QCD Thermodynamics with Two-flavors of Wilson quarks

Shinji Ejiri (University of Tokyo)

In collaboration with

T. Hatsuda, N. Ishii, Y. Maezawa, N. Ukita (University of Tokyo)

S. Aoki, K. Kanaya, Y. Taniguchi (University of Tsukuba)

東京大学 THE UNIVERSITY OF TOKYO 筑波大学 University of Tsukuba

Tokyo-Tsukuba hot and dense QCD Collaboration

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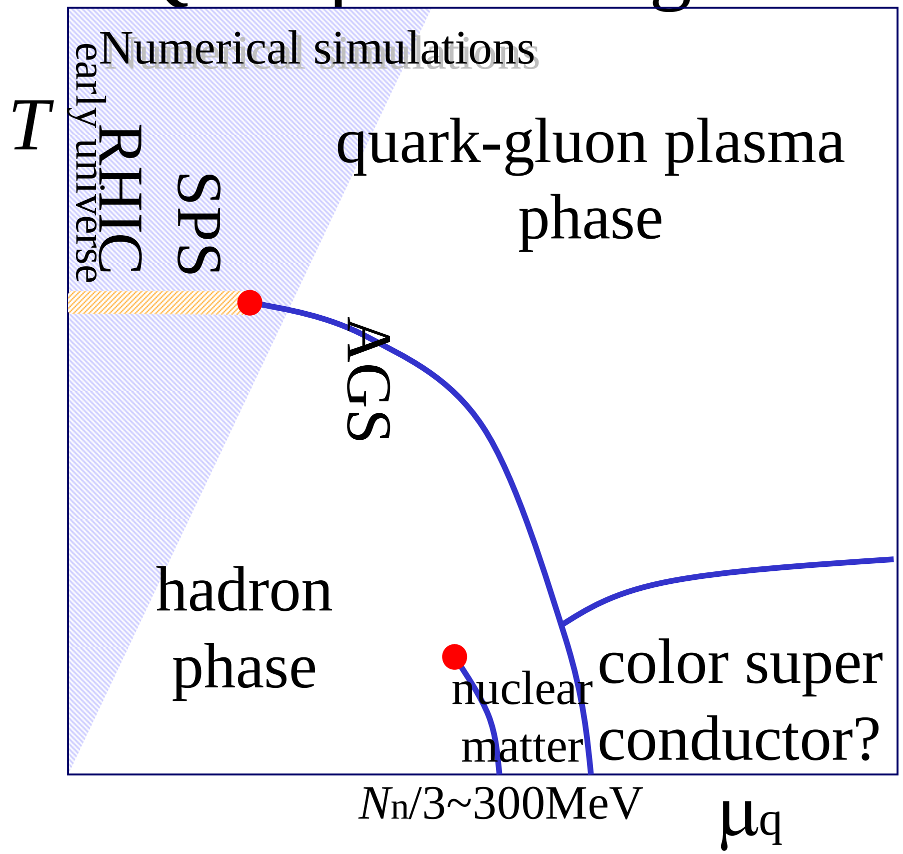
QCD Thermodynamics with Wilson type quarks

- Numerical study of full QCD at high temperature and density is very important.
- However, most of studies are performed using staggered-type quark action with the 4th root trick of the quark determinant.
- Studies by a different lattice formulation is necessary to confirm the reliability of the results from lattice QCD simulations.
- We study the QCD thermodynamics using a Wilson type quark action systematically.

QCD phase diagram



Study of QCD thermodynamics with Wilson type fermions

- Systematic study has been done using Iwasaki (RG) improved gauge action + N_f=2 Clover improved Wilson action by CPPACS Collaboration (1999-2001).
  - T=0, μ_q=0: light hadron spectrum, line of chiral limit etc.
  - $T\neq0$, μ_q=0: phase structure, T_c, O(4) scaling, equation of state, etc.

- However, previous $T\neq0$ study: N_t=4 and 6, only μ_q=0.
- Technical progress for $\mu_q\neq0$ has obtained in the last 6 years.
- It is important to continue this project.
- Especially, the extension to finite density QCD is important.
- Also, small lattice spacing (large N_t) and small quark mass.

Iwasaki improved gauge action + Clover improved Wilson action

- Partition function

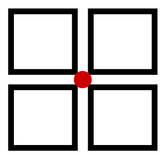
$$Z = \int \prod_{x,\mu} dU_{\mu}(x) \left(\det M\right)^{N_f} \mathrm{e}^{-S_g},$$

$$S_g = -\beta\left\{c_0 \sum_{x,\mu>\nu} W_{\mu\nu}^{1\times1}(x) + c_1 \sum_{x,\mu\neq\nu} W_{\mu\nu}^{1\times2}(x)\right\}$$

$W_{\mu\nu}^{n\times m}$: nxm wilson loop

$$\beta = 6/g^2, \quad c_1 = -0.331, \quad c_0 = 1-8c_1,$$

$$M_{x,y} = \delta_{x,y} - K\sum_{i=1}^{3}\left[(1-\gamma_i)U_i\delta_{x+\hat{i},y} + (1+\gamma_i)U_i^{+}\delta_{x-\hat{i},y}\right]$$
$$-K\left[e^{\mu_q a}(1-\gamma_4)U_4\delta_{x+\hat{4},y} + e^{-\mu_q a}(1+\gamma_4)U_4^{+}\delta_{x-\hat{4},y}\right] + \delta_{x,y}c_{SW}K\sum_{\mu<\nu}\sigma_{\mu\nu}F_{\mu\nu}$$



Clover term

μ_q: quark chemical potential

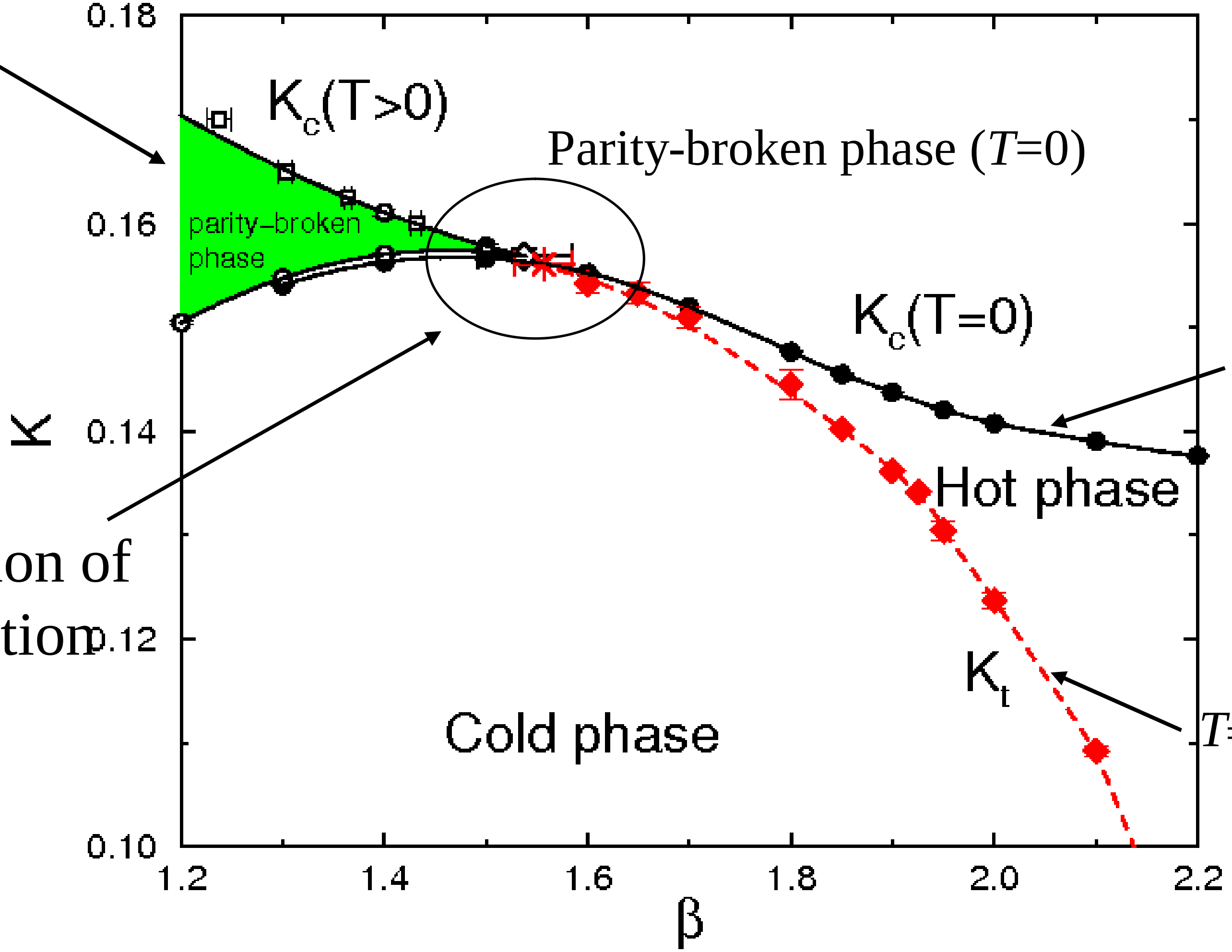
$$c_{SW} = (1-0.8412\beta^{-1})^{-3/4}$$

