Electroweak theory on the lattice with exact gauge invariance and its applications

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“Domain wall Fermions at Ten Years” @ BNL
**Electroweak theory**  (without SU(3) color int.)

- a chiral gauge theory with $SU(2)_L \times U(1)_Y$
- gauge symmetry breaking via Higgs mechanism
- baryon number violation due to chiral anomaly
- etc.

with LGT, what one can do to study these aspects of the electroweak theory?
Plan of this talk

★ a gauge-invariant construction of ElectroWeak theory
  ★ use of DW, Overlap (the Ginsparg-Wilson relation)
    cf. U(1) lattice chiral gauge theory with exact gauge invariance

★ possible applications of the lattice EW theory
  ★ a computation of the effect of quarks, leptons to the sphaleron rate
  ★ a construction of a model of dynamical EW symmetry breaking
  ★ etc.
Weyl fermions (quarks & leptons) on the lattice

chiral fermion bound to Domain wall “Wilson” fermion

Kaplan(1992)

+\n\mu_0

-\n\mu_0

\psi(x, t) = \Psi_-(x) \phi_0(t)

\phi_0(t) = \begin{cases} 
(1 + \{\sum_{\mu} 2 \sin^2(\frac{p_\mu a}{2}) - \mu_0\})^{|t|} & t \geq 0 \\
(1 + \{\sum_{\mu} 2 \sin^2(\frac{p_\mu a}{2}) + \mu_0\})^{-|t|} & t \leq -1 
\end{cases}

only for \(p_\mu\) s.t.

\[\sum_{\mu} 2 \sin^2\left(\frac{p_\mu a}{2}\right) - \mu_0 < 0 \quad (0 < \mu_0 < 2)\]

chiral det. as an “overlap” of two vacua

Narayanan-Neuberger(1993)

\[H_+ = -\ln T\] or \(H_w (\mp \mu_0)\)

\[\{v_i(x) \mid H-v_i(x) = |\epsilon_i|v_i(x) \ (i = 1, \cdots, N_-)\}\]

\[\{\bar{v}_i(x) \mid \bar{v}_i(x)H_+ = |\epsilon_i|\bar{v}_i(x) \ (i = 1, \cdots, \bar{N}_-)\}\]

\[Z = \langle v + |v- \rangle = \det(\bar{v}_k v_j)\]

“overlap formula”
overlap Dirac op. / the GW rel.

Neuberger(1997,98)

\[
D = \frac{1}{2a} \left( 1 + \gamma_5 \frac{H_w}{\sqrt{H_w^2}} \right)
\]

\[\gamma_5 D + D\gamma_5 = 2a D\gamma_5 D\]

chiral operator

Luscher; Hasenfratz, Niedermayer(1998)

\[
\hat{\gamma}_5 \equiv \gamma_5 (1 - 2a D) = -\frac{H_w}{\sqrt{H_w^2}}
\]

chiral fermion

\[
\hat{\gamma}_5 \psi_\pm(x) = \pm \psi_\pm(x)
\]

\[
\bar{\psi}_\pm(x) \gamma_5 = \mp \bar{\psi}_\pm(x)
\]

Path Integral Quantization

\[
\psi_-(x) = \sum_i v_i(x) c_i
\]

\[
\bar{\psi}_-(x) = \sum_i \bar{c}_i \bar{v}_i(x)
\]

\[
\{ v_i(x) | \hat{\gamma}_5 v_i(x) = -v_i(x) \ (i = 1, \cdots, N_-) \}
\]

\[
\{ \bar{v}_i(x) | \bar{v}_i(x) \gamma_5 = +\bar{v}_i(x) \ (i = 1, \cdots, \bar{N}_-) \}
\]

\[
Z = \int \mathcal{D}[\psi_-] \mathcal{D} [\bar{\psi}_-] e^{-a^4 \sum_x \bar{\psi}_- D \psi_-(x)}
\]

\[= \int \prod_i d c_i \prod_j d \bar{c}_j e^{-\sum_{ij} \bar{c}_j M_{ji} c_i} \]

\[= \det M_{ji} \quad M_{ji} = a^4 \sum_x \bar{v}_j D v_i(x) \]

“overlap formula”
Overlap Dirac op. / the GW rel.

