Deconfinement and Chiral Symmetry Restoration in Large N Gauge Theories

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QCD under Extreme conditions

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Why large N QCD?

- Large N phenomenology is interesting. It is usually close to QCD with three colors.

- Careful lattice studies have shown that it is possible to extract physical quantities in the large N limit, particularly in the pure gauge sector.

- Strongly connected to string theories. Predictions from string theories should be compared to large N results in QCD.

- There are N^2 gauge degrees of freedom but only N fermion degrees of freedom per fermion flavor if the fermions are in the fundamental representation

- There is no back reaction from the fermions in the 't Hooft limit: The number of colors, N goes to infinity at a fixed 't Hooft coupling, $\lambda = g^2 N$ with a finite number of fermion flavors in the fundamental representation.

- Fermions are naturally quenched in the 't Hooft limit.

A central gauge invariant observable

- We will consider large N gauge theories on a torus of size l. On the lattice, we will have L and b, with $b = 1/g^2N$ and L going to infinity such that l is held fixed.

- Let $W \in SU(N)$ denote the parallel transporter around a closed loop C (Wilson loop) or a closed loop that winds around the torus (Polyakov loop).

- The eigenvalues $e^{i\theta_k}$, $k = 1, \cdots, N$ of W are gauge invariant and independent of the point where the loop is opened.

- Consider the quantity $\rho(\theta)d\theta$ which is the probability of finding an eigenvalue $e^{i\theta_k}$ in the range $\theta < \theta_k < \theta + d\theta$ for some k.

- The above observable will help us understand all the transitions we are interested in. It contains information about traces of arbitrary powers of W. In this sense, it is a non-local observable.

Transition in the plaquette distribution

- Consider the $\rho(\theta)$ associated with an elementary plaquette.

- $\rho(\theta)$ has no gap at lattice strong coupling and develops a gap around $\theta = \pi$ as the coupling gets weaker on the lattice.

- This is a bulk transition on the lattice. Only the phase with the gap has a continuum limit. We call this the “cold” phase and denote it by “c”. The unphysical phase is the “hot” phase and is denoted by “h”.

- This transition depends on the lattice action and is related to the cross-over seen in lattice simulations at $N = 2$ and $N = 3$.

- It is the third order Gross-Witten transition in QCD$_2$.

- This transition is first order in $d = 3$ and $d = 4$.

Phases of continuum large N QCD

- Consider $\rho(\theta)$ associated with the Polyakov loops in different directions.

- If none of the $U(1)$ symmetries are broken, $\rho(\theta)$ will be uniform.

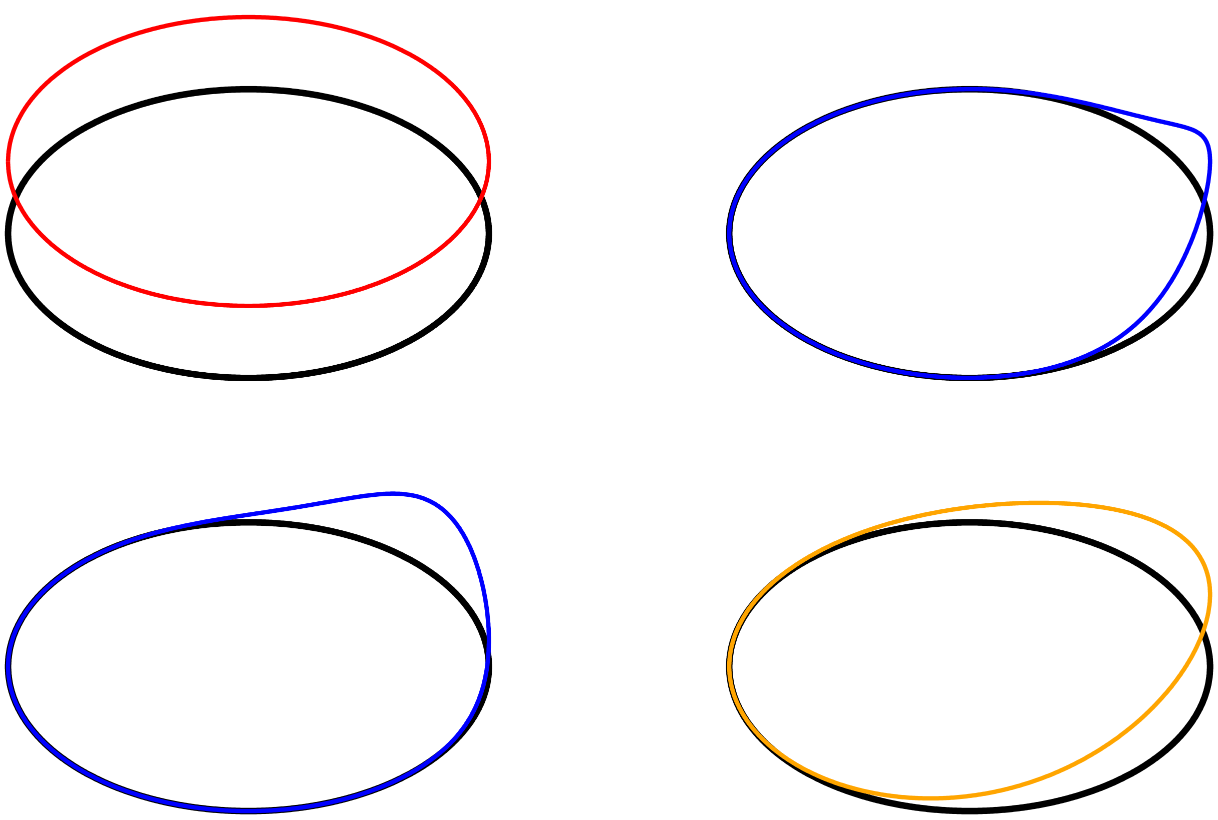
- A peak at some θ in the distribution of $\rho(\theta)$ indicates breaking of the $U(1)$ symmetry in the corresponding direction.

