Electroweak theory on the lattice with exact gauge invariance and its applications

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"Domain wall Fermions at Ten Years" @ BNL

✳ **Electroweak theory** (without SU(3) color int.)

- a chiral gauge theory with $SU(2)_L \times U(1)_Y$
- gauge symmetry breaking via Higgs mechanism
- baryon number violation due to chiral anomaly
- etc.

with LGT, what one can do to study these aspects of the electroweak theory ?

Plan of this talk

✳ a gauge-invariant construction of ElectroWeak theory

★ use of **DW, Overlap (the Ginsparg-Wilson relation)**

cf. U(1) lattice chiral gauge theory with exact gauge invariance

✳ possible applications of the lattice EW theory

★ a computation of the effect of quarks, leptons to the sphaleron rate

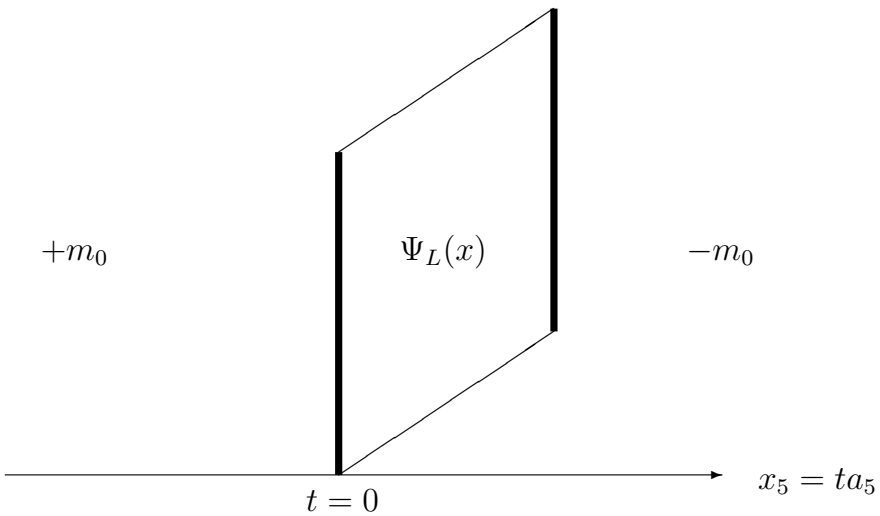
★ a construction of a model of dynamical EW symmetry breaking

★ etc.

Weyl fermions (quarks & leptons) on the lattice

chiral fermion bound to Domain wall "Wilson" fermion

Kaplan(1992)



$$\psi(x,t) = \Psi_-(x)\,\phi_0(t)$$

$$\phi_0(t) = \begin{cases} \left(1 + \left\{\sum_\mu 2\sin^2(\frac{p_\mu a}{2}) - m_0\right\}\right)^{|t|} & t \geq 0 \\ \left(1 + \left\{\sum_\mu 2\sin^2(\frac{p_\mu a}{2}) + m_0\right\}\right)^{-|t|} & t \leq -1 \end{cases}$$

only for $\boldsymbol{p_\mu}$ s.t.

$$\sum_\mu 2\sin^2\left(\frac{p_\mu a}{2}\right) - m_0 < 0 \qquad (0 < m_0 < 2)$$

chiral det. as an "overlap" of two vacua

Narayanan-Neuberger(1993)

$$H_\mp = -\ln T \text{ or } H_{\rm w}\ (\mp m_0)$$

$$\{v_i(x) \mid H_- v_i(x) = |\epsilon_i| v_i(x)\ (i = 1, \cdots, N_-)\}$$
$$\{\bar{v}_i(x) \mid \bar{v}_i(x) H_+ = |\varepsilon_i| \bar{v}_i(x)\ (i = 1, \cdots, \bar{N}_-)\}$$

$$Z = \langle v+ | v- \rangle = \det(\bar{v}_k v_j)$$

"overlap formula"

overlap Dirac op. / the GW rel.

Neuberger(1997,98)

$$D = \frac{1}{2a}\left(1 + \gamma_5 \frac{H_{\rm w}}{\sqrt{H_{\rm w}^2}}\right)$$

$$\gamma_5 D + D\gamma_5 = 2aD\gamma_5 D$$

chiral operator

Luscher; Hasenfratz, Niedermayer(1998)

$$\hat{\gamma}_5 \equiv \gamma_5(1 - 2aD) = -\frac{H_{\rm w}}{\sqrt{H_{\rm w}^2}}$$

chiral fermion

$$\hat{\gamma}_5 \psi_\pm(x) = \pm\,\psi_\pm(x)$$

$$\bar{\psi}_\pm(x)\gamma_5 = \mp\,\bar{\psi}_\pm(x)$$

Path Integral Quantization

$$\psi_-(x) = \sum_i v_i(x) c_i$$

$$\bar{\psi}_-(x) = \sum_i \bar{c}_i \bar{v}_i(x)$$

$$\{v_i(x) \mid \hat{\gamma}_5 v_i(x) = -v_i(x)\ (i = 1, \cdots, N_-)\}$$

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$$Z = \int \mathcal{D}[\psi_-]\mathcal{D}[\bar{\psi}_-]\, \mathrm{e}^{-a^4 \sum_x \bar{\psi}_- D \psi_-(x)}$$

$$= \int \prod_i dc_i \prod_j d\bar{c}_j\, \mathrm{e}^{-\sum_{ij} \bar{c}_j M_{ji} c_i}$$

$$= \det M_{ji} \qquad M_{ji} = a^4 \sum_x \bar{v}_j D v_i(x)$$

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the space of chiral fermion depends on gauge fields !!