Phase structure of QCD with Wilson-type quarks

CP-PACS, PRD63, 034502(2000)

Parity-broken phase ($T \neq 0$)

$N_t = 4$



Chiral limit at $T=0$

$m_q = 0$

Critical region of chiral transition

$T \neq 0$ pseudo-critical line

(crossover)

O(4) scaling test for N_f=2 Wilson fermion

Universality class: 3-d O(4) spin model?

Iwasaki improved + Wilson (Iwasaki et.al., Phys. Rev. Lett., 78, 179 (1997))

- $\langle \bar{\psi}\psi \rangle_{sub}$ defined by axial Ward identities (Bochichio et al., NPB262, 331 (1985))

$$\langle \bar{\psi}\psi \rangle_{sub} = 2 m_q a (2K)^2 \sum_x \langle \pi(x)\pi(0) \rangle$$

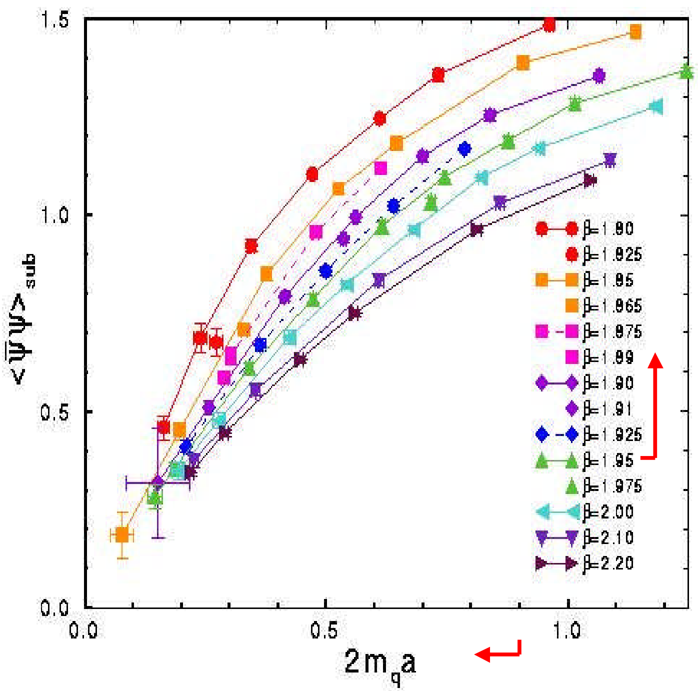
- Scaling relation is satisfied with O(4) exponents

$$M / h^{1/\delta} = f(t/h^{1/\beta\delta}) \qquad M = \langle \bar{\psi}\psi \rangle_{sub}, \quad h = 2m_q a, \quad t = \beta - \beta_{ct}$$

⇒ Consistent with the sigma model prediction.

(On the other hand, it is difficult to confirm for staggered type fermions.)

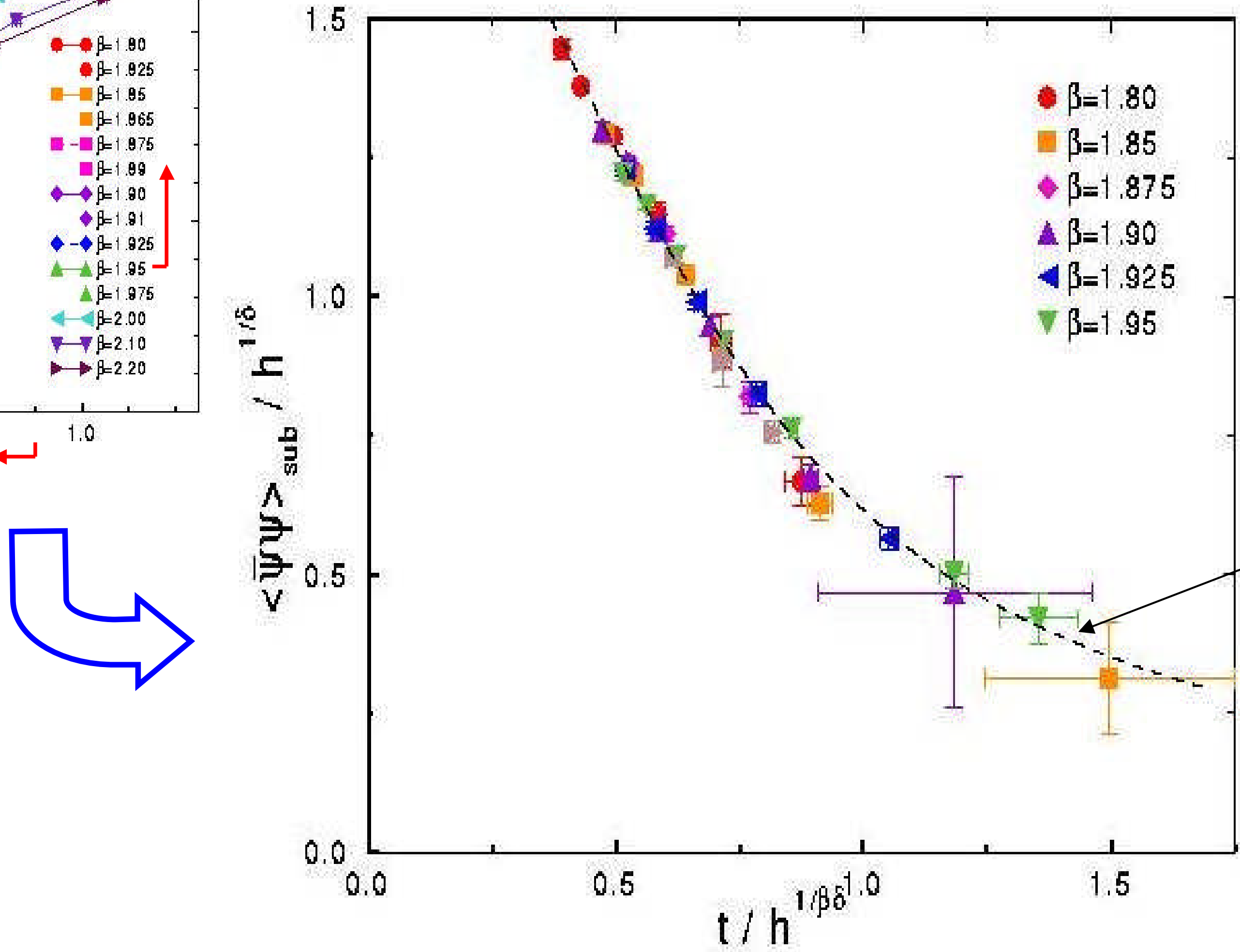
RG+Clover improved Wilson (CP-PACS, Phys. Rev. D63.034502(2000))