- Two dimensions: There is only the 0h and the 0c phase. Polyakov loops are not broken and Eguchi-Kawai reduction holds on the lattice

- Three and four dimensions. There are several phases. There is the usual 0h phase and the 0c phase. But we also have 1c, 2c and 3c phases in three dimensions and in addition a 4c phase in four dimensions. The number of the phase corresponds to the number of directions along which Polyakov loops are broken.

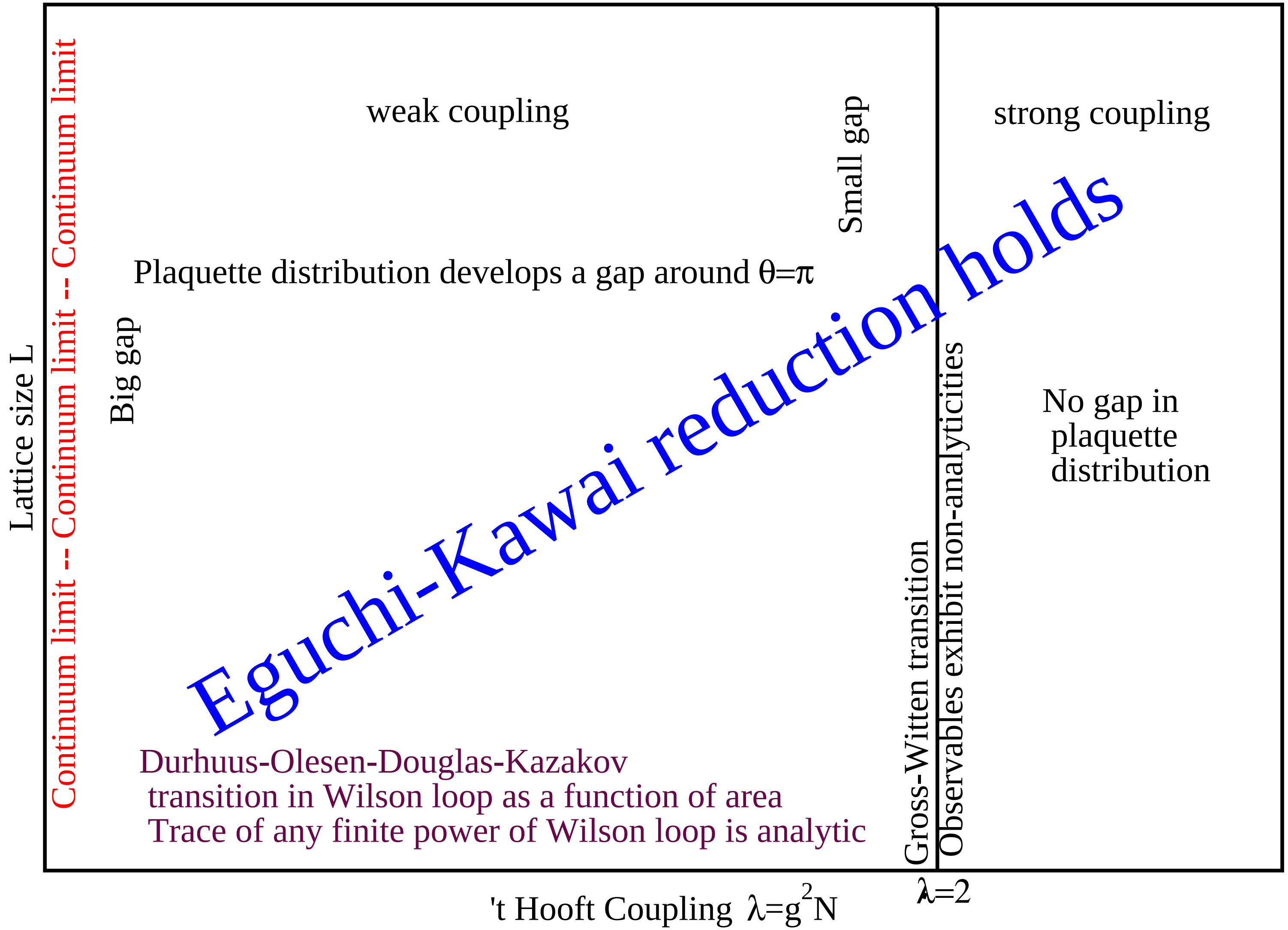
- There is a physical torus size associated with each one of these transitions.

