BNL 31/7– 2/8

THE PROBLEM OF CONFINEMENT IN LATTICE GAUGE THEORY

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PLAN OF THE TALK

CONFINEMENT

ORDER OF THE DECONFINEMENT PHASE TRANSITION

General viewpoint
Results for Sp(N) Yang-Mills theory

POTENTIAL MECHANISM FOR CONFINEMENT

Topological objects:
Abelian monopoles
and `t Hooft flux vortices

G(2) GAUGE THEORIES

pure Yang-Mills
Yang-Mills + Higgs

Svetitsky-Yaffe conjecture

Spatial Wilson loops always have an area law:
finite correlation length, no massless modes

S-Y: integrate out the spatial degrees of freedom

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complicated, local, effective scalar action for the Polyakov loop

IF deconfinement transition is 2nd order universal description $T \approx T_c$

YM theory: G		scalar theory: C(G)
(d+1)	DIMENSIONALITY	d
confined phase	CENTER SYMMETRY	disordered phase
deconfined phase	~~CENTER SYMMETRY~~	ordered phase

Overview: the various gauge groups

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Many groups

Holland, Pepe, Wiese
Nucl.Phys.B694 (2004) 35

Pepe, Nucl.Phys.B PS 141 (2005) 238

$SU(N) \rightarrow \mathbb{Z}(N)$

$Sp(N) \rightarrow \mathbb{Z}(2)$

$SO(N) \xrightarrow{\text{Spin}(N)}$
- $\mathbb{Z}(2)$ — N odd
- $\mathbb{Z}(4)$ — N=4k+2
- $\mathbb{Z}(2) \otimes \mathbb{Z}(2)$ — N=4k

exceptional groups
- G(2), F(4), E(8) → trivial center
- E(6) → $\mathbb{Z}(3)$
- E(7) → $\mathbb{Z}(2)$

- (2+1)-d strong fluctuations

SU(2), SU(3), SU(4) → 2nd order p.t. — SU(N), N=5,6,12,24,48 → 1st order p.t.

Ising — 3-state Potts — Ashkin-Teller (?)

- (3+1)-d SU(2) → 2nd order p.t. — SU(N), N=3, 4, 6, 8 → 1st order p.t.

Sp(N) → explore a different direction w.r.t. SU(N)

Sp(N) → Ising universality class available

G(2) → no center symmetry breaking down

- We expect groups with the same algebra ⇒ same behavior

SO(3) ↔ SU(2): Gavai et al.; de Forcrand and Jahn; Müller-Preussker et al.

$Sp(2)_{10}$ Yang-Mills theory

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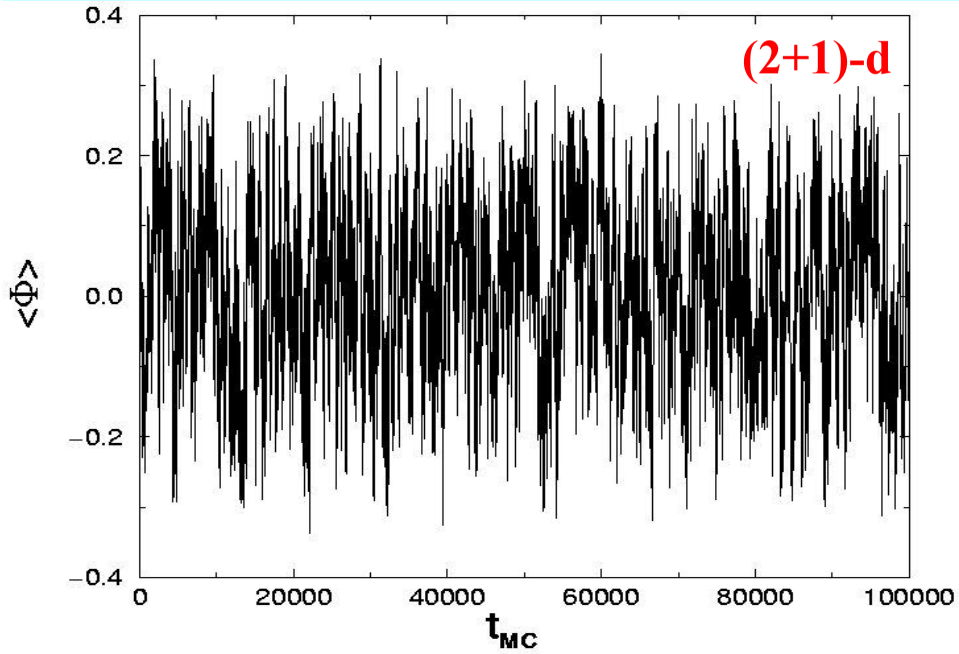

