Hadron Scattering from LQCD

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Nuclear physics

- Connect Nuclear physics to QCD
- Yet a smaller scale
- Nuclear binding energy ~ MeV
- Does it look hopeless?
- Not really!

Nuclear Physics Landscape

Mean Field Models

Density Functional

Shell Model(s)

Effective Interactions

Microscopic

Ab Initio

χEFT, EFT($\not\pi$)

^{3}He ^{4}He

QCD

p

d

^{3}H

n

QCD Vacuum

Quark-Gluon Interaction

Proton Number

1

10

1

5

10

50

100

Neutron Number

chart adapted from G. Henning

figure by H. Greisshammer

NUCLEAR PHYSICS: WHAT CAN WE DO?

- EFT description of nuclear forces
- Need low energy constants
- Use experiment
- Why not use lattice instead?

Hadronic Interactions

- Effective field theory description of few nucleon systems
- Use lattice QCD to extract the low energy constants needed
  - Decay constants: f_π, f_K
  - Axial couplings: g_A, $g_{N\Delta}$, $g_{\Sigma\Sigma}$, $g_{\Xi\Xi}$, $g_{\Sigma\Lambda}$, ...
  - Scattering lengths: NPLQCD
- Lattice Nuclear physics [Lee et al., Borasoy et al.]
- Lattice offers flexibility!
  - Study quark mass dependence
  - Compute experimentally inaccessible quantities (Hyperons)

The hybrid action program

- Domain wall fermions for valence (with hyp smeared links)
  - Chiral symmetry ($O(a^2)$ errors better scaling)
  - Ward Identities (renormalization, power divergent mixing)
  - Match the pion mass to the staggered Goldstone pion mass

- Kogut-Susskind 2+1 Dynamical flavors
  - Improved KS action (Asqtad: $O(a^4, g^2 a^2)$) [KO, Sugar, Toussaint '99]
  - MILC has generated lattices

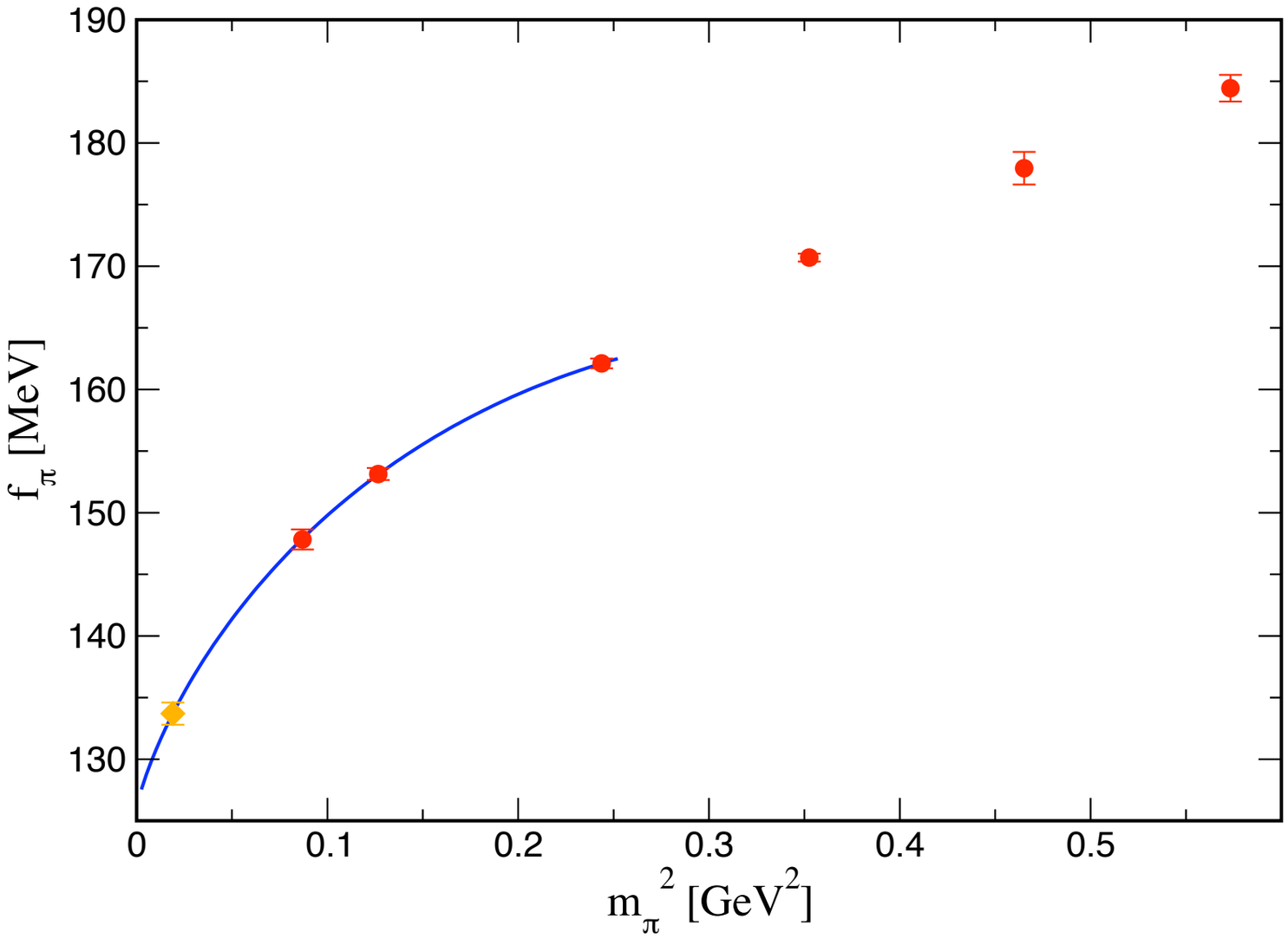
- Light quark masses: Lightest pion $m_\pi \sim 250$MeV

- Volumes: 2.6 to 3.2 fm

- Continuum extrapolation
  - MILC lattice spacings: a=0.125fm, 0.09fm
  - a=0.06fm in 1 - 2 years (..?)

- Problem: "Rooted" fermions? (Bernard, Golterman, Shamir, Sharpe, Durr, Creutz, Hassenfratz....)

Pion Decay constant



$$f_\pi = f\left[1 - \frac{m_\pi^2}{8\pi^2 f^2} log\left(\frac{m_\pi^2}{\mu^2}\right) + c(\mu^2) m_\pi^2\right]$$

- Fit the lower 3 points
- Extrapolation to physical point: 133.7(9)(3.0)MeV χ^2/d.o.f. ~ .5
- Mixed action χPT should reduce systematics (Baer et. al.)

F_K/F_π

Beane, Bedaque, KO, Savage hep-lat/0606023

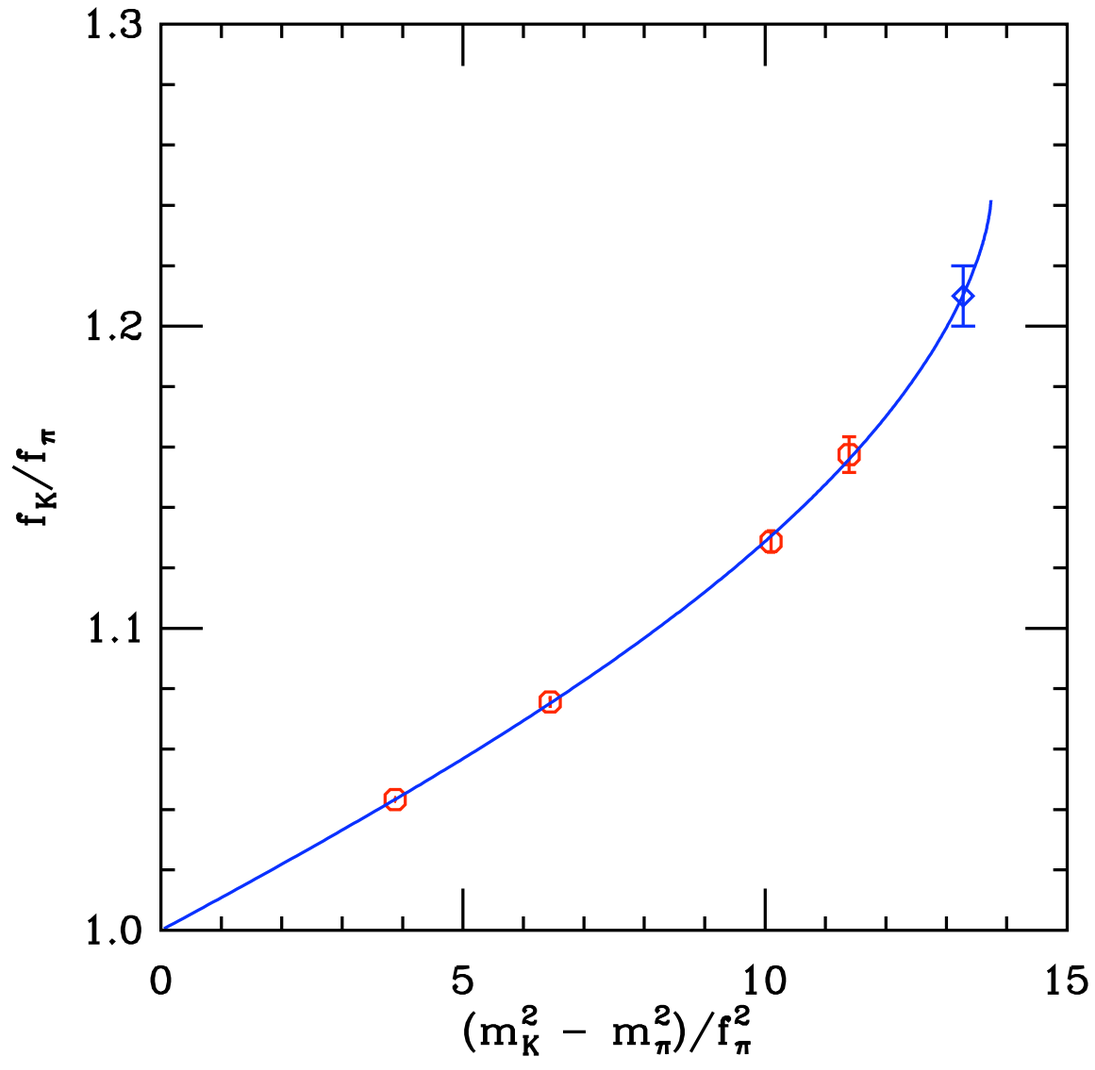
Gasser-Leutwyler:

$$\frac{f_K}{f_\pi} = 1 + \frac{5}{4} l_\pi(\mu) - \frac{1}{2} l_K(\mu) - \frac{3}{4} l_\eta(\mu) + \frac{8}{f^2}\left(m_K^2 - m_\pi^2\right) L_5(\mu)$$

$$l_i(\mu) = \frac{1}{16\pi^2}\frac{m_i^2}{f^2}\log\left(\frac{m_i^2}{\mu^2}\right)$$

$$f_K/f_\pi = 1.218 \pm 0.002\ {}^{+0.011}_{-0.024}$$

$$\left.\frac{f_K}{f_\pi}\right|_{\text{exp.}} = 1.223(12)$$



Result comparable with MILC

$$\left.\frac{f_K}{f_\pi}\right|_{\text{MILC}} = 1.210(4)(13)$$

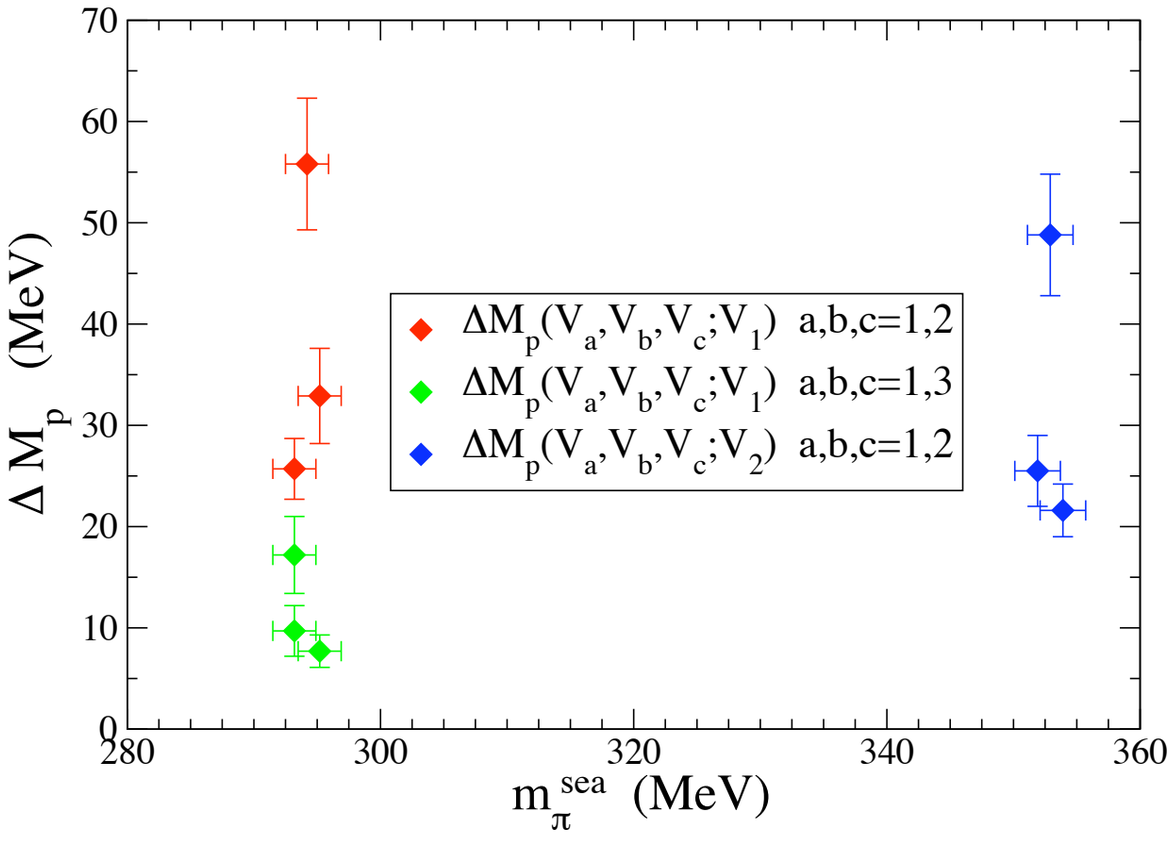
Need much higher precision to see effects of Mixed χPT Baer et.al.'05

Neutron-Proton Mass difference

Beane, KO, Savage hep-lat/0605015

$$M_n - M_p|^{d-u} = \frac{2}{3}\left(2\overline{\alpha} - \overline{\beta}\right)\left(\frac{1-\eta}{1+\eta}\right) m_\pi^2$$

MILC: $\eta = m_u/m_d = 0.43(1)(8)$



Extraction	$M_n - M_p\|^{d-u}$ (MeV) at $m_\pi^{\text{phys.}}$
LO $\mathcal{O}(m_q)$	$1.96 \pm 0.92 \pm 0.37$
NLO $\mathcal{O}(m_q^{3/2})$	$2.26 \pm 0.57 \pm 0.42$

Extraction	$\frac{1}{3}\left(2\overline{\alpha} - \overline{\beta}\right)$ (l.u.)	$\overline{\alpha} + \overline{\beta}$ (l.u.)	g_1	$\lvert g_{\Delta N}\rvert$	χ^2/dof
LO $\mathcal{O}(m_q)$	0.198 ± 0.093	2.07 ± 0.08	$--$	$--$	0.56
NLO $\mathcal{O}(m_q^{3/2})$	0.229 ± 0.058	3.4 ± 1.1	-0.10 ± 0.35	0.60 ± 0.66	0.21

Exp. value: $M_n - M_p = 1.2933317(5)$ MeV

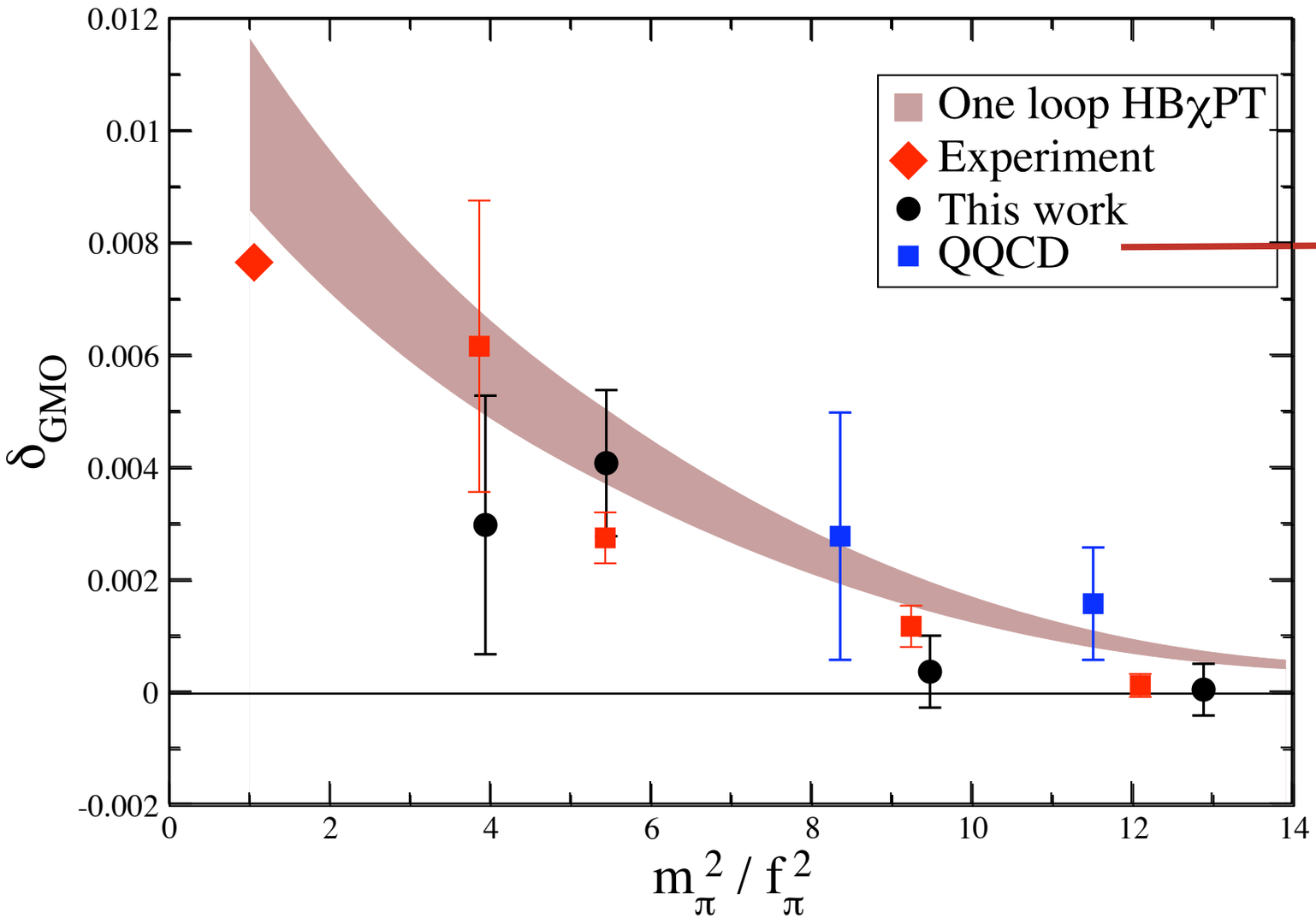
minus EM part

Gasser Leutwyler '82

$M_n - M_p = 2.05(30)$ MeV

GMO RELATION

Beane, KO, Savage hep-lat/0604013



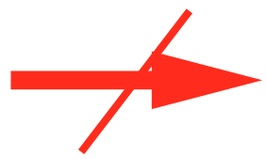
Bhattacharya *et al*. hep-lat/9512021

$$\delta_{\rm GMO} = \frac{M_\Lambda + \frac{1}{3}M_\Sigma - \frac{2}{3}M_N - \frac{2}{3}M_\Xi}{\frac{1}{8}M_\Lambda + \frac{3}{8}M_\Sigma + \frac{1}{4}M_N + \frac{1}{4}M_\Xi}$$

$$G^{\rm GMO}(t) = \frac{C_\Lambda(t) \quad C_\Sigma(t)^{1/3}}{C_N(t)^{2/3} \quad C_\Xi(t)^{2/3}} \quad \rightarrow \quad e^{-(M_\Lambda + M_\Sigma/3 - 2M_N/3 - 2M_\Xi/3)t}$$

Hadronic Interactions

- Scattering processes from Lattice QCD are not straight forward
- Miani-Testa no-go theorem (‘90) [and C. Michael ‘89]
- Infinite Volume:

  Euclidean  Minkowski

- Finite volume: discrete spectrum
  - Avoids Miani-Testa no-go theorem [M. Luscher]