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$\det \tilde{Q}$ **complex phase !**

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"overlap formula"

the gauge-field dependence must be fixed ...

Luscher(98)

1. **locality ?** *(cf. Hernandez, Jansen, Luscher(98))*
2. **gauge invariance ?**
3. **integrability ?** **topology of the space of gauge fields non-trivial due to Admissibility cond.**

* different situation from Dirac fermions in Vector-like theories, like QCD

variation of effective action & gauge anomaly

$$\Gamma_{\rm eff} = \ln\det(\bar{v}_k D v_j) \qquad \delta_\eta U(x,\mu) = i\eta_\mu(x) U(x,\mu)$$

$$\delta_\eta \Gamma_{\rm eff} = {\rm Tr}\left\{(\delta_\eta D)\hat{P}_- D^{-1} P_+\right\} + \sum_i (v_i, \delta_\eta v_i)$$

$$= i{\rm Tr}\omega\gamma_5\,(1-D) - i\sum_i (v_i, \delta_\omega v_i) \qquad \eta_\mu(x) = -i\nabla_\mu \omega(x)$$

gauge anomaly!

Construction of SU(2)xU(1) Electroweak theory (I)

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infinite volume case

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$$\eta_\mu(x) = \eta_\mu^{(2)}(x) \oplus \eta_\mu^{(1)}(x) \qquad U_t(x,\mu)^{(1)} = \mathrm{e}^{itA_\mu(x)} \;\; t \in [0,1]$$

$$\begin{aligned}\mathfrak{L}_\eta &= i\sum_i (v_i, \delta_\eta v_i) = \sum_x \eta_\mu(x) j_\mu(x) \\ &= i\int_0^1 dt\, \mathrm{Tr}\left\{\hat{P}_-[\partial_t \hat{P}_-, \delta_\eta \hat{P}_-]\right\} + \delta_\eta \int_0^1 dt \sum_{x\in\mathbb{Z}^4} \{A_\mu(x)\, k_\mu(x)\}\end{aligned}$$

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gauge anomaly cancellation

analysis of U(1) with SU(2) fixed

$$q(x) = \mathrm{tr}\,\{\gamma_5(1-aD)(x,x)\}|_{U^{(1)},U^{(2)}} \qquad x \in \mathbb{Z}^4$$

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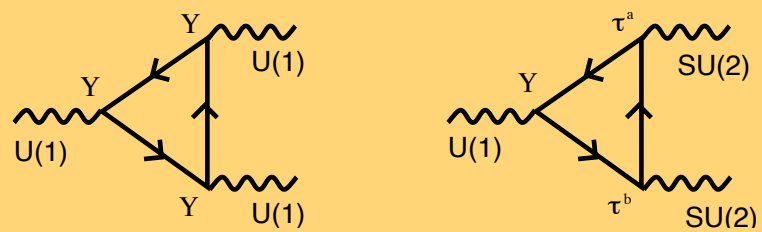

$$\sum_L Y^3 - \sum_R Y^3 = 0 \qquad \sum_{\mathrm{doublet(L)}} Y = 0, \qquad \sum_{\mathrm{singlet(R)}} Y = 0$$

$$\begin{aligned}&\sum_\alpha Y_\alpha q(x)|_{U^{(1)}\to\{U^{(1)}\}^{Y\alpha}} \\ &= \sum_\alpha Y_\alpha q(x)|_{U^{(2)}} + \sum_\alpha Y_\alpha^3 \frac{1}{32\pi^2}\epsilon_{\mu\nu\lambda\rho}F_{\mu\nu}(x)F_{\lambda\rho}(x+\hat{\mu}+\hat{\nu}) + \partial_\mu^* k_\mu(x) \\ &= \partial_\mu^* k_\mu(x)\end{aligned}$$

cf. Luscher(98)
(in 4dim.)

cf. Nakayama-YK(00)
(in 6dim.)

Construction of SU(2)xU(1) Electroweak theory (I)

infinite lattice case

- a local counter term constructed non-perturbatively
- the first gauge-invariant regularization of EW theory

  (cf. dimensional reg.)
- may be used in the perturbation theory

  ex. for computations of higher order EW contr. to g-2 (?)