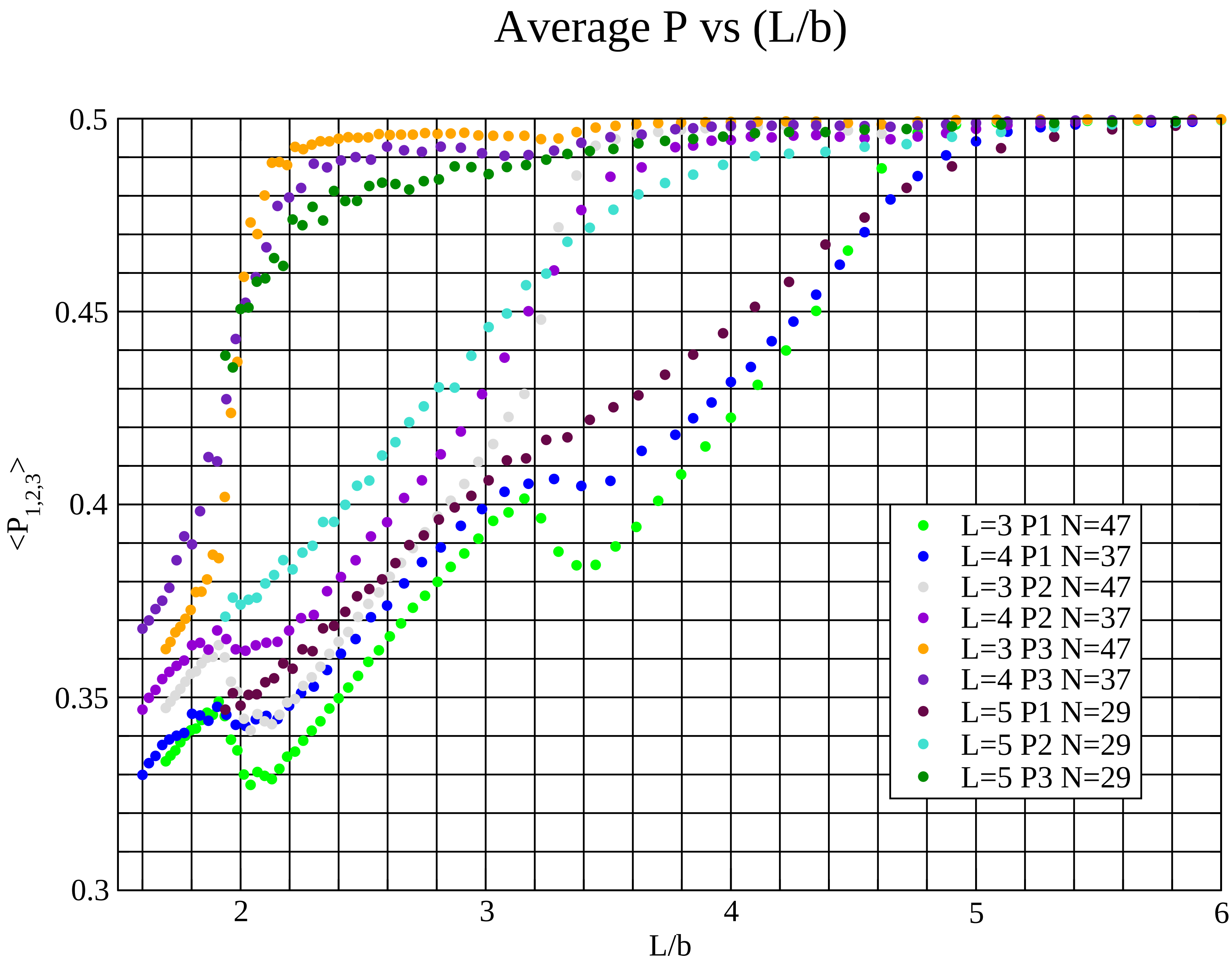
Unbroken and broken Polyakov loops



't Hooft model (2D Large N QCD)