Luscher Formula

Energy level shift in finite volume:

$$\Delta E_n \equiv E_n - 2m = 2\sqrt{p_n^2 + m^2} - 2m$$

p_n solutions of:

$$p\cot\delta(p) = \frac{1}{\pi L}\mathbf{S}\left(\frac{p^2L^2}{4\pi^2}\right) \qquad \mathbf{S}(\eta) \equiv \sum_{\mathbf{j}}^{|\mathbf{j}|<\Lambda} \frac{1}{|\mathbf{j}|^2-\eta} - 4\pi\Lambda$$

$$p_n\cot\delta(p_n) = \frac{1}{a} + \cdots \qquad \frac{1}{a} = \frac{1}{\pi L}S\left(\frac{p_0^2L^2}{4\pi^2}\right) + \cdots$$

Expansion at p~0 :

$$\Delta E_0 = -\frac{4\pi a}{mL^3}\left[1 + c_1\frac{a}{L} + c_2\left(\frac{a}{L}\right)^2\right] + \mathcal{O}\left(\frac{1}{L^6}\right)$$

a is the scattering length

c_1 and c_2 are universal constants

PION I=2 SCATTERING LENGTH

S. Bean P. Bedaque KO and M. Savage hep-lat/0506013

$$C_{\pi^+}(t) \;=\; \sum_{\mathbf{x}} \langle \pi^-(t,\mathbf{x})\; \pi^+(0,\mathbf{0}) \rangle$$

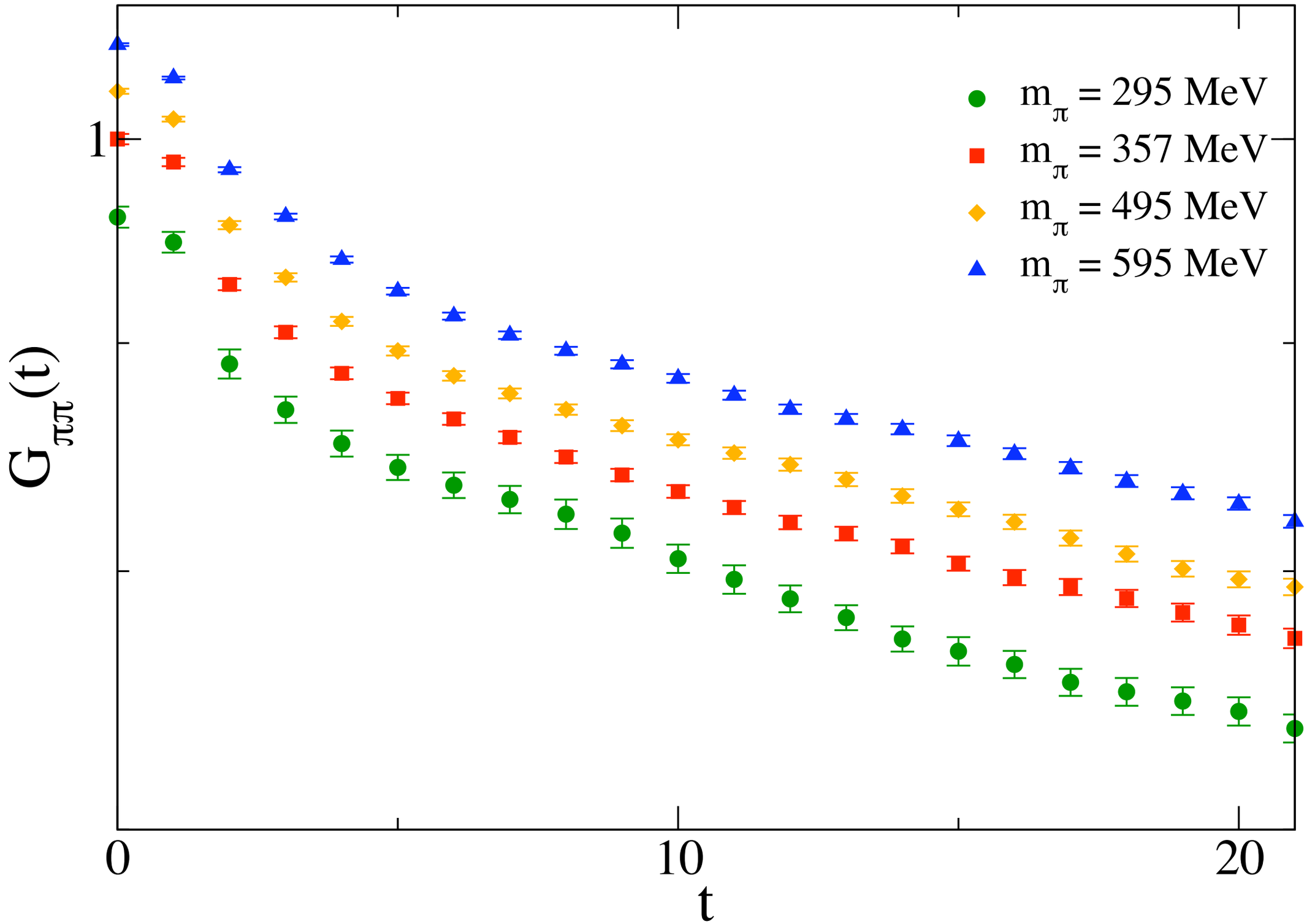
$$C_{\pi^+\pi^+}(p,t) \;=\; \sum_{|\mathbf{p}|=p} \sum_{\mathbf{x},\mathbf{y}} e^{i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} \langle \pi^-(t,\mathbf{x})\; \pi^-(t,\mathbf{y})\; \pi^+(0,\mathbf{0})\; \pi^+(0,\mathbf{0}) \rangle$$

$$G_{\pi\pi}(p,t) \;\equiv\; \frac{C_{\pi\pi}(p,t)}{C_\pi(t)^2} \;\rightarrow\; \sum_{n=0}^{\infty} \mathcal{A}_n\; e^{-\Delta E_n\; t}$$

Quenched
Sharpe etal '92
Gupta etal '93
Kuramashi etal '93
Fugugita etal '94
C. Liu etal '02
J. Junk RBG '03
CP-PACS

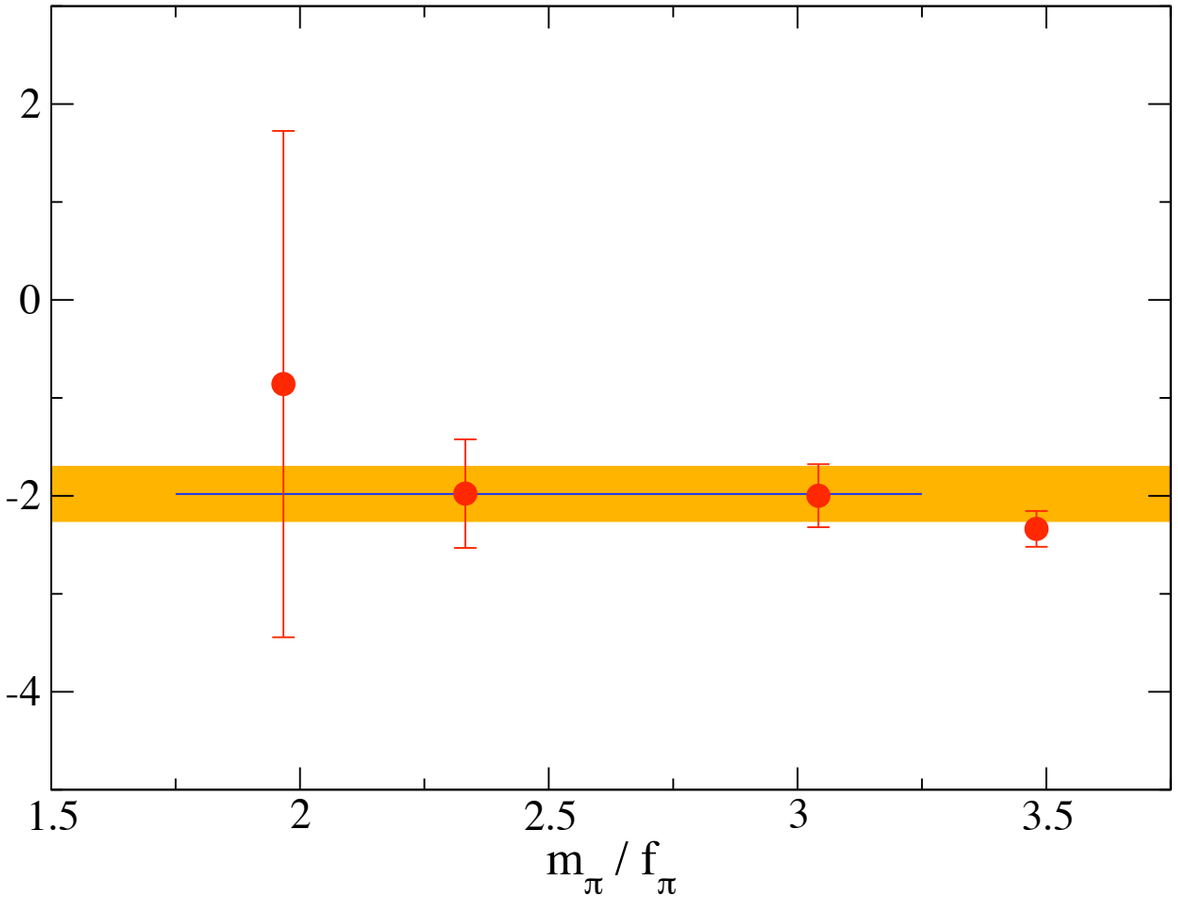
Dynamical
CP-PACS '04 (Wilson)
NPLQCD '05 (Hybrid)