$$M/h^{1/\delta} = f(t/h^{1/\beta\delta})$$

$\beta_{ct} = 1.469\,(73)$

$\chi^2/N_{df} = 0.816$



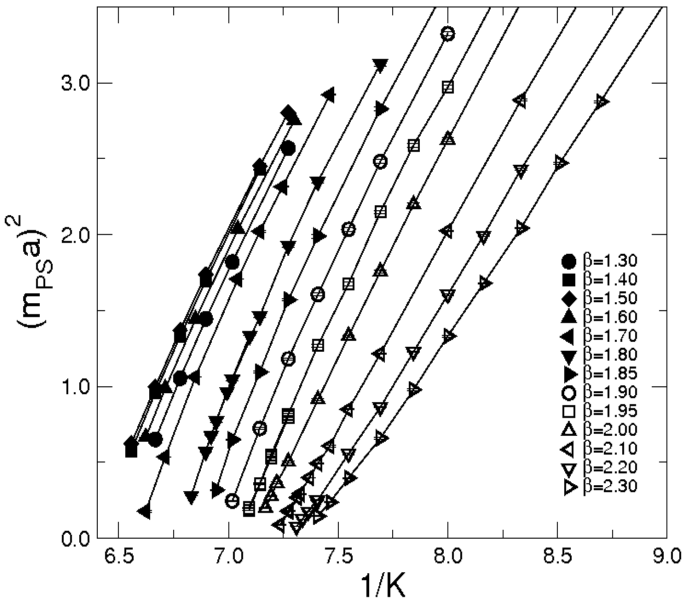
$16^3 \times 4$ lattice

O(4) scaling function (Toussaint,'97)

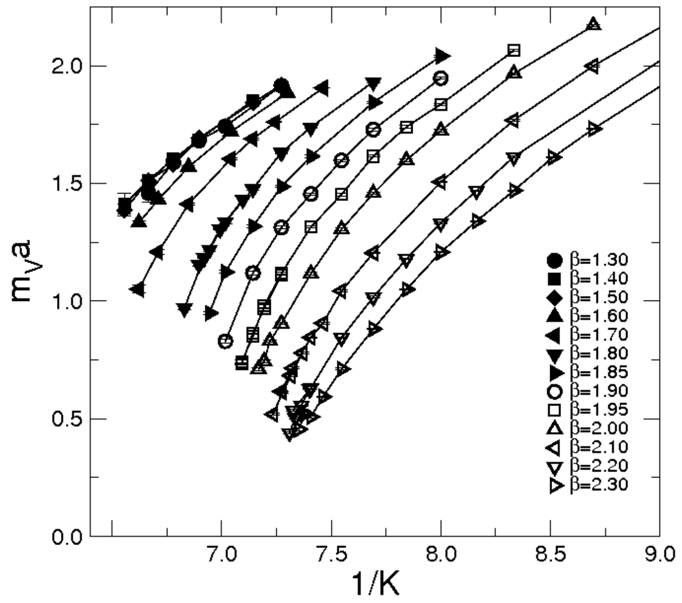
O(4) scaling relation is well satisfied.

Lines of constant physics (LCP) and temperature (T) in the (β,K) plane

Pion mass (m_{ps}) at T=0



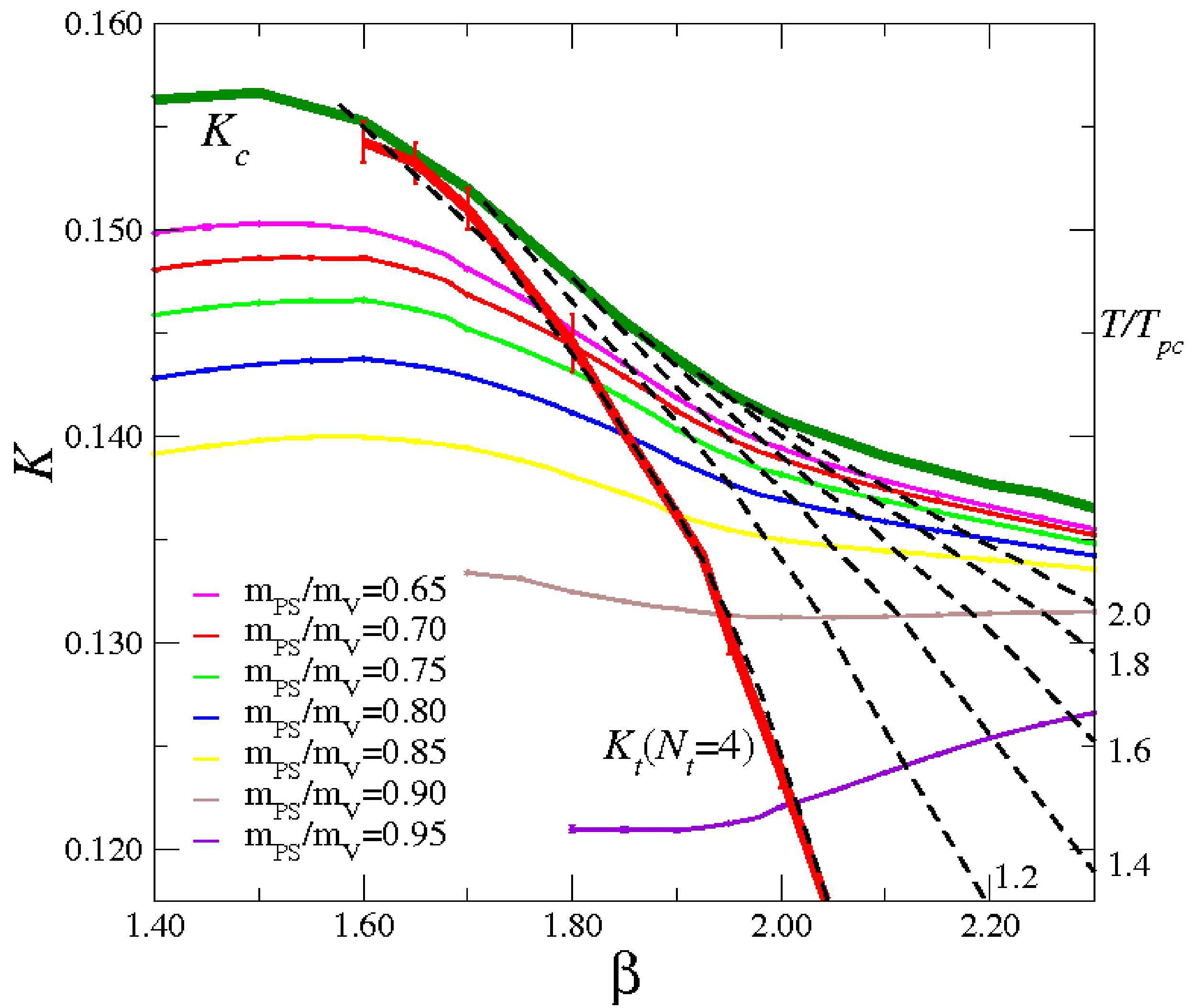
ρ meson mass (m_V) at T=0



CP-PACS Collab.,
PRL85, 4674 (2000);
PRD63, 034502 (2000);
PRD64, 074510 (2001);
PRD65, 054505 (2001)