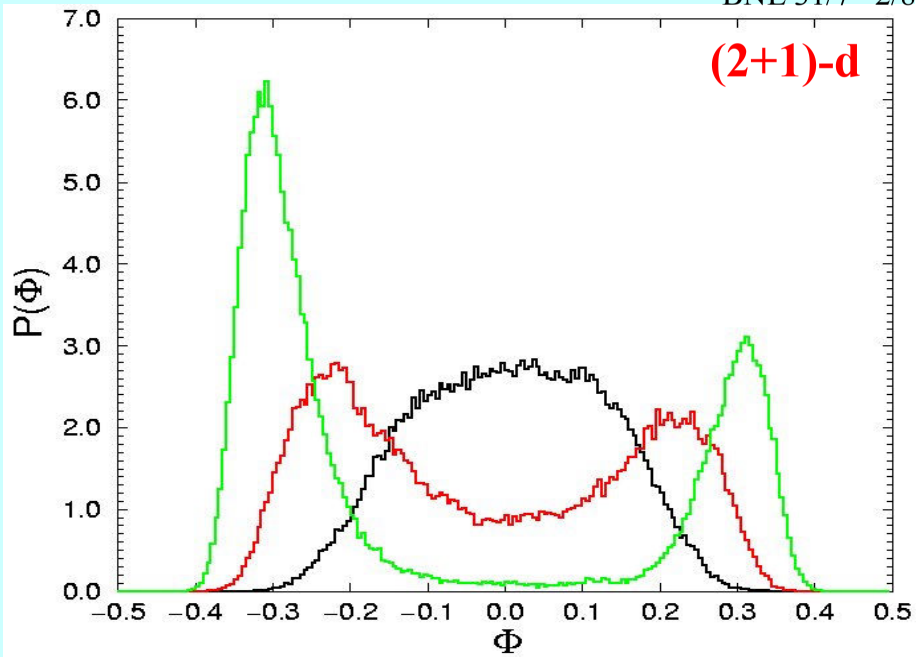

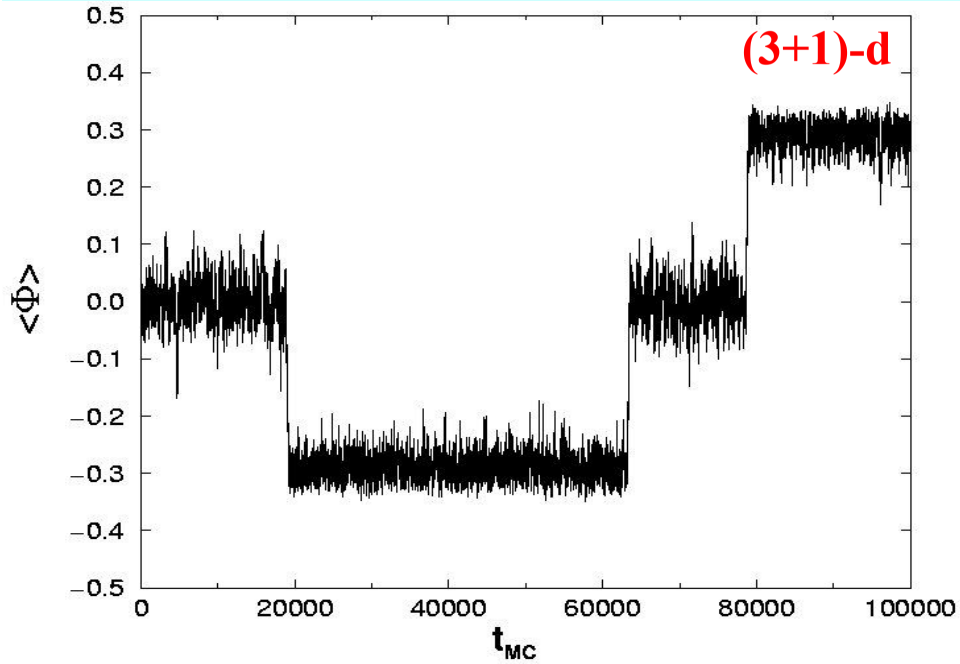

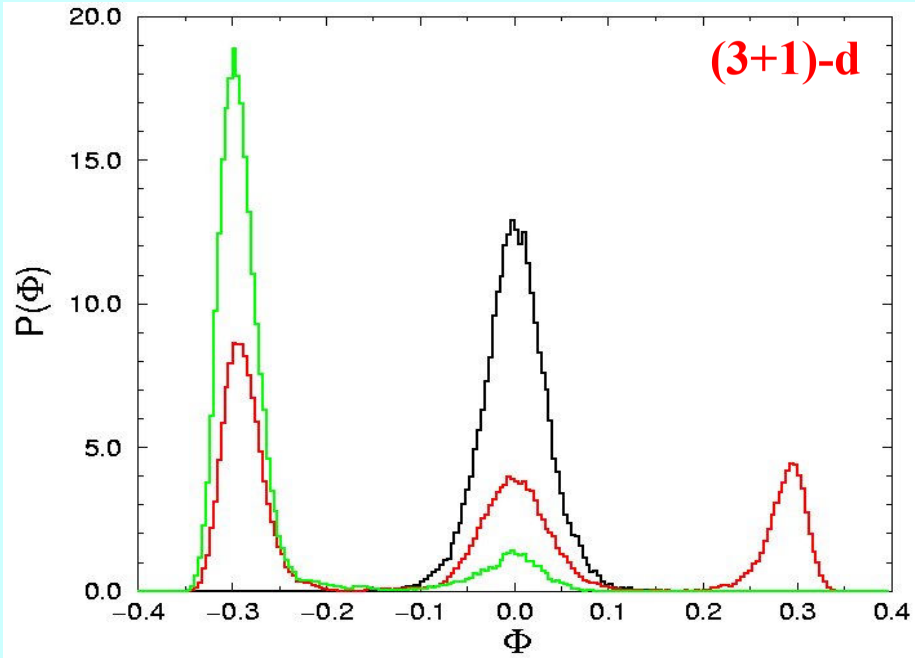

$Sp(3)_{21}$ Yang-Mills theory

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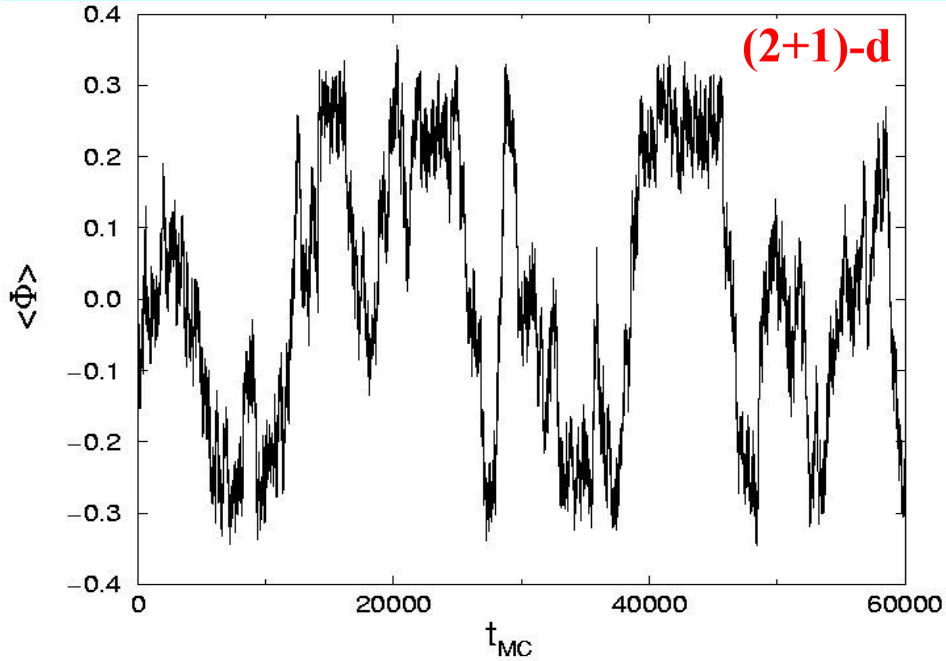
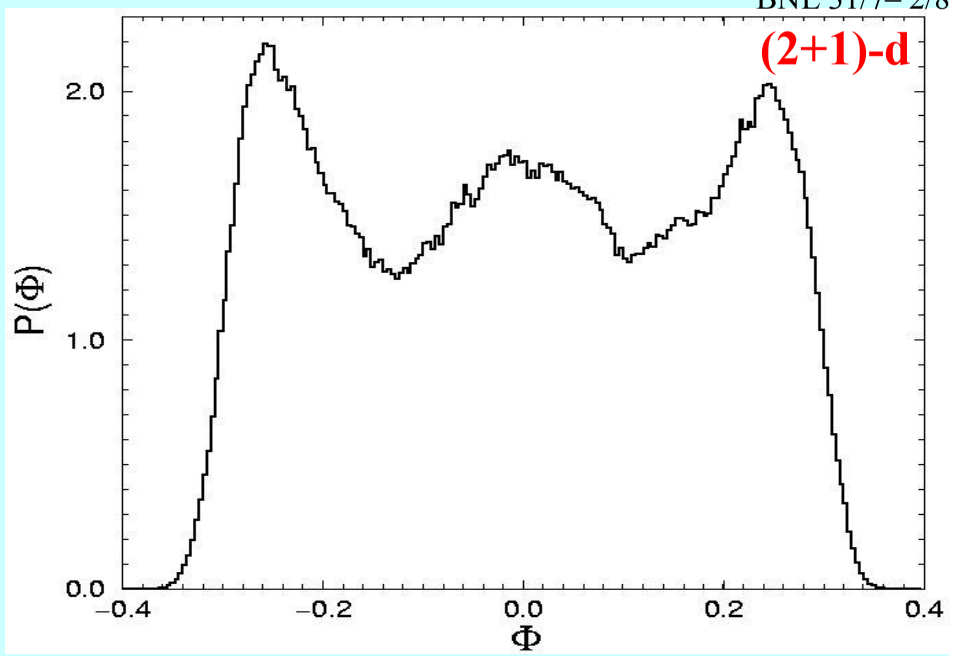
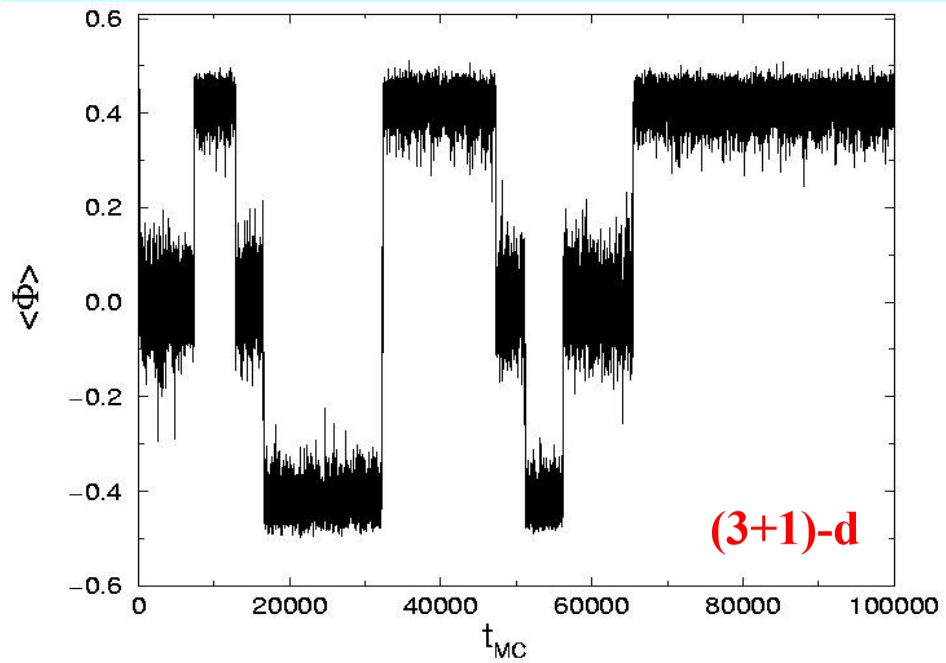
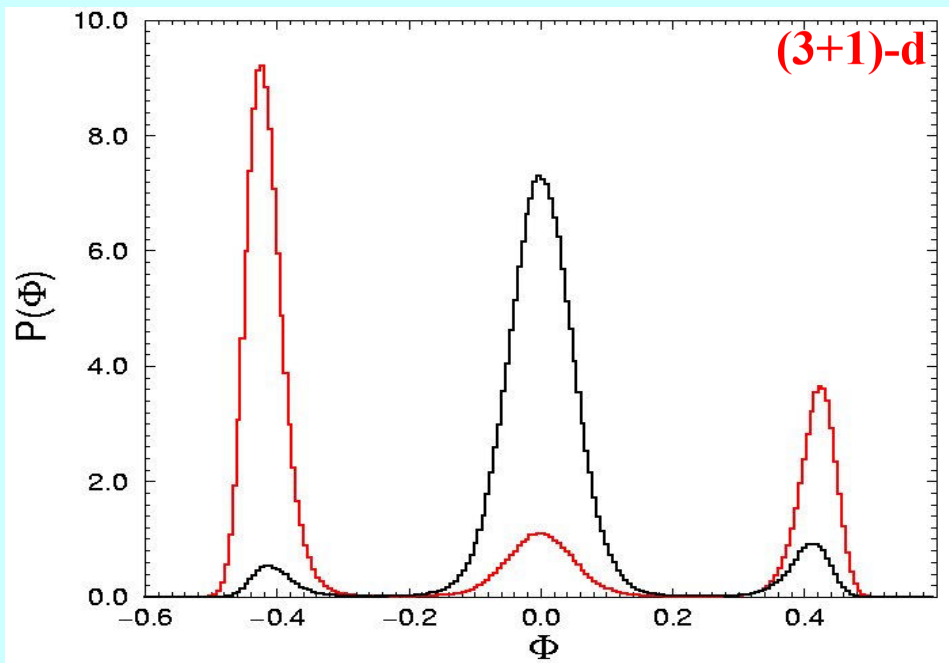