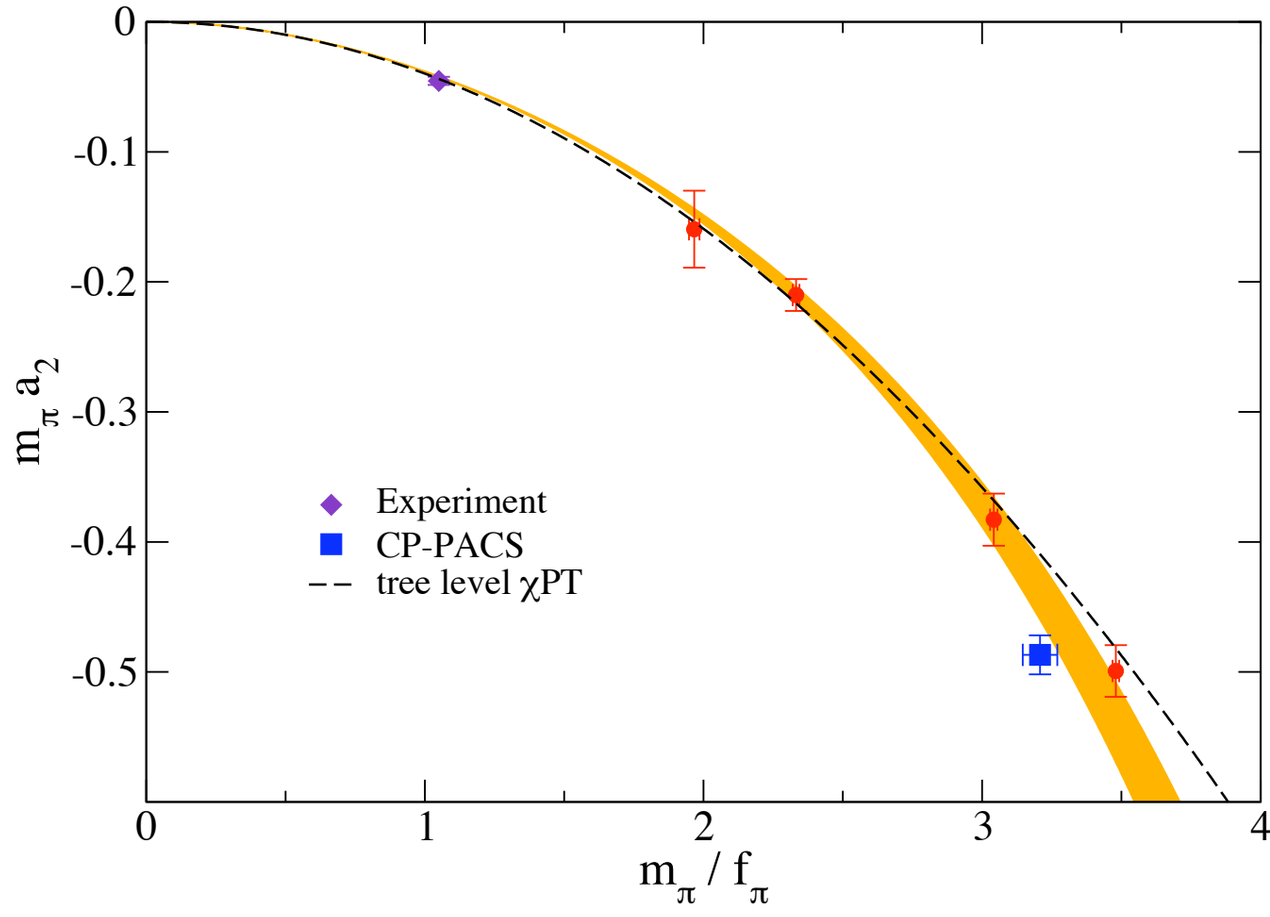
CORRELATOR RATIO



I=2 Pion Scattering

NPLQCD



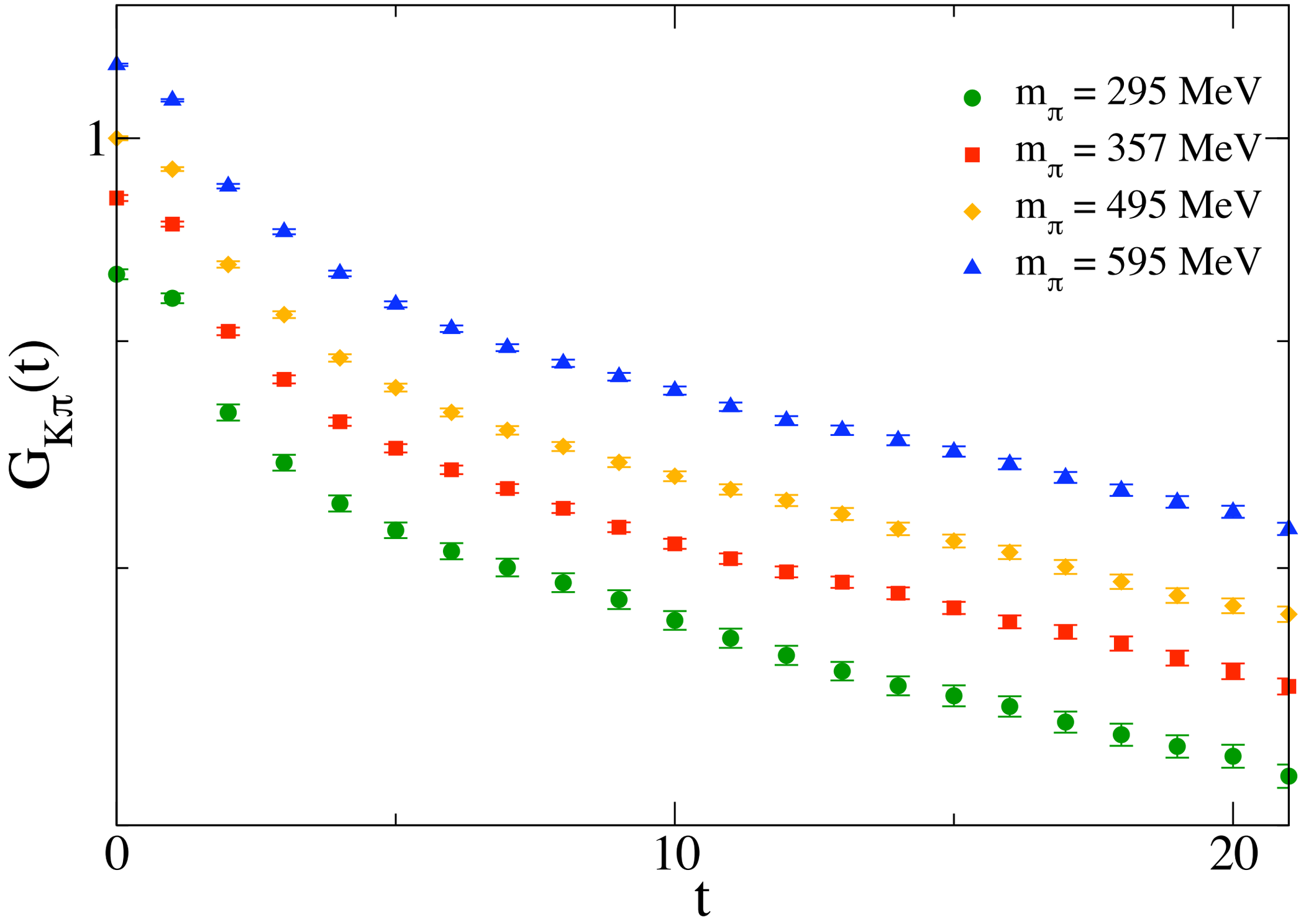


$$m_\pi a_2 = -\frac{m_\pi^2}{8\pi f_\pi^2}\left[1 + \frac{3m_\pi^2}{16\pi^2 f_\pi^2}\left(\log\frac{m_\pi^2}{\mu^2} + l_{\pi\pi}(\mu)\right)\right]$$

[Gasser-Leutwyler '84]
[Colangelo et al. '01]

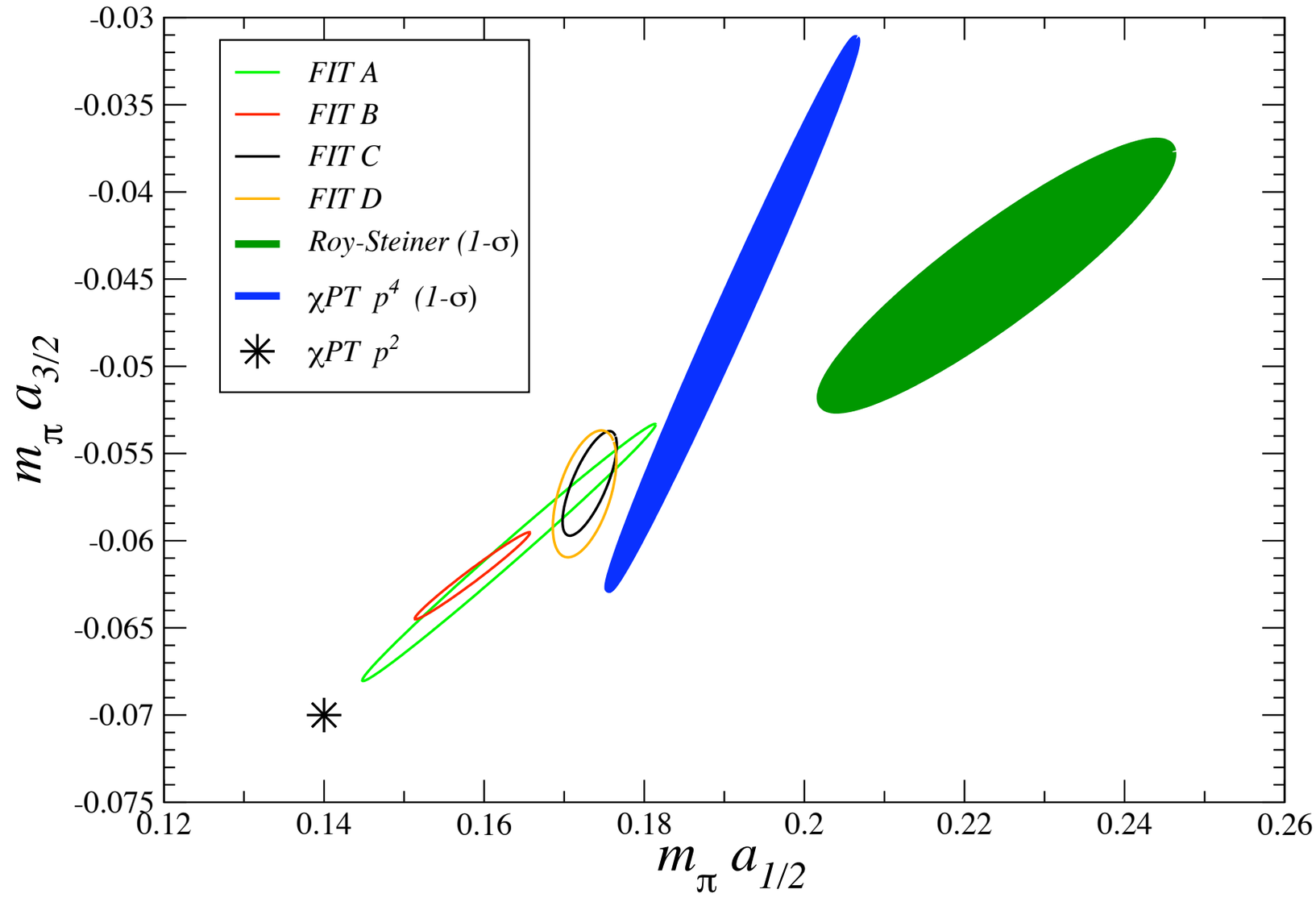
- $m_\pi a_2 = -0.0422(3)(18)$
- Experiment: $m_\pi a_2 = -0.0454(31)$
- SχPT has insignificant effect to the result [Chen et al. '05]