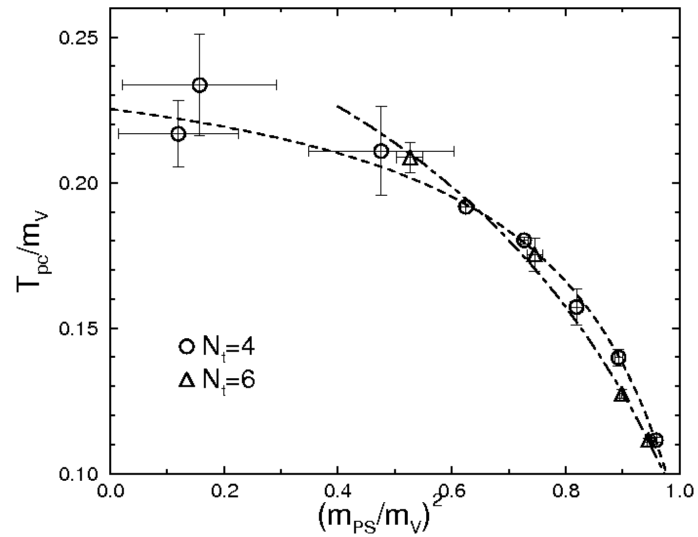
- Interpolating these data of m_{PS} and m_V, we determine lines of constant m_{PS}/m_V as lines of constant physics.
- Temperature is estimated by ρ meson mass m_V and normalized by T_{pc}/m_V for each LCP.

$$T/m_V = (N_t m_V a)^{-1}$$



Pseudo-critical temperature as a function of m_{PS}/m_V

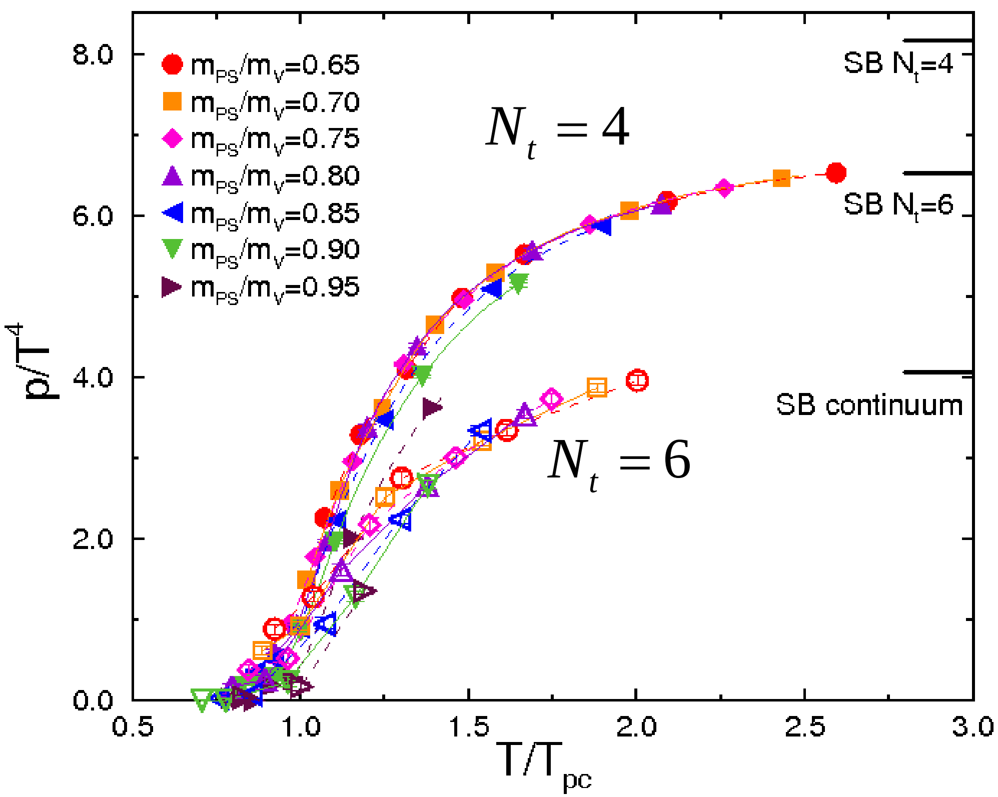
(CP-PACS, PRD64, 07510 (2001))

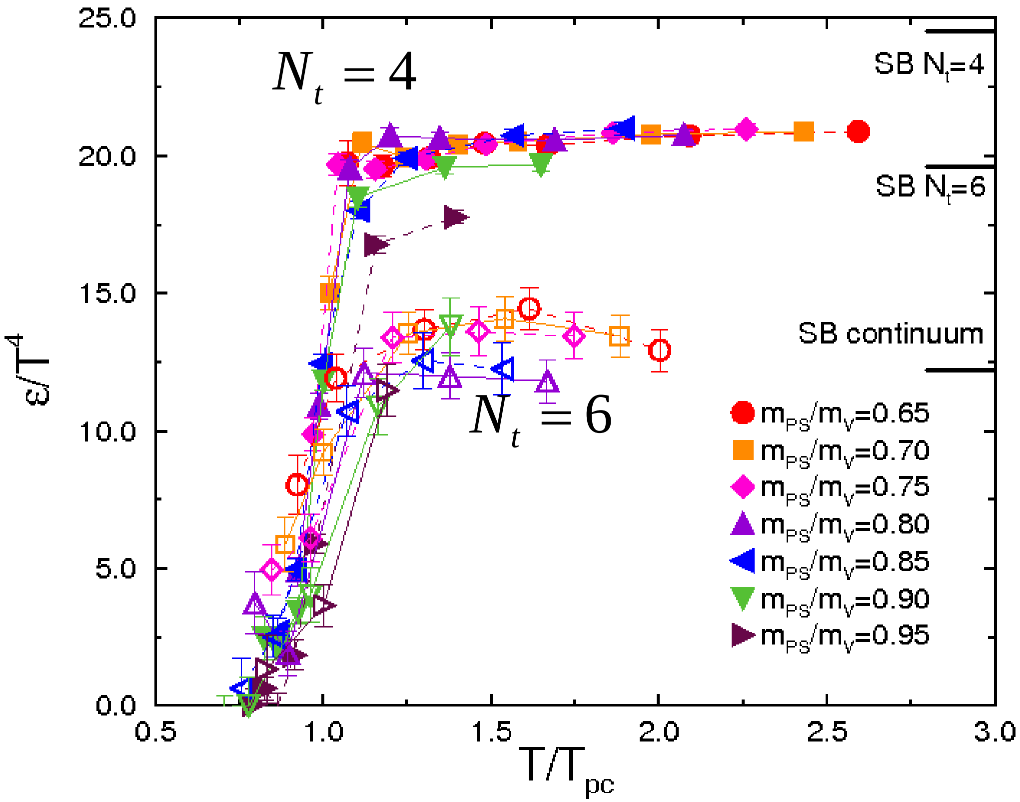


- Colored lines: line of constant m_{PS}/m_V (LCP)
- Green line (K_c): chiral limit, line of m_{PS}=0
- Red line (K_t): finite temperature pseudo-critical line
- Dashed lines: lines of constant T/T_{pc}

Pressure and Energy density at μ=0

(CP-PACS, PRD64, 074510 (2001))





- Pressure is computed by the integral method for μ=0.
- Small m_{PS}/m_V dependence. Large N_t dependence.
- Simulations with large N_t and also with chemical potential μ_q: important.