Outcome: a conjecture

Sp(N): increase the size of the group keeping the center $\mathbb{Z}(2)$ fixed

(3+1)-d: only Sp(1)=SU(2) YM theory has a 2nd order deconfinement p.t.

center: no information about the order of the deconfinement transition

conjecture

confined phase $\longleftrightarrow$ deconfined phase

(glueball states) (gluon plasma)

size of the group determines the order of the p.t.

What about confinement in YM theory with a gauge group with trivial center?

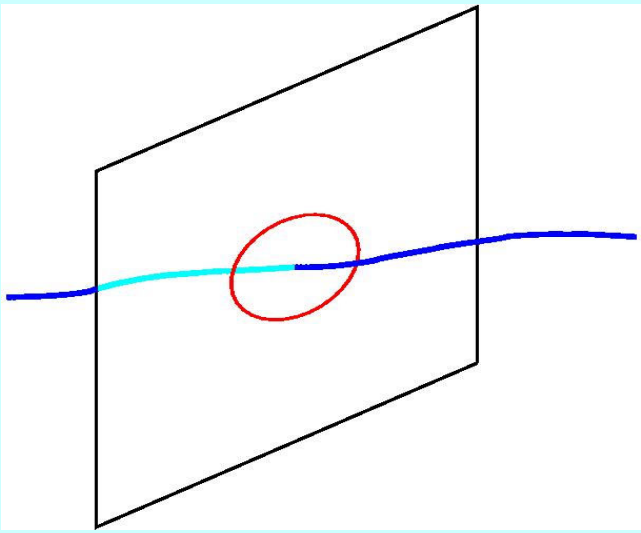
$G(2)_{14}$, $F(4)_{52}$, $E(8)_{248}$

K.Holland, P. Minkowski, M. Pepe, U.J. Wiese
Nucl.Phys.B668 (2003) 207

Mechanism of confinement

- Potential relevance of topological objects in the mechanism of confinement in non-Abelian gauge theories. Possible candidates: ’t Hooft flux vortices.

`t Hooft, Mandelstam
Suzuki et al., Negele et al.
Di Giacomo et al., Greensite et al.
Reinhardt et al.
de Forcrand and Pepe
. . . . . .



$$\pi_1(G/\mathcal{C}(G)) \neq \{\mathbb{1}\}$$

$$A_\theta(r,\theta) = \frac{1}{r}\frac{\sigma_3}{2};\ A_r(r,\theta) = 0$$

$$W = \exp\left[i\int A_\theta(r,\theta)\, r\, dr\, d\theta\right] = -\mathbb{1}$$

- Gauge theories without ’t Hooft flux vortices: study how confinement shows up.

What about confinement in YM theory with a gauge group with trivial center?

- G(2) is the simplest group such that: $\pi_1(G(2)/\{\mathbb{1}\}) = \{\mathbb{1}\}$

$$G(2) \longleftrightarrow SU(3)$$

G(2): generalities

- $G(2) \subset SO(7)$ [rank = 3; generators = 21]

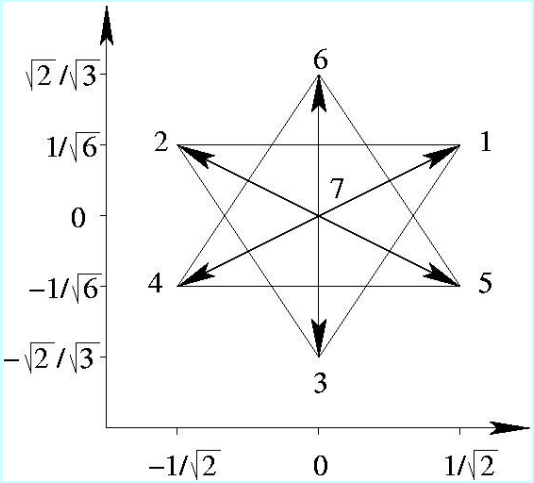
$$\det \Omega = 1 \; ; \qquad \delta_{ab} = \delta_{a'b'}\, \Omega_{a\,a'}\, \Omega_{b\,b'}$$

$$T_{a\,b\,c} = T_{a'\,b'\,c'}\, \Omega_{a\,a'}\, \Omega_{b\,b'}\, \Omega_{c\,c'} \; ; \quad T \text{ is antisymmetric}$$

14 generators; real representations (fundamental 7×7)

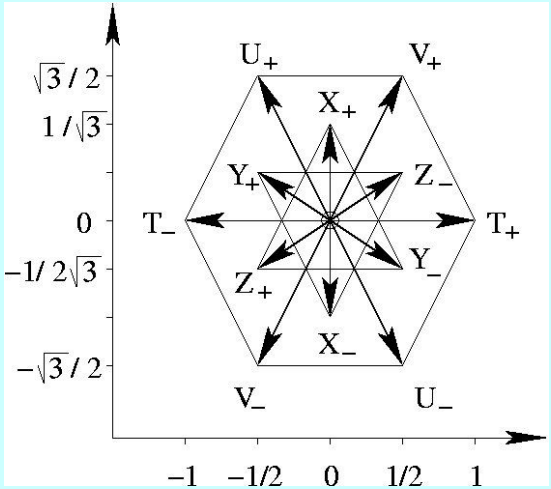
G(2)-"quarks" ~ G(2)-"antiquarks"

- G(2) has rank 2



$$\{7\} \xrightarrow[SU(3)]{} \{3\}\oplus\{\overline{3}\}\oplus\{1\}$$

- $G(2) \supset SU(3)$ in a real rep.



$$\{14\} \xrightarrow[SU(3)]{} \{8\}\oplus\{3\}\oplus\{\overline{3}\}$$

- G(2): its own universal covering group

rank 2

$\mathcal{C}(G(2)) = \{\mathbb{1}\}$

$\pi_1(\, G(2) \,/\, \{\mathbb{1}\}\,) = \{\mathbb{1}\}$

$G(2) \supset SU(3)$

- ~~"triality"~~ : all reps mix together in the tensor product decomp.

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string breaking without dynamical G(2)-"quarks"

$$\{7\}\otimes\{14\}\otimes\{14\}\otimes\{14\} = \{1\}\oplus \ldots$$

- Interesting homotopy groups

$\pi_3(\, G(2)\,) = \mathbb{Z}$ instantons

$\pi_2(\, G(2)/U(1)^2) = \mathbb{Z} \otimes \mathbb{Z}$ monopoles

} like SU(3)

$\pi_1(\, G(2) \,/\, \{\mathbb{1}\}\,) = \{\mathbb{1}\}$ no 't Hooft flux vortices unlike SU(3)

G(2) Yang-Mills

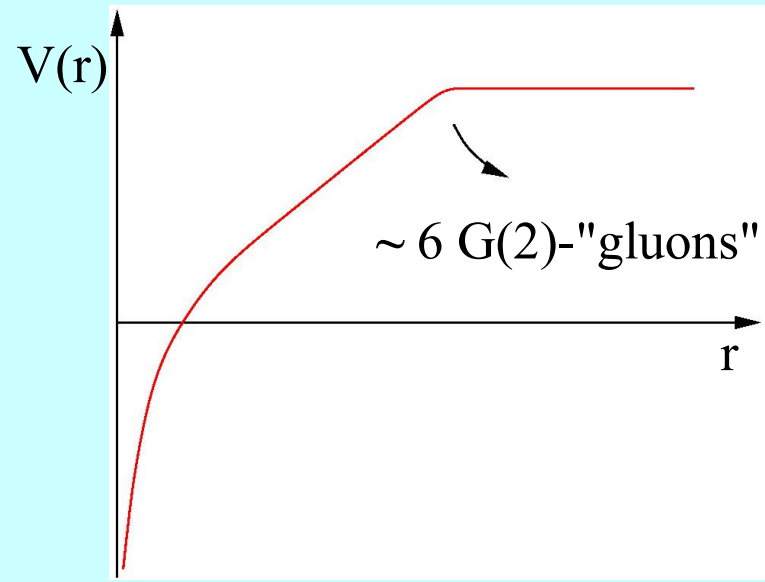
- Pure gauge: 14 G(2)-"gluons"

$$\{14\} \xrightarrow[\text{SU(3)}]{} \{8\}\oplus\{3\}\oplus\{\overline{3}\}$$

6 G(2)-"gluons" explicitly break $\mathbb{Z}(3) \Rightarrow C(\mathrm{G}(2)) = \{\mathbb{1}\}$

~quarks for SU(3)

