Dense 2-colour quark matter

In collaboration with Simon Hands, Seyong Kim, Peter Sitch
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Extreme QCD, Brookhaven, 2 August 2006

Outline

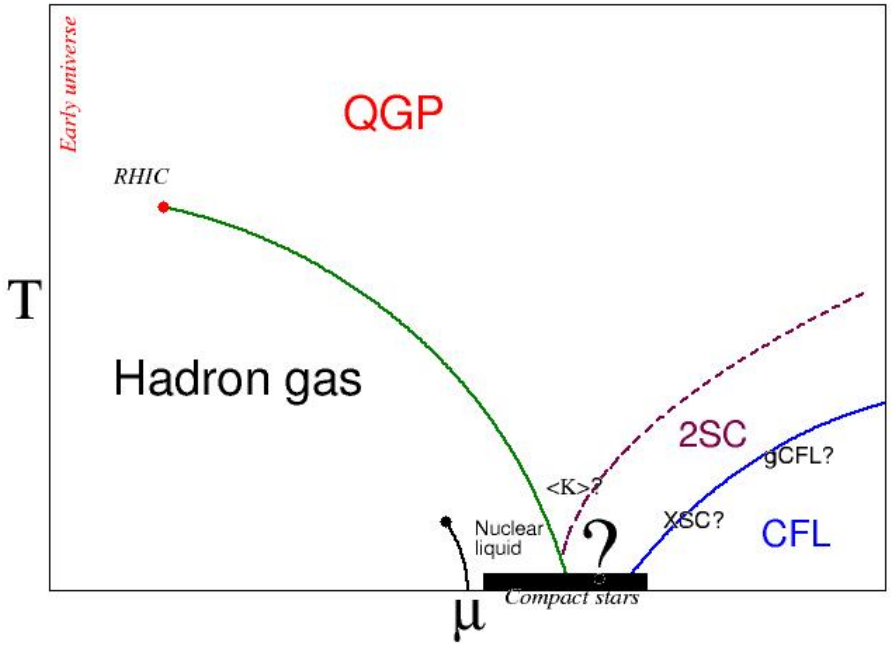
Background
- Global symmetries of QC_2D
- QC_2D vs QCD
- Models

Lattice formulation

Results
- Onset and deconfinement
- Bulk thermodynamics
- Gluon and quark propagator

Summary and outlook

Background



- A plethora of phases at high μ, low T
- Based on models and perturbation theory
- Details depend on diquark gaps and strange quark mass

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So standard Monte Carlo importance sampling can not be used!
We can still gain some insight:

- Approach from high(er) temperature
- Effective theories where problem is absent or reduced: HDET, NJL,...
- QCD-like theories without a sign problem
- QC_2D studies by Hands&Morrison; Muroya, Nakamura, Nonaka; Kogut&Sinclair; Allés, d'Elia, Lombardo; Chandrasekharan&Jiang,...

Global symmetries of QC_2D

Quarks and antiquarks are in the same representation

Anti-unitary symmetry: $\mathcal{K}\mathcal{M}\mathcal{K}^{-1} = \mathcal{M}^*$ with $\mathcal{K} \equiv C\gamma_5\tau_2$

$$\mathcal{L} = \overline{\psi}(\gamma_\nu D_\nu - \mu\gamma_0 + m)\psi$$
$$= i\Psi^\dagger\sigma_\nu(D_\nu - \mu B_\nu)\Psi + \frac{1}{2}m\Psi^T\sigma_2\tau_2\hat{M}\Psi$$
$$\Psi = \begin{pmatrix} \psi_L \\ \sigma_2\tau_2\psi_R^* \end{pmatrix}, \quad B_\nu = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\delta_{\nu 0}, \quad \hat{M} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$m = \mu = 0$: global $SU(2N_f)$ symmetry

