Some aspects of the perturbative QCD Odderon

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Outline

Framework: Regge limit in pQCD and LLA (small x physics)

Weak field regime $\longrightarrow$ linear evolution in rapidity.

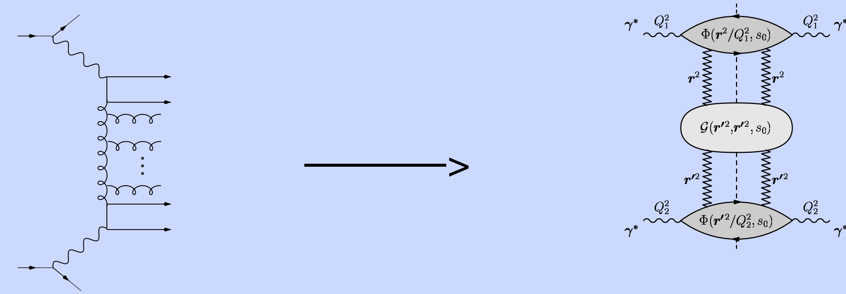
- The dynamics associated to the exchange of many reggeized gluon is strongly affected by the *gluon reggeization*.
- Möbius vs AF space in relation to the bootstrap property.
- Evolution kernel for many reggeized gluons: BKP kernel
  Bootstrap, Odderon and general descendent states.
- NLL Odderon?

BFKL resummation

<u>High energy factorization:</u>

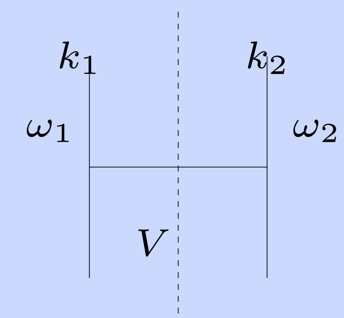
Impact factors $+$ *BFKL Green's function*

Resummation



$$\boxed{G = \sum_n a_n^{LL} \left(\alpha_s \ln s\right)^n + \sum_n a_n^{NLL} \alpha_s \left(\alpha_s \ln s\right)^n + \cdots}$$

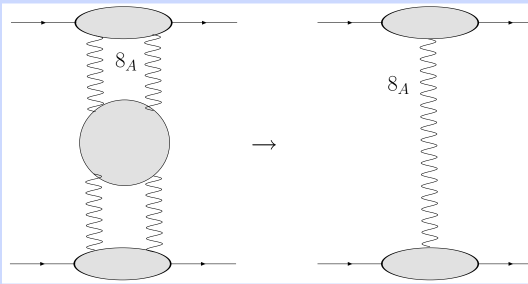
In general one has <u>non forward</u> amplitudes ($t \neq 0$)
G is associated to the linear BFKL kernel K, local in rapidity, and describes an evolution in the total rapidity interval $y \approx \ln s/s_0 \approx \ln \frac{1}{x}$:



$$\frac{\partial}{\partial y} G = \delta + K\, G\,, \qquad K = -\omega_1 - \omega_2 - V$$

Octet kernel non forward (LL, NLL) : Bootstrap equation from <u>s-channel unitarity</u> for the color octet state in the t-channel $\rightarrow$ gluon reggeizzation

$$\boxed{K^{(8)}_{BFKL} \psi_g = -\omega(\boldsymbol{k}_1 + \boldsymbol{k}_2) \psi_g}$$



Up to NLL, the impact factors in the octet channel (not related to physical scattering) are all proportional to the reggeized gluon state ψ_g

Boostrap strongly influences the dynamics

LL case much more investigated (LLA, GLLA, EGLLA)

- At LL octet 8_A and 8_S states have similar behavior: bootstrap works for both in the same way.

- Impact factor side: depending on the constituents of the external particles, n gluon t-channel states may be reduced. Fewer impact factors are fundamental.

- Green's function dynamics may be reduced, look at kernel eigenstates. First case seen is the Odderon channel which starts from 3 interacting gluons. Large N_c multigluon states are also affected. There is a chain of descendent states: physical reason is reggeization.

- Coupling of onium $(q\bar{q})$ to $2, 3, 4, 5, 6$ gluon t-channel states has been analyzed. It is reduced by bootstrap.
  Verteces do appear naturally: $V_{2\to 4} \to$ 3P and $V_{2\to 6} \to$ POO.

- There is reggeization also at NLL. There are some results but more investigation is needed. Difference between 8_A and 8_S states. Does 8_S reggeize? What about Odderon and multigluon states in NLLA?