Construction of SU(2)xU(1) Electroweak theory (II)

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finite volume case

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$$\| \, 1 - U_\square^{(2)} \, \| \leq \ \epsilon \qquad \| \, 1 - \{U_\square^{(1)}\}^{6Y} \, \| \leq \ \epsilon \qquad \epsilon < \frac{1}{30}$$

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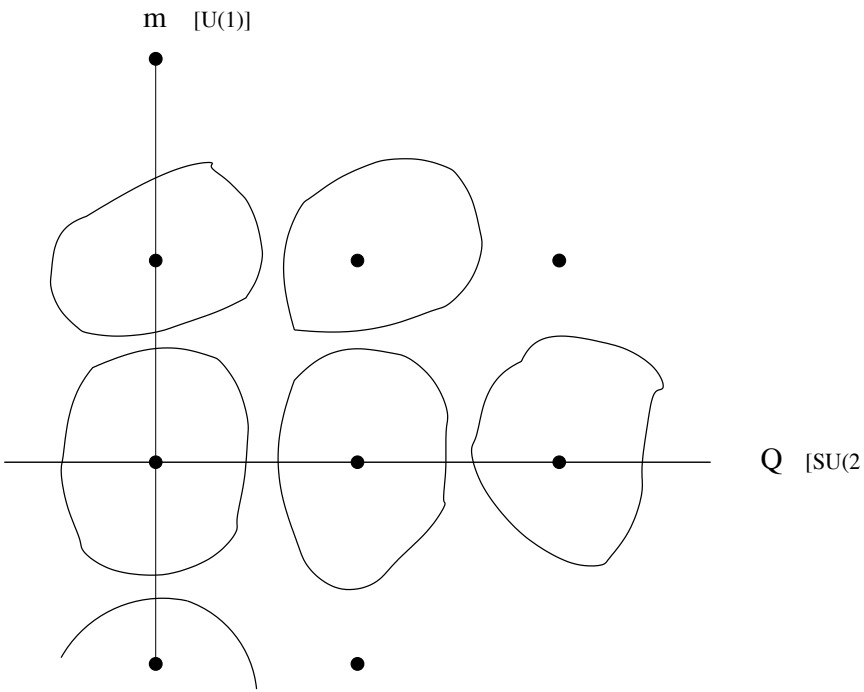

$$m_{\mu\nu} = \frac{1}{2\pi i} \sum_{s,t} \ln U_{\mu\nu}^{(1)}(x + s\hat{\mu} + t\hat{\nu})$$

$$Q = \sum_{x \in \Gamma_4} \mathrm{tr}\{\gamma_5 (1 - D)(x, x)\}|_{U^{(2)}}$$

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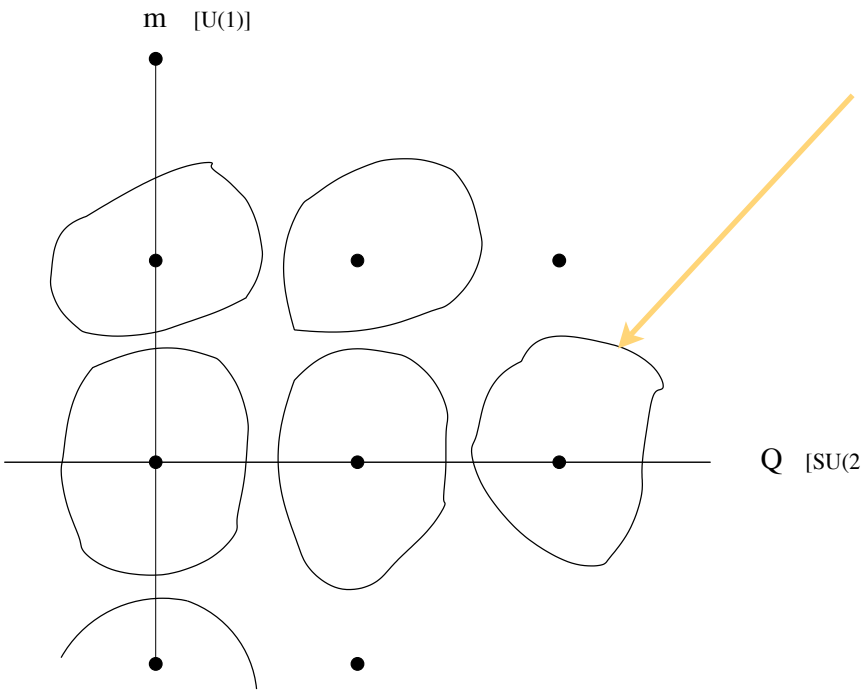
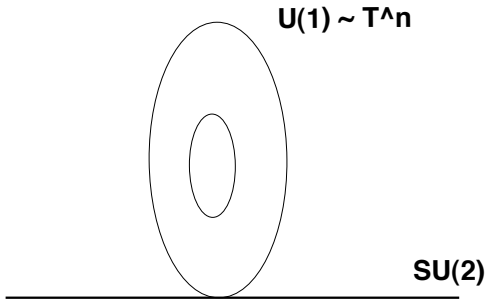


