ODDBALL REGGE TRAJECTORY AND A LOW ODDERON INTERCEPT

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Odderon Searches
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- **Effective QCD gluon Hamiltonian**

$$H^g_{eff} = Tr \int d\mathbf{x} \, [\mathbf{\Pi}^a(\mathbf{x}) \cdot \mathbf{\Pi}^a(\mathbf{x}) + \mathbf{B}^a_A(\mathbf{x}) \cdot \mathbf{B}^a_A(\mathbf{x})] \\ - \frac{1}{2} \int d\mathbf{x} d\mathbf{y} \, \rho^a_g(\mathbf{x}) V(\mathbf{x}, \mathbf{y}) \rho^a_g(\mathbf{y})$$

- **color charge density** $a, b, c = 1, 2, ...8$

$$\rho^a_g(\mathbf{x}) = f^{abc} \mathbf{A}^b(\mathbf{x}) \cdot \mathbf{\Pi}^c(\mathbf{x})$$

- **Abelian magnetic fields**

$$\mathbf{B}^a_A = \boldsymbol{\nabla} \times \mathbf{A}^a$$

- **gauge fields** $\mathbf{A}^a$**, conjugate momenta** $\mathbf{\Pi}^a = -\mathbf{E}^a$

$$\mathbf{A}^a(\mathbf{x}) = \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} [\mathbf{a}^a(\mathbf{q}) + \mathbf{a}^{a\dagger}(-\mathbf{q})] e^{i\mathbf{q}\cdot\mathbf{x}}$$

$$\mathbf{\Pi}^a(\mathbf{x}) = -i \int \frac{d\mathbf{q}}{(2\pi)^3} \sqrt{\frac{\omega_k}{2}} [\mathbf{a}^a(\mathbf{q}) - \mathbf{a}^{a\dagger}(-\mathbf{q})] e^{i\mathbf{q}\cdot\mathbf{x}}$$

- **bare gluon Fock operators**

$$a^a_\mu(\mathbf{q}) \;\; \textbf{\textit{momentum}} \;\; \mathbf{q} \;, \;\; \textbf{\textit{spin}} \; \mu = 0, \pm 1$$

- **Coulomb gauge transverse condition**

$$\boldsymbol{\nabla} \cdot \mathbf{A}^a = 0 \Longrightarrow \mathbf{q} \cdot \mathbf{a}^a(\mathbf{q}) = (-1)^\mu q_\mu a^a_{-\mu}(\mathbf{q}) = 0$$

- **Bogoliubov-Valatin canonical transformation (BCS rotation) to dressed gluon operators**

$$\alpha_\mu^a(\mathbf{q}) = \cosh\Theta(q)\, a_\mu^a(\mathbf{q}) + \sinh\Theta(q)\, a_\mu^{a\dagger}(-\mathbf{q})$$

- **satisfy the transverse commutation relations**

$$[\alpha_\mu^a(\mathbf{q}), \alpha_\nu^{b\dagger}(\mathbf{q}')] = \delta_{ab}(2\pi)^3\delta^3(\mathbf{q}-\mathbf{q}')D_{\mu\nu}(\mathbf{q})$$

$$D_{\mu\nu}(\mathbf{q}) = \left(\delta_{\mu\nu} - (-1)^\mu \frac{q_\mu q_{-\nu}}{q^2}\right)$$

- **PQCD vacuum** $|0> \Longrightarrow$ **BCS vacuum** $|\Omega>_{\mathrm{BCS}}$

- **minimize ground state energy**

$$\frac{\delta}{\delta\Theta}\left(\frac{\langle\Omega|H_{eff} - E|\Omega\rangle}{\langle\Omega|\Omega\rangle}\right) = 0$$

- **generates a gap equation**

$$\omega(q)^2 = q^2 - \frac{3}{4}\int \frac{d\mathbf{k}}{(2\pi)^3}\hat{V}(|\mathbf{q}-\mathbf{k}|)(1+(\hat{\mathbf{q}}\hat{\mathbf{k}})^2)\left(\frac{w(k)^2 - w(q)^2}{w(k)}\right)$$

