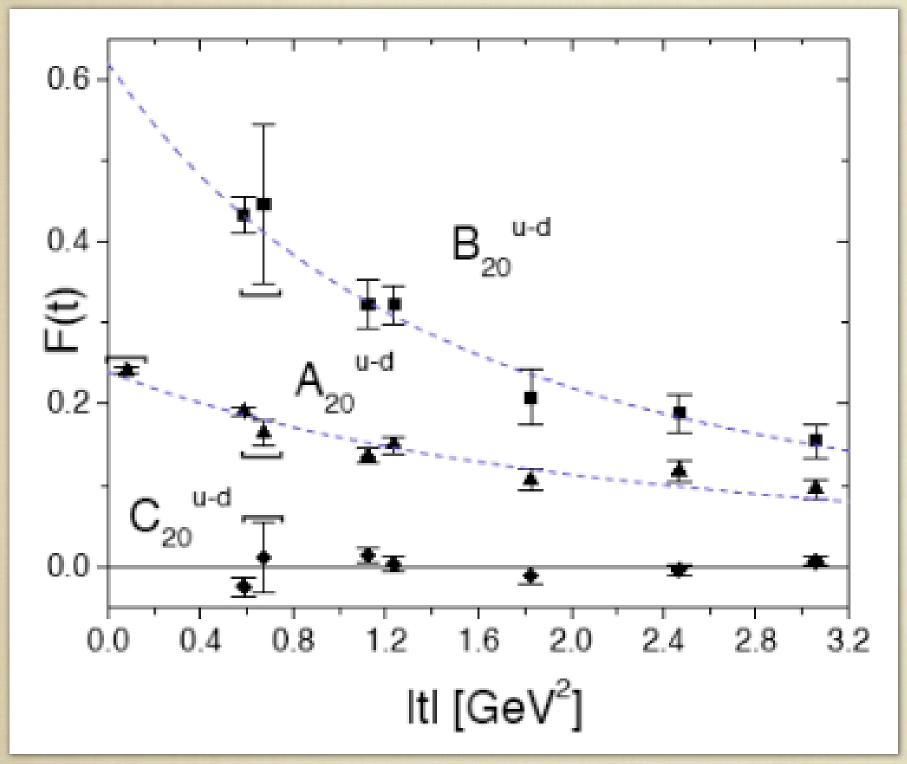


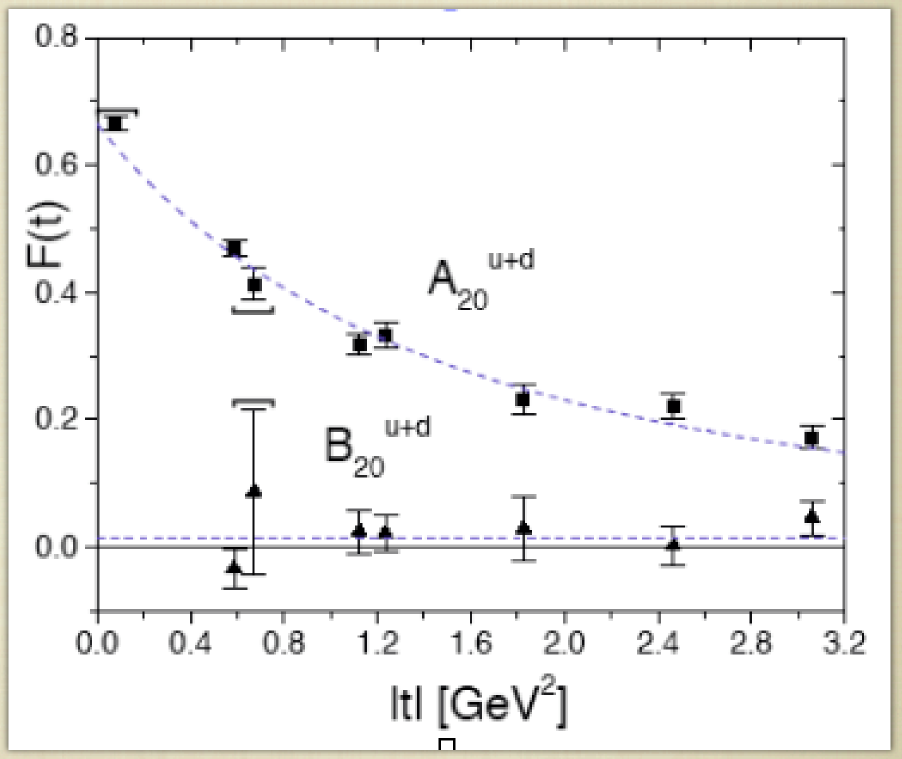


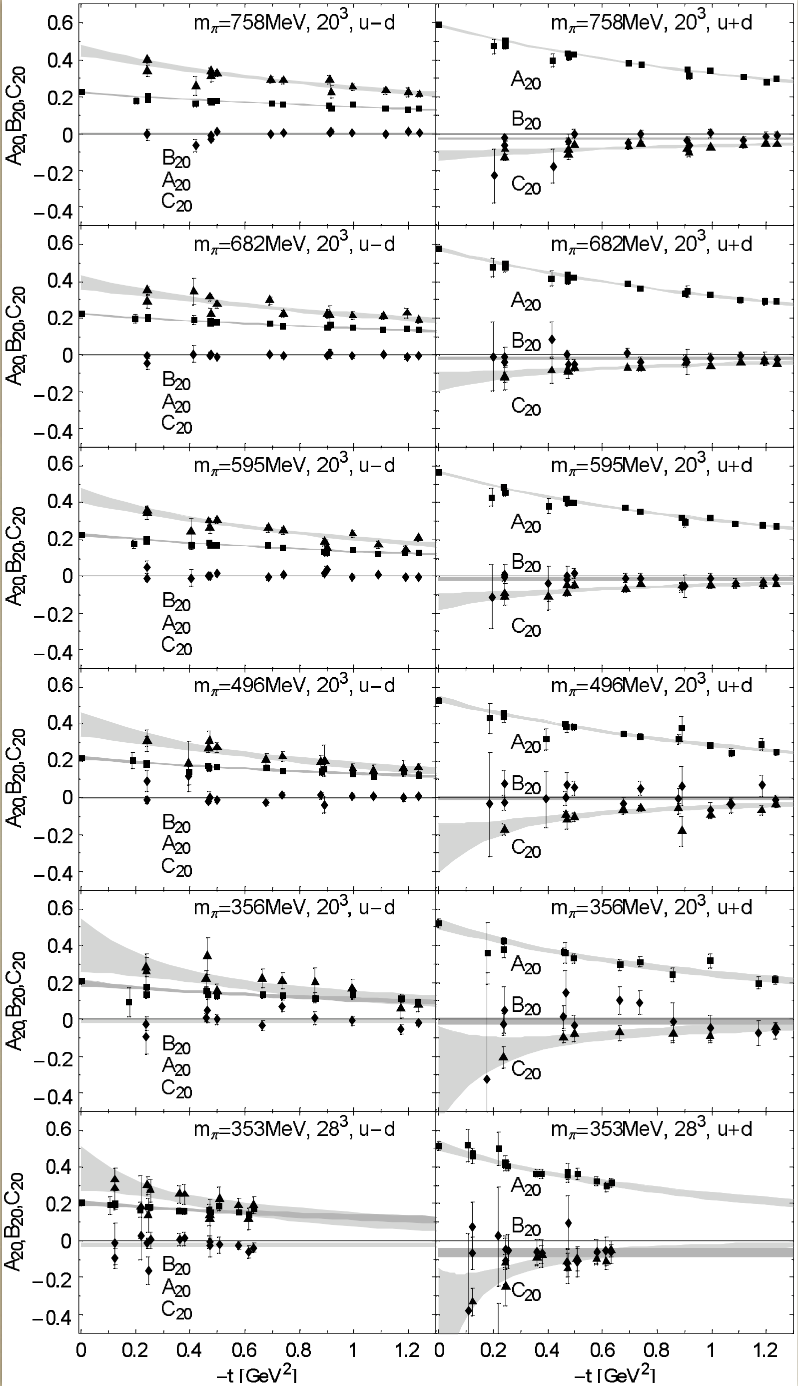
First x moments: A_{20}, B_{20}, C_{20}

m_π = 897 MeV

LHPC hep-lat/0304018







First x moments:

$$A_{20}, B_{20}, C_{20}$$

Consistent with large N behavior [Goeke et. al.]

$$\begin{aligned} |A_{20}^{u+d}| &> |A_{20}^{u-d}| \\ |B_{20}^{u-d}| &> |B_{20}^{u+d}| \\ |C_{20}^{u+d}| &> |C_{20}^{u-d}| \end{aligned}$$

Origin of nucleon spin

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"Spin crisis" - only ~ 30% arises from quark spins

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quark spin contribution $\frac{1}{2}\Delta\Sigma = \frac{1}{2}\langle 1 \rangle_{\Delta u + \Delta d}$