view from the lattice

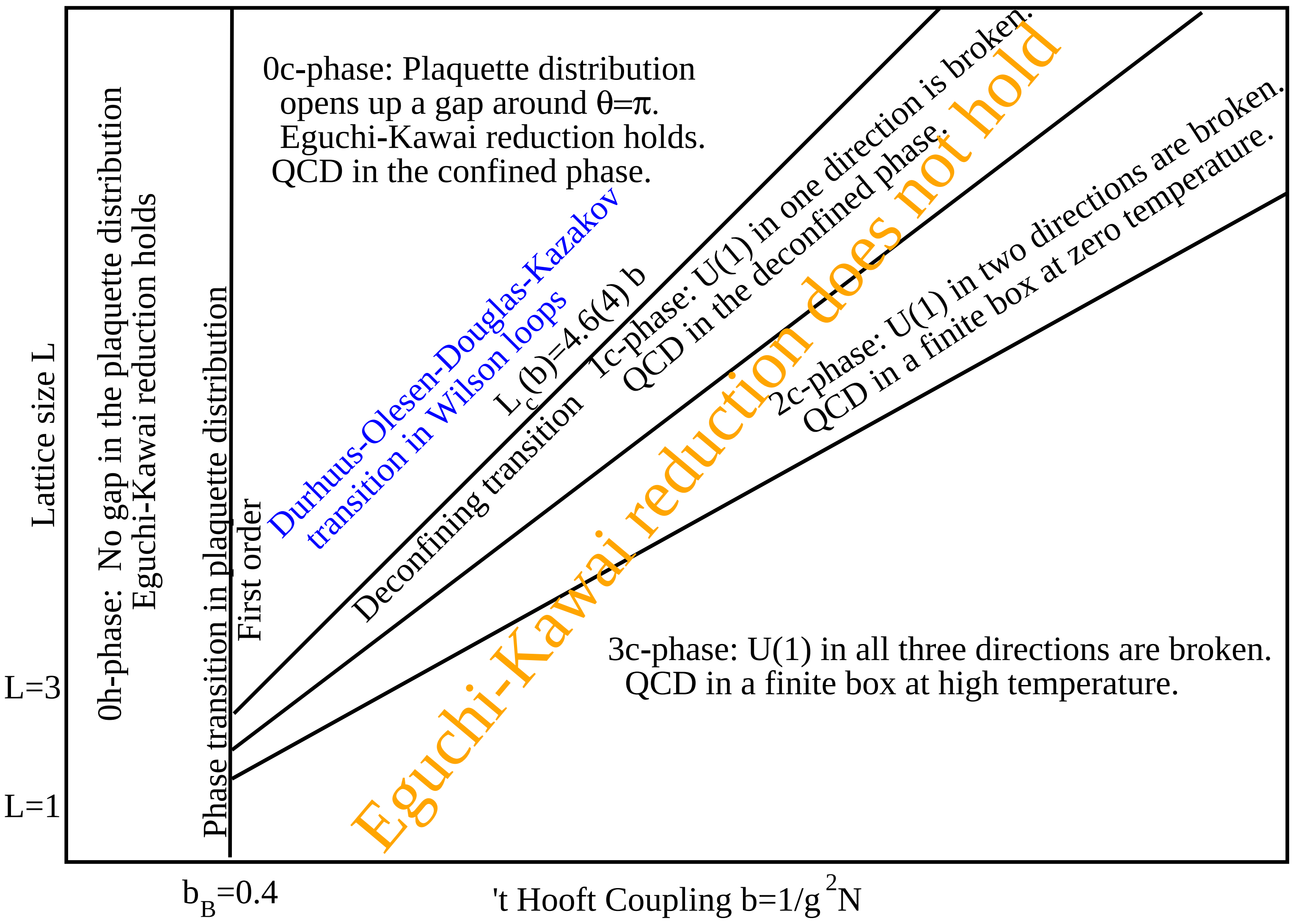




Average P vs (L/b)
0.5
0.45
0.4
0.35
0.3
2
3
4
5
6
L/b
<P1,2,3>
L=3 P1 N=47
L=4 P1 N=37
L=3 P2 N=47
L=4 P2 N=37
L=3 P3 N=47
L=4 P3 N=37
L=5 P1 N=29
L=5 P2 N=29
L=5 P3 N=29

