Partially quenched QCD with a chemical potential

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Partially quenched QCD with a chemical potential

- “partially quenched” in the sense of different chemical potentials in the sea and valence quarks

$$\langle O \rangle = \frac{1}{Z} \int \mathrm{d}[A] \; \mathrm{e}^{-S_g} \; \det[D(\mu_1) + m_1] \; O(D(\mu_0))$$

- will consider the Dirac eigenvalues here

$$O(X) = \delta^2(z_1 - X) \; \{\delta^2(z_2 - X) \ldots\}$$

Calculating eigenvalue density (method 1)

- from partially quenched partition functions

$$Z_{n_f, n_b} = \int \mathrm{d}[A] \mathrm{e}^{-S_g} \frac{\prod_{f=1}^{n_f} \det[D(\mu_f) + m_f]}{\prod_{b=n_f+1}^{n_f+n_b} \det[D(\mu_b) + m_b]}$$

- eigenvalue density given by

$$\left[\partial_{\bar{z}} \partial_z Z_{3,2}(\{m_1, z, \bar{z} | m_b, \bar{m}_b\}, \{\mu_1, \mu_0, -\bar{\mu}_0 | \mu_b, -\bar{\mu}_b\})\right]_{m_b = z,\, \mu_b = \mu_0}$$

Microscopic partition functions from chiral Lagrangians

- in the low energy limit (ϵ-regime)

$$Z_{n_f,0} = \int_{U(n_f)} dU \, \exp\left(\frac{\Sigma V}{2} \mathrm{Tr}\, M[U+U^{-1}] - \frac{F^2 V}{4} \mathrm{Tr}[U,B][U^{-1},B] \right)$$

- some integrals can be evaluated directly
- can also calculate expressions for some $Z_{0,2}$
- however partially quenched partition functions have not (yet) been evaluated directly from the chiral Lagrangian

Microscopic partition functions from chiral Random Matrix Theory

- in the low energy limit (ϵ-regime)

$$Z_{n_f,n_b} = \int dA\, dB\, \mathrm{e}^{-\alpha N \mathrm{Tr}(A^H A + B^H B)} \frac{\prod_{f=1}^{n_f} \det[D(\mu_f)+m_f]}{\prod_{b=n_f+1}^{n_f+n_b} \det[D(\mu_b)+m_b]}$$

- with Dirac matrix

$$D(\mu) = \begin{pmatrix} 0 & i\,a(\mu)A + b(\mu)B \\ i\,a(\mu)A^H + b(\mu)B^H & 0 \end{pmatrix}$$

Normalizing the chiral Random Matrix Theory

- chRMT Dirac matrix

$$D(\mu) = \begin{pmatrix} 0 & i\,a(\mu)A + b(\mu)B \\ i\,a(\mu)A^H + b(\mu)B^H & 0 \end{pmatrix}$$

- at T>0 *a* and *b* general functions of T and μ.
- at T=0
  - matching to chiral Lagrangian gives

$$\sqrt{\alpha} = \Sigma V/2\mathrm{N}, \quad b(\mu) \approx \mu F \sqrt{V/2\mathrm{N}}$$

  - μ invariance of the partition function gives

$$a^2(\mu) = 1 + b^2(\mu)$$

Calculating eigenvalue density (method 2)

- directly diagonalizing chRMT

$$Z = \int dA\, dB\, \mathrm{e}^{-\alpha N \mathrm{Tr}(A^H A + B^H B)} \quad \det[D(\mu_1) + m_1]$$