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quark spin contribution $\frac{1}{2}\Delta\Sigma = \frac{1}{2}\langle 1 \rangle_{\Delta u + \Delta d}$

total quark contribution (spin plus orbital)

$$J_q = \frac{1}{2}[A_{20}^{u+d}(0) + B_{20}^{u+d}(0)] = \frac{1}{2}[\langle x \rangle_{u+d} + B_{20}^{u+d}(0)]$$

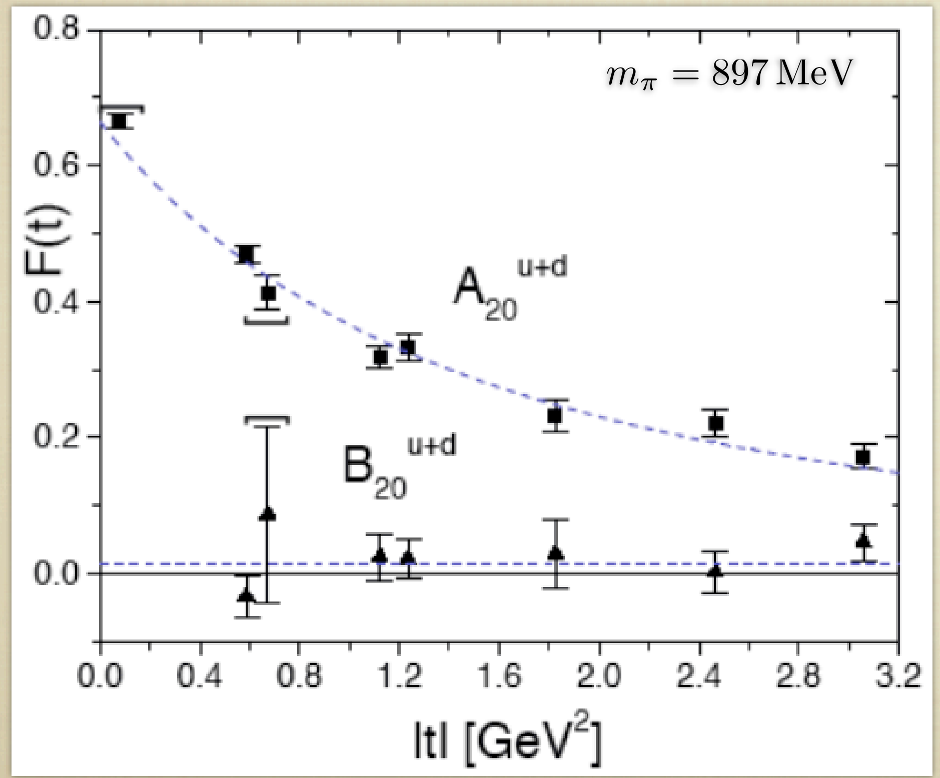
Origin of nucleon spin

"Spin crisis" - only ~ 30% arises from quark spins

quark spin contribution $\frac{1}{2}\Delta\Sigma = \frac{1}{2}\langle 1\rangle_{\Delta u+\Delta d} \quad \sim \frac{1}{2}0.682(18)$

total quark contribution (spin plus orbital)

$$J_q = \frac{1}{2}[A_{20}^{u+d}(0) + B_{20}^{u+d}(0)] = \frac{1}{2}[\langle x\rangle_{u+d} + B_{20}^{u+d}(0)] \quad \sim \frac{1}{2}0.675(7)$$



Origin of nucleon spin

"Spin crisis" - only ~ 30% arises from quark spins

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total quark contribution (spin plus orbital)