- Real hadrons are more complicated. No more pertubative tools available!
  Anyway bootstrap is expected to survive at non-perturbative level at high energies.
  Will it give dynamical insights?

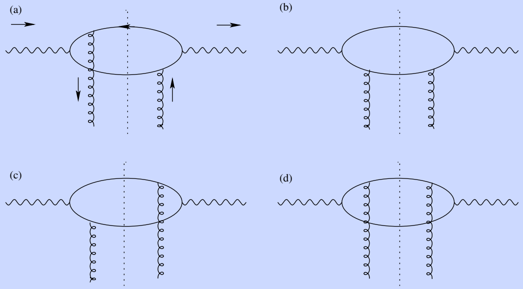
BFKL kernel in LLA

The BFKL kernel in the color R-representation $K_2^{(R)} = H_{ij}^{(R)} = -\omega_i - \omega_j - V_{ij}^{(R)}$
($V_{ij}^{(1)} = 2V_{ij}^{(8a)} = 2V_{ij}^{(8s)}$ and $\omega_i = \omega(\boldsymbol{k}_i)$)
In the octet state the bootstrap relation is $H_{ij}^{(8)} \otimes \psi_g = -\omega(\boldsymbol{q})\psi_g$ with $\psi_g = \psi_g(\boldsymbol{q}) \sim 1$

$$\boxed{H_{ij}^{(1)} \otimes 1 = -2\omega(\boldsymbol{q}) + \omega(\boldsymbol{k}_i) + \omega(\boldsymbol{k}_j) = \frac{\alpha_s N_c}{2\pi} \log \frac{\boldsymbol{q}^4}{\boldsymbol{k}_i^2 \boldsymbol{k}_j^2}}$$

where $q = k_i + k_j$.
In order to construct the Green's function G one has to study the Schrödinger like equation. The physical amplitudes are constructed from matrix elements between colorless impact factors

$$A = \langle \Phi_A | G | \Phi_B \rangle = \langle \Phi_A | e^{y H_{12}} | \Phi_B \rangle$$



Gauge invariance and Möbius space: The colorless impact factors satisfy a Ward identity in the gluon lines. $\Phi(\boldsymbol{k}_1, \boldsymbol{k}_2) \to 0\,,\, \boldsymbol{k}_i \to 0$
In particular at the Born level the coordinate form of the two gluon propagator $1/\boldsymbol{k}_1^2 \boldsymbol{k}_2^2$ can be transformed, adding terms $\sim \delta^{(2)}(\boldsymbol{k}_i)$:

$$\log |\rho_{11'}|^2 \log |\rho_{22'}|^2 \to 2 \log \frac{|\rho_{11'}|\,|\rho_{22'}|}{|\rho_{12'}|\,|\rho_{21'}|} \log \frac{|\rho_{11'}|\,|\rho_{22'}|}{|\rho_{12}|\,|\rho_{1'2'}|}\,,$$

explicit invariant under a coordinate Möbius transformation.

BFKL kernel: Möbius vs AF representation

In the Möbius (M) space the color singlet BFKL kernel and its eigenstates can be redefined according to Möbius invariance (h are the conformal weights):

$$\tilde{H}_{12} = h_{12} + \bar{h}_{12} = \sum_h \int d^2\rho_0 \, \frac{N_h}{|\rho_{12}|^2} \langle \rho | h \rangle \, \chi_h \, \langle h | \rho' \rangle$$

$$\langle \rho | h \rangle = E_h(\rho_{10}, \rho_{20}) = \left(\frac{\rho_{12}}{\rho_{10}\rho_{20}} \right)^h \left(\frac{\rho_{12}^*}{\rho_{10}^*\rho_{20}^*} \right)^{\bar{h}}$$

- But $\tilde{H}_{12} \otimes \psi_g = 0$! It comes from: $\langle h | \psi_g \rangle = 0$.

  The reason is the presence of the term added thanks to gauge freedom.

$$\langle \rho | h \rangle = \langle \rho | h^A \rangle + \langle \rho | h^\delta \rangle$$

- Let's go back (removing the δ's) and define (on the Analytic Feynman (AF) space)

$$H_{12} = \sum_h \tilde{N}_h \, | h^A \rangle \, \chi_h \, \langle h^A |$$

which is meromorphic. Well behavied on both colorless and colored impact factors. Compatible with the bootstrap relation.

