Dynamics of Phase Transitions: $SU(3)$ Lattice Gauge Theory

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Introduction
RHIC
The Order Parameter
The Structure Factor

Dynamics of Phase Transition
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Gauge Systems
The Linear Theory
The Debye Screening Mass
The Energy and Pressure Density
Polyakov loop correlations

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Introduction: RHIC

Central collision of heavy ions followed by formation of fireballs

Fire-tunnel evolution (target rest frame)
Introduction: Setup

- $SU(3)$ pure gauge theory on a $N_T N_s^3$ lattice
- For a system in equilibrium notion of time is lost, so it has to be reintroduced
- We study the Glauber (heatbath) dynamics under a heating quench driving the system from the disordered into the ordered phase
Non-equilibrium studies performed along these lines include:

- Pioneering study of $SU(2)$ and $SU(3)$ pure gauge
- $q$-state Potts models, scaling in the infinite volume limit
- $SU(3)$ pure gauge theory, scaling in the infinite volume limit
- $SU(3)$ pure gauge theory, spatial expansion
- $SU(3)$ pure gauge theory, the finite volume continuum limit
The Order Parameter

The Polyakov loop is defined as

\[ I(\vec{x}) = \text{tr} \prod_{m=0}^{N_t-1} U_{\vec{x}+m\hat{t},0} \]  

where \( U \) are \( SU(3) \) matrices on the links of a hypercubic lattice. Its average value serves as the order parameter of the theory.

- The symmetry group of the order parameter in the \( SU(3) \) gauge theory is \( Z_3 \)
- Symmetric (confined) phase \( \langle I \rangle = 0 \)
- Broken (deconfined) phase \( \langle I \rangle \neq 0 \)
- The transition is weak 1st order
- Quarks smooth out this behavior, the transition becomes a rapid crossover
The Structure Factor

Two-point correlation function of the Polyakov loops ($\langle \ldots \rangle_L$ is the lattice average)

$$\langle l(0) l^\dagger(\vec{j}) \rangle_L = \frac{1}{N^3_\sigma} \sum_{\vec{i}} l(\vec{i}) l^\dagger(\vec{i} + \vec{j})$$  \hspace{1cm} (2)

The structure factor $F(\vec{p})$ is a Fourier transform of (2). After discretization and using periodicity of the boundary conditions one arrives at the expression

$$F(\vec{p}) = \frac{a^3}{N^3_\sigma} \left| \sum_{\vec{i}} e^{-i \vec{k} \cdot \vec{i}} l(\vec{i}) \right|^2$$  \hspace{1cm} (3)
Scenarios of Phase Transitions

- **Nucleation:**
  - instability against finite amplitude
  - localized fluctuations
  - has an activation barrier
  - metastable region
  - dominated by the growth of the largest clusters

- **Spinodal decomposition:**
  - instability against infinitesimal amplitude
  - nonlocalized fluctuations
  - no activation energy
  - unstable region
  - signaled by exponential growth of the structure functions
Geometrical vs. Fortuin-Kasteleyn clusters in 3-state 3D Potts model on a $40^3$ lattice
Spin Systems: The Structure Factor

The first structure factor mode on $N_{\sigma}^3$ lattices for 3-state 3D Potts model
The first structure factor mode on $4 \times N^3_\sigma$ lattices in $SU(3)$
Infinite volume limit: \( N_{\tau} = \text{const}, \ N_{\sigma} \rightarrow \infty \)

Finite volume continuum limit: \( N_{\tau}/N_{\sigma} = \text{const}, \ N_{\sigma} \rightarrow \infty \), we study \( N_{\tau} = 4, 6, 8 \)

Rescale time axis so that all maxima coincide

\[
t' = \frac{t}{\lambda_t(N_{\tau}, T_f/T_c)},
\]

\( \lambda_t(N_{\tau}, 1.25) \) are 1:2.655:5.457 and \( \lambda_t(N_{\tau}, 1.57) \) are 1:2.768:6.362

To overcome the renormalization problem of (bare) Polyakov loop correlations divide all structure factors by their equilibrium values at \( T_f \)
The first structure factor mode on different lattices with the same physical volume for $T_f = 1.25 T_c$. 