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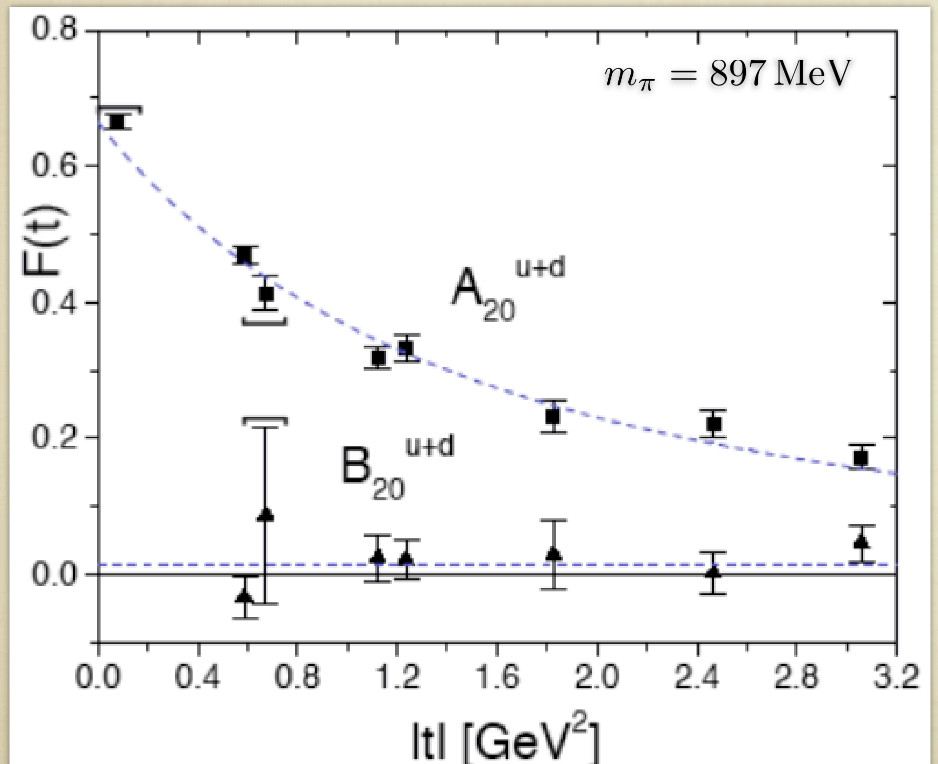