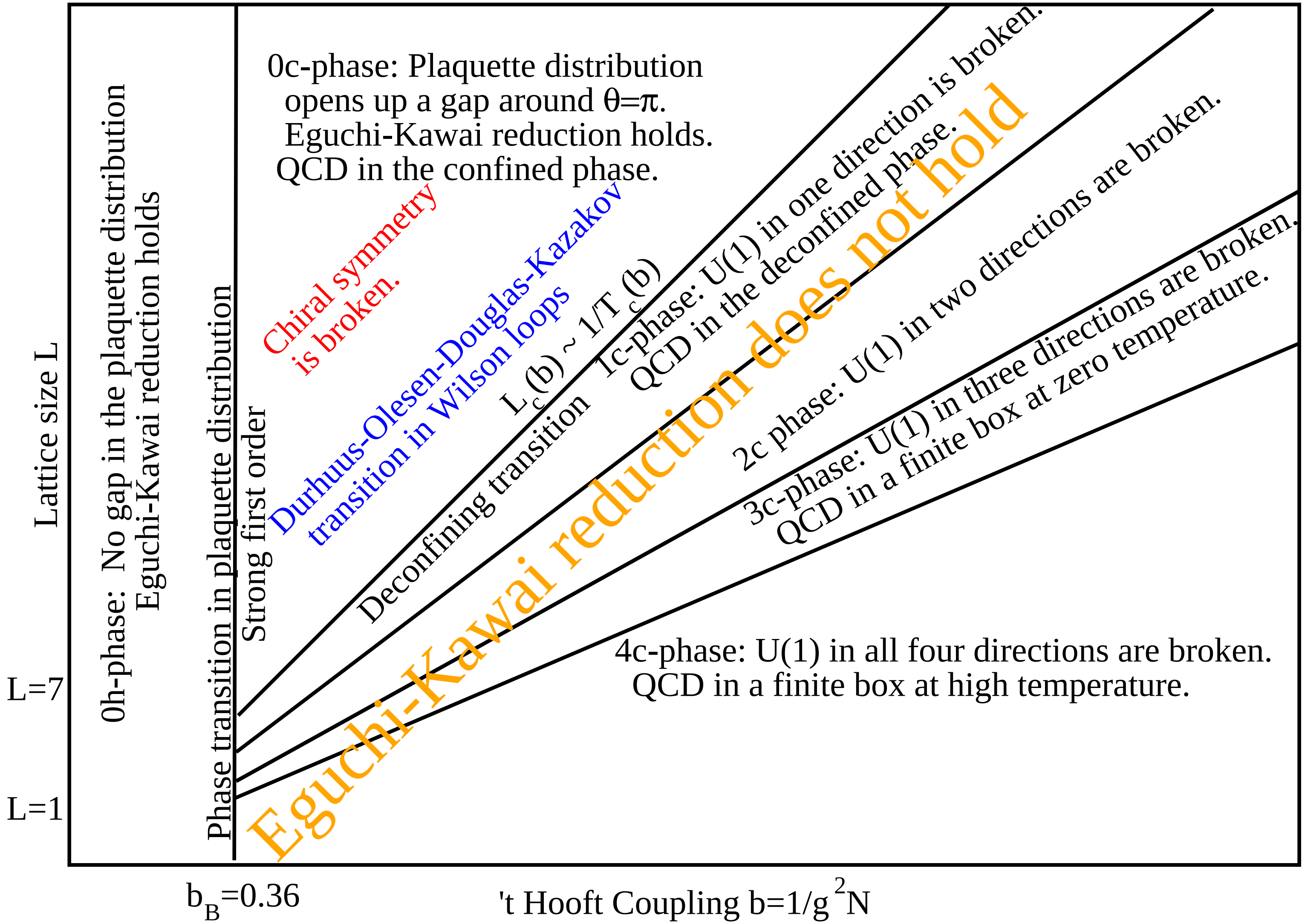
Large N QCD in three dimensions

View from the lattice



Large N QCD in four dimensions

View from the lattice



Continuum reduction

- There exists a critical size l_c that separates the 0c phase ($l > l_c$) from the 1c phase ($l < l_c$).
- Continuum reduction holds in the 0c phase and the theory does not depend on l if $l > l_c$. This theory is the confined phase off large N QCD,
- Chiral symmetry is broken in the 0c phase in the large N limit of QCD$_4$ and
$$\frac{l_c^3}{N}\{\bar\psi\psi\} \approx (0.65)^3.$$
- Consistent with chiral symmetry breaking, $m_\pi^2 \propto m_q$ and
$$f_\pi l_c \approx 0.269.$$
- $l_c \approx 1/T_c$ and theory does not feel temperatures less than T_c.
- The theory in the 1c phase behaves like finite temperature large N QCD in the deconfined phase.
- There is a finite latent heat associated with the 0c to 1c transition (J. Kiskis, hep-lat/0507003)

Fermion's role in the 1c phase

- Fermions do matter in the 1c phase even in the 't Hooft limit in the usual sense that boundary conditions of fermions matter in the temperature direction.

- Let θ be the phase associated with the $U(1)$ that defines the boundary condition with respect to the phase of the Polyakov loop in the broken direction. Let $\theta = 0$ define anti-periodic boundary conditions.

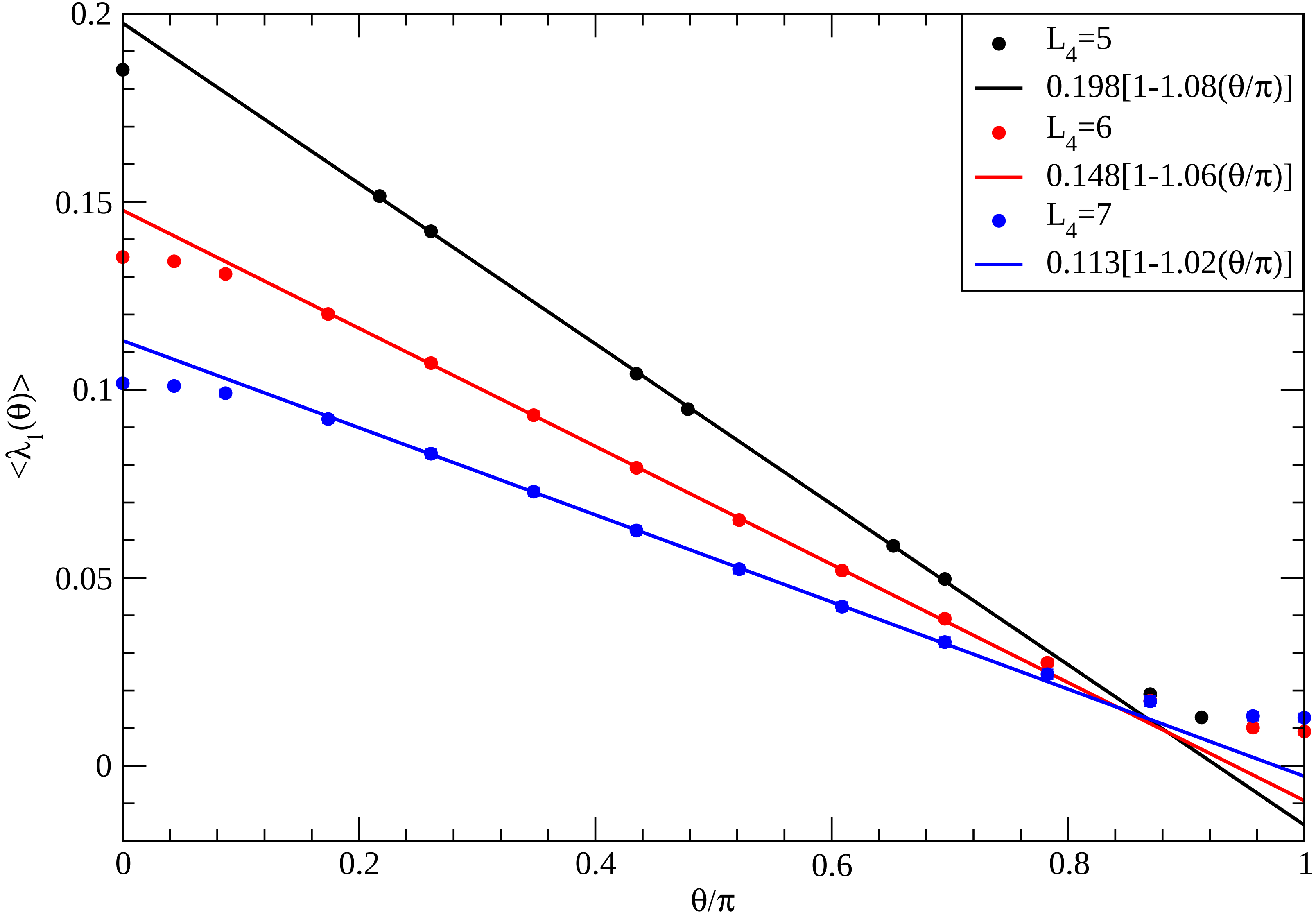
- The fermion determinant will depend on θ and dynamics should pick $\theta = 0$.