Chiral symmetry breaking

Chiral condensate

$$\overline{\psi}\psi = -\frac{1}{2}\Psi^T \sigma_2 \tau_2 \hat{M} \Psi + h.c.$$

$\langle\overline{\psi}\psi\rangle \neq 0$ breaks $SU(2N_f) \longrightarrow Sp(2N_f)$

$$\Rightarrow N_f(2N_f - 1) - 1 \text{Goldstone modes}$$

$N_f = 2$: 5 modes

$\overline{\psi}\vec{\sigma}\gamma_5\psi$ pion $\quad \psi^T \epsilon\tau_2 C\gamma_5\psi, \quad \overline{\psi}\epsilon\tau_2 C\gamma_5\overline{\psi}^T$ scalar diquark

Note: Staggered fermions have different symetry breaking pattern!

Diquark condensation

Diquarks are colour singlets in QC_2D

$\longrightarrow$ superfluidity rather than colour superconductivity

$\longrightarrow$ exact Goldstone mode from breaking of $U(1)_B$ symmetry

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Condensation of tightly bound diquarks (Goldstone baryons)

$\leftrightarrow$ Chiral perturbation theory

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Bardeen–Cooper–Schrieffer:

Pairing of quarks near the Fermi surface

$$\langle\psi\psi\rangle \propto \Delta\mu^2$$

QC_2D vs QCD

What we cannot learn

- Chiral dynamics
- Nuclear matter EOS
- Colour superconductivity
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What we might learn

- Gluodynamics
- Checks on model studies
- Qualitative features of deconfinement
- Generic features of gauge theories at high densities

Issues of interest

Spectrum

- ρ mass $\rightarrow$ Brown–Rho scaling?
- Pseudoscalar diquark $\rightarrow$ related to $U(1)_A$ anomaly?
- Compare with χPT and RMT predictions

Gluodynamics — SU(2) and SU(3) very similar?

- Dynamical fermion effects
- Deconfinement — static quark potential, Polyakov loop
- Gluon propagator $\rightarrow$ essential input into gap equation

Quark propagator

- Dynamical quark masses $\rightarrow$ effective strange quark mass?
- Direct determination of diquark gap?

Bulk thermodynamics

Phase diagram in (μ, T) plane?

Models: Chiral perturbation theory

Tightly bound, weakly interacting, pseudo-Goldstone bosons condense at $\mu \geq \mu_o = m_\pi/2$.

$$\langle qq \rangle \propto \sqrt{1 - \frac{\mu_o^4}{\mu^4}}$$

$$n_q = \begin{cases} 0\,, & \mu < \mu_o, \\ 8N_f F^2 \mu \left(1 - \frac{\mu_o^4}{\mu^4}\right) \approx 32 N_f F^2 (\mu - \mu_o)\,, & \mu \geq \mu_o. \end{cases}$$

$$p = \int n_q d\mu = 4N_f F^2 \left(\mu^2 + \frac{\mu_o^4}{\mu^2} - 2\mu_o^2\right)$$

$$\varepsilon_q = -p + \mu n_q = 4N_f F^2 \left(\mu^2 - 3\frac{\mu_o^4}{\mu^2} + 2\mu_o^2\right).$$

Models: Free, massless quarks

Stefan–Boltzmann law for number and energy density, pressure:

$$n_q = \frac{N_f N_c}{3\pi^2}\mu^3$$
$$\varepsilon_q = 3p = \frac{N_f N_c}{4\pi^2}\mu^4$$