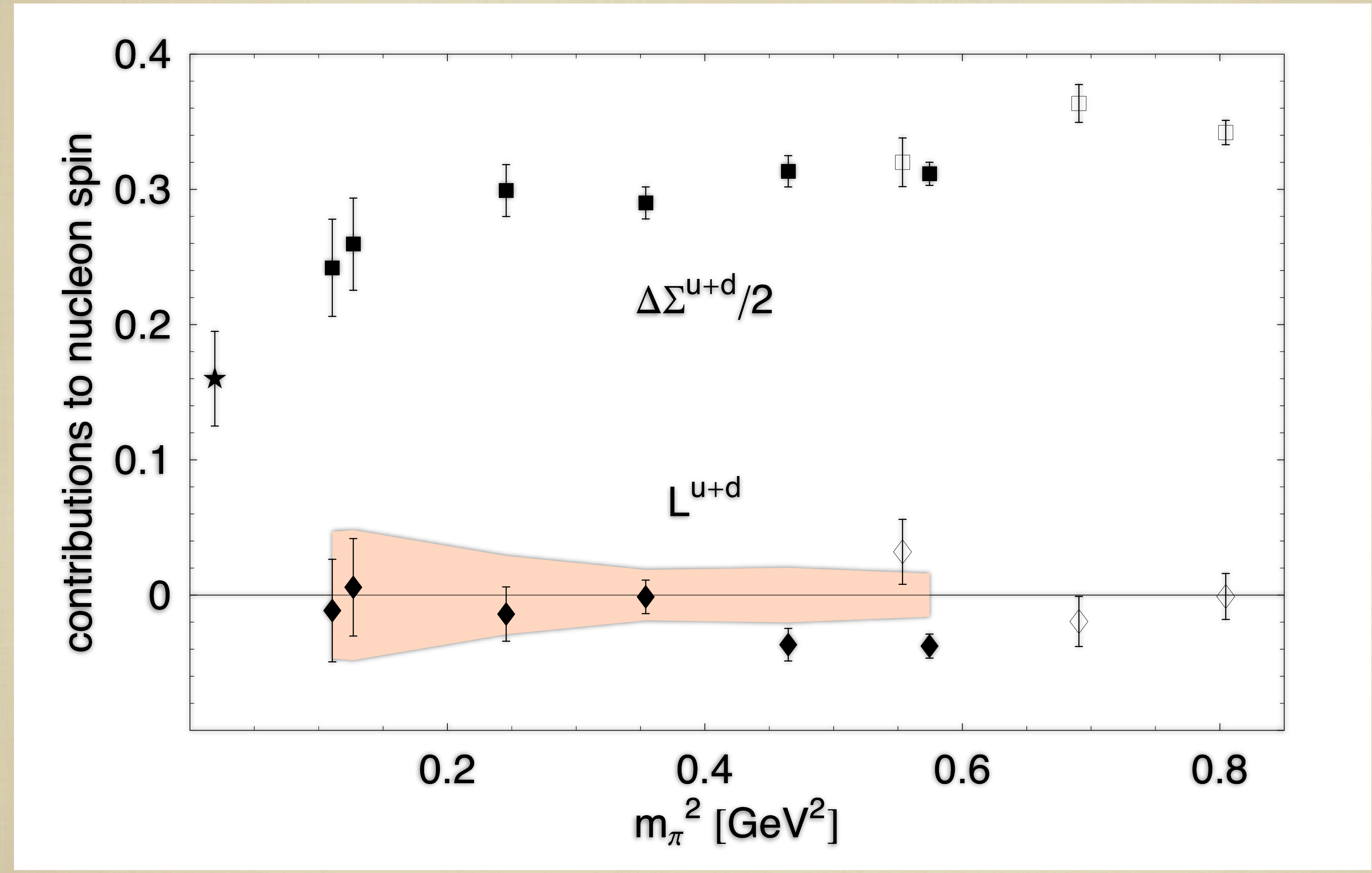
Spin Inventory

68% quark spin

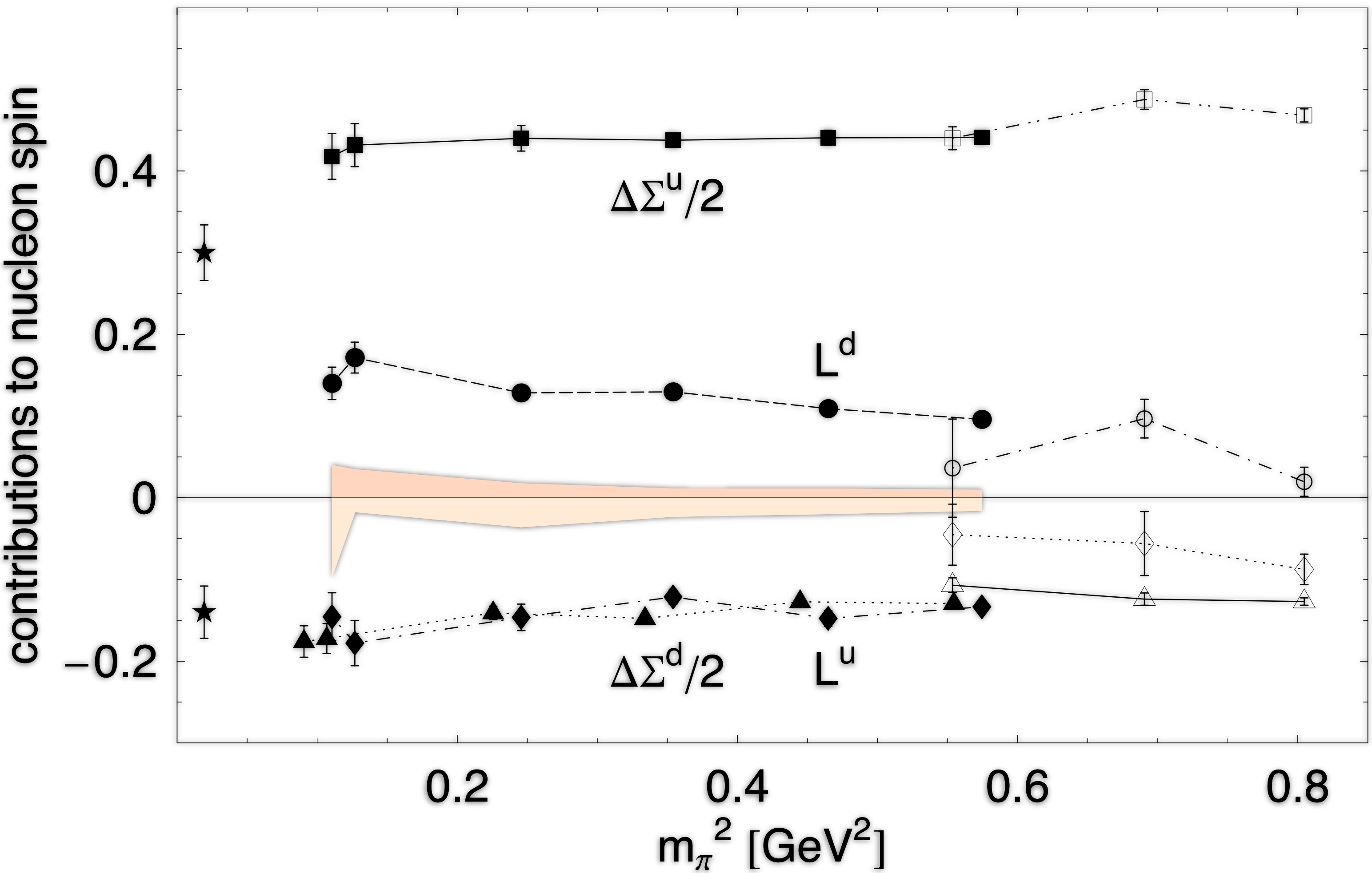
0% quark orbital

32% gluons

Nucleon spin decomposition