- Consider the lowest eigenvalue of the overlap Dirac operator as a measure of the fermion determinant and look at this as a function of θ.

- The data shows a gap in the spectrum for all θ as long as $T > T_c$. This shows strong interaction in the color space.

- The gap is the biggest for $\theta = 0$.

- The gap is linear in θ indicating free-field like behavior and the effect of the interactions in color space is to lower the effective temperature.



0.2
0.15
0.1
0.05
0
0
0.2
0.4
0.6
0.8
1
θ/π
<λ1(θ)>
L4=5
0.198[1-1.08(θ/π)]
L4=6
0.148[1-1.06(θ/π)]
L4=7
0.113[1-1.02(θ/π)]

Chiral symmetry restoration in the 1c phase

- Work on a $L^3 \times L_4$ lattice for several couplings b such that they are all in the 1c phase. Note that it is not necessary to pick $L_4 = L$. This freedom enables us to get several temperatures for the same L on the lattice.

- Define the gap, G, to the average of the lowest eigenvalue of the overlap Dirac operator.

- Use $L_c(b)$ to define a dimensionless gap, $g = GL_c(b)$, and a dimensionless temperature, $t = L_4 L_c(b)$.

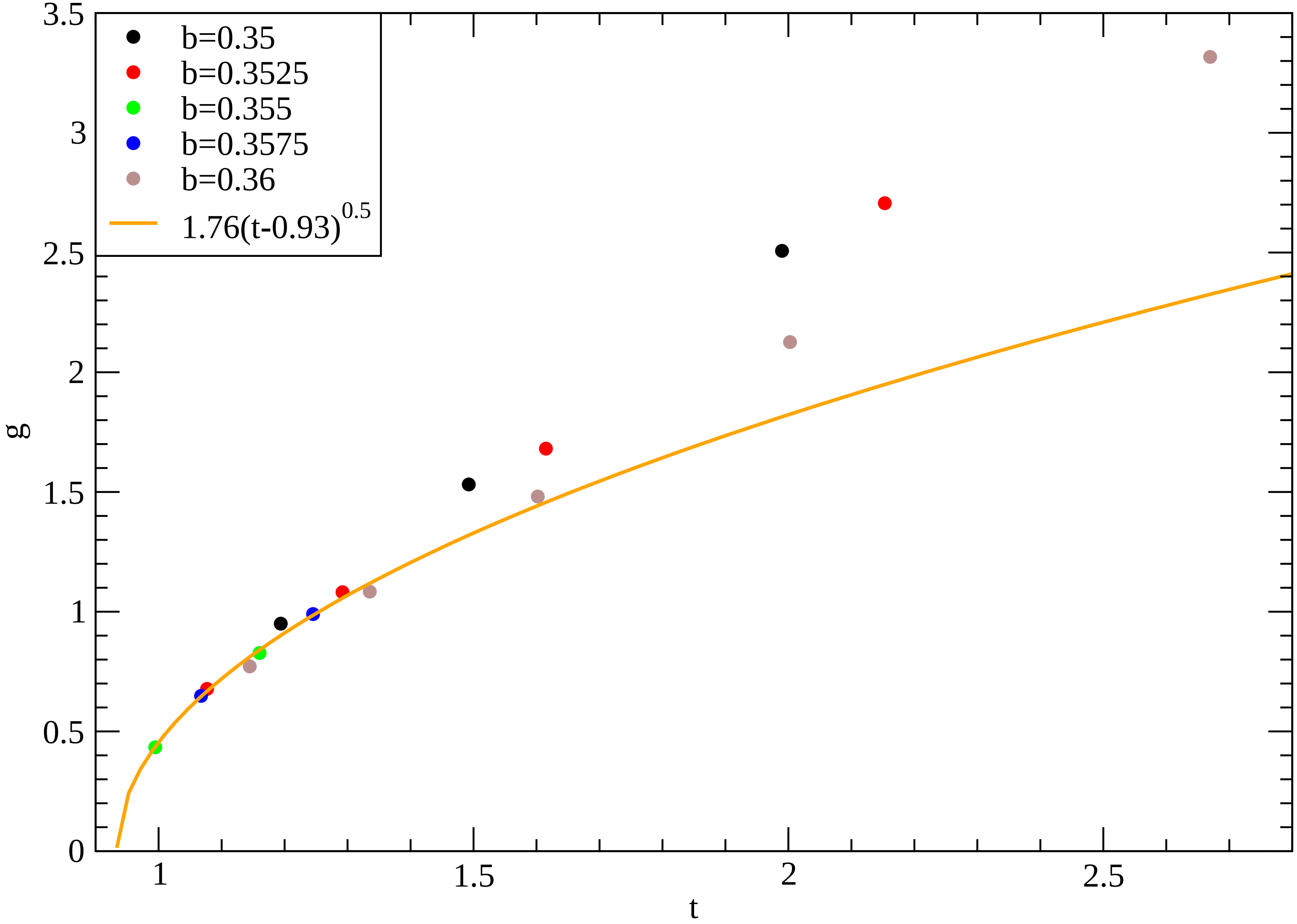
- A plot of g vs t shows that the data fall on a universal curve for small lattice spacing.

