Wilson lines and deconfinement at $T \neq 0$

1. Helsinki program

Braaten & Nieto '96.
Kajantie, Laine, Rummukainen, & Schröder '00, '02, & '03.
Giovannangeli '05. Laine & Schröder '05 & '06.
Di Renzo, Laine, Miccio, Schröder, & Torrero '06.

2. (Some) large gauge transformations

Diakonov & Oswald '03 & '04.
Megias, Ruiz Arriola, & Salcedo '03.
Holland, Minkowski, Pepe, & Wiese '03. (Center? What center?)

3. Wilson lines, and their electric field

Aharony, Marsano, Minwalla, Papadodimas, & Van Raamsdonk '03 & '05
RDP '00...Dumitru, Lenaghan, & RDP '04.
Banks & Ukawa '83. Bialas, Morel, & Petersson'04. RDP '06.

4. Deconfinement as an (adjoint) Higgs effect

vis-a-vis Vuorinen & Yaffe '06.
Weiss '82, Polchinski '91, Schaden '04.
Karsch & Wyld '86.

Helsinki program

Match original theory in 4D, to effective theory in 3D, for r > 1/T

$$\mathcal{L}^{eff} = \frac{1}{2}\,\mathrm{tr}\,G_{ij}^2 \;+\; \mathrm{tr}\,|D_i A_0|^2 + \; m_D^2\,\mathrm{tr}\,A_0^2 \;+\; \kappa\,\mathrm{tr}\,A_0^4$$

$m^2_D \sim g^2 T^2$, $\kappa \sim g^4$, series in g^2 (not g).

Optimal resummation of pert. thy, valid for *small* A_0

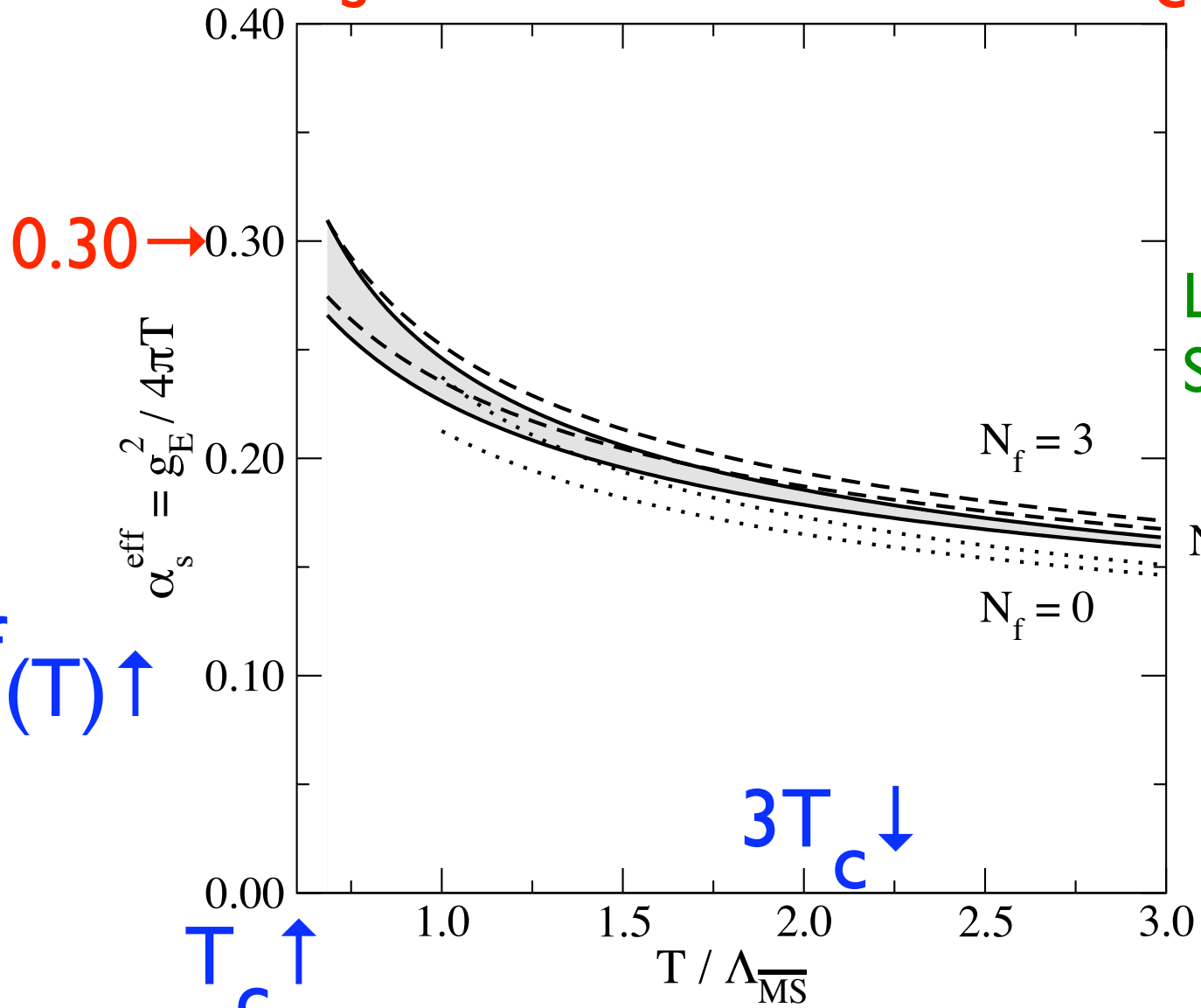
How does α_s^{eff} run? $\alpha_s^{eff}(2\,\pi\,T)$?