- G(2)-YM is asymptotically free



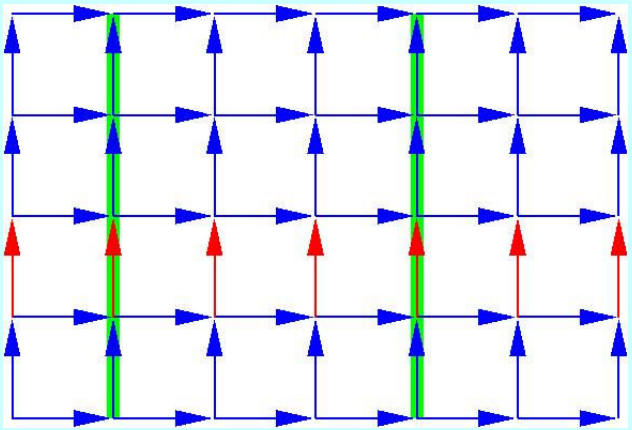
at low energies:

- confinement
- string breaking: σ=0 (QCD)

- G(2)-"laboratory": confinement similar to QCD without complications related to fermions.

- Wilson loop $\longrightarrow$ perimeter law

- Finite temperature: different behaviour than SU(3)-YM



$$\phi(\vec{x}) \longrightarrow z\phi(\vec{x})$$

$$\langle \phi(0)\phi(r)^* \rangle \xrightarrow[r \to \infty]{} \langle |\phi| \rangle^2$$

$\mathbb{Z}(3)$ unbroken $\qquad$ $\mathbb{Z}(3)$ broken

$$\langle \phi \rangle = 0, \ \sigma \neq 0 \qquad \langle \phi \rangle \neq 0, \ \sigma = 0$$

- In SU(3)-YM there is a global symmetry that breaks down.
  In G(2)-YM no symmetry ⇒ no 2nd order phase transition

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1st order $\qquad$ or $\qquad$ crossover ?

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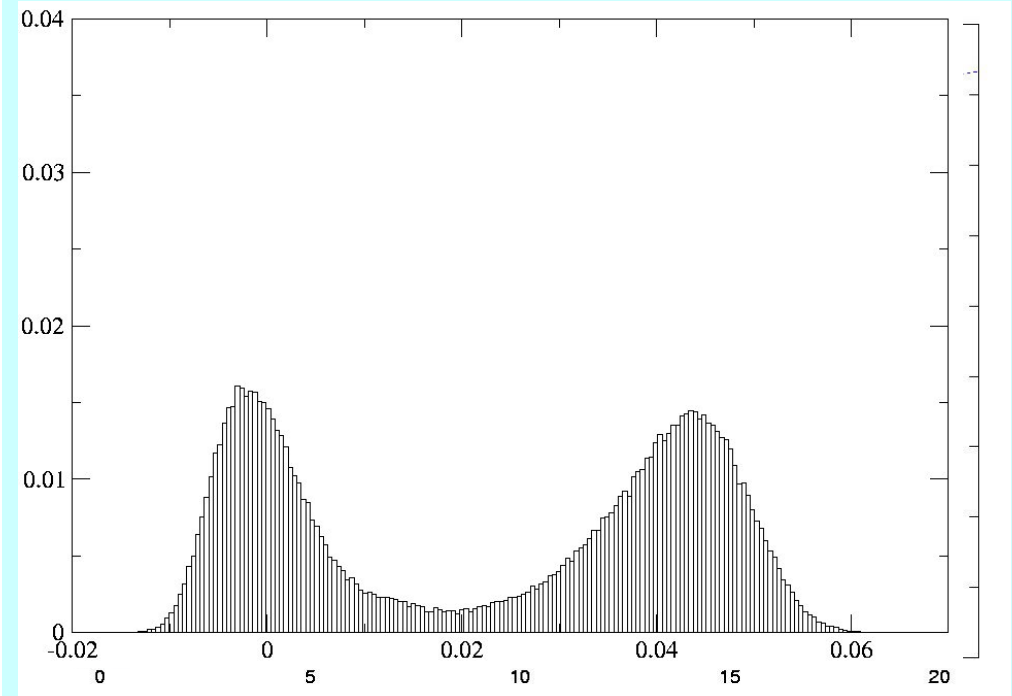
Conjecture: Sp(2) has 10 generators and it has 1st deconfinement p.t.
We expect G(2) YM to have also a 1st deconfinement p.t.

dynamical issue: numerical simulations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10
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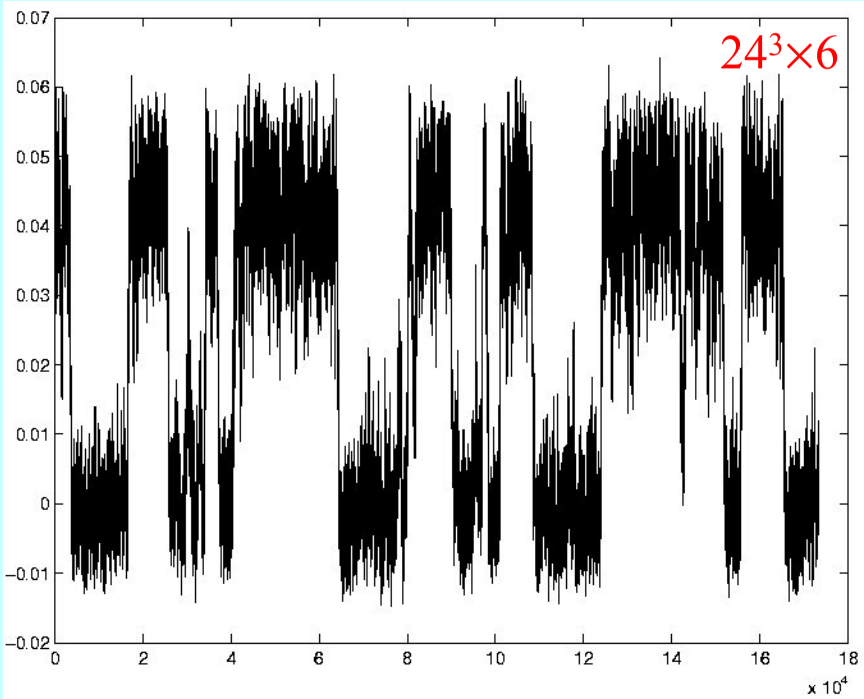
20