Gauge Systems: The Structure Factor
The first structure factor mode on different lattices with the same physical volume for $T_f = 1.57 T_c$
Dynamical generalization of the Landau-Ginzburg theory in the linear approximation results in the following equation for a structure factor:

\[
\frac{\partial F(\vec{p}, t)}{\partial t} = 2\omega(\vec{p}) F(\vec{p}, t) \tag{5}
\]

with the solution

\[
F(\vec{p}, t) = F(\vec{p}, 0) \exp \left(2\omega(\vec{p}) t\right), \tag{6}
\]

\[
\omega(\vec{p}) > 0 \text{ for } |\vec{p}| > p_c
\]

Rescale \(\omega'(\vec{p}) = \lambda_t(N_T, T_f/T_c) \omega(\vec{p})\) so \(\omega'(\vec{p})t' = \omega(\vec{p})t\)
The Critical Momentum

SU(3) determination of $p_c$ for $T_f/T_c = 1.25$
The Critical Momentum

SU(3) determination of $p_c$ for $T_f/T_c = 1.57$
The Debye Screening Mass

Fit results for $p_c/T_c$

<table>
<thead>
<tr>
<th>$T_f/T_c$</th>
<th>$N_T = 4$</th>
<th>$N_T = 6$</th>
<th>$N_T = 8$</th>
<th>$N_T = \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.25</td>
<td>1.613 (18)</td>
<td>1.424 (26)</td>
<td>1.37 (10)</td>
<td>1.058 (79)</td>
</tr>
<tr>
<td>1.568</td>
<td>2.098 (19)</td>
<td>2.058 (22)</td>
<td>2.29 (15)</td>
<td>2.006 (73)</td>
</tr>
</tbody>
</table>

The critical momentum $p_c$ is related\(^1\) by

$$m_D = \sqrt{3} p_c \quad (7)$$

to the Debye screening mass at the final temperature $T_f$ after the quench:

$$m_D = 1.83 (14) T_c \quad \text{for} \quad T_f/T_c = 1.25, \quad (8)$$

$$m_D = 3.47 (13) T_c \quad \text{for} \quad T_f/T_c = 1.568 \quad (9)$$

The gluonic energy and pressure density, $T_f = 1.25 T_c$

$\varepsilon/T^4, N_\tau=4$
$\varepsilon/T^4, N_\tau=6$
$\varepsilon/T^4, N_\tau=8$
$3p/T^4, N_\tau=4$
$3p/T^4, N_\tau=6$
$3p/T^4, N_\tau=8$

The histogram for the order parameter (Polyakov loop) at the first time step on $6 \times 24^3$ lattice, $T_f = 1.57 T_c$
The histogram for the order parameter (Polyakov loop) at the time step where the structure function reaches maximum on $6 \times 24^3$ lattice, $T_f = 1.57 T_c$
The histogram for the order parameter (Polyakov loop) at the last time step (equilibrium) on $6 \times 24^3$ lattice, $T_f = 1.57 T_c$
Polyakov loop correlations

\[ C_o(d, t) = \langle l(0, t) l(d, t) \rangle_L - \left( \langle |l(0, t)| \rangle_L \right)^2 \]  \hspace{1cm} (10)
Open Questions

- No natural physical time scale
- Quarks: no heatbath dynamics
- Initial heating is not instantaneous
The Structure Factor in Effective Model

The first structure factor mode on $64^3$ lattice, $T_f = 1.25 T_c$

Conclusion

- The phase transition proceeds through the spinodal decomposition scenario
- Domains of different *triality* slow down the equilibration
- The energy and pressure density evolve to equilibrium values
- The critical momentum is related to the Debye screening mass
- Correlations in equilibrium are weaker than in the out-of-equilibrium state
- Physical time scale can be set in effective models