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Path Integral Quantization

\[ Z = \int \mathcal{D}[\psi_-] \mathcal{D}[\bar{\psi}_-] e^{-a^4 \sum_x \bar{\psi}_- D \psi_-(x)} \]
\[ = \int \prod_i dc_i \prod_j d\bar{c}_j e^{-\sum_{ij} \bar{c}_j M_{ji} c_i} \]
\[ = \det M_{ji} \]
\[ M_{ji} = a^4 \sum_x \bar{v}_j D v_i(x) \]

"Overlap formula"

depend on gauge fields!!

the space of chiral fermion depends on gauge fields!!
overlap Dirac op. / the GW rel.

Neuberger (1997, 98)

\[ D = \frac{1}{2a} \left( 1 + \gamma_5 \frac{H_w}{\sqrt{H_w^2}} \right) \]

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Path Integral Measure depends on gauge fields!

chiral operator

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Path Integral Quantization

\[ Z = \int \mathcal{D}[\psi_-] \mathcal{D}[\bar{\psi}_-] e^{-a^4 \sum_x \bar{\psi}_- D \psi_- (x)} \]
\[ = \int \prod_i d c_i \prod_j d \bar{c}_j e^{- \sum_{ij} \bar{c}_j M_{ji} c_i} \]
\[ = \det M_{ji} \quad M_{ji} = a^4 \sum_x \bar{v}_j D v_i (x) \]

the space of chiral fermion depends on gauge fields!!

\{ \psi_i (x) \mid \hat{\gamma}_5 \psi_i (x) = - \psi_i (x) \ (i = 1, \cdots, N_-) \} \\
\{ \bar{v}_i (x) \mid \bar{\psi}_i (x) \gamma_5 = + \bar{\psi}_i (x) \ (i = 1, \cdots, \bar{N}_-) \} \]

"overlap formula"
The overlap Dirac op. / the GW rel.

Neuberger(1997, 98)

\[ D = \frac{1}{2a} \left( 1 + \gamma_5 \frac{H_w}{\sqrt{H_w^2}} \right) \]

\[ \gamma_5 D + D \gamma_5 = 2a D \gamma_5 D \]

Path Integral Measure depends on gauge fields!

Path Integral Quantization

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chiral fermion

\[ \hat{\gamma}_5 \psi_{\pm}(x) = \pm \psi_{\pm}(x) \]

\[ \overline{\psi}_{\pm}(x) \gamma_5 = \mp \overline{\psi}_{\pm}(x) \]

Path Integral Quantization

\[ Z = \int \mathcal{D}[\psi_-] \mathcal{D}[\overline{\psi}_-] e^{-a^4 \sum_x \overline{\psi}_- D \psi_-(x)} \]

\[ = \int \prod_i dc_i \prod_j d\overline{c}_j e^{-\sum_{ij} \overline{c}_j M_{ji} c_i} \]

\[ = \det M_{ji} \]

\[ M_{ji} = a^4 \sum_x \overline{v}_j D v_i(x) \]

“overlap formula”
overlap Dirac op. / the GW rel.  
**Neuberger(1997,98)**

\[
D = \frac{1}{2a} \left(1 + \gamma_5 \frac{H_w}{\sqrt{H^2_w}} \right)
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Path Integral Measure depends on gauge fields!

**chiral operator**

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**Path Integral Measure depends on gauge fields !**  

**Path Integral Measure**

\[
Z = \int \mathcal{D}[\psi_-] \mathcal{D}[\bar{\psi}_-] e^{-a^4 \sum_x \bar{\psi}_- D \psi_-(x)}
\]

\[
= \int \prod_i dc_i \prod_j d\bar{c}_j e^{-\sum_{ij} \bar{c}_j M_{ji} c_i}
\]

\[
= \text{det} \tilde{Q} M_{ji}
\]

\[
M_{ji} = a^4 \sum_x \bar{v}_j D v_i(x)
\]

“overlap formula”
the gauge-field dependence must be fixed...

Luscher(98)

1. locality ?  
   (cf. Hernandez, Jansen, Luscher(98))

2. gauge invariance ?

3. integrability ?  
   topology of the space of gauge fields
   non-trivial due to Admissibility cond.