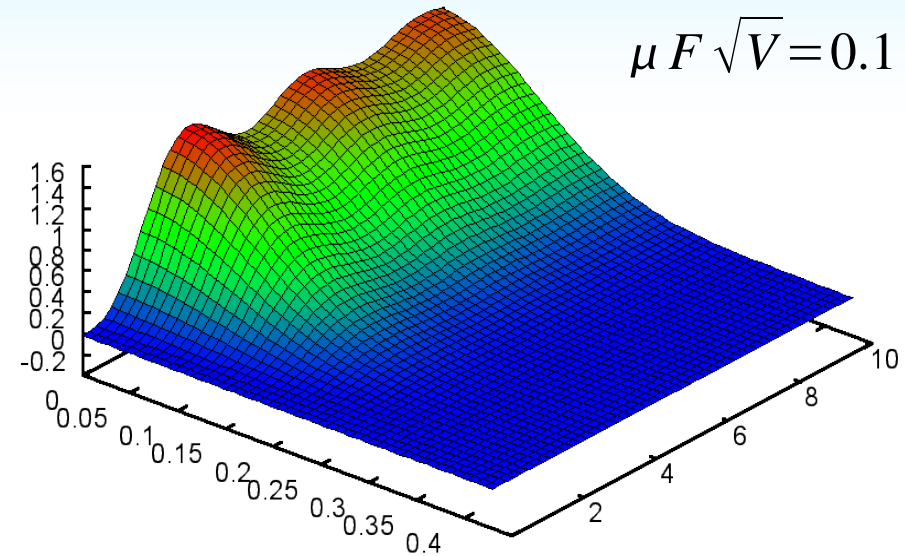
$$= \int du\, w_u(u) \quad \int |\Delta(z^2)|^2 \quad \prod_{k=1}^{N} w(u, z_k)\, dz_k$$

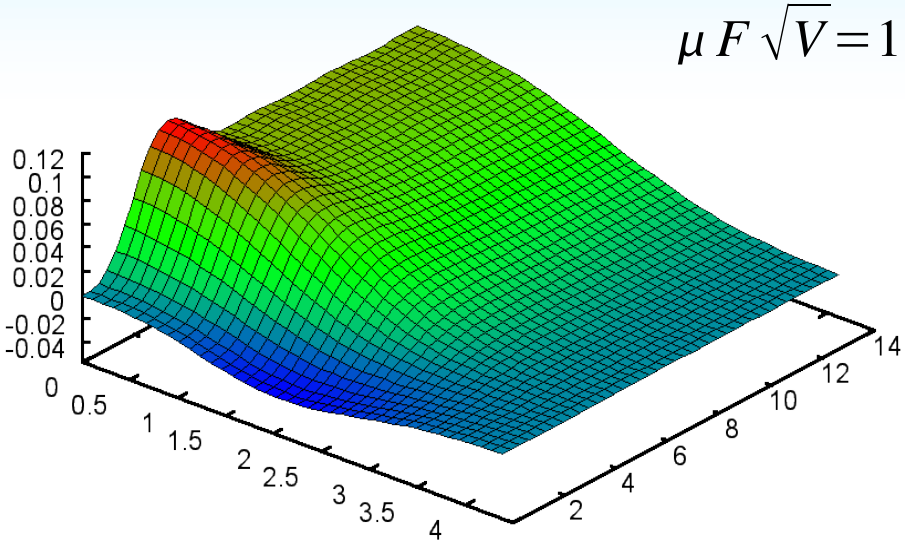
where z_k are eigenvalues of $D(\mu_0)$

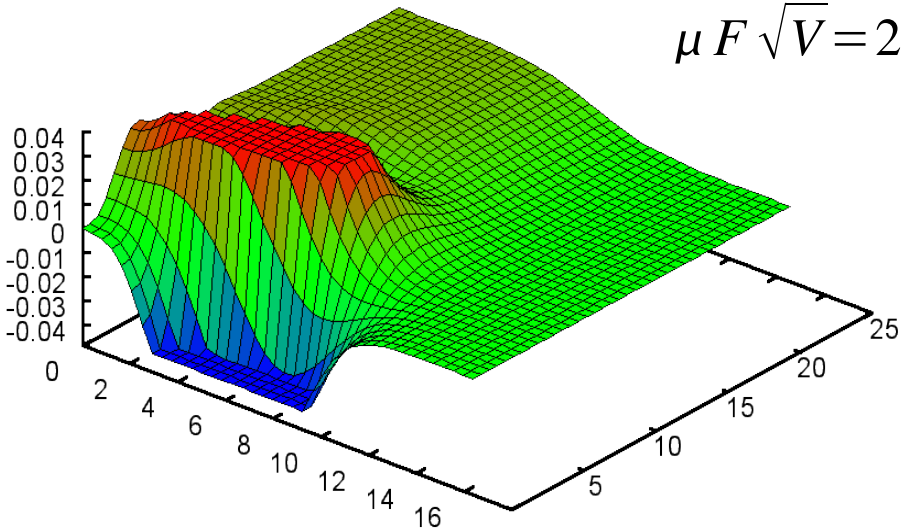
- can calculate all eigenvalue correlations