$$T^n[U(1)] \times M[SU(2)]$$



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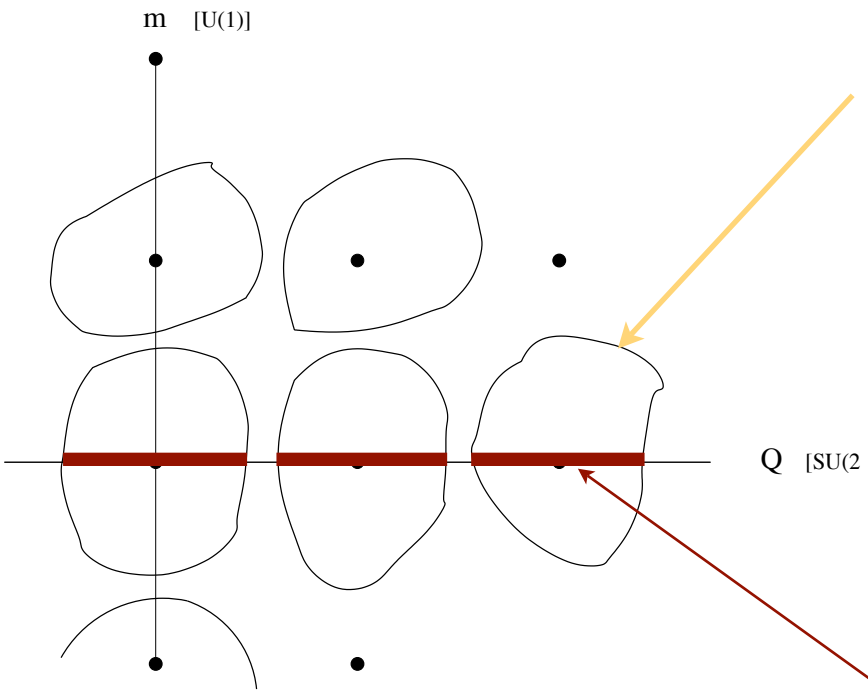

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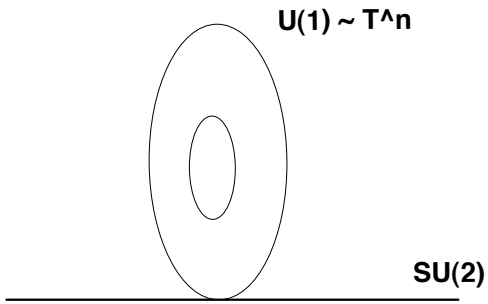

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a pair of doublets (a,b) **measure defined globally !**

$$v_j^{(a)}(x) = v_j(x)$$

$$v_j^{(b)}(x) = \left(\gamma_5 C^{-1} \otimes i\sigma_2\right) \, [v_j(x)]^*$$

cf. Nuberger(98) Bar-Campos (00)

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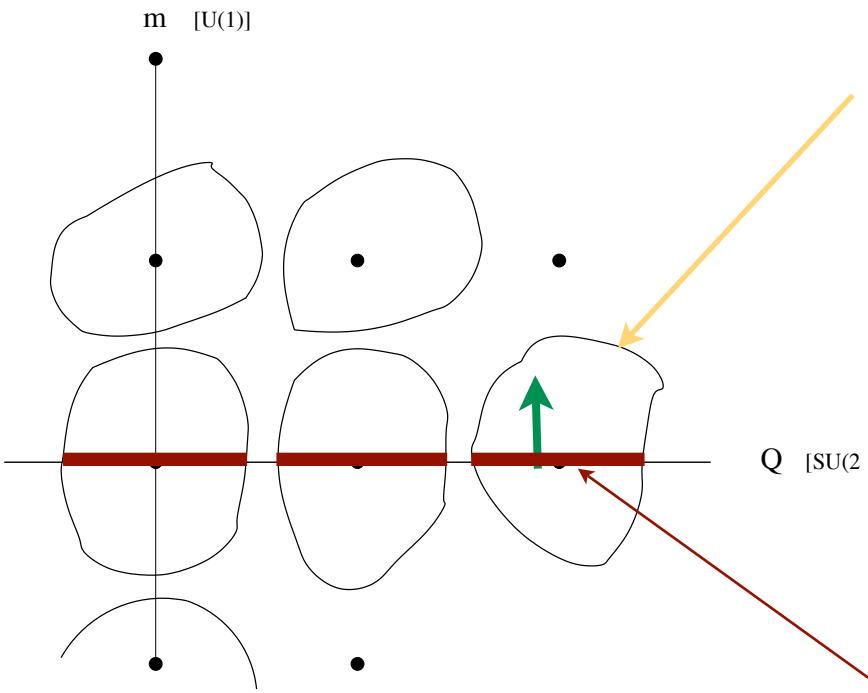

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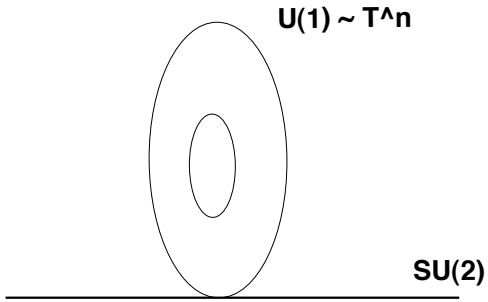