$24^3\times6$
0.07
0.06
0.05
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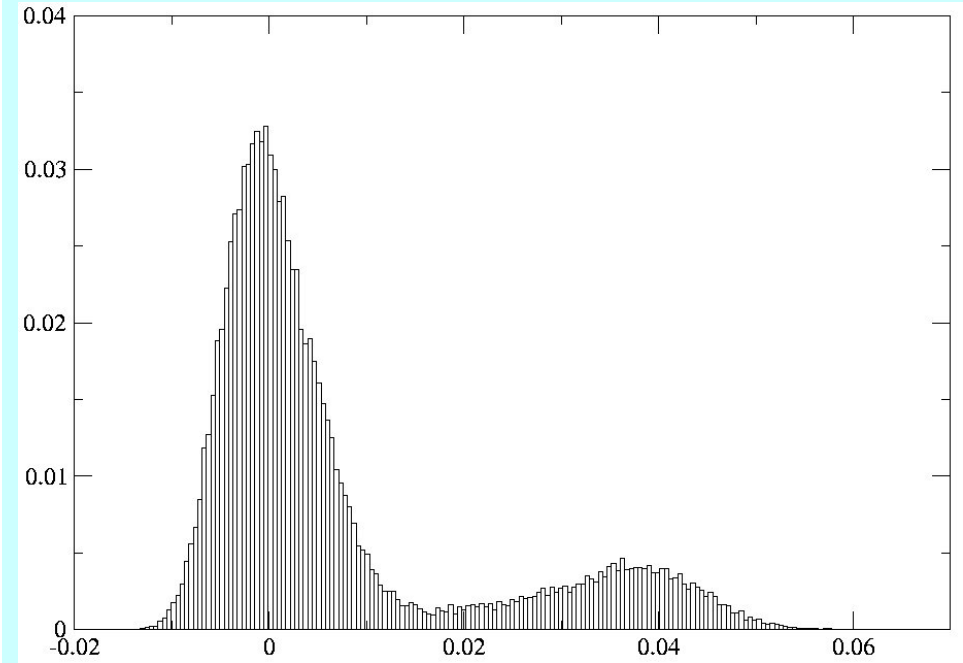
x 10^4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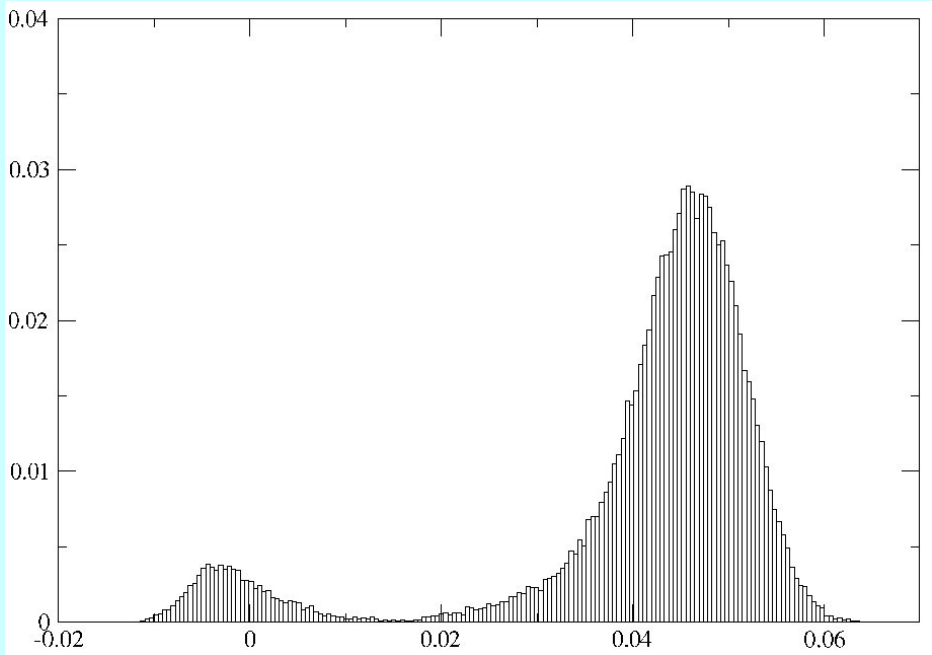
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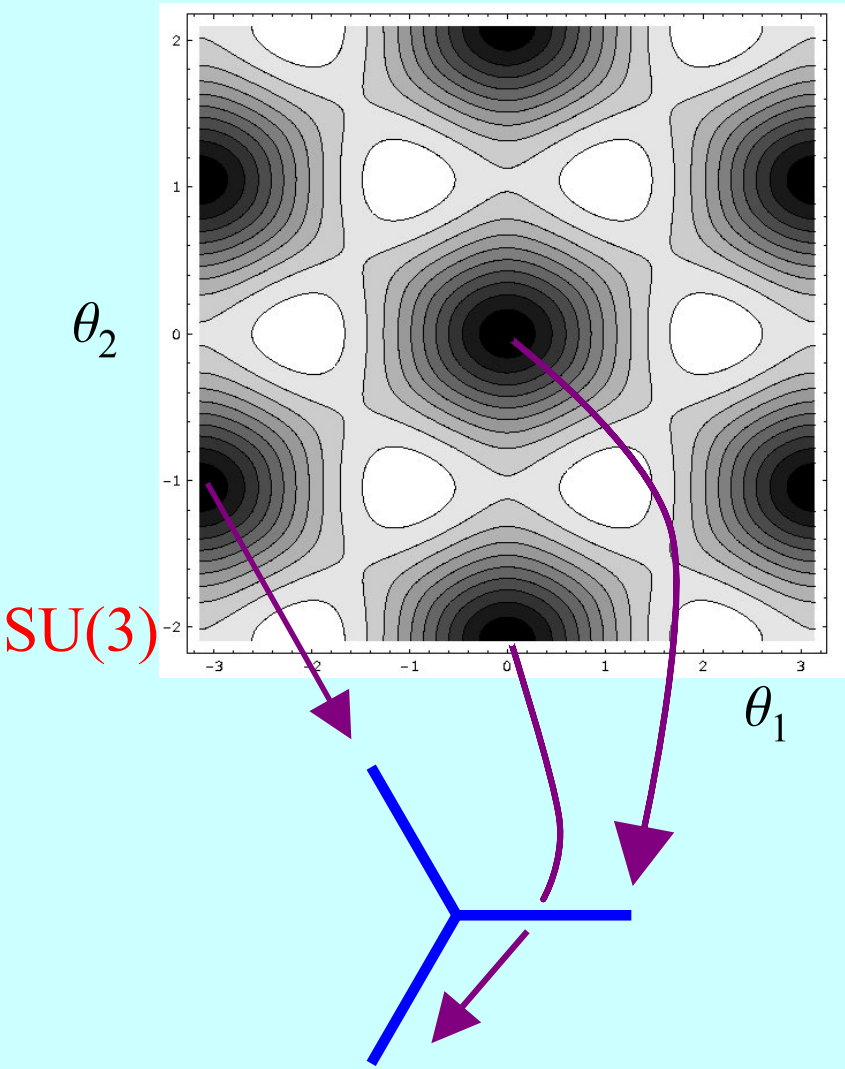
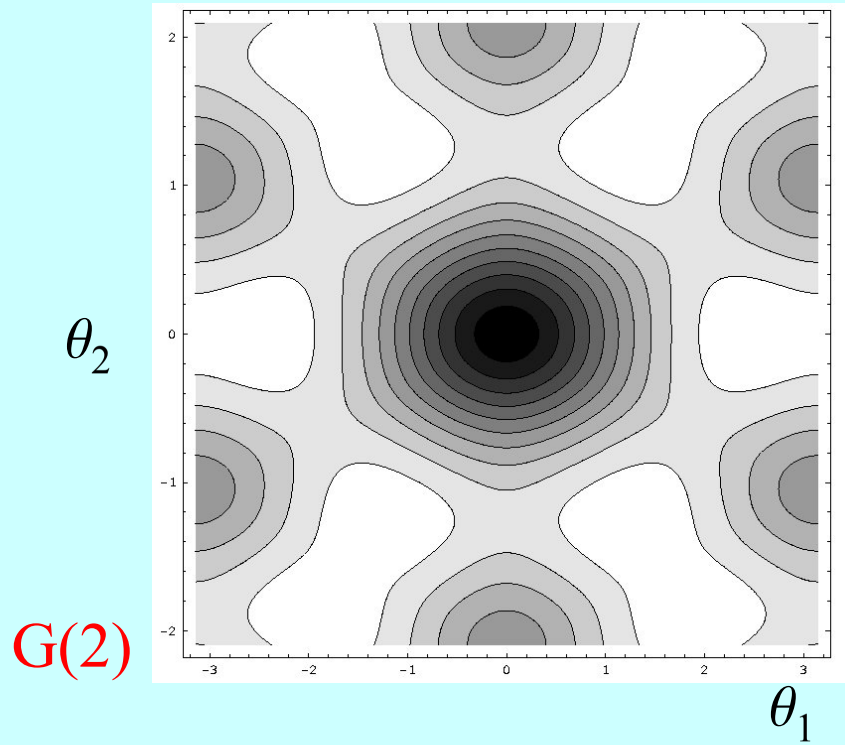
High temperature effective potential

- 1-loop expansion of the effective potential for the Polyakov loop

$\phi = \mathrm{diag}(e^{i(\theta_1+\theta_2)}, e^{i(-\theta_1+\theta_2)}, e^{-2i\theta_2})$ $\tilde{\phi}(\theta_1, \theta_2) = (\phi, \phi^*, 1)$





G(2) Yang-Mills + Higgs {7}

- Higgs {7}: $G(2) \longrightarrow SU(3)$ $\langle \phi \rangle = v$

$$\{14\} \xrightarrow[SU(3)]{} \{8\} \oplus \{3\} \oplus \{\overline{3}\}$$

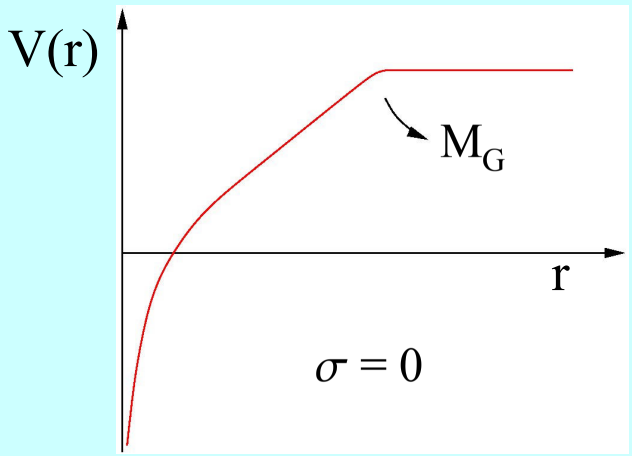
6 G(2)-"gluons" pick up a mass $M_G \propto v$

