An Introduction to Resummation and Intrinsic p_T

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The influence of partonic transverse momentum on single-particle inclusive cross sections has been a subject of interest for some time. The standard factorized cross section for $A+B \to C+X$ is a convolution in momentum fractions only,

$$E_C \frac{d\sigma_{AB\to C+X}}{d^3 p_C} = \sum_{abc} f_{a/A}(x_a,\mu^2) \otimes f_{b/B}(x_b,\mu^2) \otimes D_{C/c}(z,\mu^2) \otimes \hat{\sigma}(x_a p_A, x_b p_B, p_C/z, \mu) . \quad (1)$$

Transverse momenta $k_T < \mu$ (relative to the incoming and outgoing directions) are absorbed into parton distributions f and fragmentation functions D, while larger transverse momenta are included in the partonic hard scattering $\hat{\sigma}$. Corrections associated with this organization of states appear systematically in higher-order corrections to $\hat{\sigma}$. An important example is direct photon production, which is of special relevance to the determination of the gluon distribution. Data on direct, or prompt, photons is available over a wide range of $\sqrt{s}$ and p_T, generally with isolation cuts at collider energies. For some time, many, although not all [1], calculations of direct photon production based on Eq. (1) have fallen significantly below the data at low p_T, suggesting the need for a "k_T-smearing", perhaps reflecting the intrinsic transverse momentum of partons in hadrons [2].

An alternative factorization including transverse momenta is [3]

$$\begin{aligned} E_\ell \frac{d\sigma_{AB\to\gamma(\ell)+X}}{d^3\ell} &= \frac{1}{S^2} \sum_{ab} \int dx\, dy\, d^2\mathbf{q}\, d^2\mathbf{q}'\, \mathcal{P}_{a/A}(x, p_A\cdot n, \mathbf{q})\, \mathcal{P}_{b/B}(y, p_B\cdot n, \mathbf{q}') \\ &\quad \times \Omega_{ab\gamma}\left(\frac{s'}{\mu^2}, \frac{t'}{\mu^2}, \frac{u'}{\mu^2}, \alpha_s(\mu^2)\right) , \end{aligned} \quad (2)$$

with a modified hard-scattering function Ω, which depends on kinematic variables s', t', u' that include parton transverse momenta. The wave functions $\mathcal{P}$ are related to the light-cone wave functions f in Eq. (1) by

$$\mathcal{P}_{a/A}(x, p\cdot n, \mathbf{q}) = \sum_c \int d\lambda\, f_{c/A}(\lambda,\mu^2) \int d^2\mathbf{b}\, e^{-i\mathbf{q}\cdot\mathbf{b}}\, C_{a/c}\left(\frac{x}{\lambda}, |\mathbf{b}|\mu, \alpha_s(\mu^2)\right)\, e^{-S_a(|\mathbf{b}|, p\cdot n)} , \quad (3)$$

where the b-dependent exponent, which matches perturbative and nonperturbative contributions [4], has the effect of introducing a Gaussian k_T-smearing into the cross section (2). n^μ is an arbitrary gauge-fixing vector, so that $p\cdot n$ increases with the center-of-mass energy. The exponent S produces a k_T-smearing with a width that increases like the logarithm of s, as suggested by the analyses of [2].

References

[1] P. Aurenche *et al.*, Phys. Rev. D39, 3275 (1989).

[2] J.F. Owens, Rev. Mod. Phys. 59, 465 (1987); J. Huston *et al.*, Phys. Rev. D51, 6139 (1995); H. Baer and M.H. Reno, Phys., Rev. D54, 2017 (1996); A.D. Martin *et al.*, hep-ph/9803445.

[3] H.-L. Lai and H.-n. Li, hep-ph/9802414; E. Laenen, G. Oderda and G. Sterman, in preparation.

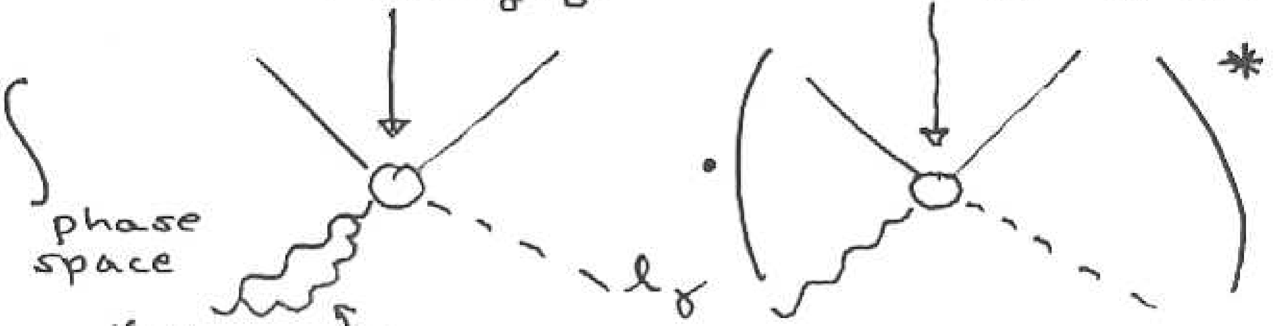
[4] J.C. Collins and D.E. Soper, Nucl. Phys. B193, 381 (1981). J.C. Collins, D.E. Soper and G. Sterman, Nucl. Phys. B250, 199 (1985).