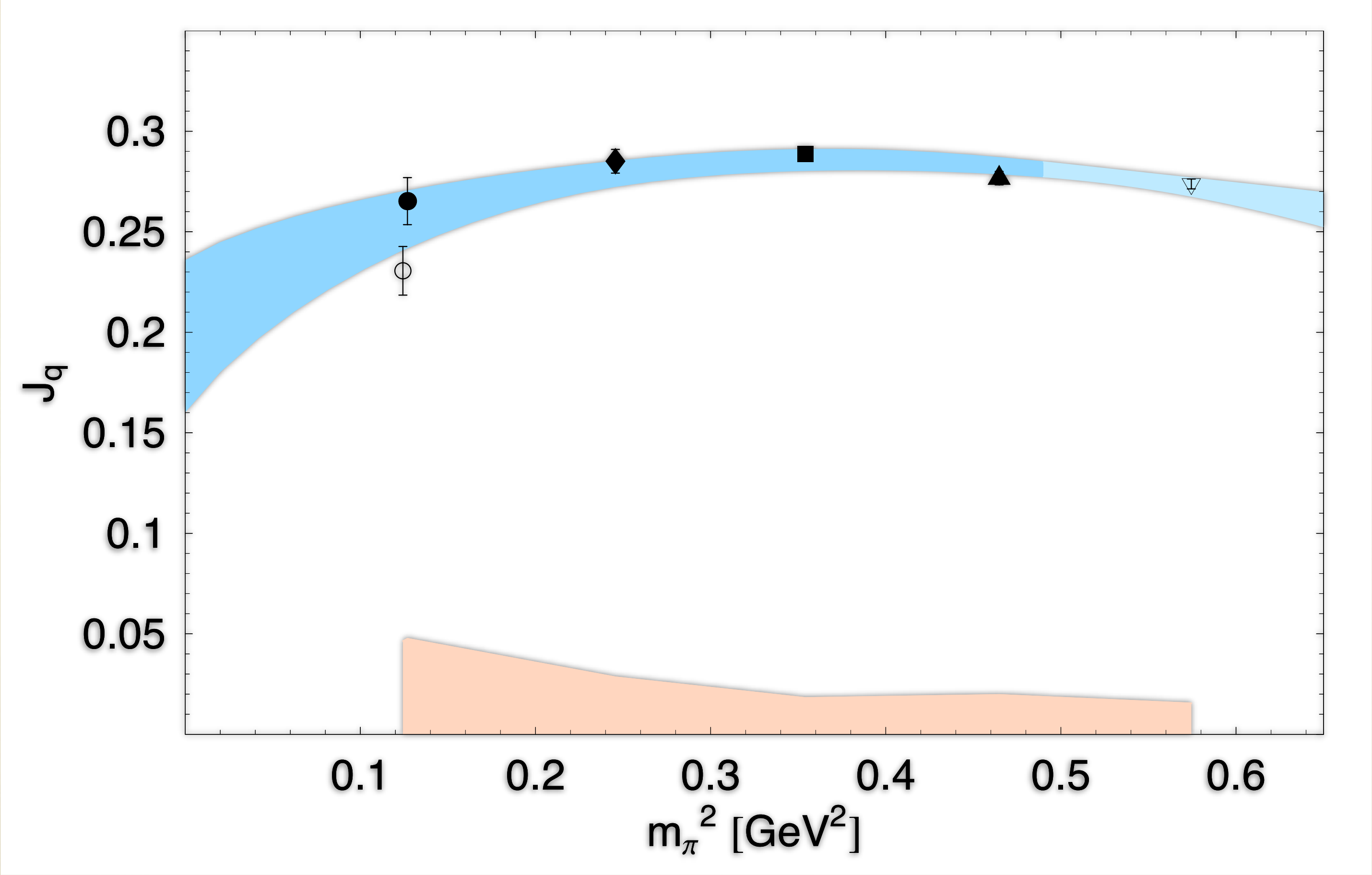
Nucleon spin decomposition



Chiral extrapolation of $J_q = \frac{1}{2}\left(A_{20}^{u+d}(0) + B_{20}^{u+d}(0)\right)$