* different situation from Dirac fermions in Vector-like theories, like QCD

variation of effective action & gauge anomaly

\[ \Gamma_{\text{eff}} = \ln \det (\bar{v}_k D v_j) \]
\[ \delta_\eta U(x, \mu) = i \eta_\mu(x) U(x, \mu) \]

\[ \delta_\eta \Gamma_{\text{eff}} = \text{Tr} \left\{ (\delta_\eta D) \hat{P}_- D^{-1} P_+ \right\} + \sum_i (v_i, \delta_\eta v_i) \]

\[ = i \text{Tr} \omega \gamma_5 (1 - D) - i \sum_i (v_i, \delta_\omega v_i) \]
\[ \eta_\mu(x) = -i \nabla_\mu \omega(x) \]

gauge anomaly!
Construction of SU(2)×U(1) Electroweak theory (I)
Construction of SU(2)xU(1) Electroweak theory (I)

infinite volume case
Construction of SU(2)\times U(1) Electroweak theory (I)

infinite volume case

\[ \eta_\mu(x) = \eta_\mu^{(2)}(x) \oplus \eta_\mu^{(1)}(x) \quad U_t(x, \mu)^{(1)} = e^{itA_\mu(x)} \quad t \in [0, 1] \]

\[ \mathcal{L}_\eta = i \sum_i (v_i, \delta_\eta v_i) = \sum_x \eta_\mu(x) j_\mu(x) \]

\[ = i \int_0^1 dt \text{Tr} \left\{ \hat{P}_- [\partial_t \hat{P}_- , \delta_\eta \hat{P}_- ] \right\} + \delta_\eta \int_0^1 dt \sum_{x \in \mathbb{Z}^4} \{ A_\mu(x) k_\mu(x) \} \]
Construction of SU(2)xU(1) Electroweak theory (I)

infinite volume case

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\[ = i \int_0^1 dt \ Tr \left\{ \hat{P}_- [\partial_t \hat{P}_-, \delta_\eta \hat{P}_-] \right\} + \delta_\eta \int_0^1 dt \sum_{x \in \mathbb{Z}^4} \{ A_\mu(x) k_\mu(x) \} \]

cf. U(1) case

Luscher(98)

Neuberger(01)
Construction of SU(2)xU(1) Electroweak theory (I)

infinite volume case

\[ \eta_\mu(x) = \eta_\mu^{(2)}(x) \oplus \eta_\mu^{(1)}(x) \quad \quad U_t(x, \mu)^{(1)} = e^{itA_\mu(x)} \quad t \in [0, 1] \]

\[ L_\eta = i \sum_i (v_i, \delta \eta v_i) = \sum_x \eta_\mu(x) j_\mu(x) \]

\[ = i \int_0^1 dt \text{Tr} \left\{ \hat{P}_- [\partial_t \hat{P}_-, \delta \eta \hat{P}_-] \right\} + \delta \eta \int_0^1 dt \sum_{x \in \mathbb{Z}^4} \{ A_\mu(x) k_\mu(x) \} \]

local counter term!

cf. U(1) case
Luscher(98)
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gauge anomaly cancellation

analysis of U(1) with SU(2) fixed

\[ q(x) = tr \{\gamma_5(1 - aD)(x, x)\} \bigg|_{U^{(1)}, U^{(2)}} \]

x \in \mathbb{Z}^4
Construction of SU(2)xU(1) Electroweak theory (I)

infinite volume case

\[ \eta_\mu(x) = \eta^{(2)}_\mu(x) \oplus \eta^{(1)}_\mu(x) \]

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gauge anomaly cancellation

analysis of U(1) with SU(2) fixed

\[ q(x) = \text{tr} \{ \gamma_5 (1 - aD)(x, x) \} |_{U(1), U(2)} \]

\[ x \in \mathbb{Z}^4 \]

\[ \sum_{\alpha} Y_\alpha q(x) |_{U(1) \rightarrow \{ U(1) \}} Y_\alpha \]

\[ = \sum_{\alpha} Y_\alpha q(x) |_{U(2)} + \sum_{\alpha} Y^3 \frac{1}{32\pi^2} \epsilon_{\mu\nu\lambda\rho} F_{\mu\nu}(x) F_{\lambda\rho}(x + \hat{\mu} + \hat{\nu}) + \partial^*_\mu k_\mu(x) \]

\[ = \partial^*_\mu k_\mu(x) \]

local counter term!

cf. U(1) case
Luscher(98)
Neuberger(01)


cf. Luscher(98)
(in 4dim.)