CORRELATOR RATIO



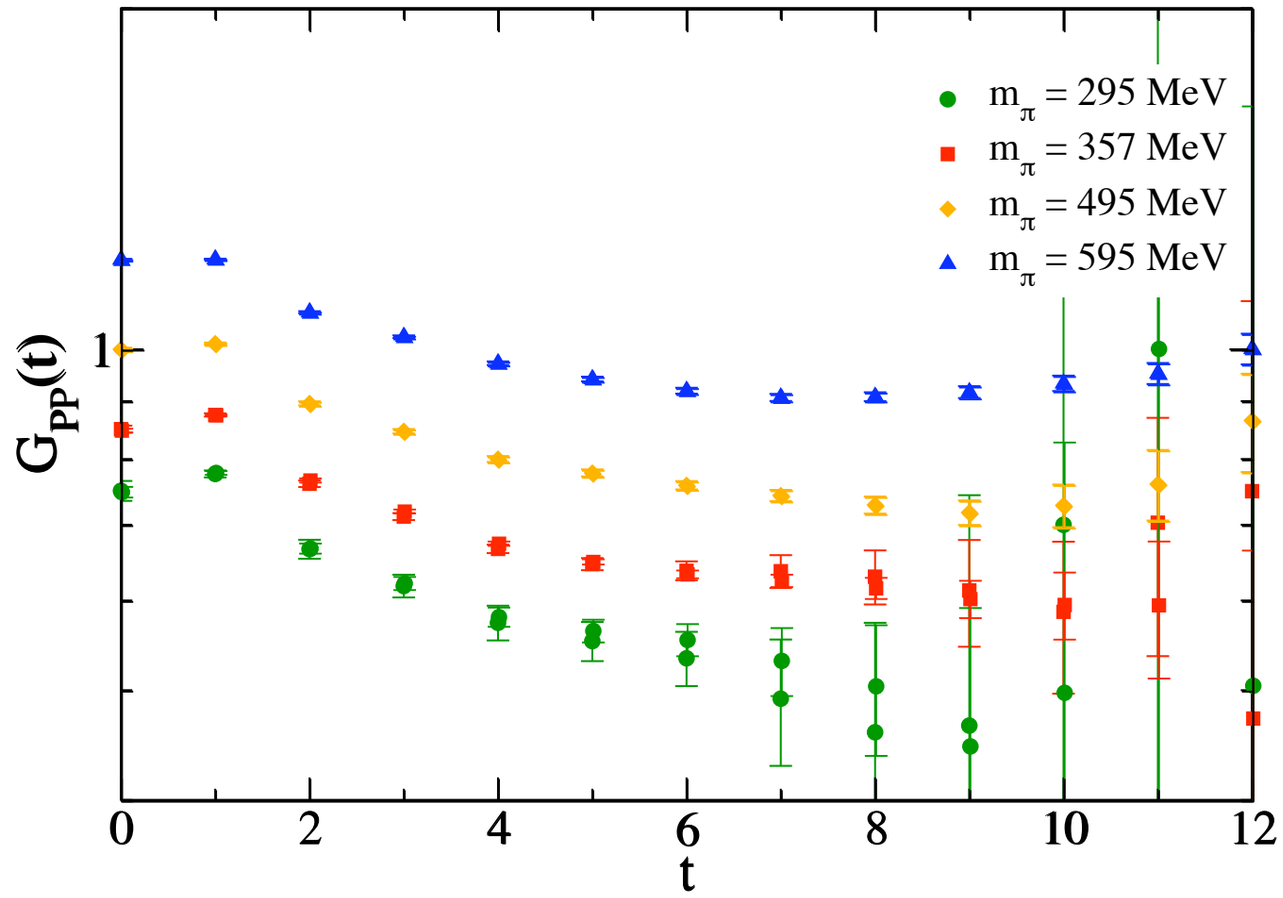
Kaon Pion Scattering Lengths

NPLQCD

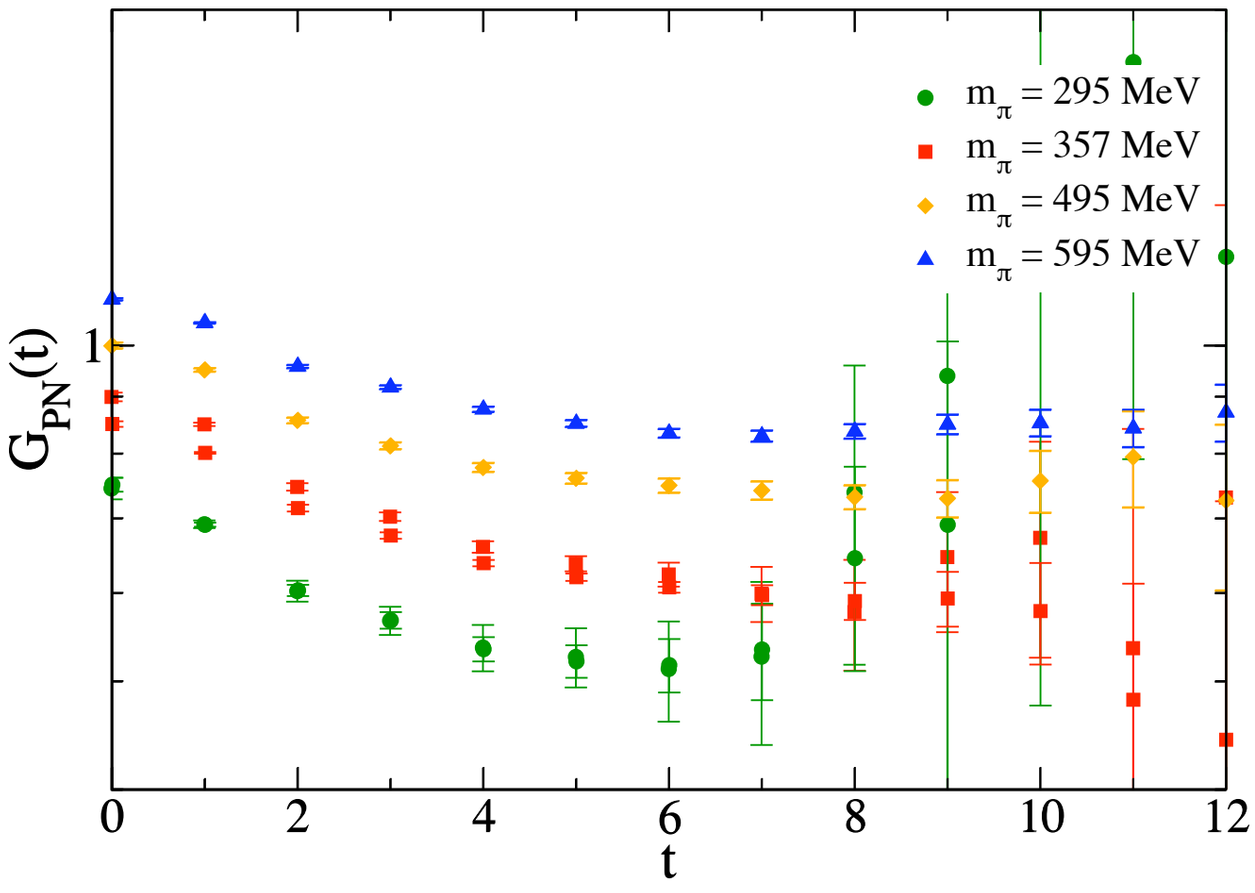


- Upcoming experiments on Kaon - pion molecules (DIRAC collab.)
- Continuum extrapolation still needed