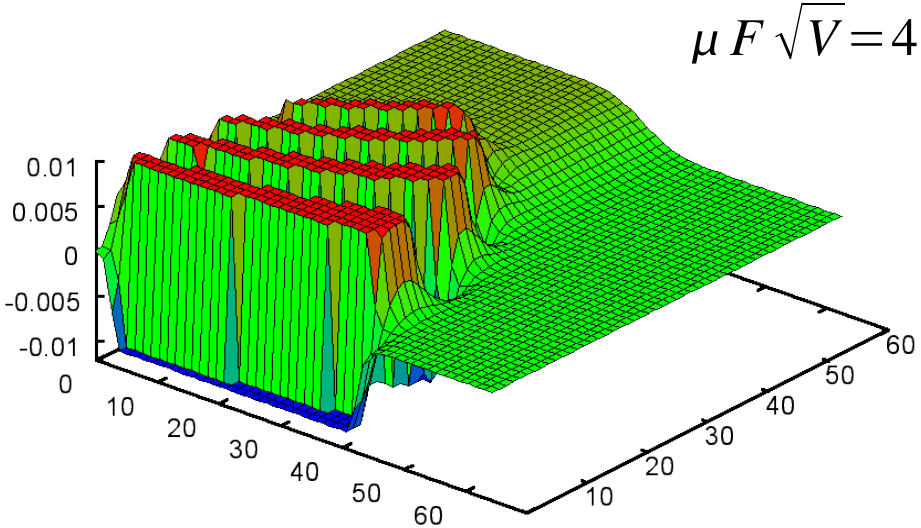
$$\rho(z_1, \ldots, z_n) \;=\; \int du\, w_u(u) \;\; \det_{i,j=1..n}\left[K(z_i, \bar{z}_j)\right]$$