Even better! Laine & Schröder '05: 2-loop calc. $\Rightarrow$ $\alpha_s^{eff}(9\,T)$!

$T_c \sim 175$ MeV: $9\,T_c \sim 1.6$ GeV, $\alpha_s^{eff}(9\,T_c) \sim 0.28$

$9\,(3\,T_c) \sim 4.8$ GeV : T_c to $\sim 3\,T_c$ *not* strong coupling

α_s^{eff} is *not* so big, even at T_c



0.30 →

Laine & Schröder '05

$\alpha_s^{eff}(T)$ ↑

$3T_c$ ↓

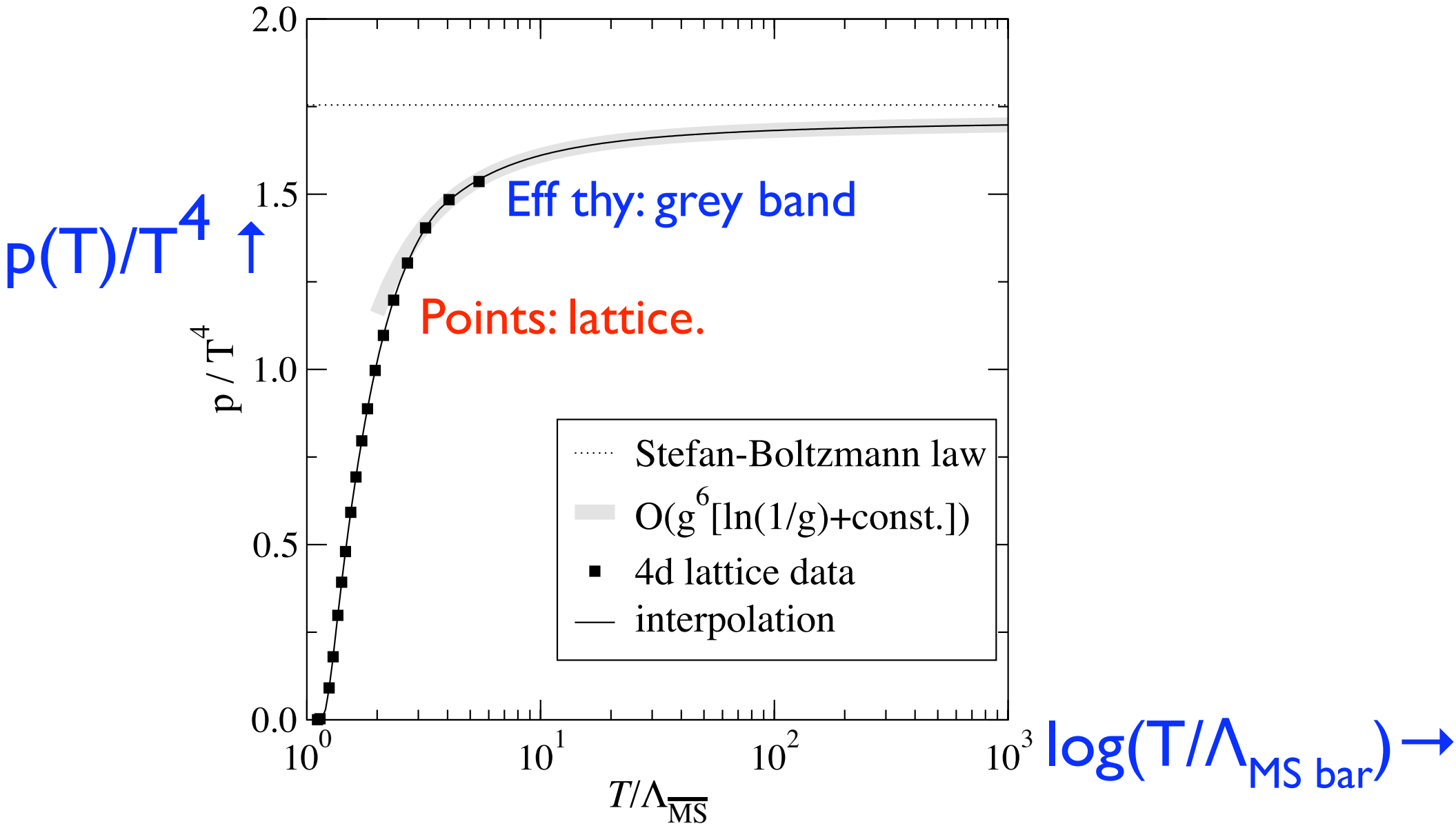
T/T_c →

T_c ↑

$\alpha_s^{eff}(c\,T)$: $c \sim 2\pi \to 9$. *Might* have been $2\pi \to 2$.

If so, then strong coupling below $3\,T_c$. *Not* what happens.