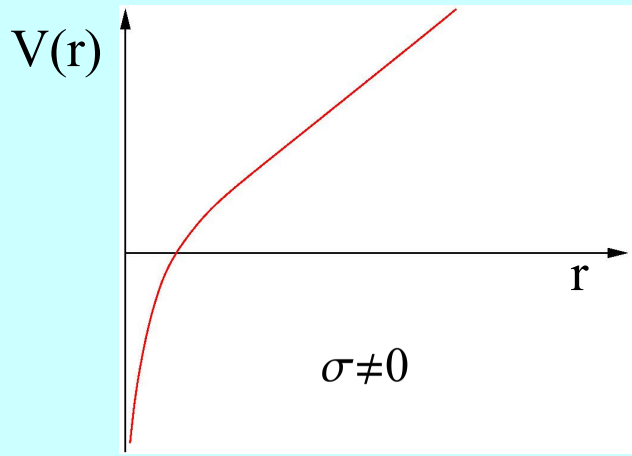
- For $M_G \lesssim \Lambda_{QCD}$ the 6 massive G(2)-"gluons" participate in the dynamics; for $M_G \gg \Lambda_{QCD}$ they decouple $\Rightarrow$ SU(3)

$\Downarrow$

Higgs {7}: handle for $G(2) \longleftrightarrow SU(3)$

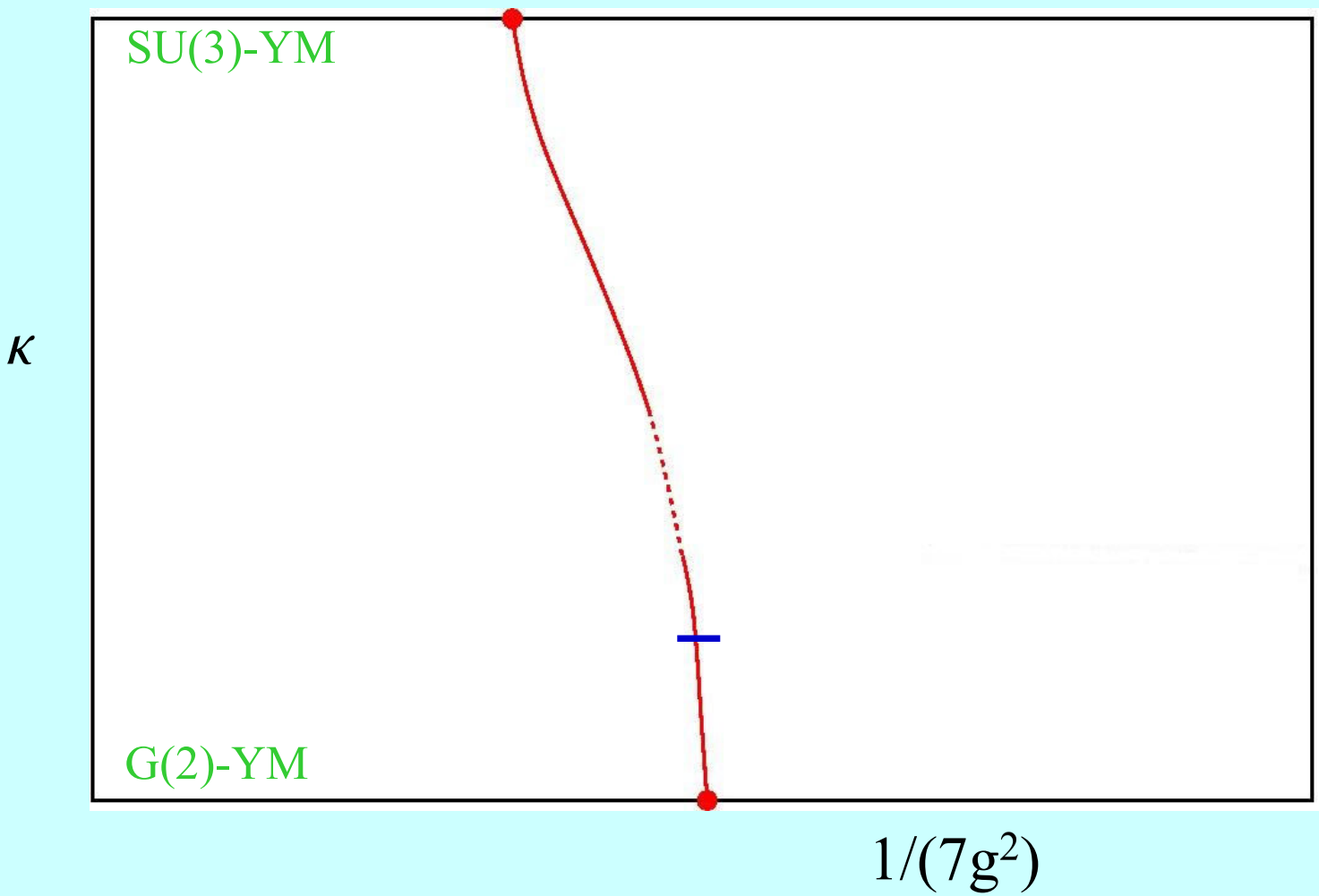
- confinement $G(2) \longrightarrow SU(3)$. 6 massive G(2)-"gluons" are {3} and $\{\overline{3}\}$ ~ quarks $\Rightarrow$ string breaking

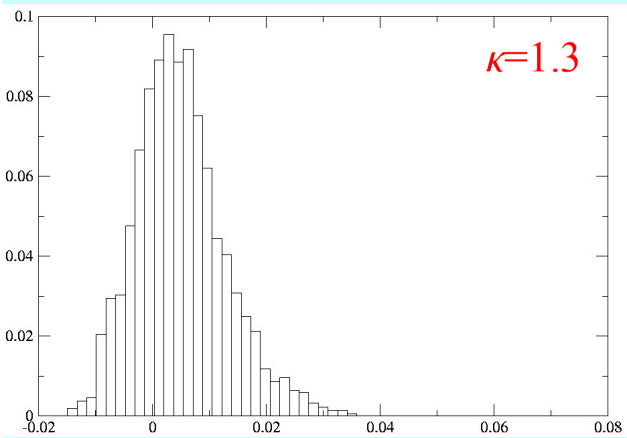




$$\mathcal{S}_{HYM} = \mathcal{S}_{YM} - \kappa \sum_{x,\mu} \phi(x)^\dagger U_\mu(x)\, \phi(x+\hat{\mu})$$