One flavor eigenvalue density





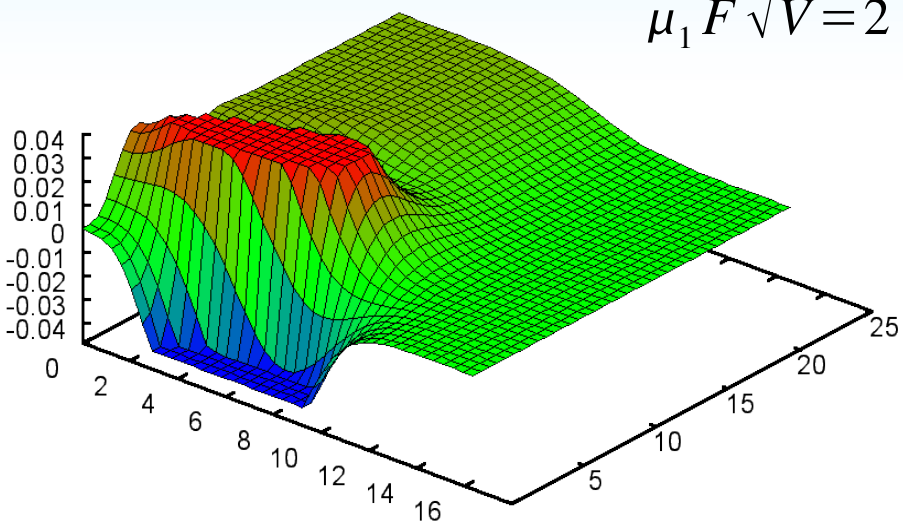




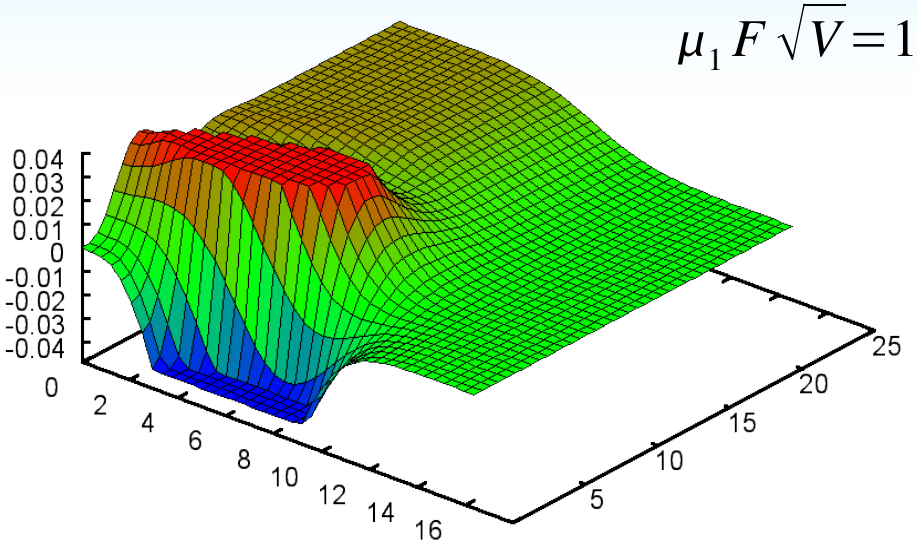
$m = 0$

One flavor eigenvalue density (partially quenched)