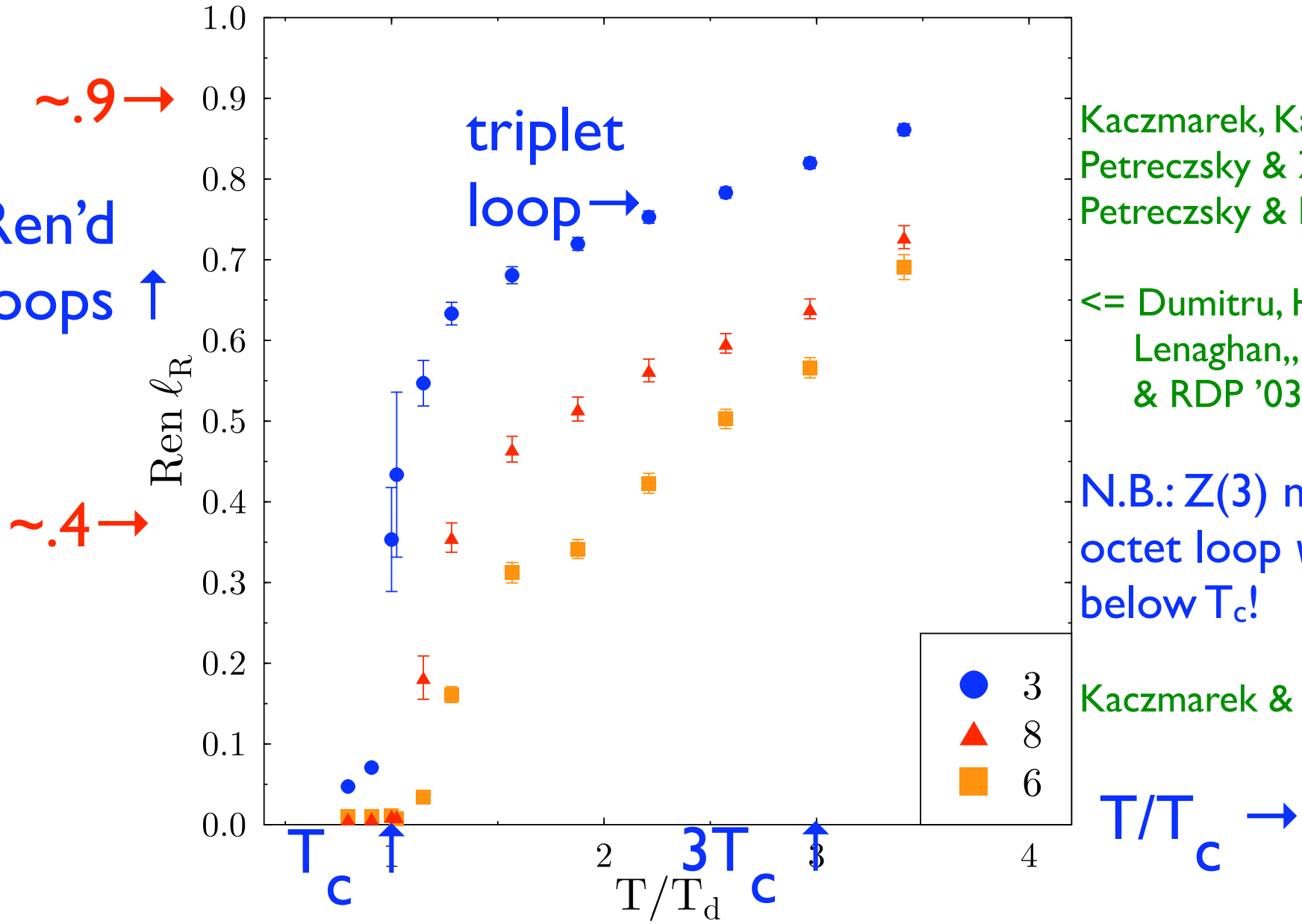
Pressure: effective theory *fails* below ~ 3 T_c



If α_s^{eff} is not so big, why *doesn't* effective thy work for the pressure?

Lattice SU(3): ren'd Polyakov Loops, T_c to $3\,T_c$

Renormalized triplet loop, $\ell_3 = \mathrm{tr}\,\mathbf{L}/\mathbf{3}$, well *below* one, T_c to ~ $3\,T_c$



Kaczmarek, Karsch, Petreczsky & Zantow '02
Petreczsky & Petrov '04

<= Dumitru, Hatta, Lenaghan,, Orginos, & RDP '03

N.B.: Z(3) neutral octet loop *very* small below T_c!

Kaczmarek & Zantow, '06

(Some) large gauge transf.'s

Ren'd loop → for T_c to ~ 3 T_c , need eff. thy. for *arbitrary* $g A_0/(2 \pi T)$.
A_0 varies slowly in space, momenta $p < T$.

Eff. thy for large A_0: invariant under static gauge transformations.

And: some time *d*ependent gauge transf.'s:

$$\mathcal{U}_c(x,\tau) = e^{2\pi i \tau T t_N} \quad , \quad t_N = \begin{pmatrix} \mathbf{1}_{N-1} & 0 \\ 0 & -(N-1) \end{pmatrix}$$

Strictly periodic in τ. Topologically non-trivial ($\pi_1(U(1)\text{'s})$).

4D inv.: $A_0^{diag} \to A_0^{diag} + 2 \pi T t_N/g$, plus time dep. rotation of A_i

But: tr $|D_i A_0|^2$ *not* invariant! $D_i A_0 = (\partial_i - i g [A_i ,) A_0$, $[A_i , t_N] \neq 0$

OK for Dogma, at small A_0 ; shift ~ $1/g$.