cf. Nakayama-YK(00)
(in 6dim.)
Construction of SU(2)xU(1) Electroweak theory (I)

infinite lattice case

- a local counter term constructed non-perturbatively
- the first gauge-invariant regularization of EW theory (cf. dimensional reg.)
- may be used in the perturbation theory
  ex. for computations of higher order EW contr. to g-2 (?)
Construction of SU(2)×U(1) Electroweak theory (II)
Construction of SU(2) x U(1) Electroweak theory (II)

finite volume case

\[ \Gamma_4 = \{ x = (x_0, \cdots, x_3) \in \mathbb{Z}^4 \mid 0 \leq x_\mu < L \} = \mathbb{L}^4 \]
Construction of SU(2)xU(1) Electroweak theory (II)

finite volume case

\[ \Gamma_4 = \{ x = (x_0, \cdots, x_3) \in \mathbb{Z}^4 \mid 0 \leq x_\mu < L \} = L^4 \]

\[ \| 1 - U^{(2)}_{\Box} \| \leq \epsilon \quad \| 1 - \{U_{\Box}^{(1)}\}^{6Y} \| \leq \epsilon \quad \epsilon < \frac{1}{30} \]
Construction of SU(2) x U(1) Electroweak theory (II)

finite volume case

\[ \Gamma_4 = \{ x = (x_0, \cdots, x_3) \in \mathbb{Z}^4 \mid 0 \leq x_\mu < L \} = \mathbb{L}^4 \]

\[ \| 1 - U_{\Box}^{(2)} \| \leq \epsilon \quad \| 1 - \{ U_{\Box}^{(1)} \}^{6Y} \| \leq \epsilon \quad \epsilon < \frac{1}{30} \]

\[ m_{\mu\nu} = \frac{1}{2\pi i} \sum_{s, t} \ln U_{\mu\nu}^{(1)} (x + s \hat{\mu} + t \hat{\nu}) \]

\[ Q = \sum_{x \in \Gamma_4} \text{tr}\{ \gamma_5 (1 - D)(x, x) \} \big|_{U^{(2)}} \]
Construction of SU(2)×U(1) Electroweak theory (II)

finite volume case

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Construction of SU(2)xU(1) Electroweak theory (II)

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\[ Q = \sum_{x \in \Gamma_4} \text{tr}\{\gamma_5 (1 - D)(x, x)\}|_{U^{(2)}} \]

\[ U_\mu(x) = e^{iA_\mu_T(x)}g(x)g(x + \hat{\mu})^{-1}U^{(w)}_{[w]}(x, \mu)V^{(m)}_{[m]}(x, \mu) \]

\[ F_{\mu \nu}(x) = \partial_\mu A^T_\nu(x) - \partial_\nu A^T_\mu(x) + \frac{2\pi m_{\mu \nu}}{L^2} \]

a pair of doublets (a,b) measure defined globally!

\[ v_j^{(a)}(x) = v_j(x) \]

\[ v_j^{(b)}(x) = (\gamma_5 C^{-1} \otimes i\sigma_2) [v_j(x)]^* \]

cf. Neuberger(98) Bar-Campos (00)
Construction of SU(2)\times U(1) Electroweak theory (II)

finite volume case

\[ \Gamma_4 = \{ x = (x_0, \ldots, x_3) \in \mathbb{Z}^4 \mid 0 \leq x_\mu < L \} = \mathbb{L}^4 \]

\[ \parallel 1 - U_{\Box}^{(2)} \parallel \leq \epsilon \quad \parallel 1 - \{ U_{\Box}^{(1)} \}^6 Y \parallel \leq \epsilon \quad \epsilon < \frac{1}{30} \]

\[ Q = \sum_{x \in \Gamma_4} \text{tr}\{ \gamma_5 (1 - D)(x, x) \} \mid_{U(2)} \]

\[ m_{\mu \nu} = \frac{1}{2\pi i} \sum_{s, t} \ln U_{\mu \nu}^{(1)}(x + s \hat{\mu} + t \hat{\nu}) \]

\[ u_{(a)}(x) = u_j(x) \]

\[ u_{(b)}(x) = (\gamma_5 C^{-1} \otimes i\sigma_2) [u_j(x)]^* \]

\[ U_{\mu}(x) = e^{iA_{\mu}^T(x)} g(x) g(x + \hat{\mu})^{-1} U_{[w]}(x, \mu) V_{[m]}(x, \mu) \]

\[ F_{\mu \nu}(x) = \partial_\mu A_{\nu}^T(x) - \partial_\nu A_{\mu}^T(x) + \frac{2\pi m_{\mu \nu}}{L^2} \]

\[ U(1) \sim T^n \times M[SU(2)] \]

a pair of doublets \((a,b)\) measure defined globally!