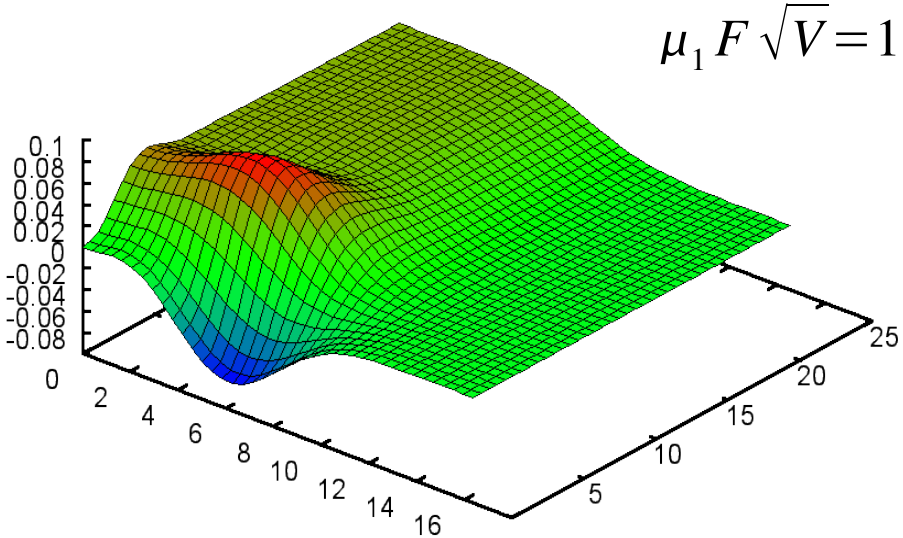
$\mu_1 F \sqrt{V} = 2$



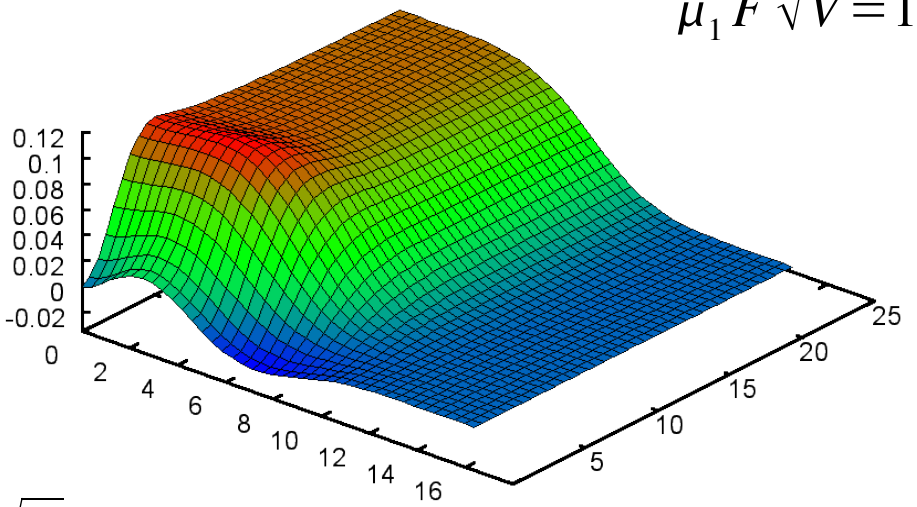
$\mu_1 F \sqrt{V} = 1.9$



$\mu_1 F \sqrt{V} = 1.5$



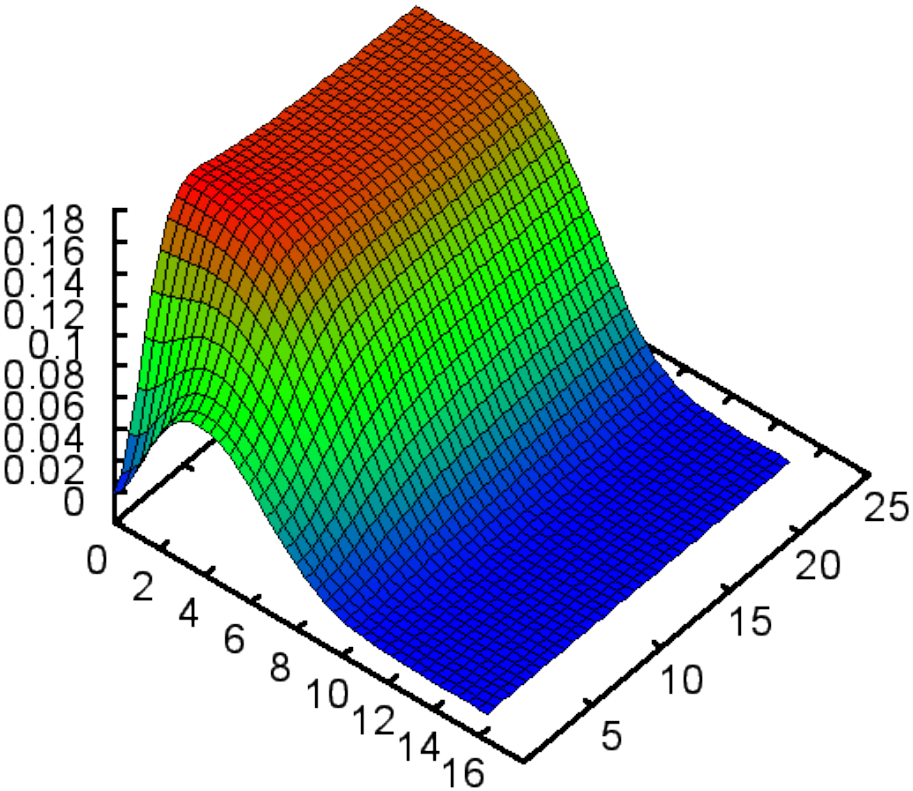
$\mu_1 F \sqrt{V} = 1$



$m_1 = 0, \quad \mu_0 F \sqrt{V} = 2$

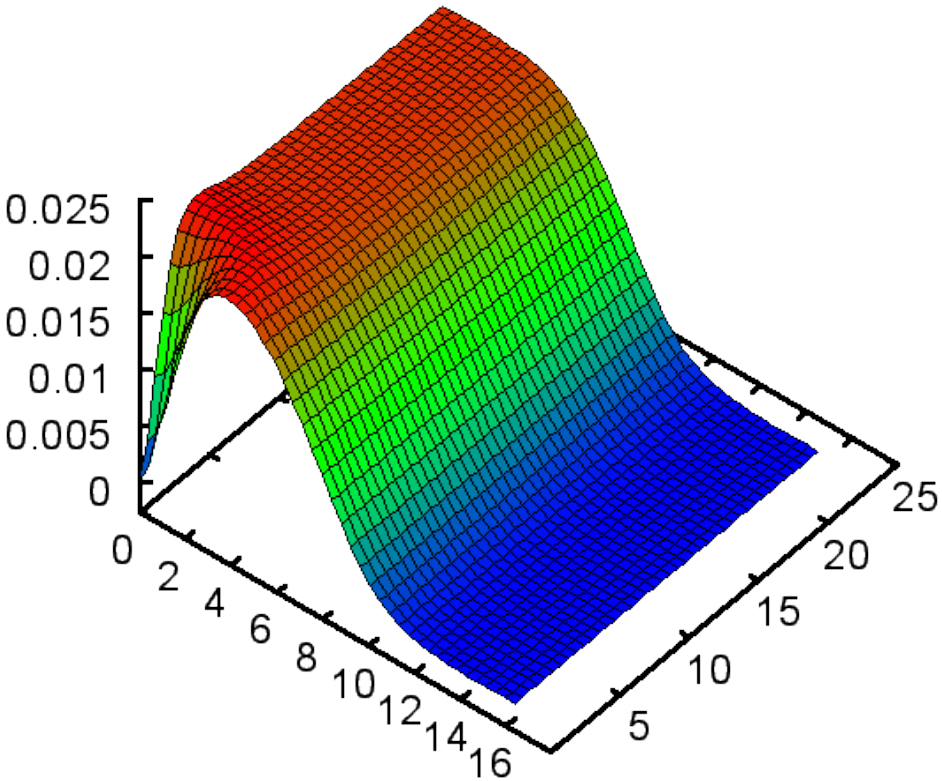
One flavor eigenvalue density (partially quenched)

$$\mu_0 F \sqrt{V} = 2$$



partially quenched

$$m_1 = 0, \quad \mu_1 F \sqrt{V} = 0$$



quenched

Applications

- fitting low energy constants (Σ,F)
  - eigenvalue density with μ sensitive to both LEC's
  - tested for quenched configurations
  - can now do dynamical at $\mu_1=0$ and fit $\mu_0\neq 0$
- imaginary chemical potential
  - eigenvalues are purely imaginary
  - can also fit LEC's (Damgaard, et. al. (unquenched))
  - now have expressions for partially quenched eigenvalue correlations

Partially quenched QCD with an imaginary chemical potential

- density and correlations for single valence chemical potential given by previous result generalized to complex chemical potential
  - not sensitive to F

- correlations of eigenvalues of different chemical potentials are sensitive to difference (and F)

$$\int \mathrm{d}[A]\mathrm{e}^{-S_g} \ \det[D(i\mu_1)+m_1] \ \ \delta(x-iD(i\mu_1)) \ \ \delta(y-iD(i\mu_0))$$