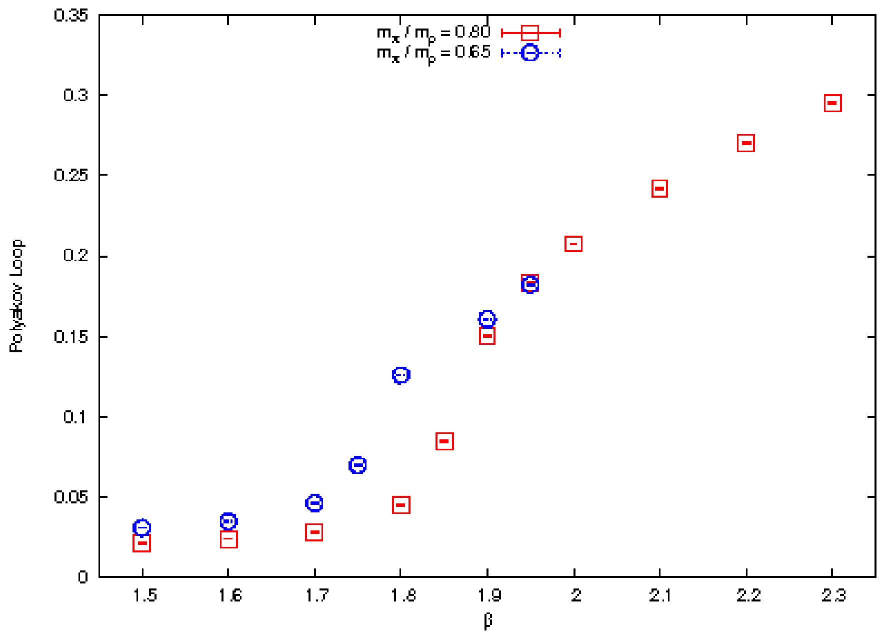
Important topics for this project

- Smaller lattice spacing (Large N_t) future plan
- Scaling study for 2+1 flavor QCD future plan
- Screening mass
  - Electric screening mass → Maezawa's talk
  - Magnetic screening mass
    - Spatial string tension → This talk
- Finite chemical potential
  - Equation of state → This talk (preliminary results)
    - Taylor expansion method
  - (T, μ_q) phase structure

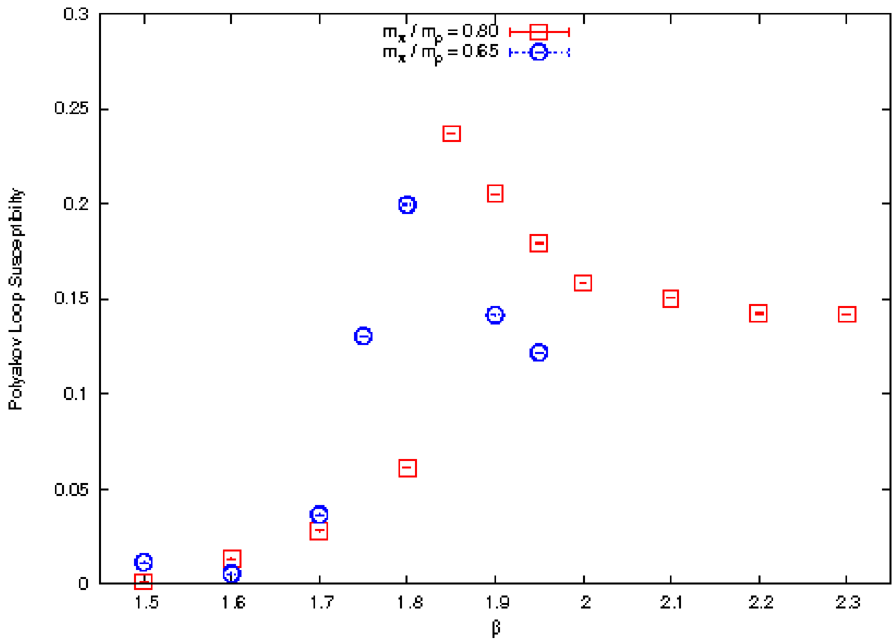
Simulation parameters

- We perform simulations for N_f=2 at m_{PS}/m_V=0.65 and 0.80.
- Iwasaki (RG) improved gauge action and clover-improved Wilson fermion action are used.
- Lattice size: $N_{\text{site}} = N_s^3 \times N_t = 16^3 \times \underline{4}$
- 500 ~ 600 configurations (5000 ~ 6000 trajectories) for each T (β).

Polyakov loop



Polyakov loop susceptibility



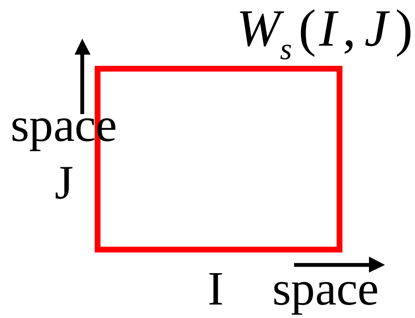
Comment on the spatial string tension

- Electric screening mass m_E is extracted from correlation functions of temporal gauge fields, assuming a Yukawa type potential in the deconfinement phase at T>Tc.
- m_E can be compare with the perturbative prediction. (Maezawa's talk)

On the other hand,

- Perturbative calculation of magnetic screening mass m_M is difficult. (Linde '80)
- m_M should be calculated by correlation functions for spatial components of gauge fields using Lattice QCD simulations.
- However a spatial string tension remains non-zero value at high T in quenched simulations. (Bali et al. '93, Boyd et al. '96)
  – If spatial gauge fields show confinement properties, the situation becomes more complicated.
- As a first step, we consider the spatial string tension for full QCD.

Spatial string tension (σ_s)



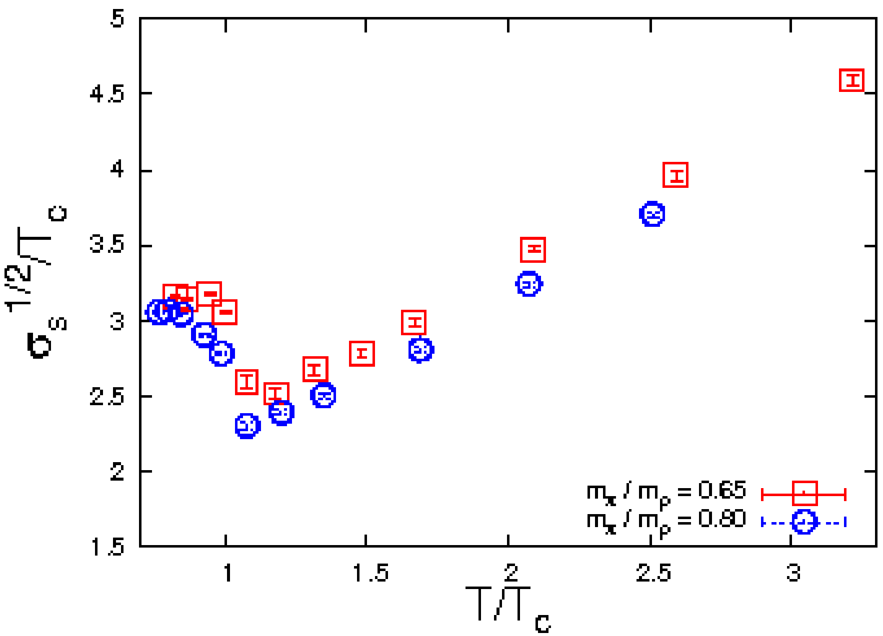
- Fitting Assumption for spatial Wilson loops *W*s(*I*,*J*):