\[ \text{cf. Nuberger(98) Bar-Campos (00)} \]
Construction of SU(2) \times U(1) Electroweak theory (II)

finite volume case

$$\Gamma_4 = \{ x = (x_0, \cdots, x_3) \in \mathbb{Z}^4 \mid 0 \leq x_\mu < L \} = \mathbb{L}^4$$

$$\| 1 - U^{(2)}_\square \| \leq \epsilon \quad \| 1 - \{U^{(1)}_\square \}^6 Y \| \leq \epsilon \quad \epsilon < \frac{1}{30}$$

$$m_{\mu\nu} = \frac{1}{2\pi i} \sum_{s,t} \ln U^{(1)}_\mu(x + s\mu + t\nu)$$

$$Q = \sum_{x \in \Gamma_4} \text{tr}\{\gamma_5(1 - D)(x, x)\}|_{U(2)}$$

$$v^{(a)}_j(x) = v_j(x)$$

$$v^{(b)}_j(x) = (\gamma_5 C^{-1} \otimes i\sigma_2) [v_j(x)]^*$$

\( U(1) \sim T^n \)

\( T^n[U(1)] \times M[SU(2)] \)

\( F_{\mu\nu}(x) = \partial_\mu A^T_\nu(x) - \partial_\nu A^T_\mu(x) + \frac{2\pi m_{\mu\nu}}{L^2} \)

a pair of doublets (a,b) \text{ measure defined globally !}

cf. Neuberger(98) Bar-Campos(00)
Construction of SU(2)xU(1) Electroweak theory (II)
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\[ \eta_\mu(x) = \eta_\mu^{(2)}(x) \oplus \eta_\mu^{(1)}(x) \quad U_t^{(1)}(x, \mu) = e^{it\tilde{A}_\mu(x)} U_{[w]}(x, \mu) \quad t \in [0, 1] \quad (m_{\mu\nu} = 0) \]

\[ \mathcal{L}_\eta = i \sum_i (v_i, \delta_\eta v_i) = \sum_x \eta_\mu(x) j_\mu(x) \]

\[ = i \int_0^1 dt \text{Tr} \left\{ \hat{P}_- [\partial_t \hat{P}_, \delta_\eta \hat{P}_-] \right\} + \delta_\eta \int_0^1 dt \sum_{x \in \Gamma_4} \left\{ \tilde{A}_\mu(x) k_\mu(x) \right\} + \mathcal{L}_\eta \big|_{U^{(1)}=U_{[w]},U^{(2)}=U_{[w],U^{(2)}}} \]

Kadoh-YK in prep.  cf. Luscher(98)
Construction of SU(2)xU(1) Electroweak theory (II)

$$\eta_\mu(x) = \eta_\mu^{(2)}(x) \oplus \eta_\mu^{(1)}(x)$$
$$U_t^{(1)}(x, \mu) = e^{it\tilde{A}_\mu(x)} U[w](x, \mu) \quad t \in [0, 1] \quad (m_{\mu\nu} = 0)$$

$$\mathcal{L}_\eta = i \sum_i (v_i, \delta_\eta v_i) = \sum_x \eta_\mu(x) j_\mu(x)$$

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local counter term!  Wilson line contr.  

Kadoh-YK in prep.  cf. Luscher(98)
Construction of SU(2)xU(1) Electroweak theory (II)

η_µ(x) = η_µ(2)(x) ⊕ η_µ(1)(x) \quad U_t^{(1)}(x, \mu) = e^{it\tilde{A}_\mu(x)} U_{[w]}(x, \mu) \quad t \in [0, 1] \quad (m_{\mu\nu} = 0)

\mathcal{L}_\eta = i \sum_i (v_i, \delta_\eta v_i) = \sum_x \eta_\mu(x) j_\mu(x)