Effective theory of Wilson lines

Instead of A_0 , use (straight, thermal) Wilson lines:

$$\mathbf{L}(x) = P\, e^{ig \int_0^{1/T} A_0(x,\tau)\, d\tau}$$

L = SU(N) matrix, diagonalize:

$$\mathbf{L}(x) = \Omega(x)^\dagger\, e^{i\lambda(x)}\, \Omega(x)$$

Under static gauge transf.'s: diagonal λ invariant, Ω is not.

Time dependent gauge transf.'s → λ's periodic. Of course!

But what is the electric field?

Guess

$$E_i(x) = \frac{T}{ig}\, \mathbf{L}^\dagger(x)\, D_i\, \mathbf{L}(x)$$

Effective electric field

Like original E_i, effective $E_i \sim L^\dagger D_i L$ is:

gauge covariant

hermitian (so not $E_i \sim D_i L$); if center symmetry exists, center sym. $\sim D_i A_0$ at small A_0 .

Eff. E_i gauge covariant "average" in τ :

$$E_i(x)/T = \int_0^{1/T} d\tau\, \mathbf{L}(x,\tau)^\dagger\, \partial_i A_0(x,\tau)\, \mathbf{L}(x,\tau) - \mathbf{L}(x,1/T)^\dagger\, [A_i(x), \mathbf{L}(x,1/T)]$$

Classically, eff. thy. =
gauged principal chiral field

$$\mathcal{L}_{cl}^{eff} = \frac{1}{2}\,\mathrm{tr}\, G_{ij}^2 + \frac{T^2}{g^2}\,\mathrm{tr}\,|D_i \mathbf{L}|^2$$

Non-renormalizable, but only eff. thy: temp. T natural UV cutoff.

Lattice: Banks & Ukawa '83...

Instanton # in 4D = winding # (L) in 3D.

Deconfinement & adjoint Higgs phase?

Diagonalize $L = \Omega^\dagger e^{i\lambda} \Omega$ Under static gauge transf.'s *U*, $e^{i\lambda}$ invariant, Ω not:

$$\Omega \to \Omega \mathcal{U} \ , \ D_i \to \mathcal{U}^\dagger D_i \mathcal{U}$$

Electric field:

$$\mathrm{tr}\ |D_i \mathbf{L}|^2 \ = \ \mathrm{tr}\ (\partial_i \lambda)^2 \ + \ \mathrm{tr}\ \left|\left[\Omega\, D_i\, \Omega^\dagger, \mathrm{e}^{i\lambda}\right]\right|^2 \ .$$

1st term same as abelian

2nd term gauge *in*variant coupling of electric & magnetic sectors.

If $\langle e^{i\lambda} \rangle = 1$, no Higgs phase.

If $\langle e^{i\lambda} \rangle \neq 1$, Higgs phase,

$$m_{ij}^2 = g^2 |\mathrm{e}^{i\lambda_i} - \mathrm{e}^{i\lambda_j}|^2$$

i, j = 1...N. Diagonal gluons massless, off diagonal massive.

Loop potential, perturbative & not.

U(N): constant L, 1 loop order:

$$\mathcal{L}^{eff}_{1\ loop} = -\frac{2\,T^4}{\pi^2}\sum_{m=1}^{\infty}\frac{1}{m^4}\,|\mathrm{tr}\,\mathbf{L}^m|^2 \ .$$

Perturbative vacuum $\langle e^{i\lambda}\rangle = 1$, stable to all orders in α_s .

SU(3) lattice: near T_c , pressure(T) ~ T^4 *and* ~T^2 .

To represent: add, *by hand:*

$$\mathcal{L}^{eff}_{non-pert.}(\mathbf{L}) = +\,B_f\,T^2\ |\mathrm{tr}\,\mathbf{L}|^2$$

$B_f \sim \# T_c^2$ "fuzzy" bag const. Non-pert., not unique.