- The data fits $1.76\sqrt{t - 0.93}$ for $1 < t < 1.5$.

- There is clear numerical evidence for a first order phase transition in the fermionic sector.

- If we could supercool in the 1c phase below $t = 1$, we would find $T_c^{\mathrm{chiral}} \approx 0.93 T_c^{\mathrm{deconfined}}$

- Holographic models usually find a first order chiral transition.


b=0.35
b=0.3525
b=0.355
b=0.3575
b=0.36
1.76(t-0.93)^0.5
g
t
0
0.5
1
1.5
2
2.5
3
3.5
1
1.5
2
2.5

Model for the restoration of chiral symmetry

- Consider a gaussian Random Matrix model for a general complex matrix C. Then consider the massless Dirac operator as $D = \begin{pmatrix} 0 & C \\ -C^\dagger & 0 \end{pmatrix}$. One can compute the joint distribution of the eigenvalues $i\lambda$ of D in this model. A single parameter, namely the chiral condensate Σ, fits QCD data to this model.

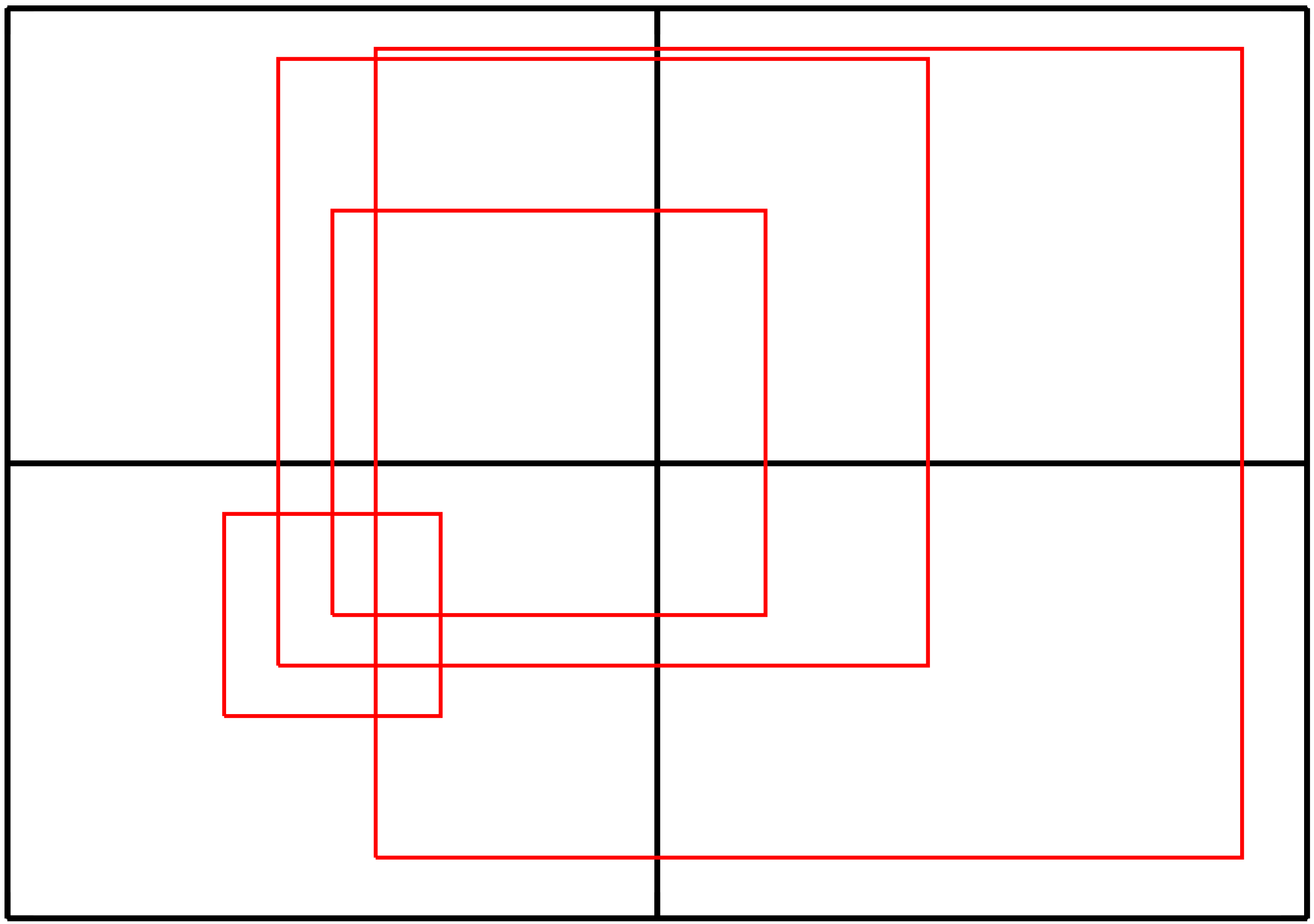
- This can be generalized to fit the data in the deconfined phase where chiral symmetry is restored.

- Consider $D = \begin{pmatrix} 0 & C + i\omega \\ -C^\dagger - i\omega & 0 \end{pmatrix}$ as the Dirac operator where ω is the lowest Matsubara frequency at a given temperature.

- This model undergoes a phase transition at some ω_c and we look at $\omega > \omega_c$ to match the data in the chirally symmetric phase.