= i \int_0^1 dt \text{Tr} \left\{ \hat{P}_- [\partial_t \hat{P}_-, \delta_\eta \hat{P}_-] \right\} + \delta_\eta \int_0^1 dt \sum_{x \in \Gamma_4} \left\{ \tilde{A}_\mu(x) k_\mu(x) \right\} + \mathcal{L}_\eta \bigg|_{U(1)=U_{[w]}, U(2)}

\text{local counter term!} \quad \text{Wilson line contr.}

Kadoh-YK in prep. \quad \text{cf. Luscher}(98)

gauge anomaly cancellation

\text{cohomological analysis in } \Gamma_4 \quad x \in \Gamma_4

q(x) = \text{tr} \left\{ \gamma_5 (1 - aD)(x, x) \right\} \bigg|_{U(1), U(2)}

\text{cohomological analysis in } \Gamma_4 \quad x \in \Gamma_4
Construction of SU(2)xU(1) Electroweak theory (II)

\[ \eta_\mu(x) = \eta^{(2)}_\mu(x) \oplus \eta^{(1)}_\mu(x) \quad U^{(1)}_t(x, \mu) = e^{i t \tilde{A}_\mu(x)} U_w(x, \mu) \quad t \in [0, 1] \quad (m_{\mu\nu} = 0) \]

\[ \mathcal{L}_\eta = i \sum_i (v_i, \delta \eta v_i) = \sum_x \eta_\mu(x) j_\mu(x) \]
\[ = i \int_0^1 dt \, \text{Tr} \left\{ \hat{P}_- [\partial_t \hat{P}_-, \delta \eta \hat{P}_-] \right\} + \delta \eta \int_0^1 dt \, \sum_{x \in \Gamma_4} \left\{ \tilde{A}_\mu(x) k_\mu(x) \right\} + \mathcal{L}_\eta \mid_{U(1) = U_w, U(2)} \]

**local counter term!**

**Wilson line contr.**

Kadoh-YK in prep.  cf. Luscher(98)

gauge anomaly cancellation

cohomological analysis in \( \Gamma_4 \)  \( x \in \Gamma_4 \)

\[ q(x) = \text{tr} \left\{ \gamma_5 (1 - aD)(x, x) \right\} \mid_{U(1), U(2)} \]

\[ \sum_\alpha Y_\alpha q(x) \mid_{U(1) \rightarrow \{U(1)\}} Y_\alpha = 0 \]

\[ \sum_L Y^3 - \sum_R Y^3 = 0 \quad \sum \text{doublet} \quad \sum \text{singlet} \]

\[ \begin{align*}
\sum_\alpha Y_\alpha q(x) \mid_{U(2)} + \sum_\alpha Y^3_\alpha \frac{1}{32\pi^2} \epsilon_{\mu\nu\lambda\rho} F_{\mu\nu}(x) F_{\lambda\rho}(x + \hat{\mu} + \hat{\nu}) + \partial^* k_\mu(x) \\
= \partial^* k_\mu(x)
\end{align*} \]

Suzuki et al. (01) Kadoh-Nakayama-YK(04)
Construction of SU(2)xU(1) Electroweak theory (II)
finite volume case

- covers all SU(2) topological sectors with vanishing U(1) magnetic fluxes
- global integrability can be proved rigorously
even number of SU(2) doublets, U(1) Wilson line parts
- explicit with two simplifications  \textit{cf.} U(1), Luscher (98)
  ★ direct proof of gauge anomaly cancellation in $\mathbb{L}^4$
  ★ separate treatment of the Wilson line
- some non-perturbative applications?

based on:
D. Kadoh and Y.K., in preparation
possible applications of lattice EW theory (I)
possible applications of lattice EW theory (I)

a computation of the effect of quarks & leptons to the sphaleron rate at finite temp. (at one-loop)
possible applications of lattice EW theory (I)

a computation of the effect of quarks & leptons to the sphaleron rate at finite temp. (at one-loop)

introduction of Higgs field & Yukawa-couplings

\[ S_{EW} = S_G + S_F + \sum_x \{ \nabla_\mu \phi^\dagger \nabla_\mu \phi + V(\phi) \} \]
\[ - \sum_x \left\{ y_t \bar{Q}_- \bar{\phi} t_+(x) + y_b \bar{Q}_- \bar{\phi} b_+(x) + c.c. \right\} \]
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sphaleron on the lattice

\[ U^{(2)}_\mu (x), U^{(1)}_\mu (x), \phi (x) (x \in \mathbb{L}^3) \]

saddle point cooling
Perez- van Baal (96)
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sphaleron on the lattice

\[ U^{(2)}_\mu(x), U^{(1)}_\mu(x), \phi(x) \ (x \in \mathbb{L}^3) \]

fermion fluctuation det.