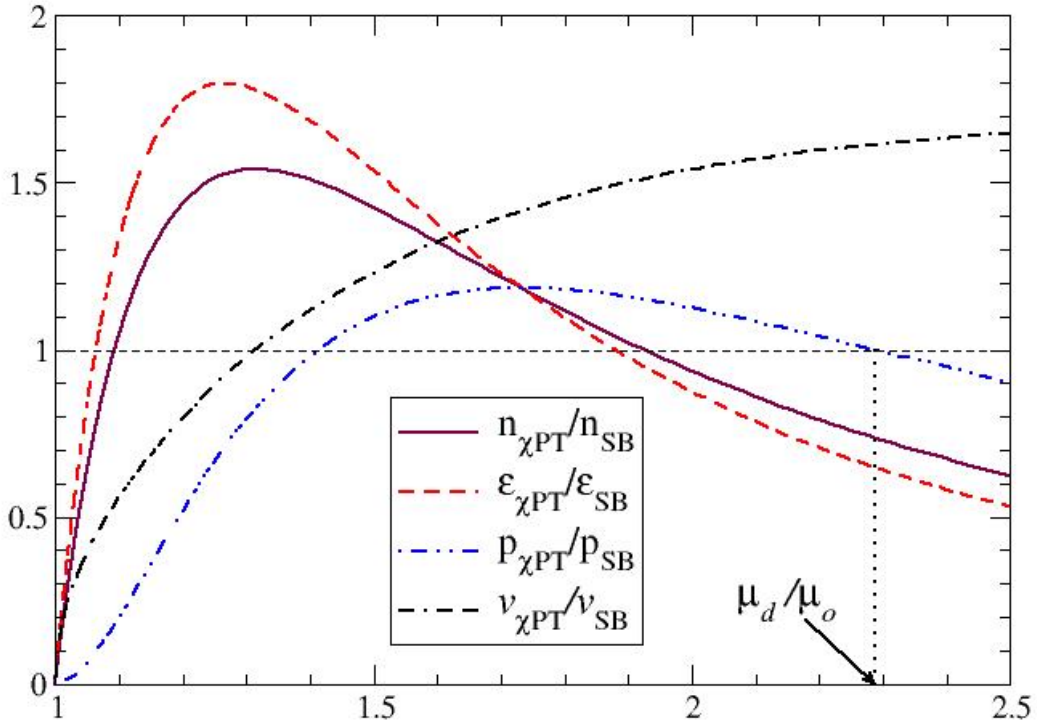
Any attractive interaction between quarks gives rise to BCS condensation with gap Δ
Quarks condense in a shell thickness Δ around Fermi surface

$$\Longrightarrow \quad \langle qq \rangle \propto \Delta\mu^2$$

Models: Deconfinement transition

At μ_d the two model states have equal pressure

$$\Longleftrightarrow \quad \mu_d^3 - 4\pi F\sqrt{\frac{3}{N_c}}(\mu_d^2 - \mu_o^2) = 0$$



Lattice formulation

We use Wilson fermions:

- Correct symmetry breaking pattern, Goldstone spectrum
- $N_f < 4$ needed to guarantee continuum limit
- No problems with locality, fourth root trick
- Chiral symmetry buried at bottom of Fermi sea

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$$S = \bar{\psi}_1 M(\mu)\psi_1 + \bar{\psi}_2 M(\mu)\psi_2 - J\bar{\psi}_1(C\gamma_5)\tau_2\bar{\psi}_2^T + \bar{J}\psi_2^T(C\gamma_5)\tau_2\psi_1$$

$$\gamma_5 M(\mu)\gamma_5 = M^\dagger(-\mu), \quad C\gamma_5\tau_2 M(\mu) C\gamma_5\tau_2 = -M^*(\mu)$$

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Diquark source J introduced to

- lift low-lying eigenmodes in the superfluid phase
- study diquark condensation without uncontrolled approximations

Lattice formulation (II)

We can rewrite the action using transformed fields $\phi, \bar{\phi}$,

$$S = \begin{pmatrix} \overline{\psi} & \bar{\phi} \end{pmatrix} \begin{pmatrix} M(\mu) & J\gamma_5 \\ -\bar{J}\gamma_5 & M(-\mu) \end{pmatrix} \begin{pmatrix} \psi \\ \phi \end{pmatrix} \qquad \bar{\phi} = -\psi_2^T C \tau_2, \quad \phi = C^{-1} \tau_2 \overline{\psi}_2^T$$