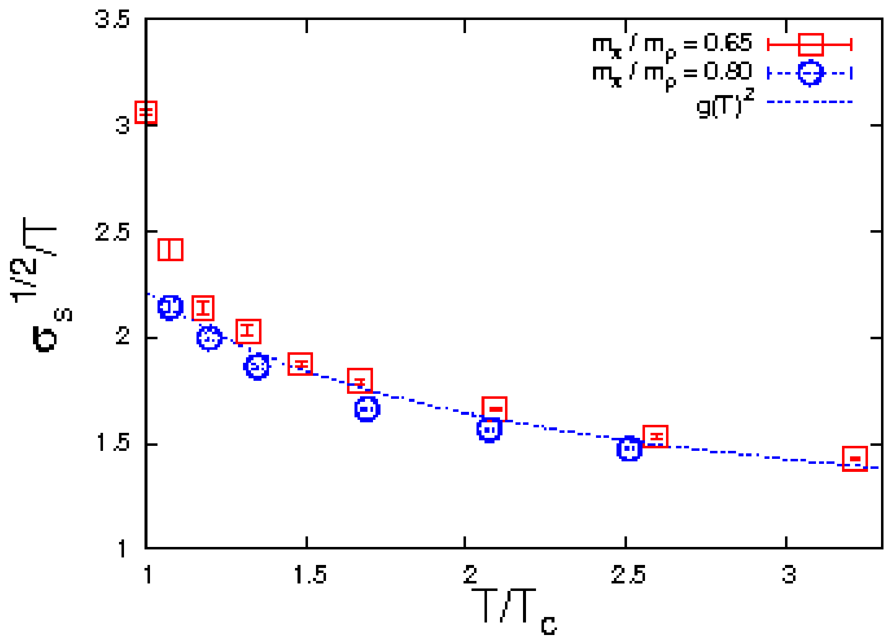
$$-\ln W_s(I,J) = \sigma_s IJ + P(2I+2J) + C.$$

(σ_s, *P*, *C*: fit parameters)

- Expectation from 3-dim effective theory: $\sqrt{\sigma_s(T)} \propto g^2(T)T$

($g^2(T)$: Temperature dependent running coupling constant.)





- Linier rising behavior is obtained at $T>T_c$.
- It suggests: the spatial string tension survives at high *T*.

Equation of state at μ≠0 by Taylor expansion

Basic thermodynamic quantities at μ≠0

- Pressure: $p = \frac{T}{V} \ln Z$ (Z: grand partition function)
- Quark number density: $n_{\mathrm{u,d}} = \frac{T}{V} \frac{\partial \ln Z}{\partial \mu_{\mathrm{u,d}}} = \frac{\partial p}{\partial \mu_{\mathrm{u,d}}}$
- Quark (Baryon) number susceptibility: $\chi_{\mathrm{q}} = 9\chi_{\mathrm{B}} = \left(\frac{\partial}{\partial \mu_{\mathrm{u}}} + \frac{\partial}{\partial \mu_{\mathrm{d}}} \right) (n_{\mathrm{u}} + n_{\mathrm{d}}) = \frac{\partial^2 p}{\partial \mu_{\mathrm{q}}^2}$
- Isospin susceptibility: $\chi_{\mathrm{I}} = \left(\frac{\partial}{\partial \mu_{\mathrm{u}}} - \frac{\partial}{\partial \mu_{\mathrm{d}}} \right) (n_{\mathrm{u}} - n_{\mathrm{d}}) = \frac{\partial^2 p}{\partial \mu_{\mathrm{I}}^2}$

$$\left(\mu_{\mathrm{q}} = (\mu_{\mathrm{u}} + \mu_{\mathrm{d}})/2 \right)$$
$$\left(\mu_{\mathrm{I}} = (\mu_{\mathrm{u}} - \mu_{\mathrm{d}})/2 \right)$$

- Charge susceptibility: $\chi_{\mathrm{C}} = \left(\frac{2}{3} \frac{\partial}{\partial \mu_{\mathrm{u}}} - \frac{1}{3} \frac{\partial}{\partial \mu_{\mathrm{d}}} \right) \left(\frac{2}{3} n_{\mathrm{u}} - \frac{1}{3} n_{\mathrm{d}} \right)$

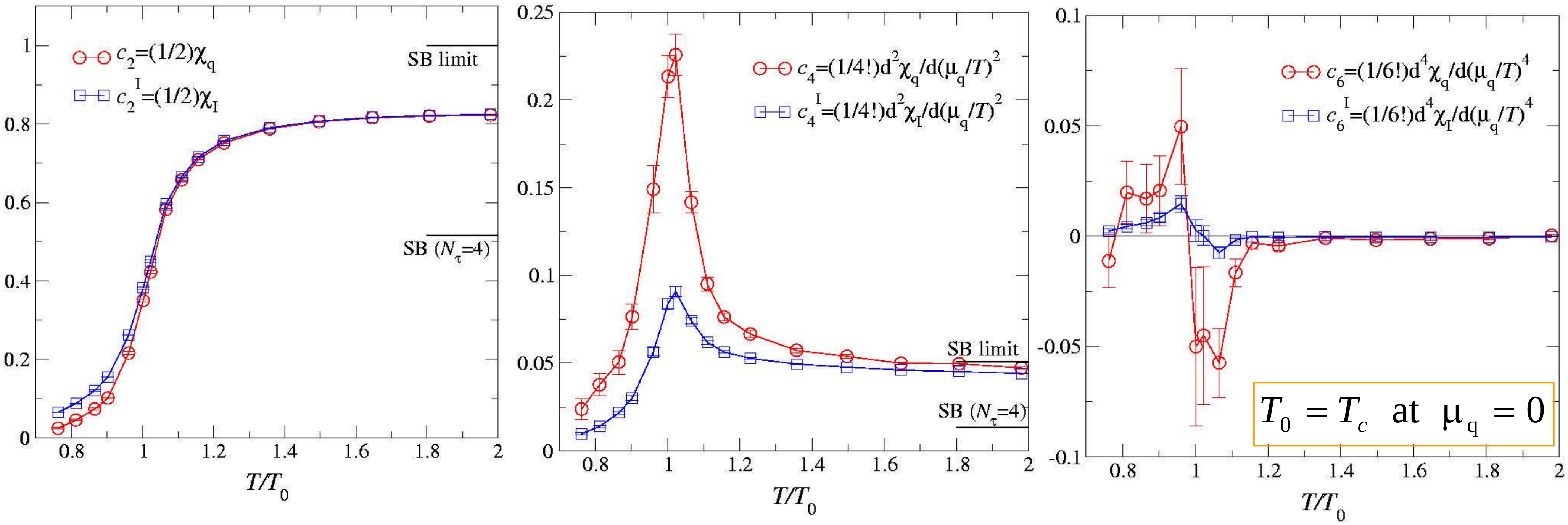
$$\frac{\chi_{\mathrm{C}}}{T^2} = \frac{1}{36} \frac{\chi_{\mathrm{q}}}{T^2} + \frac{1}{4} \frac{\chi_{\mathrm{I}}}{T^2}$$
for $\mu_{\mathrm{u}} = \mu_{\mathrm{d}} \equiv \mu_{\mathrm{q}}$

- We calculate the derivatives of partition function (Taylor expansion coefficients).

Comparison with the results by staggered

Derivatives of pressure and susceptibilities

Results of p-4 staggered (Bielefeld Swansea, PRD71, 054508 (2005))



$$\frac{\chi_q}{T^2}(\mu) = \frac{\partial^2 (p/T^4)}{\partial(\mu_q/T)^2} = 2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2 + 30c_6\left(\frac{\mu_q}{T}\right)^4 + \cdots$$