Bartels, Lipatov, Salvadore, G.P.V. (2005)

M space - AF space

- The space of function considered is associated to the scalar product

$$\langle f|g\rangle = \int d\mu f^* g\,, \quad d\mu = \boldsymbol{k}_1^2\boldsymbol{k}_2^2\delta^{(2)}(\boldsymbol{q}-\boldsymbol{k}_1-\boldsymbol{k}_2)d^2\boldsymbol{k}_1 d^2\boldsymbol{k}_2$$

This is the AF space, associated to functions with propagators (non amputated). All the objects are meromorphic in the momenta.

- We define the mapping between AF and M spaces spanned by the $|\rho^A\rangle$ and $|\rho\rangle$ basis.

$$\Phi : AF \to M, \quad \Phi = 1 + \frac{F}{1-F}P\,,$$

$$\Phi^{-1} : M \to AF, \quad \Phi^{-1} = 1 - FP\,,$$

$$F = 2^{1-h-\bar{h}}\,, \quad P = P^2$$

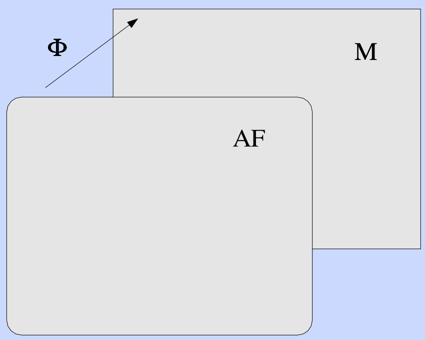
$$Pf(x_1,x_2) = \frac{1}{2}\left[f(x_1,-x_1) + f(-x_2,x_2)\right]$$

such that $\Phi\Phi^{-1} = I_M$ and $\Phi^{-1}\Phi = I_{AF}$

One can extend the action of any operator

In particular $\boxed{\vec{M}_r^{AF} = \Phi^{-1}\vec{M}_r\Phi}$.

So one can construct a deformed realization of the sl(2,C) algebra.



- There might be a general message: A transformation due to gauge freedom does not spoil the integrability or a symmetry, but may link more or less evident realizations of them, associated to different space of functions.

Odderon and multigluon states

Odderon in QCD: consider states odd under C ($A_\mu(x) \to -A_\mu^T(x)$)

$$O_{\alpha\beta\gamma}(x,y,z) = Tr\Big(\{A_\alpha(x), A_\beta(y)\}A_\gamma(z)\Big) = \frac{1}{2} d_{abc} A_\alpha^a(x), A_\beta^b(y) A_\gamma^c(z)$$

BKP kernel (GLLA) for three gluons in a singlet state:

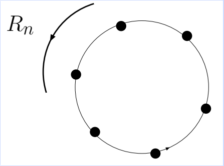
$$\boxed{H_3 = -\omega_1 - \omega_2 - \omega_3 + T_1T_2V_{12} + T_2T_3V_{23} + T_3T_1V_{31} = \frac{1}{2}\left(H_{12}^{(1)} + H_{23}^{(1)} + H_{31}^{(1)}\right)}$$

BKP kernel for n gluons in a singlet state:

$$\boxed{H_n = -\sum_i \omega_i + \sum_{i<j} T_iT_jV_{ij} = \frac{1}{N_c}\sum_{i<j} T_iT_jH_{ij}^{(1)}}$$

And for $N_c \to \infty$
cylinder topology

$$\boxed{H_n = \frac{1}{2}\left(H_{12}^{(1)} + H_{23}^{(1)} + \cdots + H_{n1}^{(1)}\right)}$$



On the space of Möbius functions: manifest holomophic factorization $H_n = h_n + h_n^*$, conformal invariance and Integrability due to $n-1$ integral of motions generated by the trasfer matrix of an integrable Heisemberg spin XXX model.

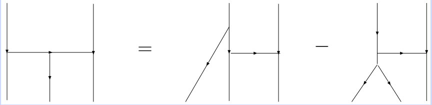
Chain of linked solutions: descendent states

The quantities conserved under the n gluon evolution (kernel H_n) are associated to the operators

$$q_r = \sum_{1\le i_1 \le i_2 \dots \le i_r \le n} \rho_{i_1 i_2}\rho_{i_2 i_3}\dots\rho_{i_r i_1}\; p_{i_1} p_{i_2} \dots p_{i_r}\;,\;\; [q_r, q_s] = 0\,,\;\; [q_r, h] = 0$$

with $q_2 = M^2$ (Casimir of Möbius algebra) and $q_n = \rho_{12}\cdots\rho_{n1} P_1 \cdots P_n$
Let us concentrate on the non trivial eigenstates of H_3 with $q_3 = 0$ for the Odderon (BLV states) or in general of H_n with $q_n = 0$.