- **gluon self-energy**

$$\omega(q) = qe^{-2\Theta(q)}$$

- **confinement via Cornell type potential**

$$V(|\mathbf{x}-\mathbf{y}|) = \sigma|\mathbf{x}-\mathbf{y}| - \frac{\alpha_s}{|\mathbf{x}-\mathbf{y}|}$$
$$\sigma = 0.18(\boldsymbol{from\, lattice}) \;\; \alpha_s = 0.42$$

- **solve gap eq. for gluon constituent mass**

$$m_g \equiv \omega(0) \cong 0.8GeV$$

- **predicted gluon condensate (cooper pairs)**

$$\langle \alpha G^a_{\mu\nu} G^{\mu\nu}_a \rangle = (433\ MeV)^4$$
$$[\boldsymbol{agrees\, with\, lattice}\ (441\ MeV)^4]$$

- **TDA two gluon glueball wavefunction**

$$|\Psi^{JPC}_{LS}\rangle = \sum_{am_1m_2} \int \frac{d\mathbf{k}}{(2\pi)^3} \Phi^{JPC}_{LSm_1m_2}(\mathbf{k}) \alpha^{a\dagger}_{m_1}(\mathbf{k}) \alpha^{a\dagger}_{m_2}(-\mathbf{k}) |\Omega\rangle$$

- **solve for glueball mass** M_{JPC}

$$H|\Psi^{JPC}_{LS}\rangle = M_{JPC}|\Psi^{JPC}_{LS}\rangle \;\; [\boldsymbol{agree\, with\, lattice}]$$

- **predict a Regge trajectory close to the pomeron**

$$\alpha_P \approx .25t + 1$$

- **variational three gluon glueball wavefunction**

$$|\Psi^{JPC}\rangle = \int d\mathbf{q}_1 d\mathbf{q}_2 d\mathbf{q}_3 \delta(\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3)$$
$$F^{JPC}_{\mu_1\mu_2\mu_3}(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3) C^{abc} \alpha^{a\dagger}_{\mu_1}(\mathbf{q}_1)\alpha^{b\dagger}_{\mu_2}(\mathbf{q}_2)\alpha^{c\dagger}_{\mu_3}(\mathbf{q}_3)|\Omega\rangle_{\rm BCS}$$

- **Bose statistics** $C^{abc} = f^{abc}$ **(**$C=1$**) or** d^{abc} **(**$C=-1$**)**
- **variational equation for the** J^{PC} **glueball**

$$\frac{\langle \Psi^{JPC}|H^g_{eff}|\Psi^{JPC}\rangle}{\langle \Psi^{JPC}|\Psi^{JPC}\rangle} = M_{JPC} \ \ [\boldsymbol{agree\ with\ lattice}]$$

- **predict leading Regge trajectory (odderon)**

$$\alpha_O^{eff} = .23t - 0.88 \ [\textbf{note low intercept}]$$

- **compare to nonrelativistic constituent model**

$$H_M = \sum_i \frac{\mathbf{q}_i^2}{2m_g} + V_0 + \sum_{i<j}[\sigma r_{ij} - \frac{\alpha}{r_{ij}} + V_{ss}\mathbf{S}_i\cdot\mathbf{S}_j]$$