N_t=6

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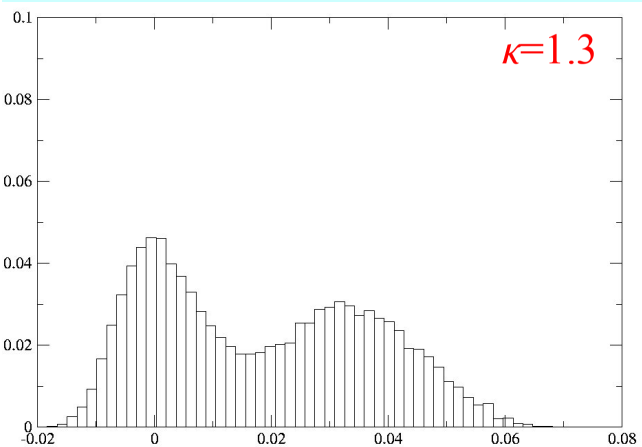
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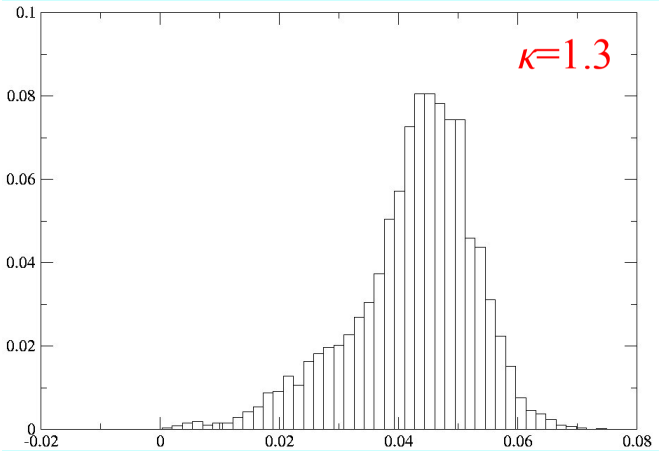
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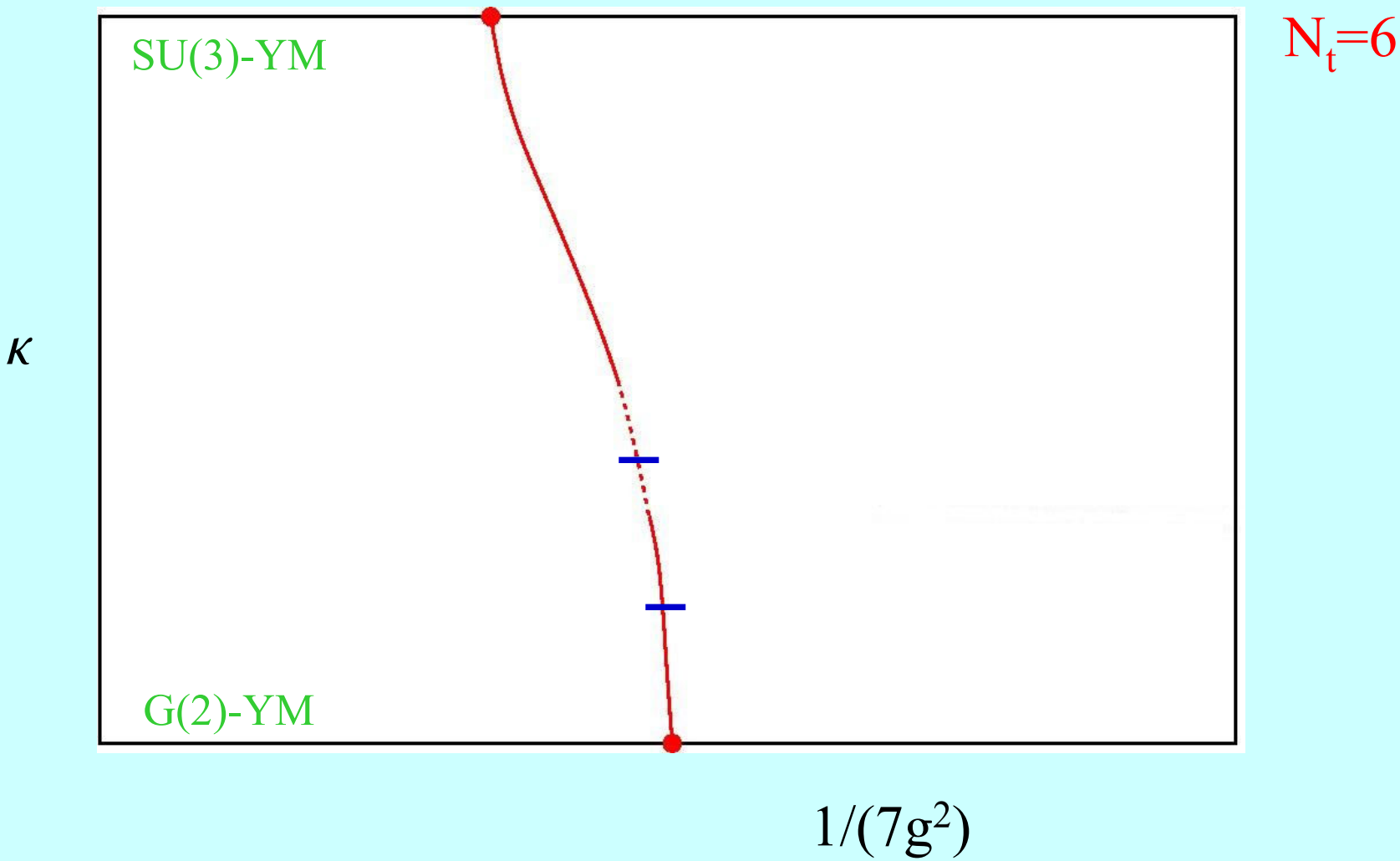
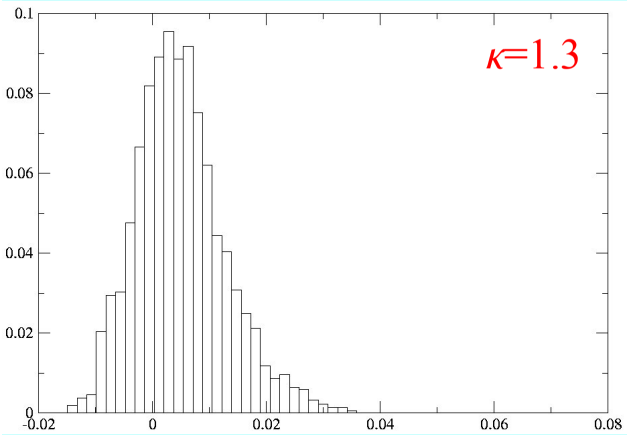