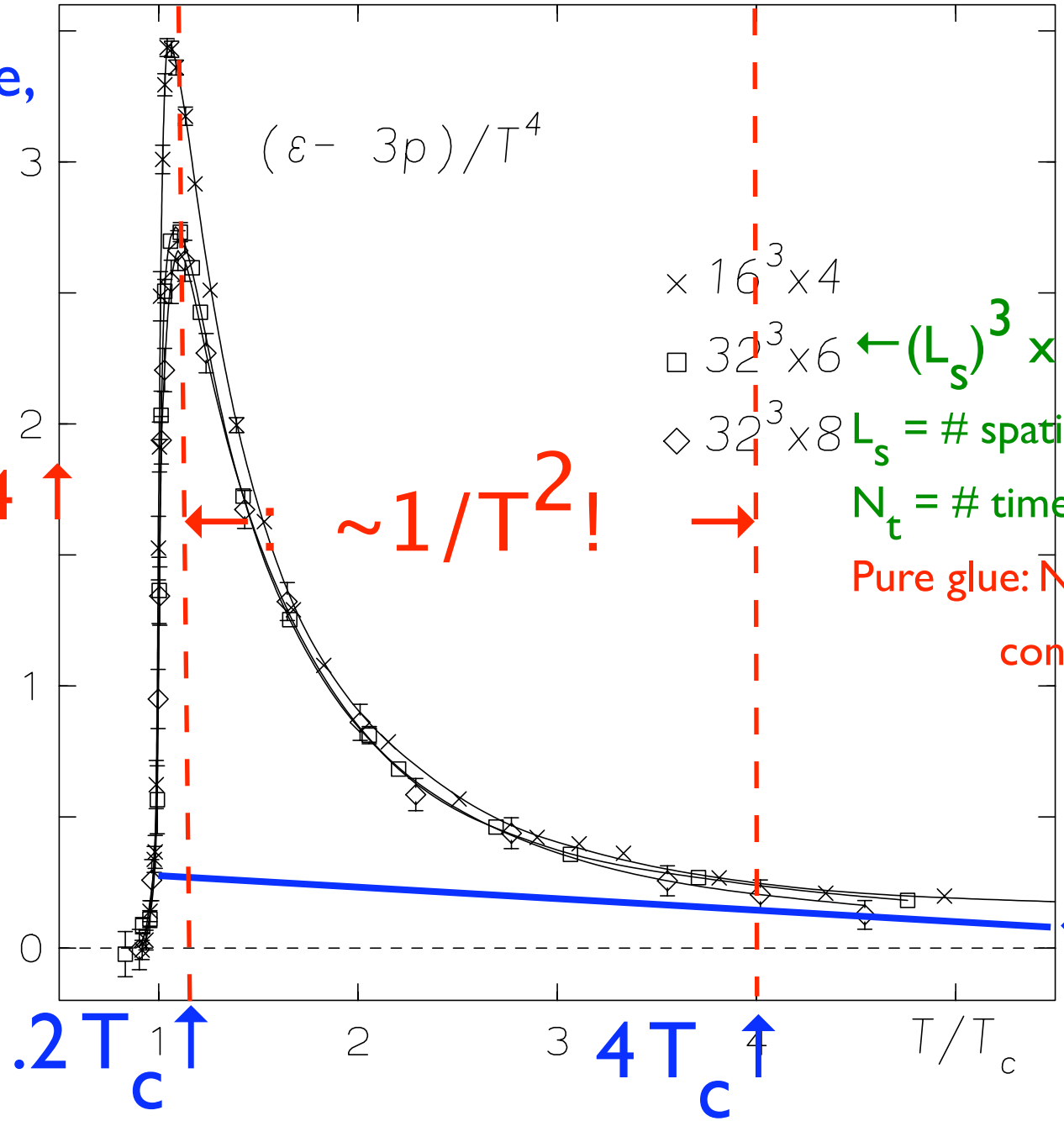
$B_f \neq 0 \Rightarrow \langle e^{i\lambda}\rangle \neq 1 \Rightarrow$ Higgs phase near $T_c \Rightarrow$

Splitting of magnetic glueballs near T_c.