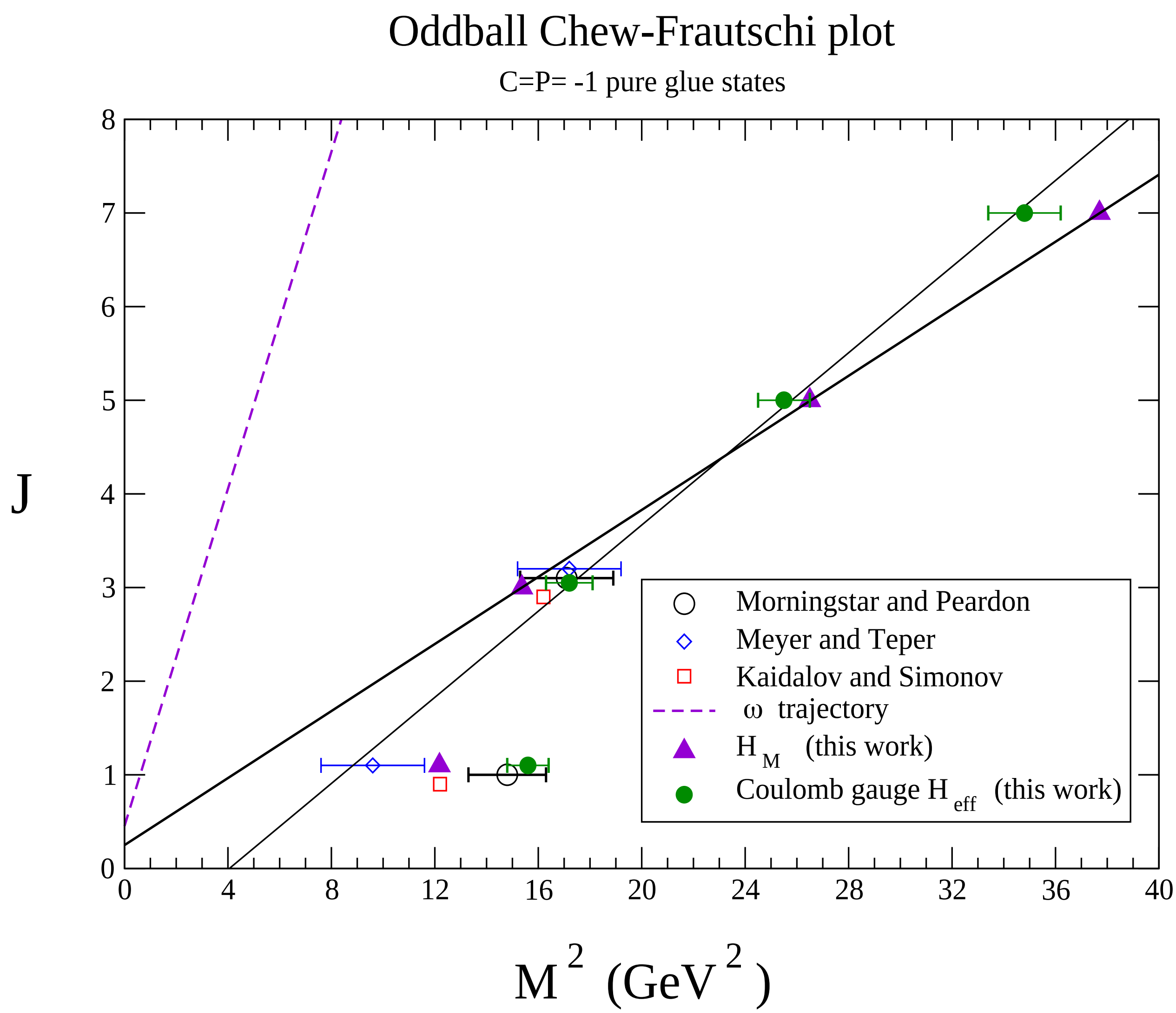
- **use** $q\bar{q}$ **funnel potential**

$V_0 =$ **-.9 GeV,** $\alpha =$ **.27,** $\sigma =$ **.25 GeV**^{2} $m_g =$ **.8 GeV and adjust** $V_{ss} \to 0.085$ **GeV to fit pomeron**

- **predict leading Regge trajectory (odderon)**

$$\alpha_O^{M} = .18t + 0.25 \ [\textbf{note low intercept}]$$

lattice, H^g_{eff} and H_M odderons vs. ω trajectory



SUMMARY

- lattice, H^g_{eff} and H_M models predict an odderon
- starts with 3^{--} (not 1^{--}, need lattice 5^{--})
- odderon slope is similar to pomeron
- odderon intercept is low, below 0.5
- search for odderon where ω trajectory is not dominant (examine $\frac{d\sigma}{dt}$, not $\sigma_{total} \propto s^{\alpha_\omega(0)-1}$)