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$$v_j^{(b)}(x) = \left(\gamma_5 C^{-1} \otimes i\sigma_2\right) \, [v_j(x)]^*$$

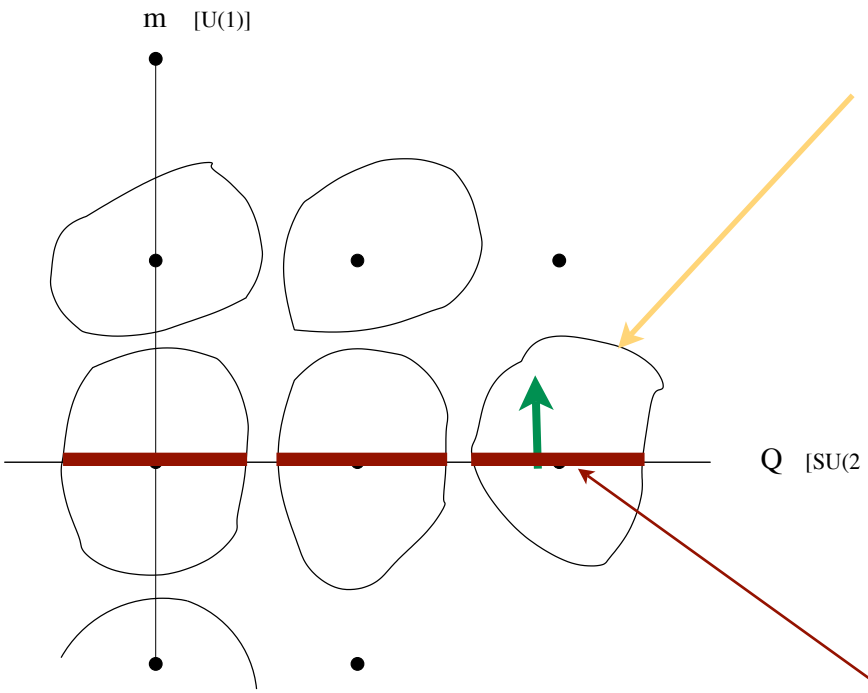
cf. Nuberger(98) Bar-Campos (00)

Construction of SU(2)xU(1) Electroweak theory (II)

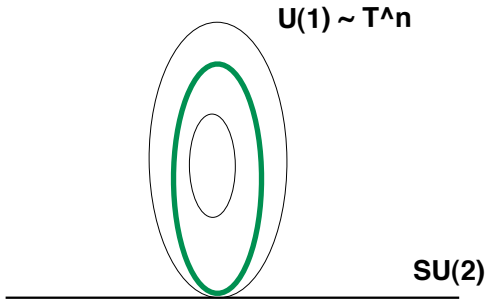
finite volume case

$$\Gamma_4 = \{x = (x_0, \cdots, x_3) \in \mathbb{Z}^4 \ \mid \ 0 \le x_\mu < L\} = \mathbb{L}^4$$

$$\| 1 - U_\square^{(2)} \| \le \epsilon \qquad \| 1 - \{U_\square^{(1)}\}^{6Y} \| \le \epsilon \qquad \epsilon < \frac{1}{30}$$



$$T^n[U(1)] \times M[SU(2)]$$



$$U_\mu(x) = \mathrm{e}^{iA_\mu^T(x)} g(x) g(x+\hat{\mu})^{-1} U_{[\mathrm{w}]}(x,\mu) V_{[\mathrm{m}]}(x,\mu)$$

$$F_{\mu\nu}(x) = \partial_\mu A_\nu^T(x) - \partial_\nu A_\mu^T(x) + \frac{2\pi m_{\mu\nu}}{L^2}$$

$$m_{\mu\nu} = \frac{1}{2\pi i} \sum_{s,t} \ln U_{\mu\nu}^{(1)}(x + s\hat{\mu} + t\hat{\nu})$$

$$Q = \sum_{x \in \Gamma_4} \mathrm{tr}\{\gamma_5 (1 - D)(x,x)\}|_{U^{(2)}}$$

a pair of doublets (a,b)

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Construction of SU(2)xU(1) Electroweak theory (II)

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$$\eta_\mu(x) = \eta_\mu^{(2)}(x) \oplus \eta_\mu^{(1)}(x) \qquad U_t^{(1)}(x,\mu) = \mathrm{e}^{it\tilde{A}_\mu(x)}\, U_{[w]}(x,\mu) \quad t \in [0,1] \ (m_{\mu\nu} = 0)$$

$$\begin{aligned}\mathfrak{L}_\eta &= i\sum_i (v_i, \delta_\eta v_i) = \sum_x \eta_\mu(x) j_\mu(x) \\ &= i\int_0^1 dt\, \mathrm{Tr}\left\{\hat{P}_-[\partial_t \hat{P}_-, \delta_\eta \hat{P}_-]\right\} + \delta_\eta \int_0^1 dt \sum_{x\in\Gamma_4} \left\{\tilde{A}_\mu(x)\, k_\mu(x)\right\} + \mathfrak{L}_\eta|_{U^{(1)}=U_{[\mathrm{w}]},U^{(2)}}\end{aligned}$$