\[ \kappa_F(v, \lambda, y_t, \cdots) \equiv \prod_q \prod_l \prod_{\omega_n} \text{det} M / \text{det} M_0 \]
\[ M_t = \begin{pmatrix} (\bar{v}_k Dv_j) & y_t (\bar{v}_k \phi u_j) \\ y_t (\bar{u}_k \phi^\dagger v_j) & (\bar{u}_k Du_j) \end{pmatrix} \]

- sum over matsubara freq.
- one-loop renormarizations
- dependence on the Higgs, Yukawa coupling
- comparison to other methods

\[ \text{cf. Bodeker et. (00)} \]

saddle point cooling

Perez- van Baal (96)
possible applications of lattice EW theory (II)
possible applications of lattice EW theory (II)

a construction of a model of
dynamical EW symmetry breaking
possible applications of lattice EW theory (II)

a construction of a model of
dynamical EW symmetry breaking

\[ \text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{U}(1)_{\text{EM}} \]
possible applications of lattice EW theory (II)

a construction of a model of dynamical EW symmetry breaking

\[ \text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{U}(1)_{\text{EM}} \]

\[ \text{SU}(2)_{\text{TC}} \]

“minimal walking technicolor model”

Dietrich, Sannino, Tuominen (05)
possible applications of lattice EW theory (II)

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\[ SU(2)_{TC} \]

“minimal walking technicolor model”

\[ \lambda^a_1, \lambda^a_2, \lambda^a_3, \lambda^a_4 \]

\[ \psi_1, \psi_2, \psi_3, \psi_4 \]

Dietrich, Sannino, Tuominen (05)
possible applications of lattice EW theory (II)

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\[
\text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{U}(1)_{\text{EM}}
\]

\[
\text{SU}(2)_{\text{TC}} \quad \text{“minimal walking technicolor model”}
\]

\[
\text{Dietrich, Sannino, Tuominen (05)}
\]

4 x SU(2)$_{\text{TC}}$ adjoint

4 x SU(2)$_{\text{TC}}$ singlet

\[
\begin{align*}
\lambda_1^a \\
\lambda_2^a \\
\lambda_3^a \\
\lambda_4^a
\end{align*}
\]

\[
\begin{align*}
\psi_1 \\
\psi_2 \\
\psi_3 \\
\psi_4
\end{align*}
\]

chiral sym. & breaking \[ \langle \lambda \lambda \rangle \]

\[
\text{SU}(4) \rightarrow \text{O}(4)
\]
possible applications of lattice EW theory (II)

a construction of a model of dynamical EW symmetry breaking

\[
\text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{U}(1)_{\text{EM}}
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\]

\text{Dietrich, Sannino, Tuominen (05)}

4 x SU(2)_{TC} adjoint \quad 4 x SU(2)_{TC} singlet

\[
\begin{align*}
\lambda^a_1 & \\
\lambda^a_2 & \rightarrow 3 \times \text{SU}(2)_L \text{ doublet} \\
\lambda^a_3 & \leftarrow \text{SU}(2)_L \text{ doublet} \\
\lambda^a_4 & \phantom{\text{SU}(2)_L \text{ doublet}}
\end{align*}
\]

\[
\psi_1 \quad \psi_2 \quad \psi_3 \quad \psi_4
\]

chiral sym. & breaking \( \langle \lambda \lambda \rangle \)

\[
\text{SU}(4) \rightarrow \text{O}(4)
\]
possible applications of lattice EW theory (II)

a construction of a model of dynamical EW symmetry breaking

\[ SU(2)_L \times U(1)_Y \quad \rightarrow \quad U(1)_{EM} \]

\[ SU(2)_{TC} \quad \text{“minimal walking technicolor model”} \]