Can PT help account for these effects?
Yes, take...

A closer look at higher orders:
'hidden logs of k_T'

At NLO: soft glue in $\hat{\sigma}_{qg \to \gamma X}$

$M_{LO}(x_g P_B)$ $\quad\quad$ $M_{LO}(x_B P_B)$



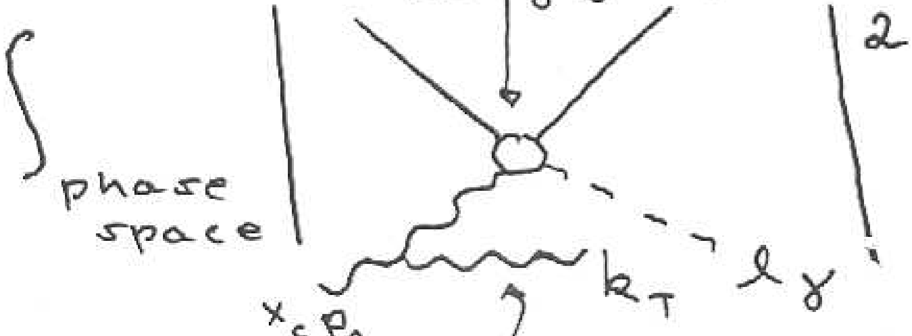
$\int_{\text{phase space}}$ $\cdot$ $(\ )^*$

$x_g P_A$ $\quad$ ℓ_γ

$- \alpha_s C_A \int \frac{dk'^2_T}{k'^2_T} \ln k'^2_T$

$C_A = N_c = 3$ $\quad$ soft

$M_{LO}(x_g P_B - k_T)$



$\int_{\text{phase space}} \Big| \quad \Big|^2$

$x_g P_A$ $\quad$ k_T $\quad$ ℓ_γ

$\alpha_s C_A \int \frac{dk^2_T}{k^2_T} \ln k^2_T$

DY: if measure $Q_T \sim k_T \sim Q$ fixed, and no cancellation
$\to \frac{1}{k_T^2} \times$ logs

sum:

$$- \alpha_s C_A |M_{LO}(x_g P_A)|^2 \int \frac{dk'^2_T}{k'^2_T} \ln k'^2_T$$

cancellation of divergence at $k_T \to 0$

$$+ \alpha_s C_A \int \frac{dk^2_T}{k^2_T} \ln k^2_T \, |M_{LO}(x_g P_A - k_T)|^2$$

$k_T \to 0$ behavior approximately:

$$\alpha_S C_A \int dk_T^2 \ln k_T^2 \left.\frac{\partial}{\partial k_T^2}\right|_{k_T^2=0} |M_{LO}(x_g p_A - k_T)|^2$$

finite integral

finite as $k_T \to 0$

- emission: redistribution of partonic 'beam'
- virtual correction: depletion of forward 'beam'
- unitarity: $\to$ sum is finite
- in NLO: calculation the k_T^2 integral is done exactly
- beyond NLO: form of $k_T^2 \to 0$ behavior known to all orders

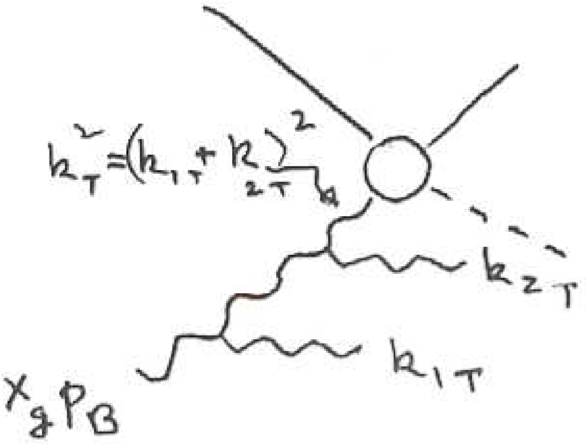
'k_T-resummation'

leading logs: $\alpha_S C_A \ln k_T^2$

$$\to \alpha_S C_A \ln k_T^2 \, e^{-\alpha_S C_A \frac{1}{2} \ln^2 \frac{k_T^2}{Q^2}}$$

$Q \sim l_T$

Why exponentiation?