Kadoh-YK in prep. *cf. Luscher(98)*

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gauge anomaly cancellation

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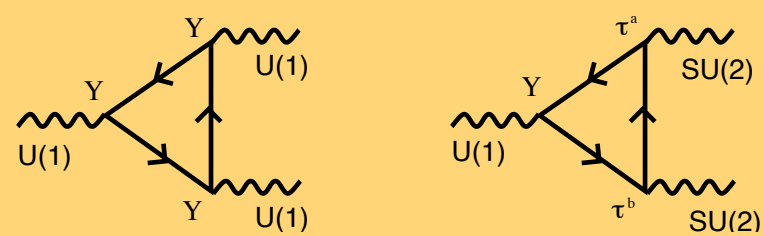

$$\sum_L Y^3 - \sum_R Y^3 = 0 \qquad \sum_{\mathrm{doublet(L)}} Y = 0, \qquad \sum_{\mathrm{singlet(R)}} Y = 0$$

$$\sum_\alpha Y_\alpha q(x)|_{U^{(1)}\to\{U^{(1)}\}^{Y\alpha}}$$

$$= \sum_\alpha Y_\alpha q(x)|_{U^{(2)}} + \sum_\alpha Y_\alpha^3 \frac{1}{32\pi^2}\, \epsilon_{\mu\nu\lambda\rho} F_{\mu\nu}(x) F_{\lambda\rho}(x+\hat{\mu}+\hat{\nu}) + \partial_\mu^* k_\mu(x)$$

$$= \partial_\mu^* k_\mu(x)$$

Suzuki et al. (01) Kadoh-Nakayama-YK(04)

Construction of SU(2)xU(1) Electroweak theory (II)

finite volume case

- covers all SU(2) topological sectors with vanishing U(1) magnetic fluxes
- global integrability can be proved rigorously

  even number of SU(2) doublets, U(1) Wilson line parts
- explicit with two simplifications *cf. U(1), Luscher (98)*

  ★ direct proof of gauge anomaly cancellation in $\mathbb{L}^4$

  ★ separate treatment of the Wilson line
- some non-perturbative applications ?

based on :
Y.~Nakayama and Y.K., Nucl. Phys. B597, 519 (2001)
D.~Kadoh, Y.~Nakayama and Y.K., JHEP 0412, 006 (2004)
D.~Kadoh and Y.K., in preparation

possible applications of lattice EW theory (I)

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a computation of the effect of quarks & leptons to the sphaleron rate at finite temp. (at one-loop)

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introduction of Higgs field
& Yukawa-couplings

$$S_{EW} = S_G + S_F + \sum_x \{\nabla_\mu \phi^\dagger \nabla_\mu \phi + V(\phi)\}$$
$$- \sum_x \Big\{ \, y_t \, \bar{Q}_- \tilde{\phi} \, t_+(x) + y_b \, \bar{Q}_- \phi \, b_+(x) + c.c. \Big\}$$

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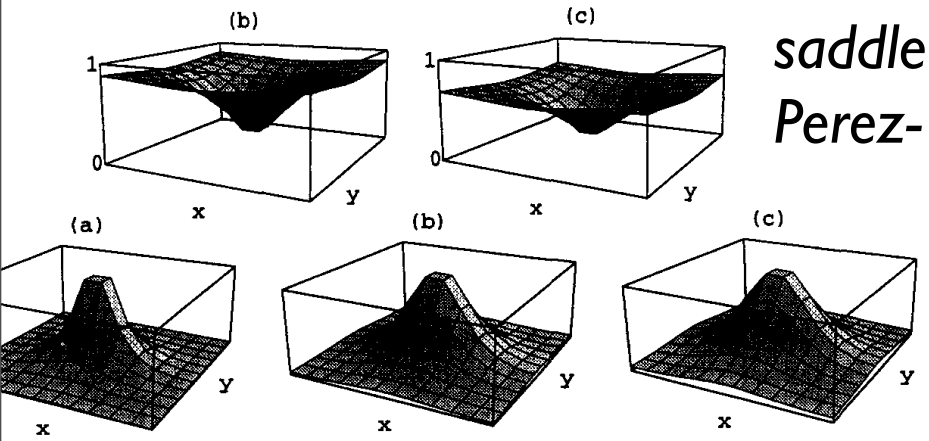
sphaleron on the lattice

$$U_\mu^{(2)}(x), U_\mu^{(1)}(x), \phi(x) \, (x \in \mathbb{L}^3)$$



saddle point cooling
Perez- van Baal (96)