ChPT including Delta (Chen and Ji)

$$J_q(m_\pi;\Delta) = J_q(m_\pi) - \frac{1}{2}\left(\frac{9}{2}b_{qN} + 3a_{q\pi} - \frac{15}{2}b_{q\Delta}\right)\frac{8g_{\pi N\Delta}^2}{9(4\pi f_\pi)^2}(m_\pi^2 - 2\Delta^2)\ln\left(\frac{m_\pi^2}{\Lambda_\chi^2}\right) + 2\Delta\sqrt{\Delta^2 - m_\pi^2}\ln\left(\frac{\Delta - \sqrt{\Delta^2 - m_\pi^2}}{\Delta + \sqrt{\Delta^2 - m_\pi^2}}\right)$$

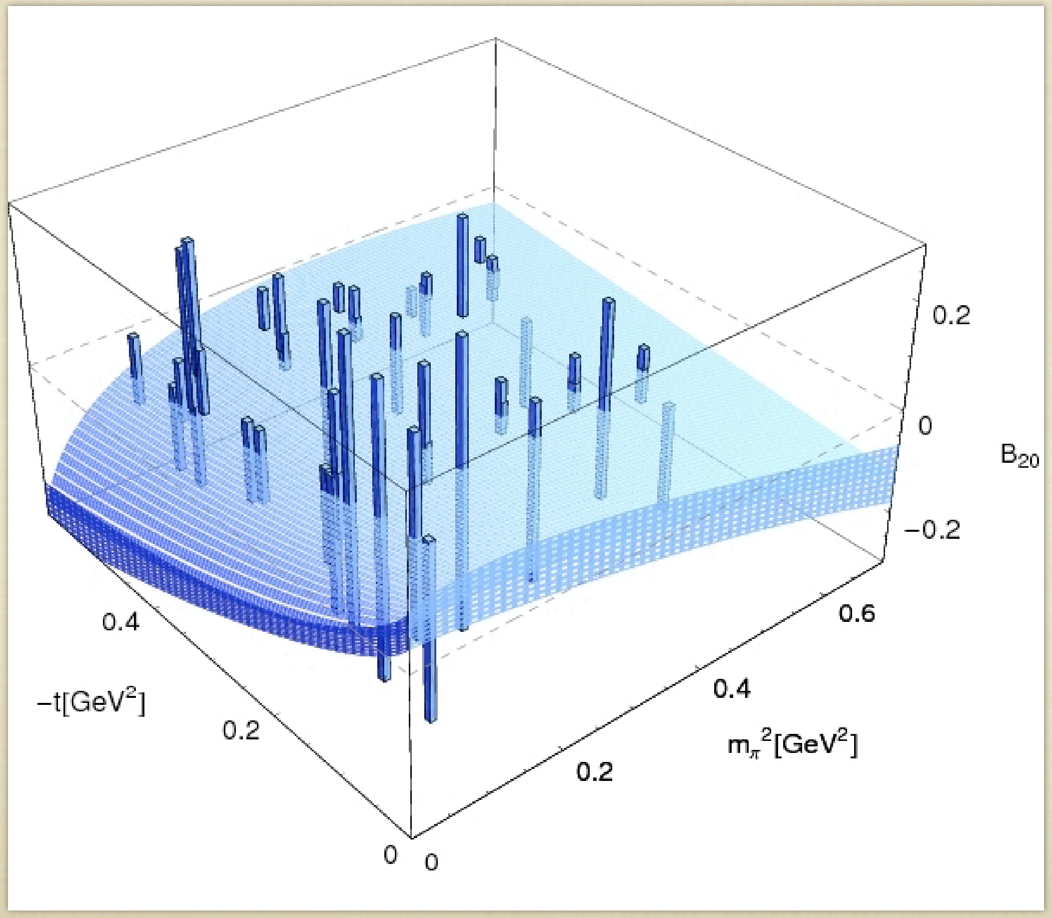


Chiral Extrapolation of $B_{20}^{u+d}(t, m_\pi)$

Chiral extrapolation O(p[4]) relativistic ChPT O(p[5]) corrections

Note: connected diagrams only (Dorati, Hemmert, et. al.)

$$B_{20}^{u-d}(t, m_\pi) = \frac{m_N(m_\pi)}{m_N}\left\{B_{20}^{0,u-d} + A_{20}^{0,u-d} g_B(t, m_\pi) + \delta_B^t\, t + \delta_B^{m_\pi}\, m_\pi^2\right\}$$