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So $\det \mathcal{M}$ is positive iff $J\bar{J}$ is real and positive.

$$\mathcal{M}^\dagger \mathcal{M} = \begin{pmatrix} M^\dagger(\mu)M(\mu) + |\bar{J}|^2 & (J - \bar{J}^*)\gamma_5 M(-\mu) \\ (J^* - \bar{J})\gamma_5 M(\mu) & M^\dagger(-\mu)M(-\mu) + |J|^2 \end{pmatrix}$$

If $\bar{J} = J^*$ this reduces to

$$\det \mathcal{M}^\dagger \mathcal{M} = \left[\det(M^\dagger(\mu)M(\mu) + \bar{J}J)\right]^2$$

- Can take square root analytically
- Can use Dirac rather than Gorkov spinors

Observables

$$n_q = \sum_i \kappa \Big\langle \overline{\psi}_i(x)(\gamma_0 - 1)e^{\mu} U_t(x)\psi_i(x+\hat{t}) + \overline{\psi}_i(x)(\gamma_0 + 1)e^{-\mu} U_t^{\dagger}(x-\hat{t})\psi_i(x-\hat{t}) \Big\rangle$$

$$\varepsilon_q = \sum_i \kappa \Big\langle \overline{\psi}_i(x)(\gamma_0 - 1)e^{\mu} U_t(x)\psi_i(x+\hat{t}) - \overline{\psi}_i(x)(\gamma_0 + 1)e^{-\mu} U_t^{\dagger}(x-\hat{t})\psi_i(x-\hat{t}) \Big\rangle - \varepsilon_q^0$$

$$\varepsilon_g \equiv \frac{1}{N_s^3 N_t} \left\langle a_t \frac{\partial S_g}{\partial a_t} \right\rangle = \frac{3\beta}{N_c} \mathrm{Tr}\langle \Box_t - \Box_s \rangle$$

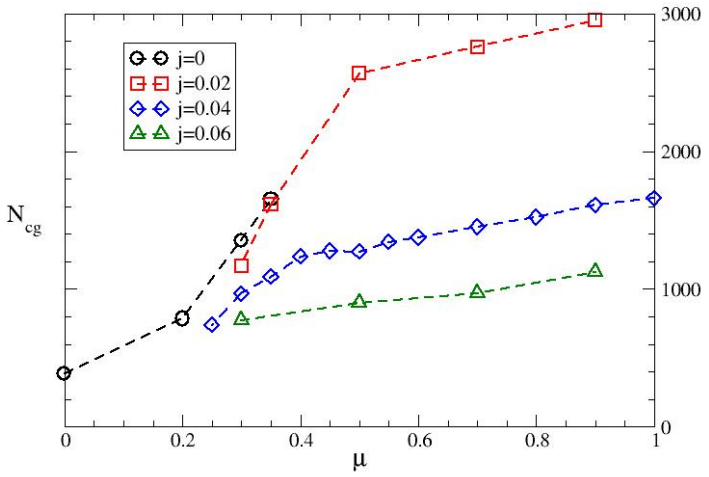
$$p(\mu) = \int^{\mu} \frac{\partial S}{\partial \mu}(\mu')d\mu' = \int^{\mu} n_q(\mu')d\mu'$$

Simulation Parameters

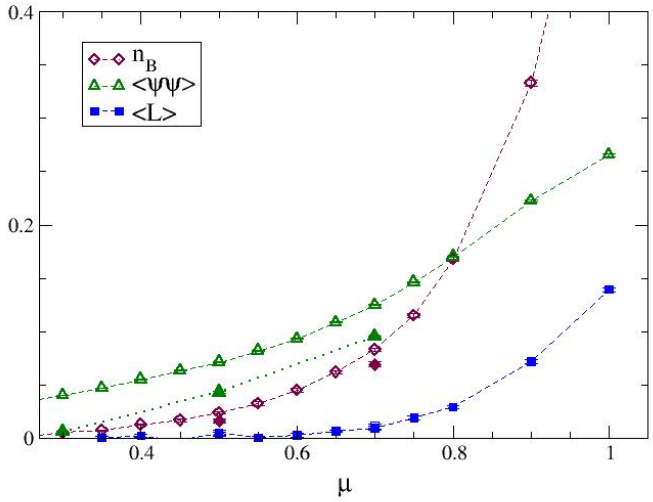
We work on $8^3 \times 16$ lattices with $\beta = 1.7, \kappa = 0.178$