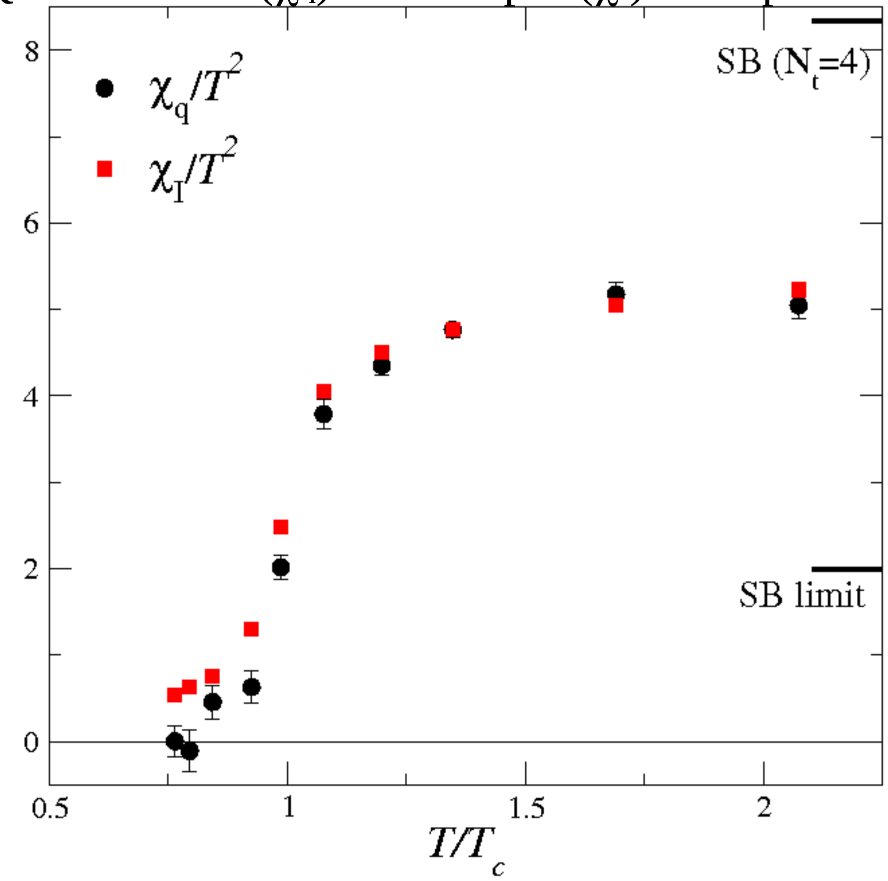
$$\frac{\chi_I}{T^2}(\mu) = \frac{\partial^2 (p/T^4)}{\partial(\mu_I/T)^2} = 2c_2^I + 12c_4^I\left(\frac{\mu_q}{T}\right)^2 + 30c_6^I\left(\frac{\mu_q}{T}\right)^4 + \cdots$$

- As a first step, we reproduce these results by a Wilson type quark action.

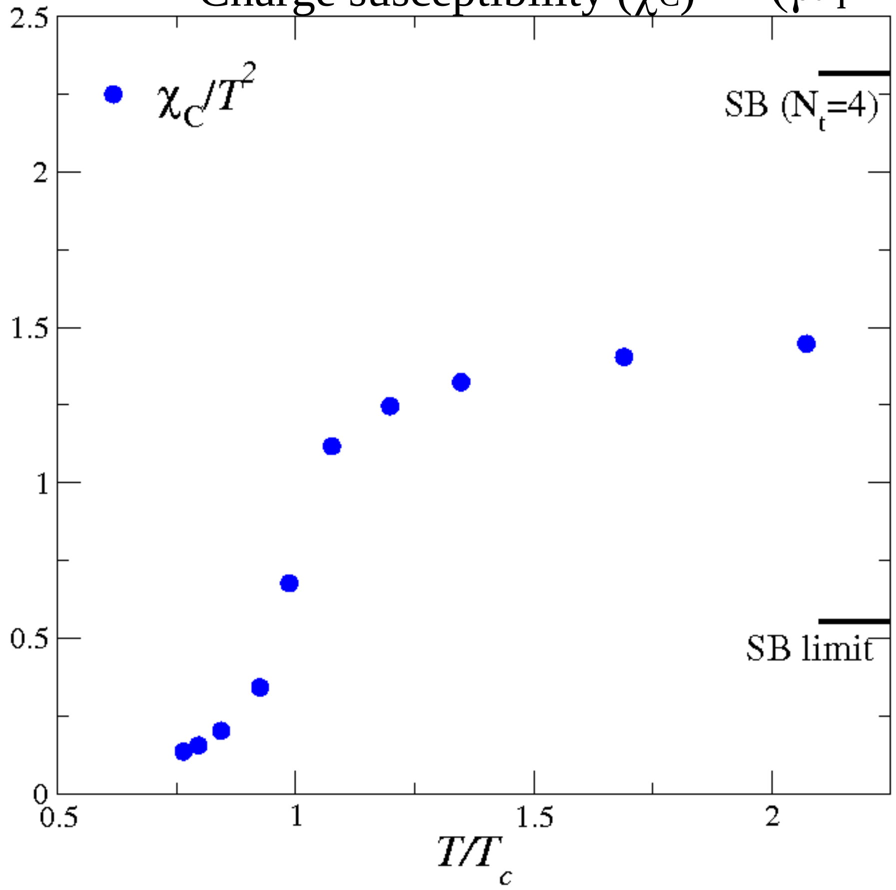
Preliminary results of susceptibilities

RG + Clover Wilson (m_{PS}/m_V=0.8)

Quark number(χ_q) and Isospin (χ_I) susceptibilities



Charge susceptibility (χ_C) (μ_q=0)



- We also measured the second derivatives of these susceptibilities (4th derivatives of pressure), however the statistical errors are too large at present.
- We use the random noise method with N_{noise}=10 for this calculation.
- The choice of the number of noise vectors may be important.

Lines of constant pressure

- It is interesting to compare the line of constant pressure (or energy density) to $T_c(\mu)$ or the chemical freeze out points.
- We estimate the line of constant p near $\mu_q=0$.

Along this line

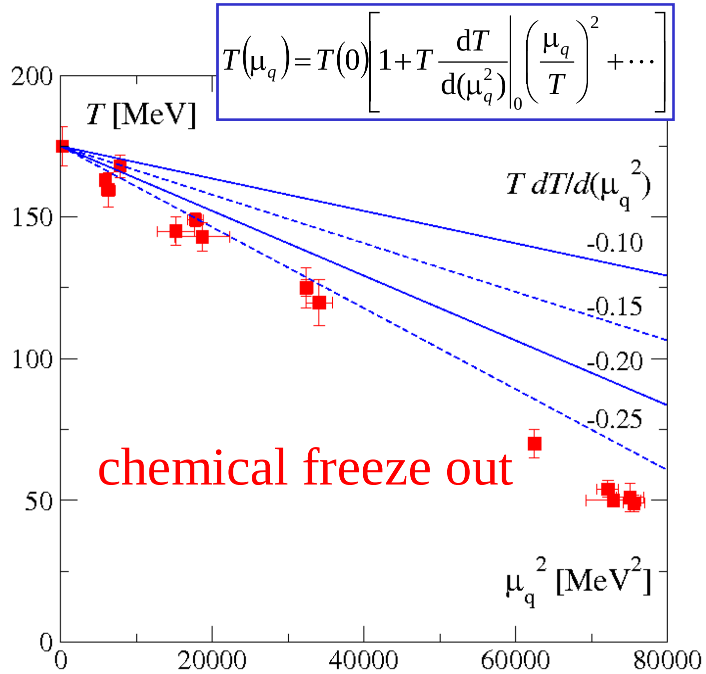
$$\Delta p = \frac{\partial p}{\partial T}\Delta T + \frac{\mathrm{d}p}{\mathrm{d}(\mu_q^2)}\Delta(\mu_q^2) = 0$$