- $k_{1T} \ll k_{2T} \rightarrow$ second emission independent of first

- independence $\rightarrow$

  (change in) $|M|^2 \propto |M|^2$

  change with k_T scale $\sim d/d\ln k_T^2$

Effect of running coupling

$\frac{1}{2}\alpha_s C_A \ln^2 k_T^2/Q^2$

$$\rightarrow C_A \int_{k_T^2}^{Q^2} \frac{d\mu^2}{\mu^2}\, \alpha_s(\mu^2) \ln \mu^2/Q^2$$

in QCD: $\alpha_s(\mu^2)$ suppresses larger transverse momentum scales

.. k_T Resummation

In NLO:

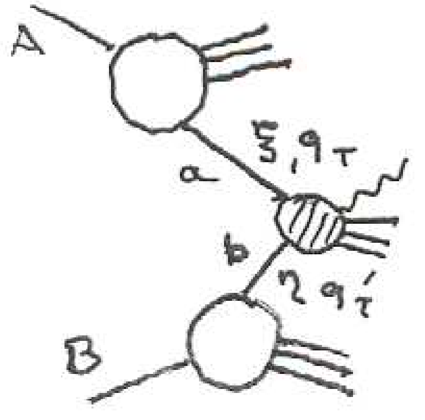
$$\int \frac{dk_T^2}{k_T^2} \to \ln(1-w)$$

$w \to 1$ for exact $qg \to q\gamma$ elastic scattering

It is possible to resum in k_T (organize all $\alpha_s^m \ln^n k_T^2$)

- higher order corrections to recoil of observed particle to soft radiation

Need alternative factorization



$\gamma(Q)$, for example (but also $p_Q, p_{\bar{Q}}$)

$$E_\ell \frac{d^3\sigma_{AB}}{d^3\ell} = \frac{1}{S^2} \sum_{ab} \int d\xi\, d\eta\, d^2q\, d^2q' \cdot P_{a/A}(\xi, q_T)\, P_{b/B}(\eta, q_T') \cdot \Omega_{ab}\left(\frac{s'}{\mu^2}, \frac{t'}{\mu^2}, \frac{u'}{\mu^2}, \alpha_s(\mu^2)\right)$$

s', t', u': full on-shell kinematics with q_T, q_T'

E. Laenen, G. Oderda, G. S[terman]; Li, Lai

$$\Omega_{ab} = \frac{S^2}{2s'} \left| M_{\text{Born}}(s't'u') \right|^2 \left(\delta(s'+t'+u') + \theta(\alpha_s) \right)$$

$$x = \frac{1}{yS + T} \left[y(-U) - 2(q+q')\cdot l_T + \cdots \right]$$

hadronic

$\mathcal{P}$'s include evolution (Collins–Soper 81)

$$\mathcal{P}(x,q) = \int d\lambda\, \phi(\lambda,\mu^2) \cdot \int d^2b\, e^{-iq\cdot b}\, C\left(\frac{\lambda}{x}, b\mu\right) \cdot \exp[-S(b,Q)]$$

- expand S in α_s, do b integral
  → recover normal factorization order-by-order, or:
- possibility of incorporating NP q_T via Collins-Soper exponent (NP B193, 381 (1981))

$$S(b) \approx -\int_{c_1 b/\sqrt{(1+b^2/b_0^2)}}^{c_2 M} \frac{d\bar{\mu}}{\bar{\mu}} \left[\ln \frac{c_2^2 M^2}{\mu^2} A(\alpha_s(\mu)) + B(\alpha_s(\mu)) + b^2 (\ln Q) F_2(b) + b^2 F_1(b,x) \right]$$

underlines match resummed series with power corrections

Schematic results:

$S(b,Q) =$ perturbation theory (PT) + non-perturbative (NP)

PT: sums $\ln^n k_T^2/Q^2$

cutoff in IR (b_0 above)

'parton shower'

NP: induced by PT

also b_0-dependent

Gaussian smearing!

$$e^{-b^2 \ln Q\, F_2 + b^2 F_1} \xrightarrow[\text{Fourier transform}]{} e^{-q_T^2/4(F_2 \ln Q + F_1)}$$

Q-dependent width

$$\langle q_T^2 \rangle \sim F_2 \ln Q + F_1 + \text{corrections}$$

Full result: convolution of resummed PT, NP with LO + modified NLO + ...

Gaussian, dominates at low E and grows with P_T