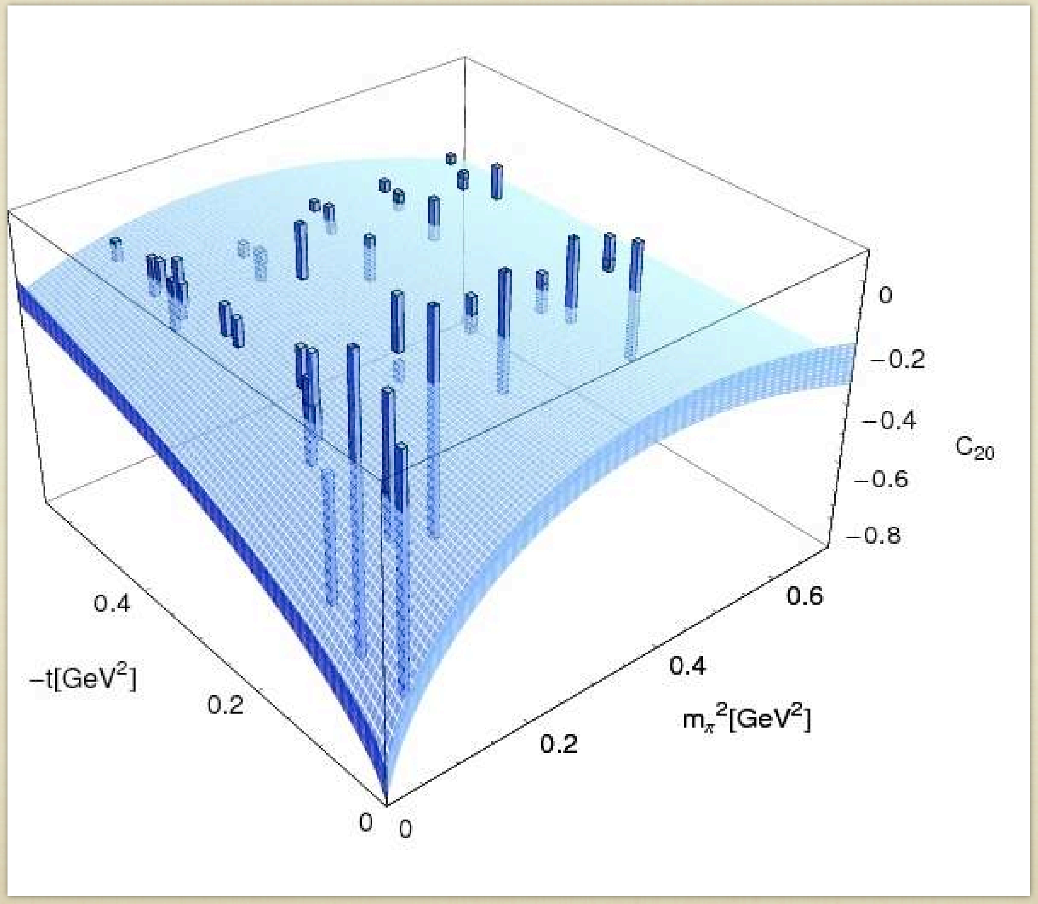


Chiral Extrapolation of $C_{20}^{u+d}(t, m_\pi)$

Chiral extrapolation O(p^4) relativistic ChPT O(p^5) corrections

Note: connected diagrams only (Dorati, Hemmert, et. al.)

$$C_{20}^{u-d}(t, m_\pi) = \frac{m_N(m_\pi)}{m_N} \left\{ C_{20}^{0,u-d} + A_{20}^{0,u-d} g_C(t, m_\pi) + \delta_C^t \, t + \delta_C^{m_\pi} \, m_\pi^2 \right\}$$



Summary

- Entering era of quantitative solution in chiral regime
  - Moments of quark distributions
  - Form factors: F_1 , F_2 , G_A , G_P
  - Generalized form factors A B C
  - Transverse structure
  - Origin of nucleon spin
- Opportunity for theory and experiment to work in consort
  - Validate by agreement with key experiments
  - GPD's: Expt. convolution, Theory moments, combine
  - Resolve experimental discrepancies
    - F_2 : 2-γ contributions to Rosenbluth, pol. transfer
    - G_A : ν vs π-electroproduction

Current effort and future challenges

- Full QCD with chiral fermions in chiral regime
  - DW proposals for configurations and hadron structure next week
- Disconnected diagrams
  - Strangeness content of nucleon
  - Flavor singlet matrix elements
- Gluon observables
- Role of diquarks in hadrons
- Unstable states
- Exotic mesons and baryons
- Hadron-hadron phase shifts, adiabatic potentials