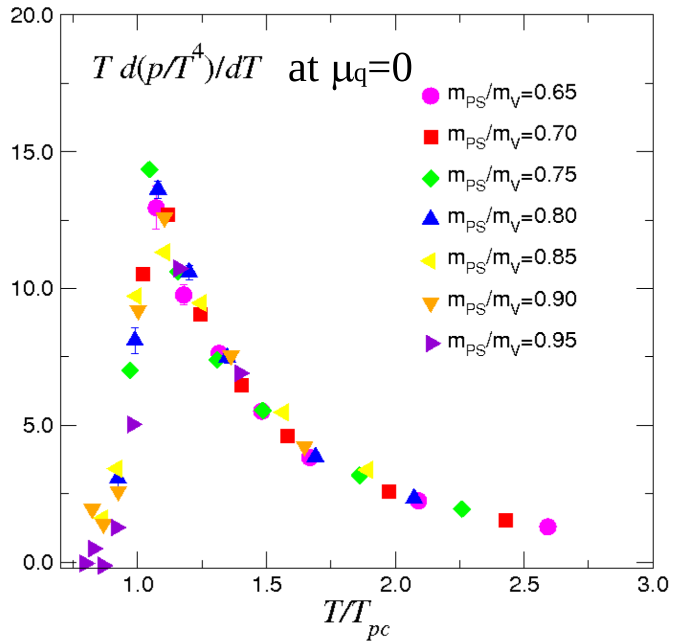
- The slope of constant p line in the (T, μ_q) plane is given by

$$T\frac{\mathrm{d}T}{\mathrm{d}(\mu_q^2)} = -\frac{\partial(p/T^4)}{\partial(\mu_q/T)^2}\Bigg/\left(T\frac{\partial(p/T^4)}{\partial T}+\frac{4p}{T^4}\right)$$

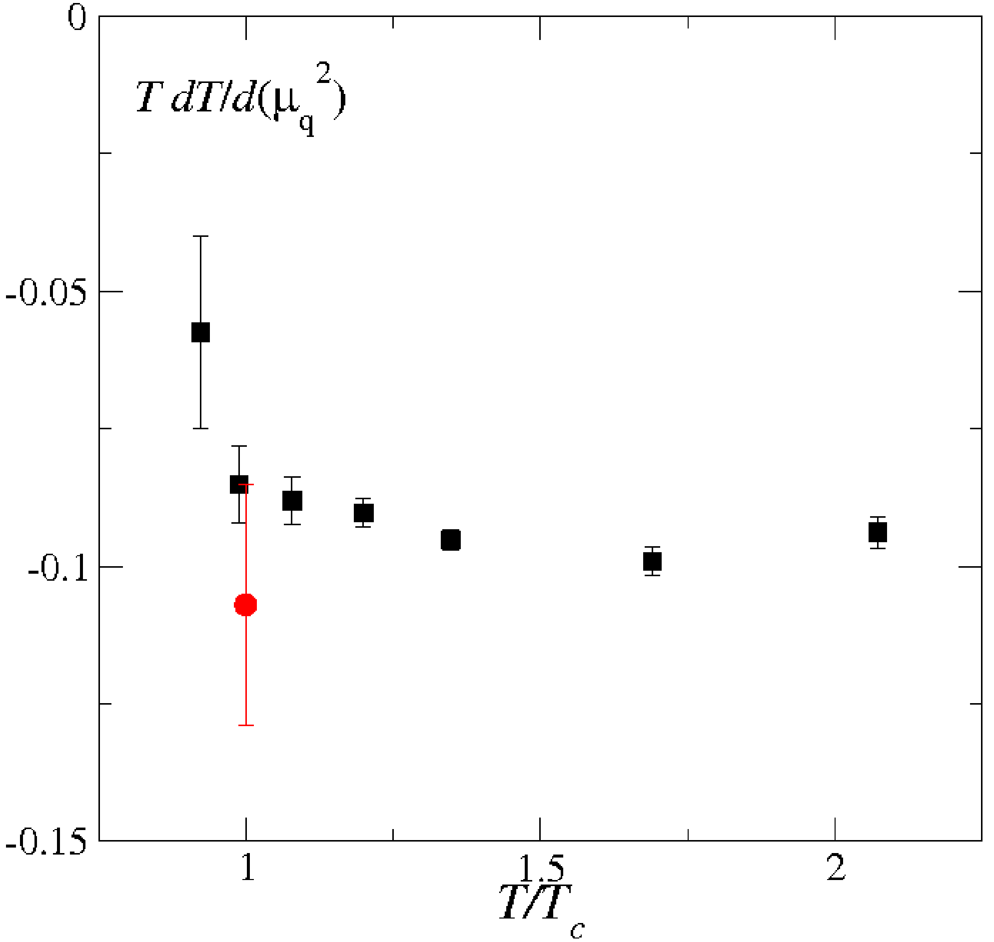
$$T\frac{\partial(p/T^4)}{\partial T} = -a\frac{\partial\beta}{\partial a}\frac{\partial(p/T^4)}{\partial\beta} - a\frac{\partial K}{\partial a}\frac{\partial(p/T^4)}{\partial K} = \frac{\varepsilon-3p}{T^4}$$

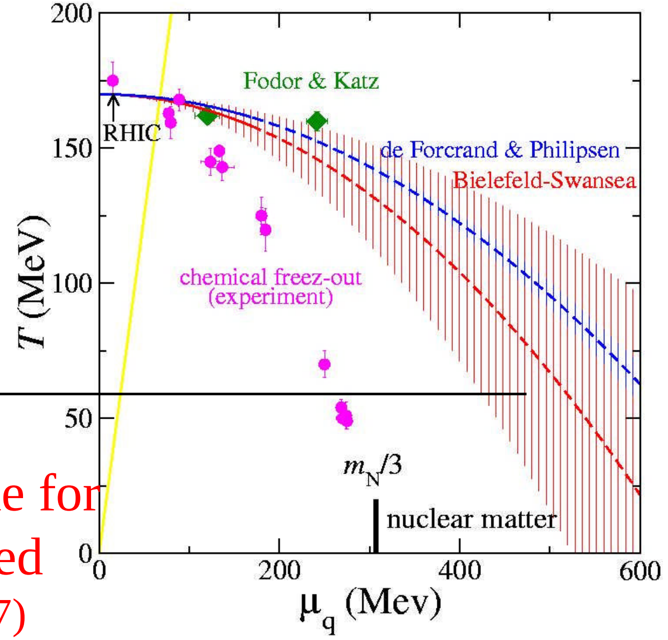
(Data in CP-PACS, PRD64, 074510 (2001))





Preliminary results of $T\frac{dT}{d(\mu_q^2)}$ at μ_q=0 for m_{PS}/m_V=0.8





Transition line for p4-staggered (m_{PS}/m_V=0.7)

- The slope at μ_q=0 is about -0.1. This is roughly consistent with the previous results by an improved staggered (Red dot: Bielefeld-Swansea, PRD66, 074507(2002), (m_{PS}/m_V=0.7)).
- Further studies at small quark mass and large N_t are necessary to compare with the experimental results.

Summary

- We report the current status of our study of QCD thermodynamics with a Wilson-type quark action.
- The Lines of constant physics, in the (β,K) plane are investigated and determined the relation between the parameters (β,K) and (T, m_{PS}/m_V).
- Simulations are performed on 16^3x4 lattice.
- We discussed the spatial string tension at high temperature.
- Derivatives of pressure with respect to μ_q and μ_I up to 2^{th} order are computed and the preliminary results are obtained.
- Fluctuations of Quark number density, isospin density and charge density are discussed.
- The line of constant pressure in the (T,μ) plane is also discussed.
- For the calculation of 4^{th} order derivatives, the choice of the number of noise vector (N_{noise}) is important. N_{noise}=10 is not enough.

We expect fruitful results in near future.

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Abstract

The studies of the QCD thermodynamics by lattice simulations can provide basic input for the analysis of the experimental signatures for QGP formation and are very important. However, most of numerical simulations have been done by using staggered type quark actions, therefore studies by a different formulation of quarks on a lattice is necessary to confirm the reliability of the results from lattice QCD simulations. We have started systematic simulations of two-flavor QCD with an improved Wilson quark action. In this talk, we report the current status of our project and show the preliminary results of the Taylor expansion coefficients of the thermodynamic grand partition function in terms of chemical potential to investigate the equation of state in the low density region. Also we discuss several characteristic properties in the high temperature phase.