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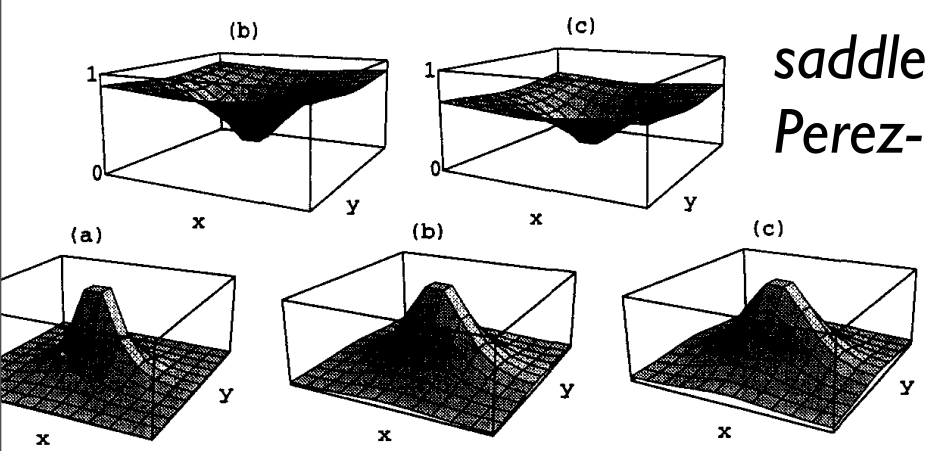

saddle point cooling
Perez- van Baal (96)

fermion fluctuation det.

cf. Moore (96)

$$\kappa_F(v, \lambda, y_t, \cdots) \equiv \prod_{q,l} \prod_{\omega_n} \det \mathcal{M} / \det \mathcal{M}_0$$

$$\mathcal{M}_t = \begin{pmatrix} (\bar{v}_k D v_j) & y_t(\bar{v}_k \tilde{\phi} u_j) \\ y_t(\bar{u}_k \tilde{\phi}^\dagger v_j) & (\bar{u}_k D u_j) \end{pmatrix}$$

- sum over matsubara freq.
- one-loop renormarizations
- dependence on the Higgs, Yukawa coupling
- comparison to other methods

cf. Bodeker et. (00)

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a construction of a model of dynamical EW symmetry breaking

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$SU(2)_{TC}$ "minimal walking technicolor model"

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4 x $SU(2)_{TC}$ adjoint	4 x $SU(2)_{TC}$ singlet
λ_1^a	ψ_1
λ_2^a	ψ_2
λ_3^a	ψ_3
λ_4^a	ψ_4

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$$\mathbf{SU(2)_L \times SU(2)_R} \longrightarrow \mathbf{SU(2)_V}$$

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with DW / overlap fermions, lattice construction is possible if Y(3,2)=0, just like EW theory

cf. N=1 SYM *Nishimura(97)* *Neuberger(98)* *Kaplan(99)*

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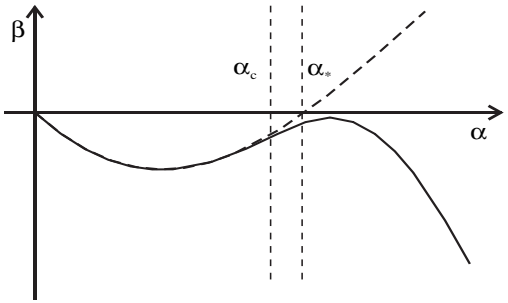
if g_Y=0, numerical simulation is possible !

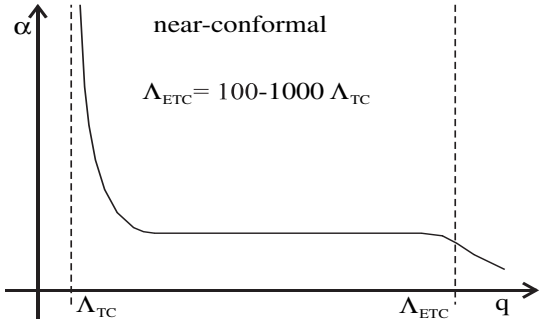
**global issue; Neuberger(98), Bar(02)*

properties claimed *Sannino et al.*

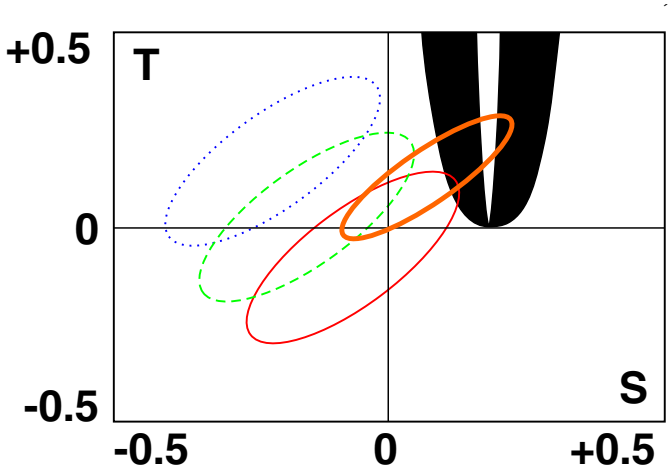


- almost conformal; "walking coupling"
- chiral symmetry breaking

  SU(4) ---> O(4)



- consistent with the (usually severe) constraints from EW precision measurements