- Require: $\boxed{|q_n|^2 E_n = 0} \Rightarrow |\rho_{12}\cdots\rho_{n1}|^2 |\partial_1|^2 \cdots |\partial_n|^2 E_n = 0 \Rightarrow$
  $\phi_n = |\partial_1|^2 \cdots |\partial_n|^2 E_n = \sum_i \delta^{(2)}(\rho_{i,i+1}) g_i$ Classify w.r.t. cyclic permutations: ansatz in momentum space (R_n is the cyclic rotation operator)
  $$\boxed{\phi_n(\boldsymbol{k}_1, \cdots, \boldsymbol{k}_n) = \sum_i (R_n)^i c_i \phi_{n-1}(\boldsymbol{k}_1 + \boldsymbol{k}_2, \boldsymbol{k}_3, \cdots, \boldsymbol{k}_n)}$$
- Rewrite the kernel as:
  $$\boxed{H_n = \frac{1}{2}\Big(H^{(1)}_{23} + \cdots + H^{(1)}_{n2}\Big) + \frac{1}{2}\Big(H^{(1)}_{12} + H^{(1)}_{n1} - H^{(1)}_{n2}\Big)}$$
- Using the properties: bootstrap and the symmetry for exchange of lines $1 \leftrightarrow 3$ of the *commutator* of the real kernel defined as
  $$W(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3|\boldsymbol{k}'_1, \boldsymbol{k}'_3) = V(\boldsymbol{k}_2, \boldsymbol{k}_3|\boldsymbol{k}'_1 - \boldsymbol{k}_1, \boldsymbol{k}'_3) - V(\boldsymbol{k}_1 + \boldsymbol{k}_2, \boldsymbol{k}_3|\boldsymbol{k}'_1, \boldsymbol{k}'_3)$$



Generalization of BLV Odderon states to n gluons.

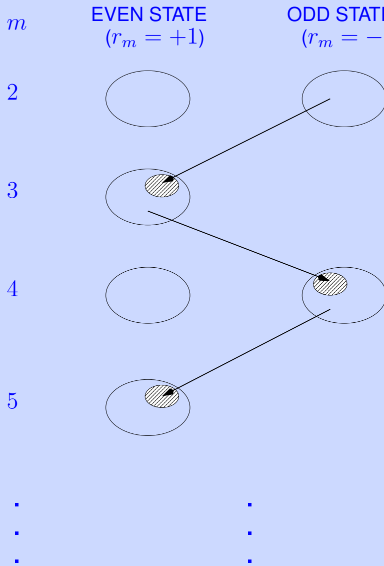
we can obtain states corresponding to n gluons from a construction starting from eigenstates with $n-1$ gluons in the large N_c limit.
If $H_{n-1}\phi_{n-1} = \chi\phi_{n-1}$ and $R_{n-1}\phi_{n-1} = r_{n-1}\phi_{n-1}$ then

$$H_n\phi_n = \chi\phi_n + \Big[\sum_i (R_n)^i \Big(1 + r_{n-1}(R_n)^{-1}\Big) c_i\Big] \int W(1,2,3)\phi_{n-1}(1,3,\ldots n)$$

Last term zero $\Rightarrow$ secular equation with eigenvalue $r_{n-1} = (-1)^n$ and eigenvector $c_i = (-1)^{(n-1)i}$

- n even: $c_i = (-1)^i$. Not physical states because not Bose symmetric (color traces even!)
- n odd: $c_i = +1$. Physical solutions ($n = 3$ corresponds to the BLV Odderon solution)

$$\boxed{E_h^{(m)}(k_1, k_2, \cdots, k_m) = \sum_{i=0}^{m-1} (R_m)^i c_i \frac{(k_1+k_2)^2}{k_1^2 k_2^2} E_h^{(m-1)}(k_1+k_2, k_3, \cdots, k_m)}$$



Note also that the constructing procedure is nilpotent and cannot be iterated.
G.P.V. (2000)

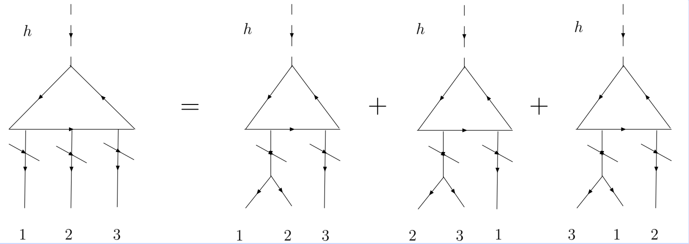
Odderon: BLV states

BLV Odderon states (intercept ≤ 1): Bartels, Lipatov, G.P.V. (2000)