Partially quenched QCD with an imaginary chemical potential

- can also be solved from chiral RMT

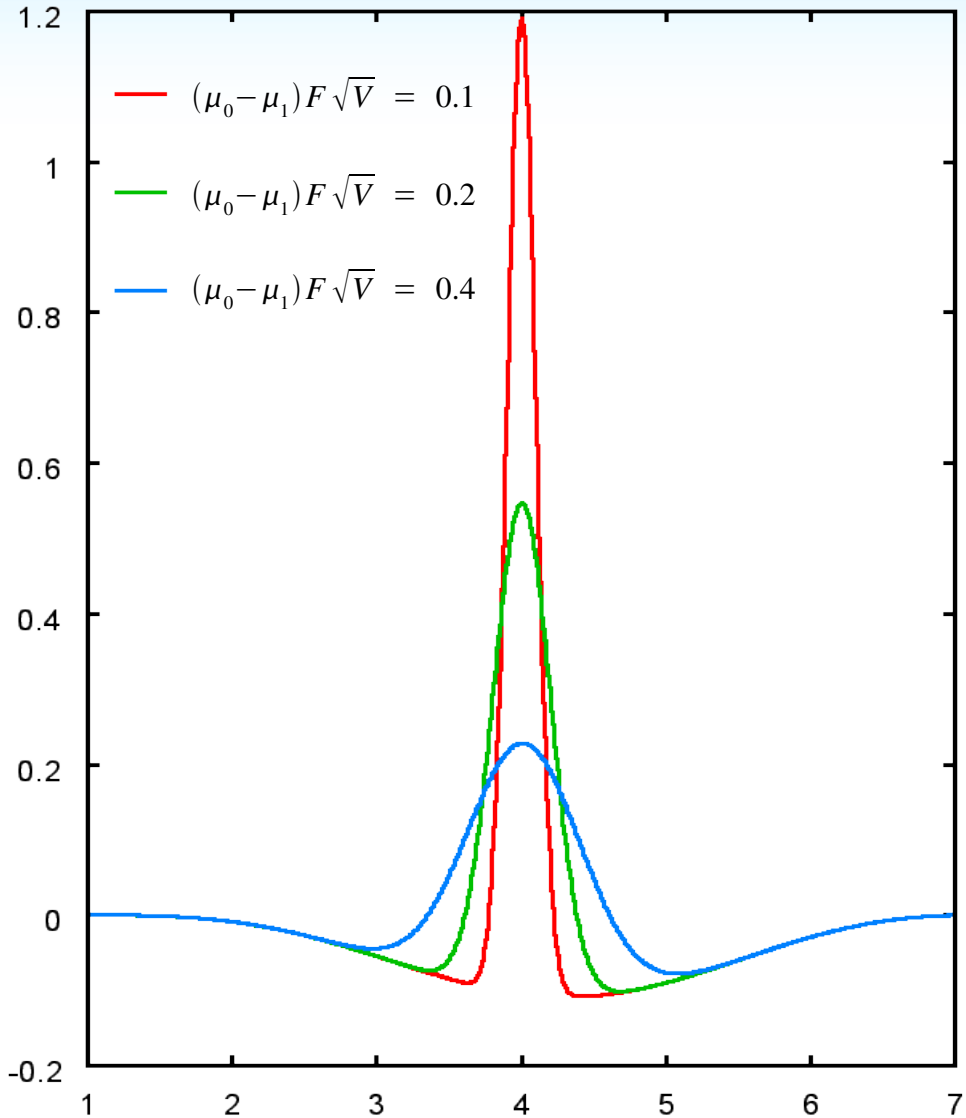
$$Z = \int dA\, dB\, \mathrm{e}^{-\alpha N \mathrm{Tr}(A^H A + B^H B)} \det[D(i\mu_1) + m_1]$$
$$= \int \Delta^2(x^2)\Delta^2(y^2) \det[w(x_i, y_j)] \prod_k (x_k^2 + m_1^2)\; d^N x\; d^N y$$