NUCLEON-NUCLEON



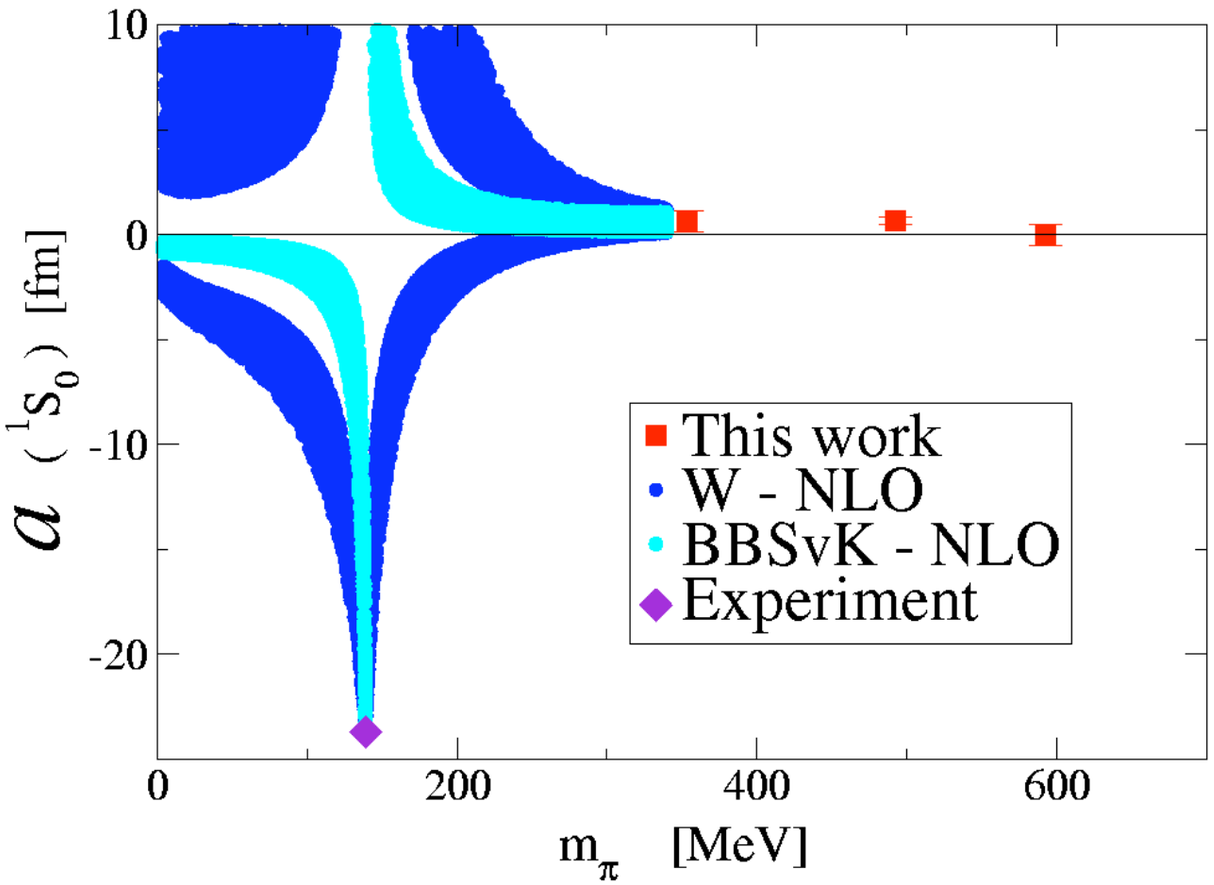
$^1S_0$ channel

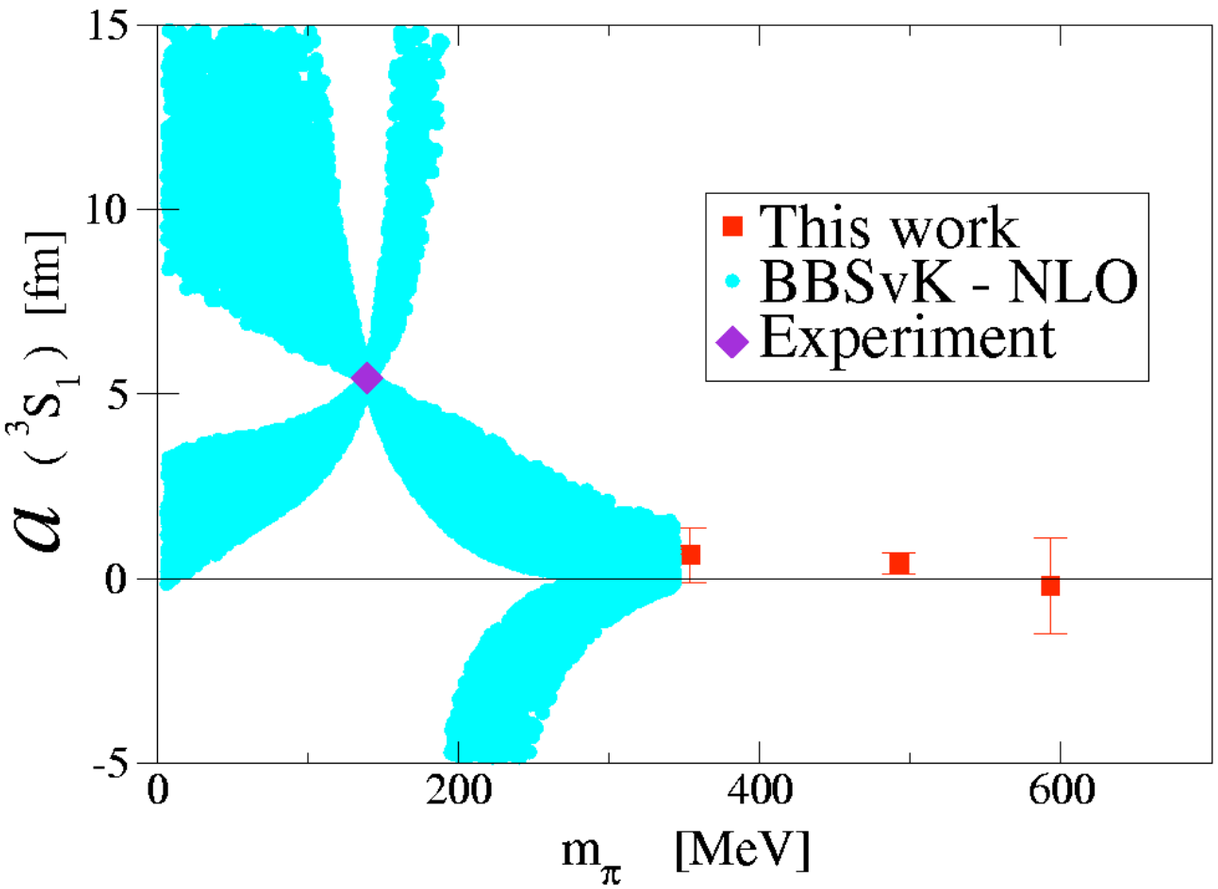


$^3S_1$ channel

Nucleon-Nucleon

NPLQCD: **Phys.Rev.Lett.97 2006**



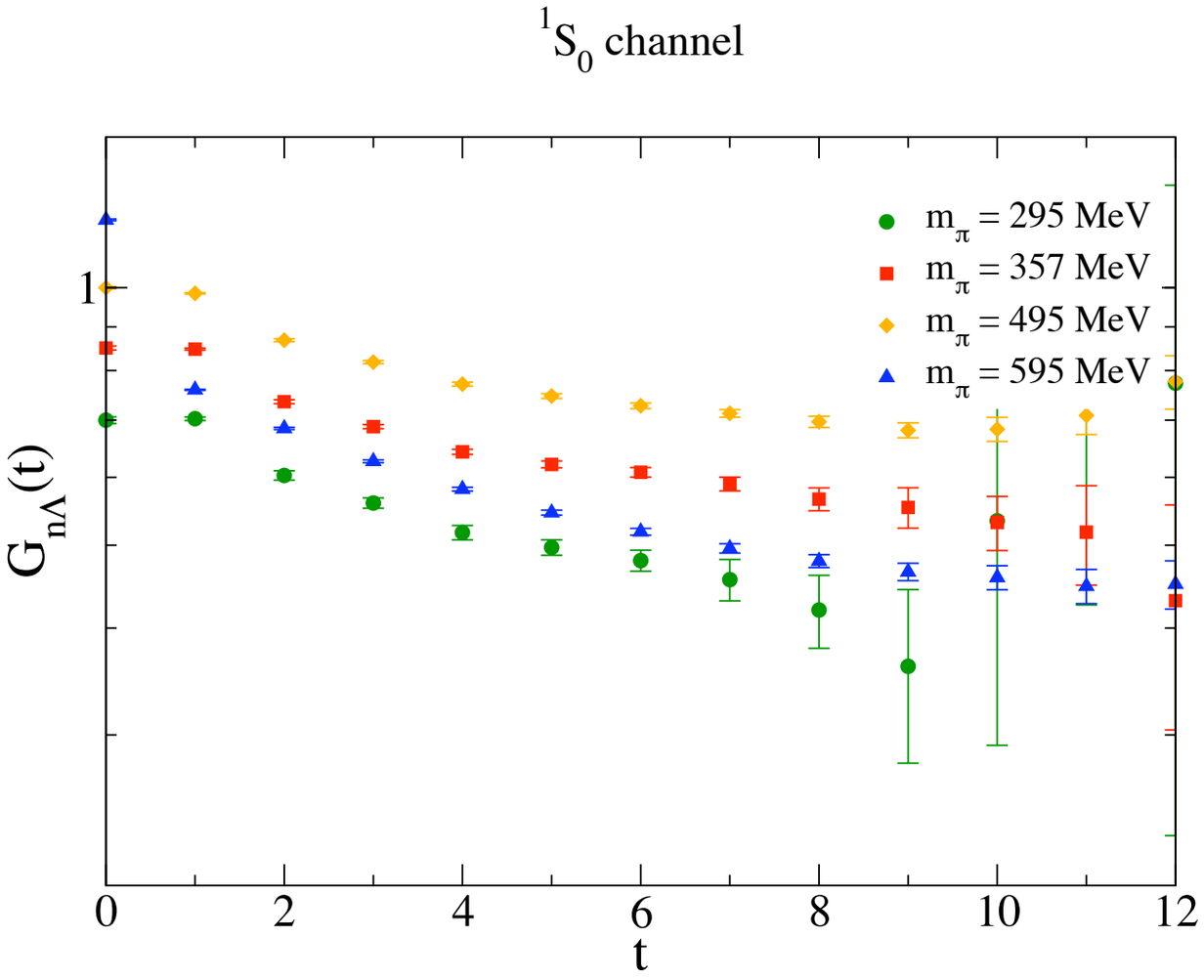


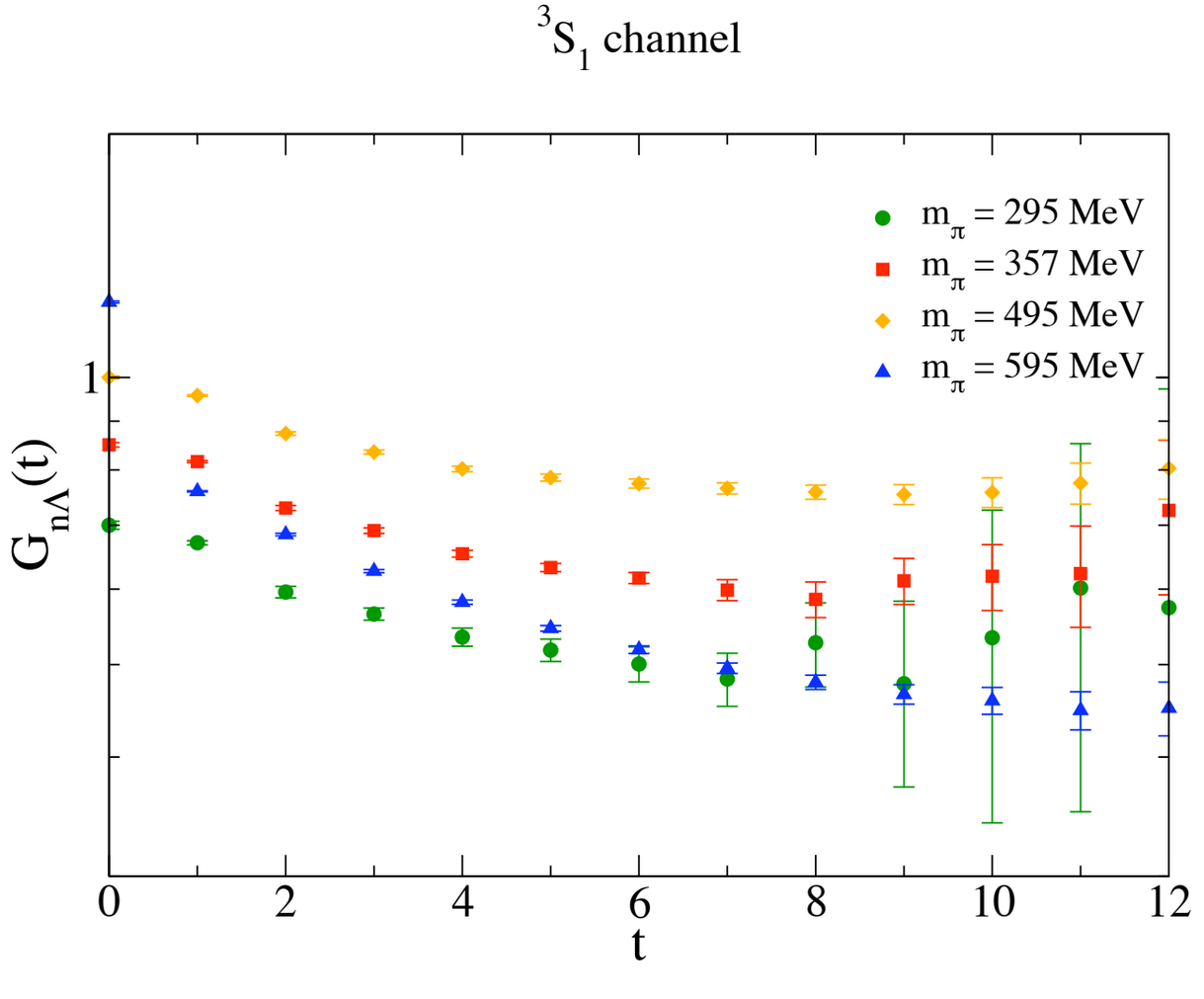
BBSvK: Beane Bedaque Savage van Kolck ‘02