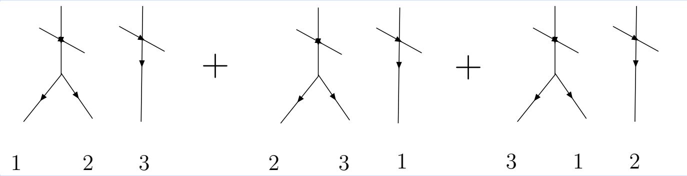
$$\boxed{\boldsymbol{k}_1^2\boldsymbol{k}_2^2\boldsymbol{k}_3^2 E_h^{(3)}(k_1, k_2, \cdots, k_m) = (\boldsymbol{k}_1 + \boldsymbol{k}_2)^2 \boldsymbol{k}_3^2 E_h(\boldsymbol{k}_1 + \boldsymbol{k}_2, \boldsymbol{k}_3) + \text{Cyclic perm.}}$$

The $E_h(\boldsymbol{k}_i, \boldsymbol{k}_j)$ are odd conformal spin BFKL Pomeron states, associated to $x_h \leq 0$ eigenvalues. Let's note that the kernel in momentum representation is meromorphic and so are its eigenstates. They belong to the AF-space.

The structure of the solution is:



So it is convenient to define the symmetrized splitting mapping S

$S \;=$ 

$\Rightarrow$ $\boxed{H_3 S\, \varphi_{odd} = S H_2\, \varphi_{odd}}$

Consider the structure of $V_{2\to6}$ for $N_c \to \infty$: $\langle h|V_{2\to6} S \otimes S|h_1\rangle \otimes |h_2\rangle \propto \langle h|V_{2\to4}|h_1\rangle \otimes |h_2\rangle$

This reduction was used to study the diffractive process $\gamma^*\, p \to \eta_c + X$. In this case the proton couples to the Pomeron, not to the Odderon, and there is much less uncertainty.

The reduced POO vertex ($V_{2\to6}$), projected onto P and O states, has been reobtained in the non linear equations for the dipole-CGC evolution.

Odderon in the NLLA?

- NLL contribution will include running coupling effects.
  The big question is: will the maximal intercept still be at 1? If not, where it will move?
- Structure of the NLL Odderon kernel:

$$\boxed{H_3^{odd} = \omega_1^{8_S} + \omega_2^{8_S} + \omega_3^{8_S} + V_{12}^{8_S} + V_{23}^{8_S} + V_{31}^{8_S} + V_{123}^{odd}}$$

- One can see that: $V_{ij}^{8_A} = 1/2 V_{ij}^{1} + \cdots \neq V_{ij}^{8_S} = 1/2 V_{ij}^{1} + \cdots$
  but V_{123}^{odd} has to be computed (not easy).
- Question: what about a bootstrapped solution for Odderon states at NLL?
  - Is there an 8_S reggeon at NLL? Associated bootstrap equation?
    There is Strong bootstrap related to impact factors in 8_A states. But for the 8_S case?
    Feature: at NLL with bootstrap no more gluons at the same transverse position.
  - To have in NLLA intercept at 1 there should probably be a more general bootstrap in the 8_s channel involving all 1, 2 and 3 gluon interactions.
- As for the BFKL pomeron, linear NLL evolution will affect any non linear evolution.
- May be useful to look at a simplified model, but still many open questions:
  In the $n_f \to \infty$ limit V_{123}^{odd} is absent (only 1 and 2 gluon dynamics)
  Still one can trace running coupling effects.

Odderon in the NLL:2

- Beyond the Green's function, associated to the Odderon states, the impact factors should be computed in the NLLA!

- For the proton is difficult to think about any perturbative model.

- The helicity flip dynamics is present at NLL.
  Just note that at the Pomeron level NLL perturbative impact factors permits to study helicity flip and then non trivial dynamical features of polarized cross sections: this comes from the properties of the basic PPR vertex.

Conclusions

- Bootstrap is an important selfconsistency requirement in the LL approach and is associated to the gluon reggeization.
- Gluon reggeization is crucial to understand the general features for the evolution of states with a fixed number of gluon as well as for the changing one.
- The Odderon states with leading intercept are a direct consequence of reggeization (at LL reggeization does not distinguish 8_A and 8_s states).
- The bootstrap mechanism can be followed at work inside the AF-space of functions.
- Odderon at NLL?