Dietrich, Sannino, Tuominen (05)

\[ 4 \times SU(2)_{TC} \text{ adjoint} \quad 4 \times SU(2)_{TC} \text{ singlet} \]

\[ \begin{array}{c}
\lambda^a_1 \\
\lambda^a_2 \\
\lambda^a_3 \\
\lambda^a_4 \\
\end{array} \quad \begin{array}{c}
\psi_1 \\
\psi_2 \\
\psi_3 \\
\psi_4 \\
\end{array} \]

\[ 3 \times SU(2)_L \text{ doublet} \quad 1 \times SU(2)_L \text{ doublet} \]

chiral sym. & breaking \[ \langle \lambda \lambda \rangle \]

\[ SU(4) \quad \rightarrow \quad O(4) \]

\[ SU(2)_L \times SU(2)_R \quad \rightarrow \quad SU(2)_V \]
possible applications of lattice EW theory (II)

a construction of a model of dynamical EW symmetry breaking

\[ \text{SU}(2)_L \times \text{U}(1)_{\text{Y}} \rightarrow \text{U}(1)_{\text{EM}} \]

\[ \text{SU}(2)_{\text{TC}} \quad \text{“minimal walking technicolor model”} \]

Dietrich, Sannino, Tuominen (05)

4 x SU(2)_{TC} adjoint

\[
\begin{align*}
\lambda_1^a \\
\lambda_2^a \\
\lambda_3^a \\
\lambda_4^a
\end{align*}
\]

0 x SU(2)_{TC} doublet

\[
\psi_1 \\
\psi_2 \\
\psi_3 \\
\psi_4
\]

with DW / overlap fermions, lattice construction is possible if \( Y(3,2) = 0 \), just like EW theory

cf. N=1 SYM Nishimura(97)
Neuberger(98) Kaplan(99)

chiral sym. & breaking \( \langle \lambda \lambda \rangle \)

\[ \text{SU}(4) \rightarrow \text{O}(4) \]

\[ \text{SU}(2)_L \times \text{SU}(2)_R \rightarrow \text{SU}(2)_V \]
possible applications of lattice EW theory (II)

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\[ \text{SU}(2)_L \times \text{U}(1)_Y \rightarrow \text{U}(1)_{\text{EM}} \]

\[ \text{SU}(2)_{\text{TC}} \]

“minimal walking technicolor model”

Dietrich, Sannino, Tuominen (05)

4 x SU(2)_{\text{TC}} adjoint

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with DW / overlap fermions, lattice construction is possible if \( Y(3,2) = 0 \), just like EW theory

\[ \langle \lambda \lambda \rangle \]

\[ \langle \lambda \lambda \rangle \]

\[ \langle \lambda \lambda \rangle \]

\[ \langle \lambda \lambda \rangle \]

chiral sym. & breaking\[ \langle \lambda \lambda \rangle \]

\[ \text{SU}(4) \rightarrow \text{O}(4) \]

\[ \text{SU}(2)_L \times \text{SU}(2)_R \rightarrow \text{SU}(2)_V \]

if \( g_Y = 0 \), numerical simulation is possible!

*global issue; Neuberger (98), Bar (02)
properties claimed \textit{Sannino et al.}

- almost conformal; “walking coupling”
- chiral symmetry breaking

\( \text{SU}(4) \rightarrow \text{O}(4) \)

- consistent with the (usually severe) constraints from EW precision measurements

- Light composite Higgs scalar

\( M_H \sim 150 \text{ GeV} \)

\textbf{Check needed by a non-perturbative method}
properties claimed \textit{Sannino et al.}

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  \[ M_H \sim 150 \text{ GeV} \]

**Check needed by a non-perturbative method**

**These are the problems familiar in lattice QCD, although tough**
properties claimed \textit{Sannino et al.}

- almost conformal; “walking coupling”
- chiral symmetry breaking
  \[ \text{SU}(4) \rightarrow \text{O}(4) \]
  matching to CRMT
  \[ \sum F_\pi \]
  \textit{cf. Toublan-Vervaarshot} (99)
  the order of finite temp. rest.
- consistent with the (usually severe) constraints from EW precision measurements

\[ \text{O}(p^4) \text{ low energy coupling } L_{10} \]
- Light composite Higgs scalar

\[ M_H \sim 150 \text{ GeV} \]

\textbf{Check needed by a non-perturbative method}

\textbf{These are the problems familiar in lattice QCD, although tough}
I hope it is not ... rice cake in a picture

“E ni Kaita Mochi” in Japanese

What is the sound of one hand clapping?