W: Weinberg ‘90;Weingberg ‘91; Ordonez et.al ‘95

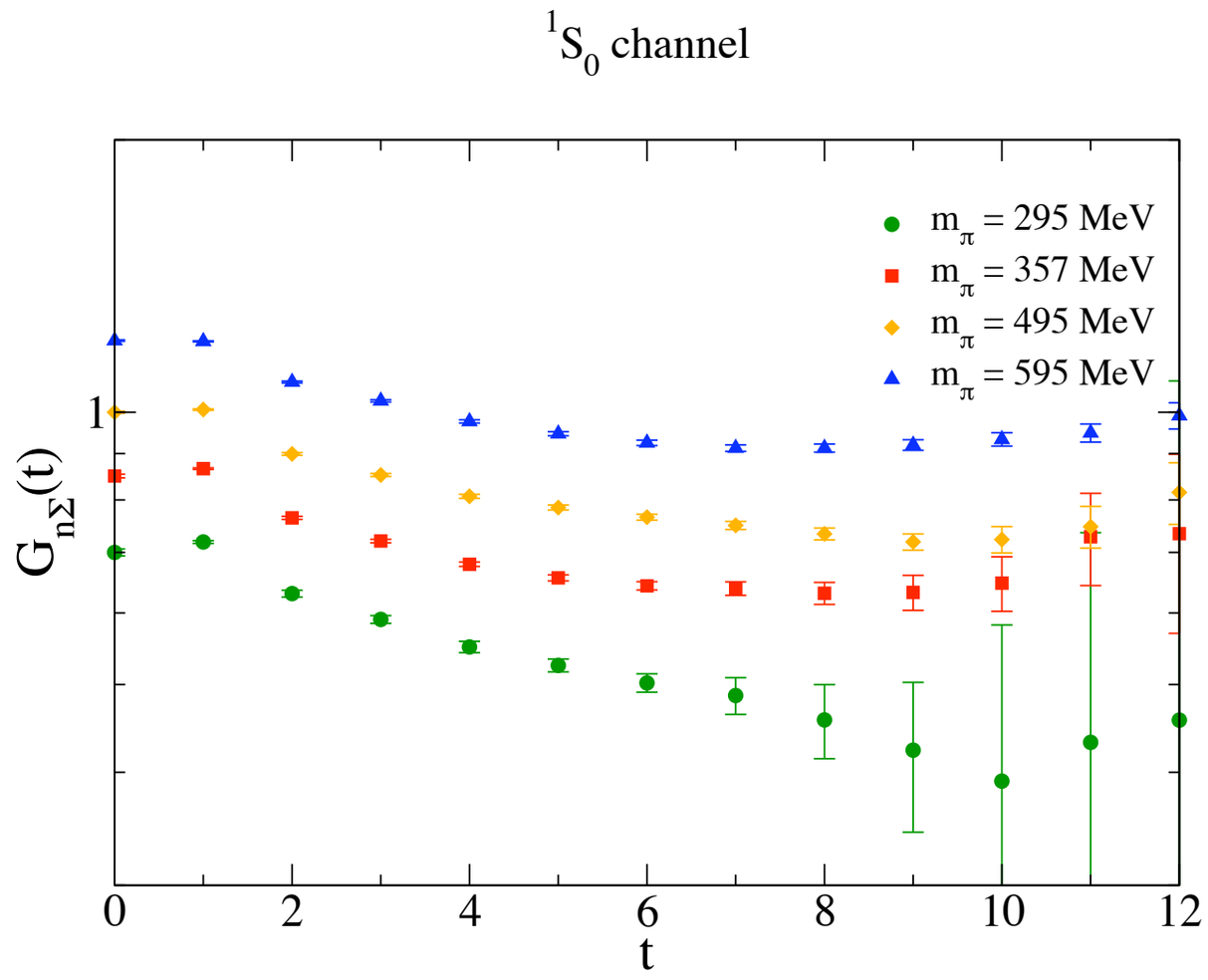
Fukugita et al. ‘95: Quenched heavy pions