SU(3)-YM
G(2)-YM
N_t=6
κ
$1/(7g^2)$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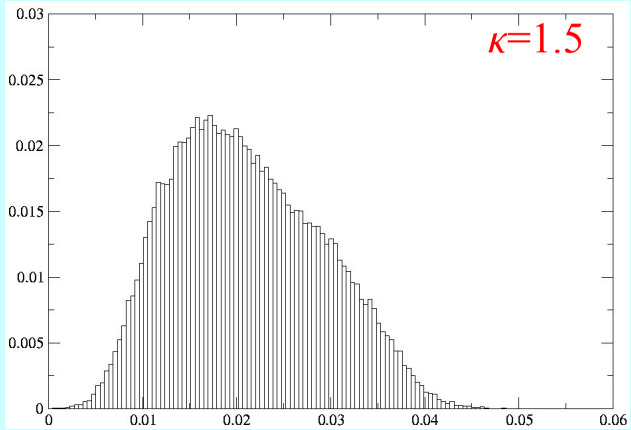
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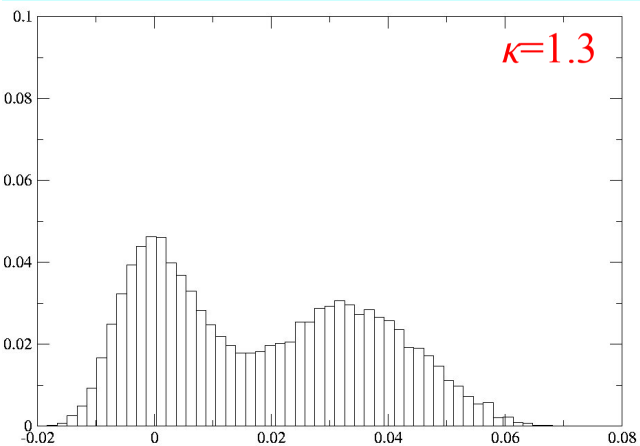
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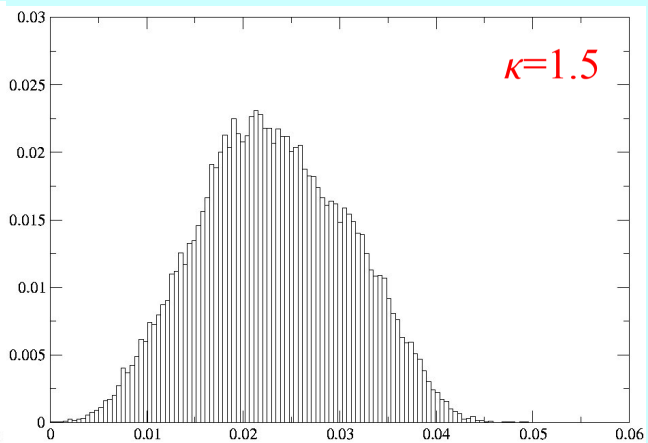
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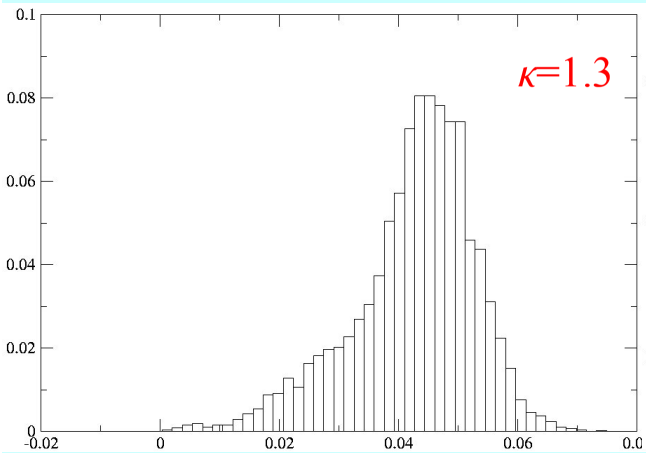
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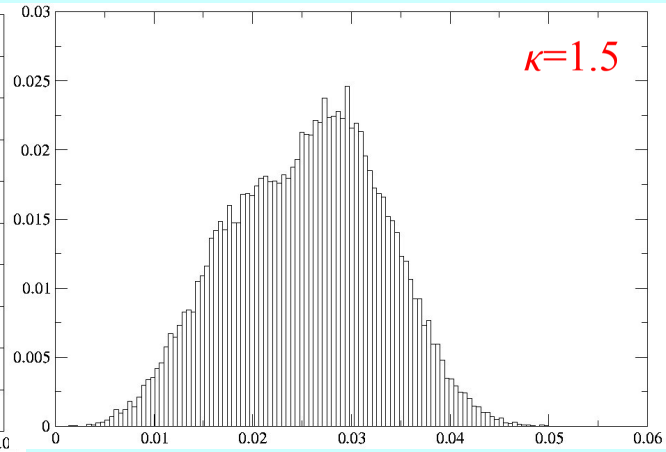
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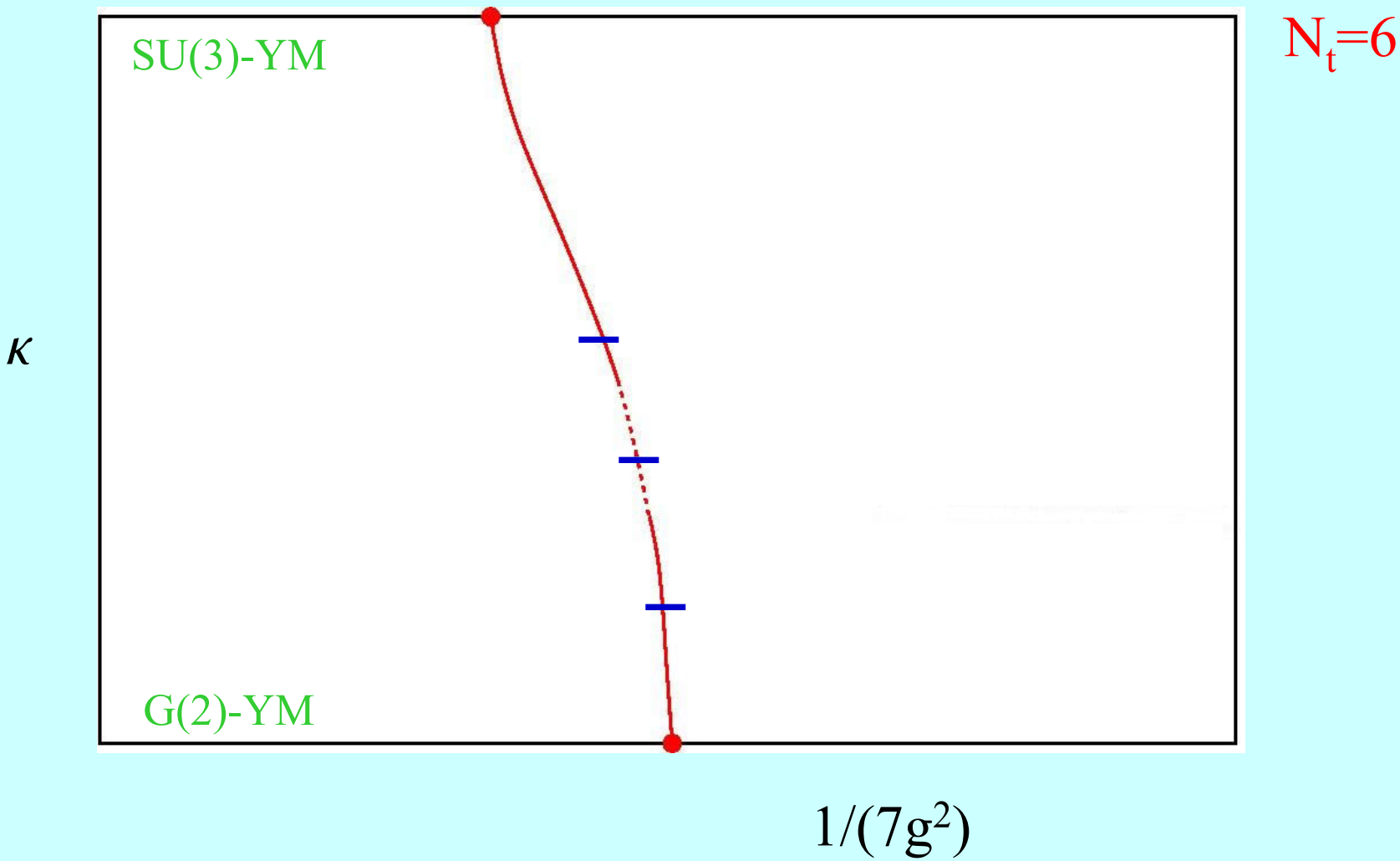
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G(2)-YM
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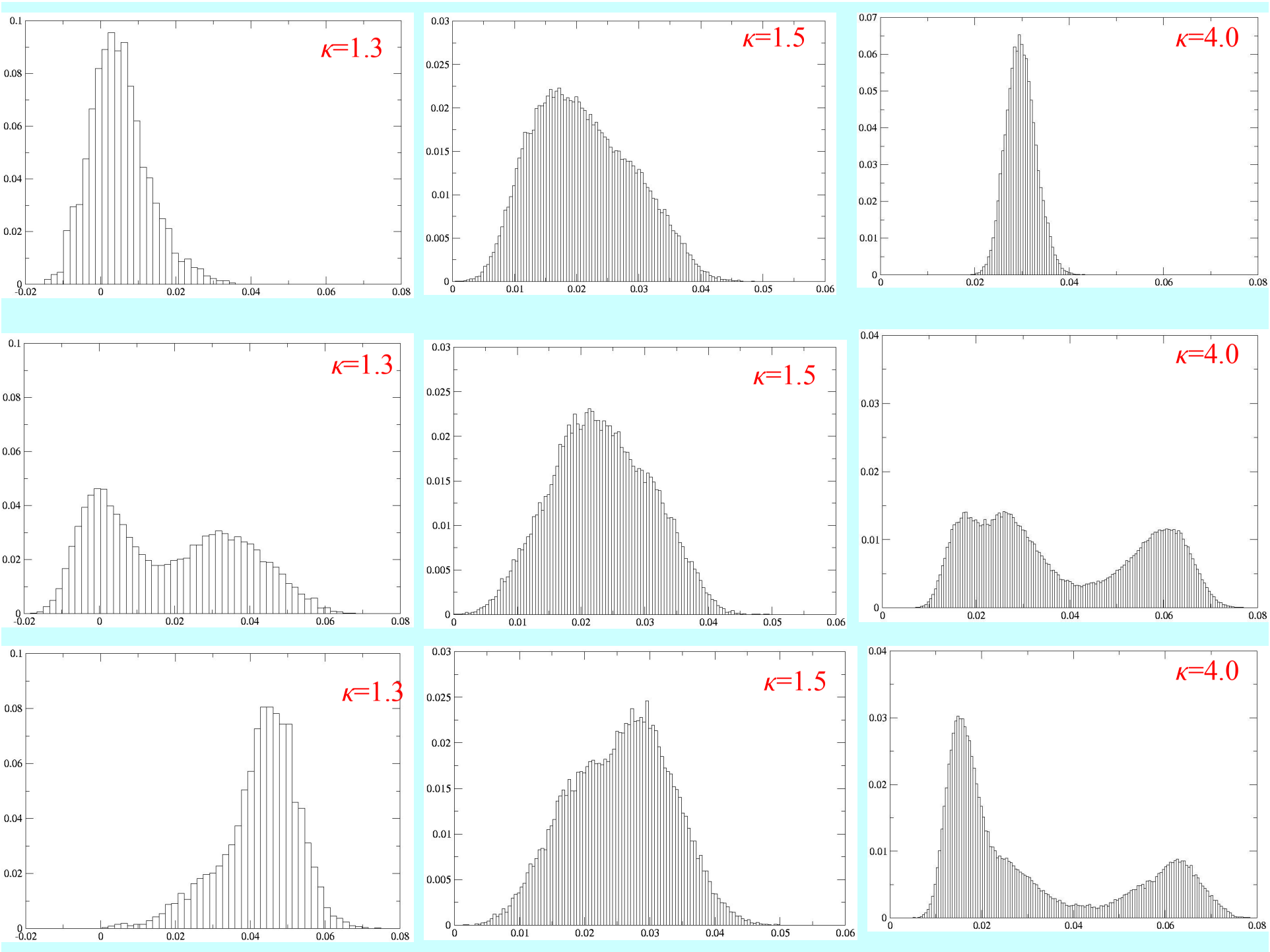
κ
$1/(7g^2)$


κ=1.3
κ=1.5
κ=4.0
κ=1.3
κ=1.5
κ=4.0
κ=1.3
κ=1.5
κ=4.0

Conclusions

- Confinement is a difficult problem: QCD $\longrightarrow$ Yang-Mills theory
- We must have a broad perspective: not only SU(N) but all Lie groups
- Conjecture: the low-temperature dynamics of Yang-Mills theory is mildly dependent on the gauge group

  * the size of the group determines the order of the deconfinement p.t.
  * the center is relevant only if the transition is 2nd order
- G(2) Yang-Mills theory: interesting theoretical laboratory to investigate SU(3) Yang-Mills theory in an unusual framework similar to QCD

(3+1)-d only $Sp(1)=SU(2)_3$ YM has a 2nd order deconfinement p.t.
$SU(3)_8$ YM weak 1st order, no known universality class available
YM with all other gauge groups have 1st order

(2+1)-d $SU(2)_3$, $SU(3)_8$, $Sp(2)_{10}$ YM has a 2nd order deconfinement p.t.,
$SU(4)_{15}$ YM: weak 1st or 2nd ?, $G(2)_{14}$ YM: not known
YM with all other gauge groups have 1st order