Such Higgs phase only for adjoint SU(N), *not* SU(N) x SU(N)

$(e-3p)/T^4 \sim 1/T^2$

SU(3) lattice,
NO quarks
Bielefeld,
lat/9602007



$(e-3p)/T^4$ ↑

$\leftarrow \sim 1/T^2! \rightarrow$

$\leftarrow (L_s)^3 \times N_t$:

L_s = # spatial lattice spacings

N_t = # time steps

Pure glue: N_t = 6,8 *close* to continuum limit

e = energy
p = pressure

$1.2\,T_c$ ↑

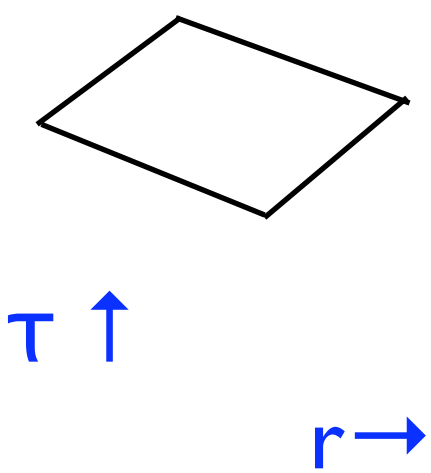
$4\,T_c$ ↑

← Perturbative cont. $\sim \alpha_s^2$

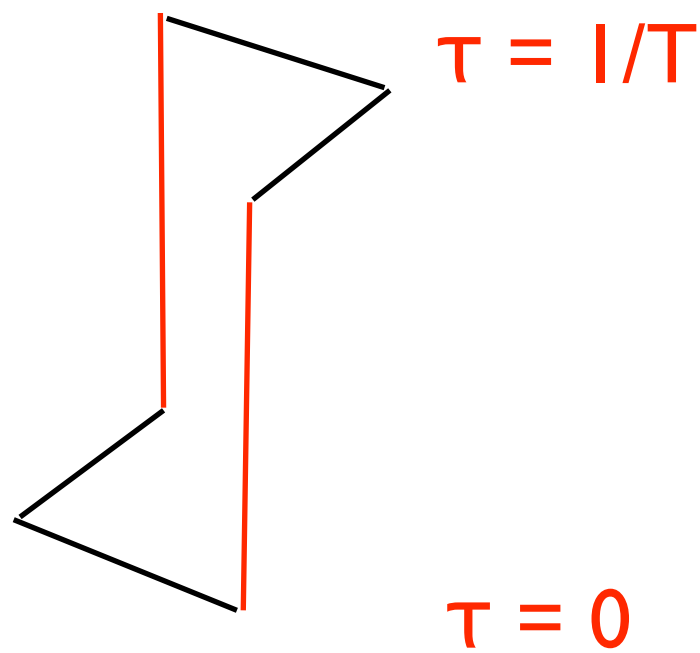
"Split" plaquettes

How to tell if in adjoint Higgs phase? Measure magnetic glueball mass from two-point function of spatial plaquette "split" in time

Usual spatial plaquette



"Split" spatial plaquette



For SU(3), surely hard to disentangle (heavy) magnetic from (light) electric states. Easier for SU(N > 3)?

(Dynamical) eigenvalue repulsion

SU(N): *Guess* for the confined vacuum:

$$\mathbf{L}_{\rm conf} = {\rm diag}(1, z, z^2 \ldots z^{N-1}) \ , \ z = {\rm e}^{2\pi i/N}.$$

Correct center symmetry:

$$\langle {\rm tr}\, \mathbf{L}^j_{\rm conf} \rangle = 0 \ , \ j = 1 \ldots (N-1)$$

But: pressure < 0! pressure(L_c) ~ 1, not N^2 => Confined vacuum at N=∞

At N = ∞, total eigenvalue repulsion. *How is it generated dynamically?*

Small volume: Vandermonde det. in measure ~random matrix model

Large volume: kinetic terms of Higgs modes in adjoint Higgs phase.

One loop (?):

$$\mathcal{L}^{\rm eff}_{1\,{\rm loop}} \sim - \sum_{a,b=1}^{N} (g^2 |e^{i\lambda_a} - e^{i\lambda_b}|^2)^{3/2}$$

Can study eff. thy, numerically, on lattice.