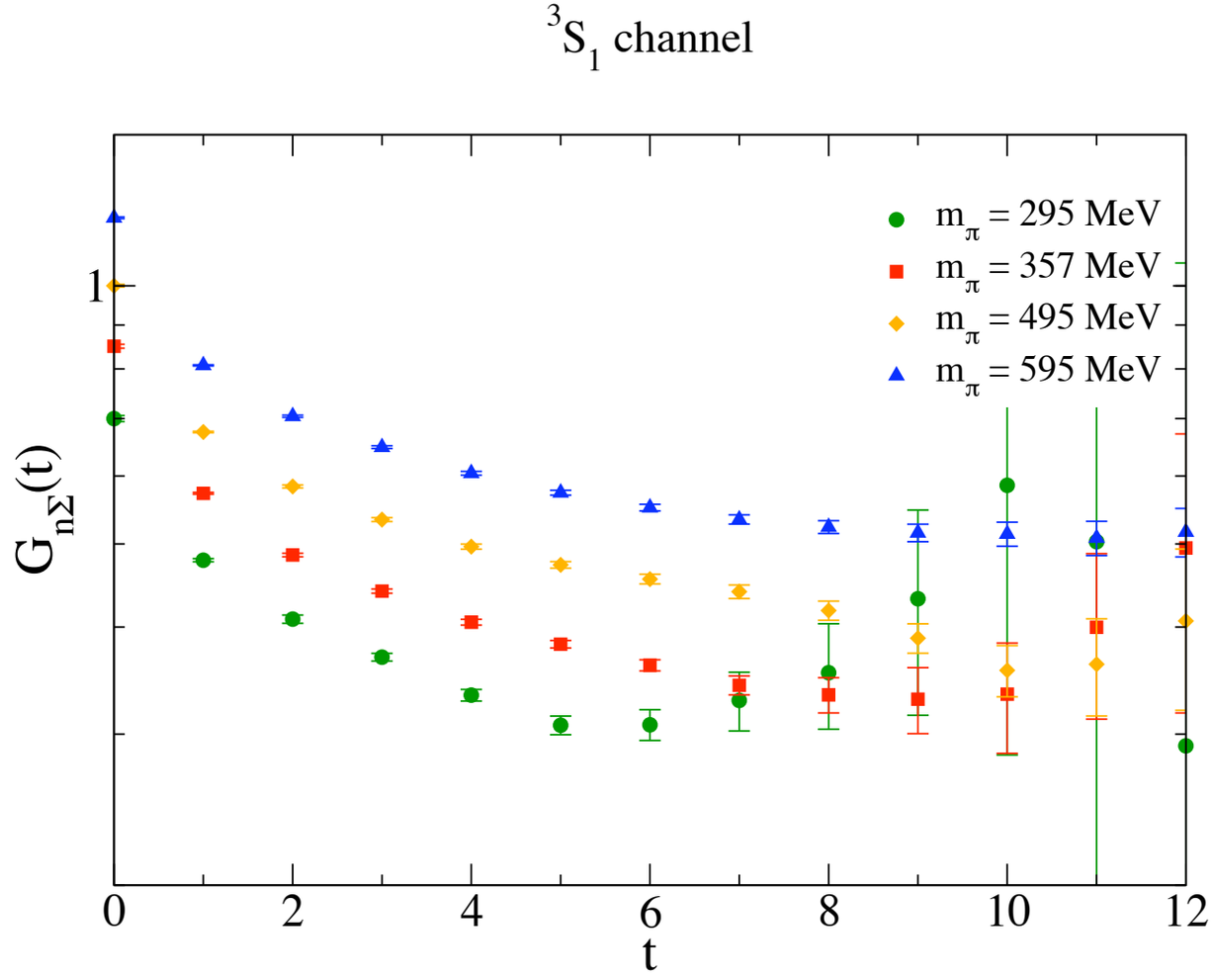
Neutron - Lambda





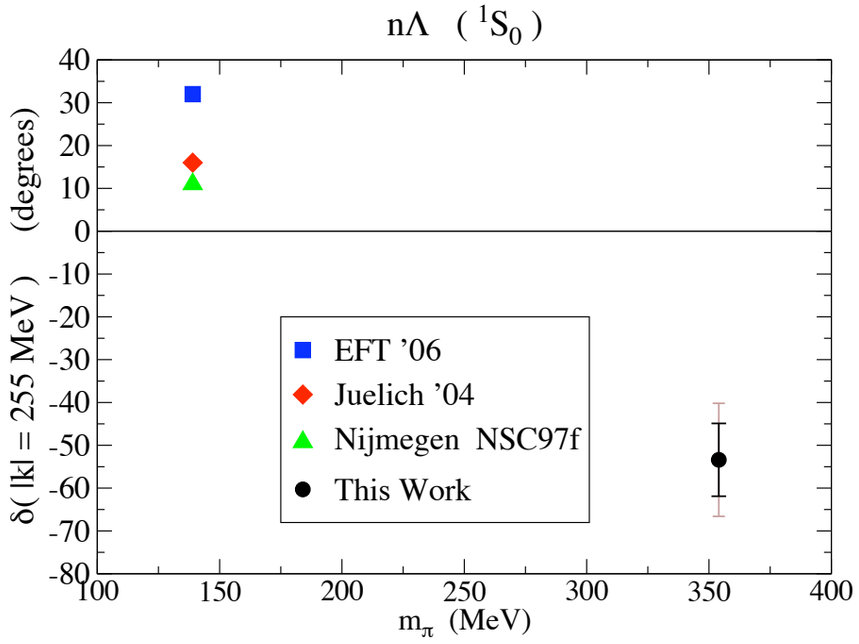
Neutron-Sigma

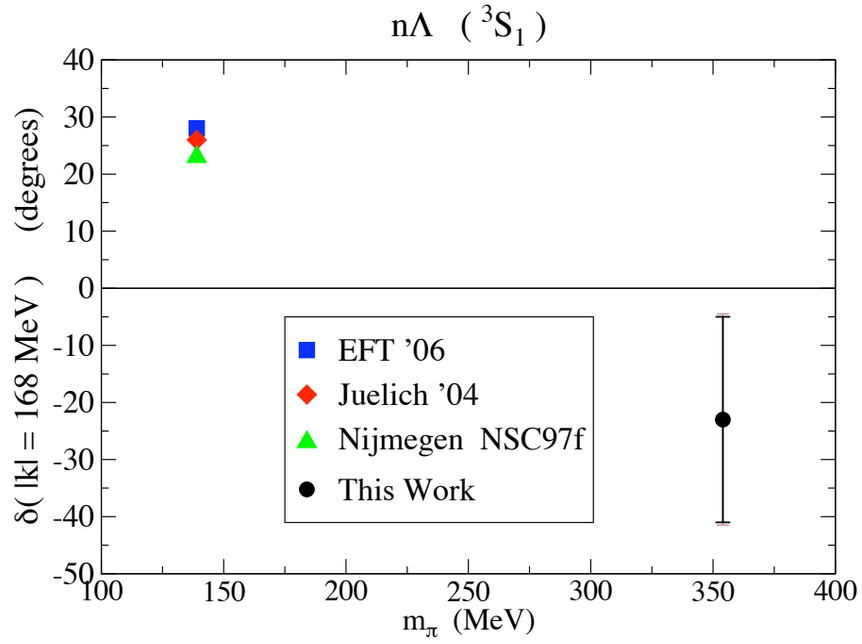


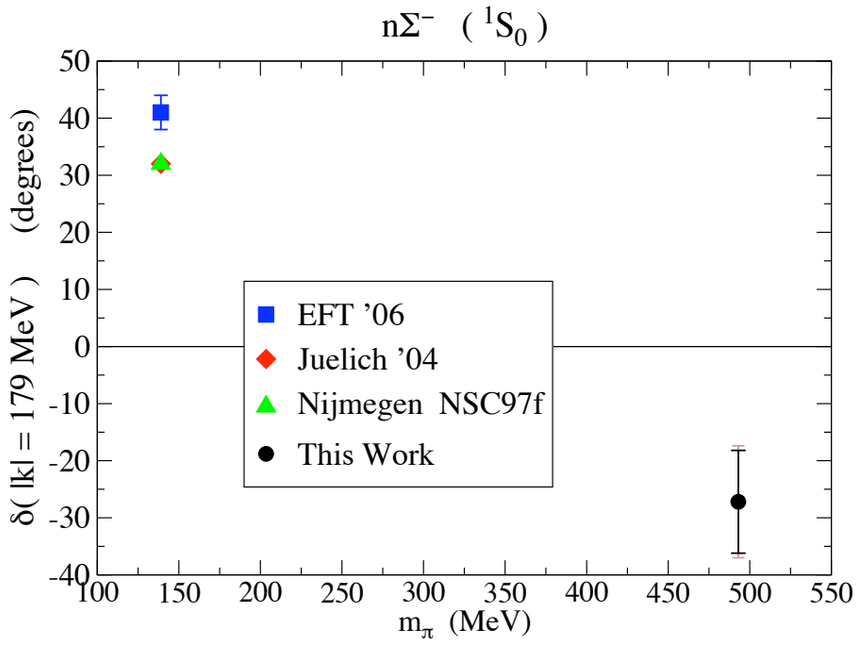


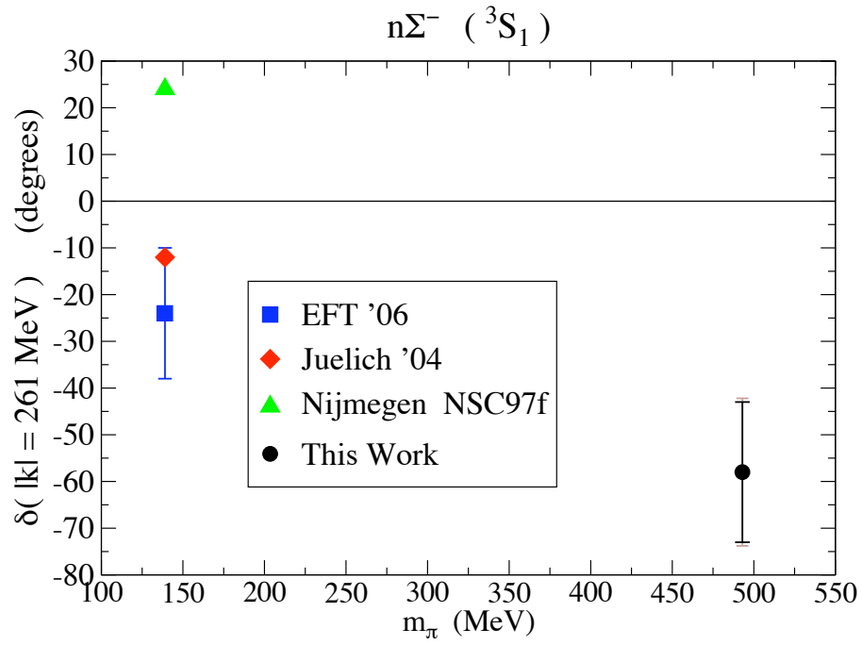
Nucleon-Hyperon

NPLQCD: **hep-lat/0612026**





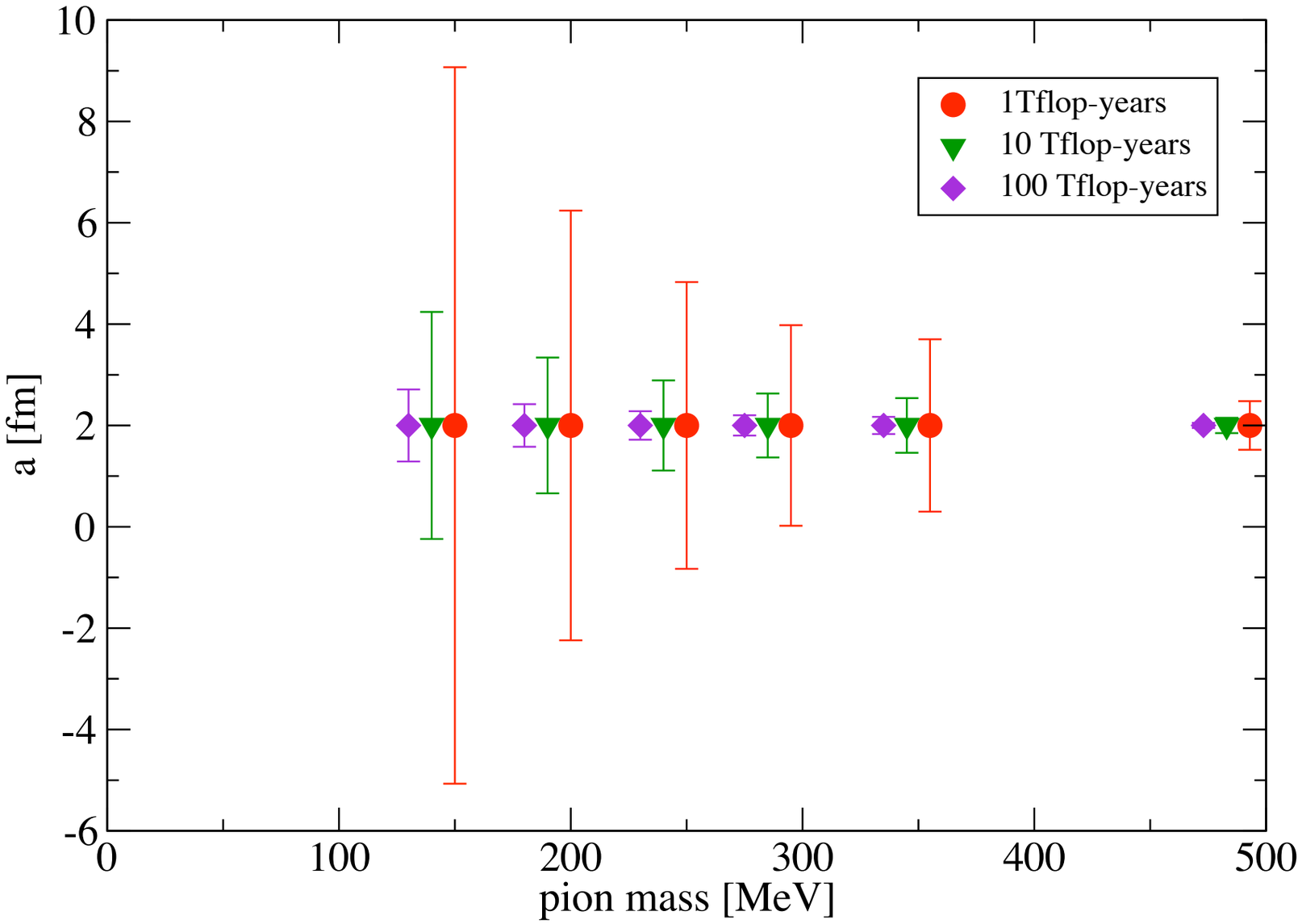




Hadronic interactions (Future)

- These calculations are the beginning of the beginning!
- Need lighter pion masses, multiple volume sizes, and lattice spacings
  - Determine if we see scattering states
- Meson baryon channels: (K-n, K-Σ ...) ----- Neutron stars
- Hyperon-Hyperon and Hyperon-Nucleon channels [NPLQCD: **hep-lat/0612026**]
  - Hyper-nuclear physics and Neutron stars
- Need to make lattices designed for this project
- Higher statistics: (JLAB spectrum program -- INCITE recent award)

Hadron Interactions: Projected errors



- Errors on scattering nucleon-nucleon scattering length as function of computational resources
- Only cost for correlation function calculation presented