- The natural quantities to compare are $\theta_i = \lambda_i - \lambda_1$ for $i > 1$ since we have a soft edge in the symmetric phase. There is evidence that the joint distribution of θ_i in the random matrix model agrees with QCD in the 1c-phase.

Wilson loops with and without folding



Durhuus-Olesen transition

- The eigenvalue distribution $\rho(\theta, A)$ of Wilson loop operators in two dimensional QCD only depends on the area and it is the Fourier transform on

$$\frac{1}{N}\langle TrW^n\rangle = \frac{1}{n}L^{(1)}_{n-1}(2An)e^{-An}$$

  This is analytic but it results in non-analytic behavior in $\rho(\theta, A)$ since it involves sum over n from 0 to ∞.

- Implicit formula exist for $\rho(\theta, A)$ in the continuum for two dimensional QCD and one finds that the distribution has a gap if $A\lambda < 4$.

- Using the notation of different phases in $D > 2$, the abobe transition is one seen in an observable within the 0c phase of QCD$_2$

- This phase transition also exists in QCD$_4$ and separates the strong coupling phase of continuum QCD from its weak coupling phase.

Renormalized Wilson loop operator

The following steps defines a renormalized Wilson loop operator for a rectangular loop on the lattice that was used to investigate the Durhuus-Olesen phase transition:

- APE smearing to eliminate perimeter and corner divergences:

$$X_\mu^{(n+1)}(x;f) = (1-f)U_\mu^{(n)}(x;f) + \frac{f}{6}\Sigma_{U_\mu^{(n)}(x;f)}$$

$$U_\mu^{(n+1)}(x;f) = X_\mu^{(n+1)}(x;f)\frac{1}{\sqrt{[X_\mu^{(n+1)}(x;f)]^\dagger X_\mu^{(n+1)}(x;f)}}$$

  - $\Sigma_{U_\mu^{(n)}(x;f)}$ is the *staple* associated with $U_\mu^{(n)}(x;f)$.
  - f is the smearing parameter and has to be in the range $0 < f < 0.75$.
  - $U_\mu^{(0)}(x,f) = U_\mu(x)$, the original link element distributed according to the standard Wilson plaquette action.

- The smeared variables define the renormalized Wilson loop operator,

$$\hat{W}[L_1, L_2; f; n = \frac{(L_1+L_2)^2}{4}],$$

  and the associated eigenvalue density $\hat{\rho}(\theta)$.

The continuum operator $\hat{W}(l, f)$

- Lattice coupling: $b = \frac{1}{g^2 N}$
- Lattice spacing in $0c$: $a(b)t_c = 1/L_c(b)$
- Pick $L_1 = L_2 = L$ (square loop) with the physical size given by $l = La(b)$ and we will measure this in units of t_c.

The continuum limit is taken at a fixed f and l by taking $a(b)$ to zero. The observable itself will be $\hat{\rho}(\theta; l, f)$. We find that

- This non-local observable undergoes a transition from being gap-less for large l (or small f) to having a gap for small l (or large f).
- There is a critical line in the (f, l) plane given by $f_c(l)$ where $\rho(\theta; l, f_c(l))$ is non-zero for all $-\pi < \theta < \pi$ but $\rho(\pm\pi; l, f_c(l) = 0$.
- $\rho(\theta; l, f)$ exhibits universal behavior according to Durhuus-Olesen formulas and the transition is continuous.

References

[1] A. V. Manohar, arXiv:hep-ph/9802419.

[2] M. Teper, PoS **LAT2005**, 256 (2006) [arXiv:hep-lat/0509019].

[3] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. **323**, 183 (2000) [arXiv:hep-th/9905111].

[4] J. Kiskis, R. Narayanan and H. Neuberger, "Does the crossover from perturbative to nonperturbative physics in QCD Phys. Lett. B **574**, 65 (2003) [arXiv:hep-lat/0308033].

[5] R. Narayanan and H. Neuberger, PoS **LAT2005**, 005 (2006) [arXiv:hep-lat/0509014].

[6] J. Kiskis, R. Narayanan and H. Neuberger, Phys. Rev. D **66**, 025019 (2002) [arXiv:hep-lat/0203005].

[7] T. D. Cohen, Phys. Rev. Lett. **93**, 201601 (2004) [arXiv:hep-ph/0407306].

[8] R. Narayanan and H. Neuberger, Nucl. Phys. B **696**, 107 (2004) [arXiv:hep-lat/0405025].

[9] J. J. M. Verbaarschot and T. Wettig, Ann. Rev. Nucl. Part. Sci. **50**, 343 (2000) [arXiv:hep-ph/0003017].

[10] R. Narayanan and H. Neuberger, Phys. Lett. B **616**, 76 (2005) [arXiv:hep-lat/0503033].

[11] D. J. Gross and Y. Kitazawa, Nucl. Phys. B **206**, 440 (1982).

[12] J. Kiskis, arXiv:hep-lat/0507003.

[13] R. Narayanan and H. Neuberger, Phys. Lett. B **638**, 546 (2006) [arXiv:hep-th/0605173].

[14] O. Aharony, J. Sonnenschein and S. Yankielowicz, arXiv:hep-th/0604161.

[15] R. Narayanan and H. Neuberger, JHEP **0603**, 064 (2006) [arXiv:hep-th/0601210].

[16] B. Durhuus and P. Olesen, Nucl. Phys. B **184**, 461 (1981).