x_k are eigenvalues of $iD(i\mu_1)$ and y_k of $iD(i\mu_0)$

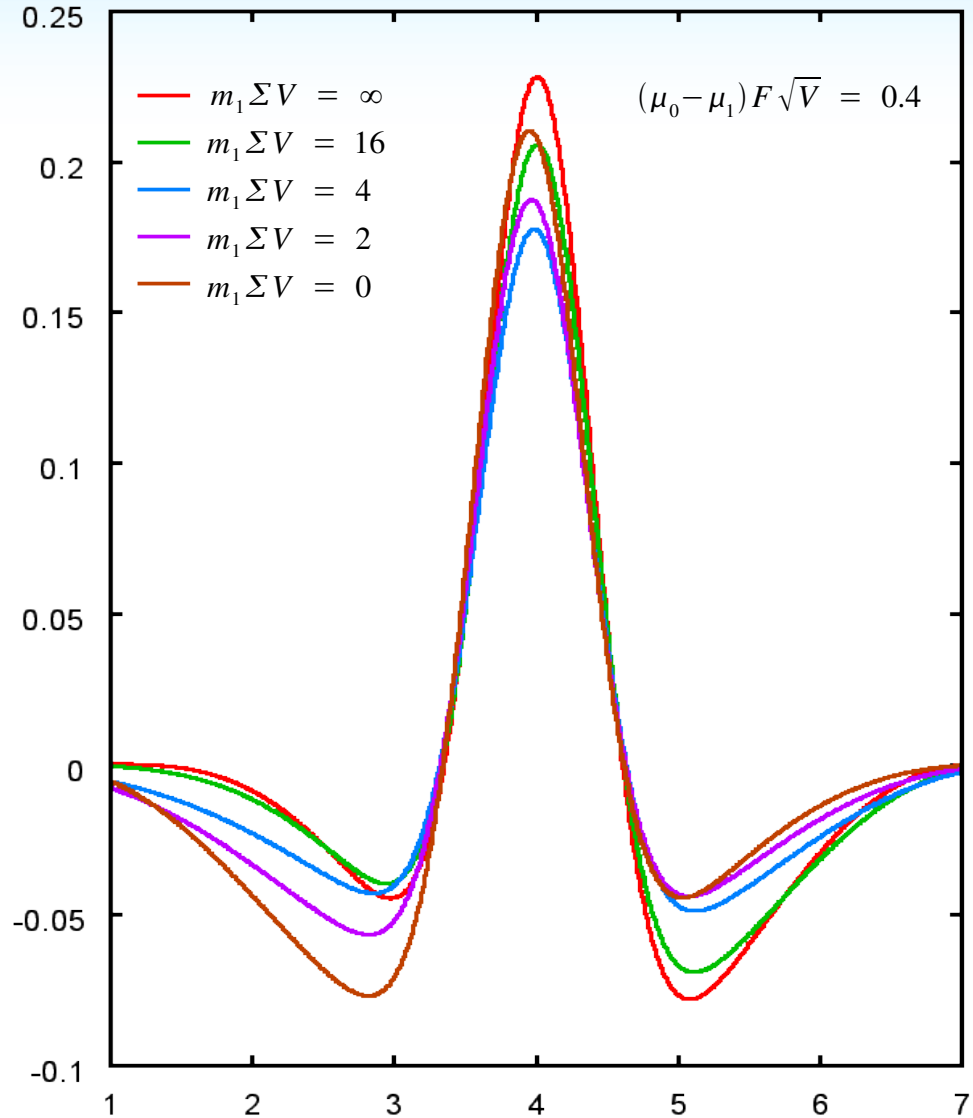
- mixed eigenvalue correlation function

$$\rho(x, y) = \langle\, \delta(x - iD(i\mu_1))\; \delta(y - iD(i\mu_0))\, \rangle - \langle\, \delta(x - iD(i\mu_1))\, \rangle \langle\, \delta(y - iD(i\mu_0))\, \rangle$$

Mixed eigenvalue correlation function



quenched ρ(4,y)



one flavor ρ(4,y)

Summary

- can now obtain formulas for eigenvalues (and partition functions) of partially quenched low energy QCD with a (complex) chemical potential from chiral RMT
- for imaginary chemical potential can calculate mixed eigenvalue correlations
- useful for fitting LEC's from configurations generated at $\mu=0$
- may also be useful in extracting other quantities (such as susceptibilities) and possibly provide information on the phase diagram