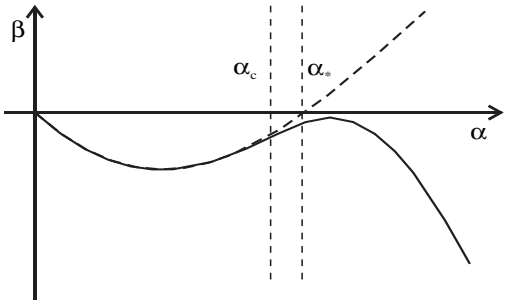


- Light composite Higgs scalar

  $M_H \sim 150$ GeV

Check needed by a non-perturbative method

properties claimed *Sannino et al.*



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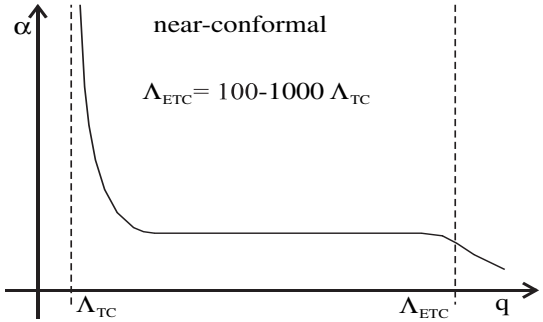

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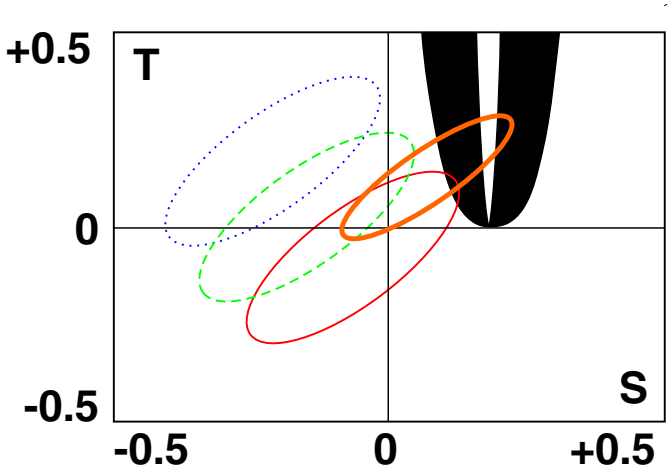

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These are the problems familiar in lattice QCD, although tough

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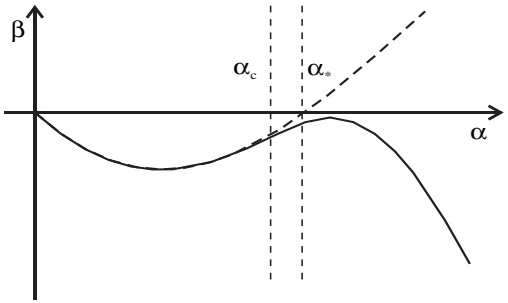

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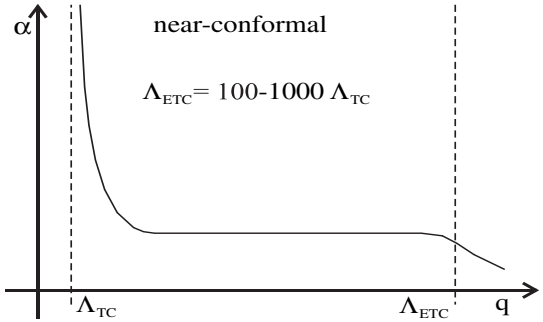


- chiral symmetry breaking

  SU(4) ---> O(4)

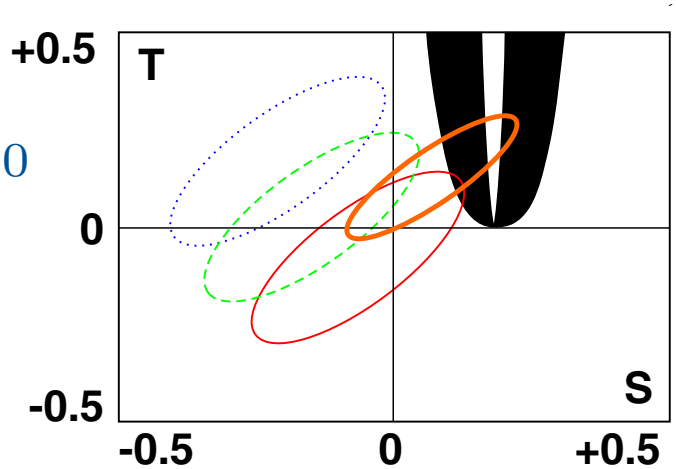
  matching to CRMT Σ F_π

  cf. Toublan-Vervaarshot (99)

  the order of finite temp. rest.

- consistent with the (usually severe) constraints from EW precision measurements

  O(p^4) low energy coupling L_{10}



- Light composite Higgs scalar

  M_H ~ 150 GeV spectrum; neutral scalar, rho, ...

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I hope it is not ...

rice cake in a picture

"E ni Kaita Mochi" in japanese

What is the sound of one hand clapping?