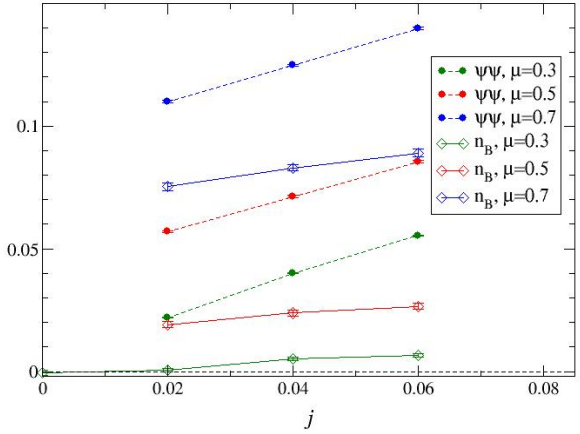
$$\Longrightarrow \quad a \approx 0.22\text{fm}, \quad am_\pi \approx 0.78, \quad m_\pi/m_\rho \approx 0.92$$

- Simulations performed with $j = J/\kappa = 0.04$ for $\mu = 0.3 - 1.0$
- 300–500 trajectories for each μ.
- Simulations with $j = 0.02, 0.06$ for $\mu = 0.3, 0.5, 0.7, 0.9$ $\rightarrow$ enable extrapolation to $j = 0$.



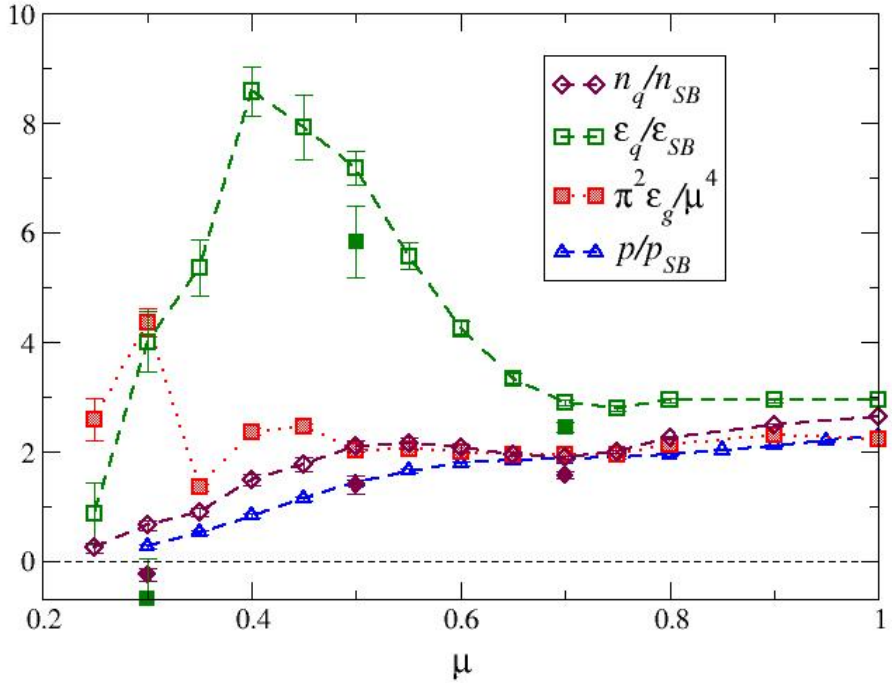
Onset and deconfinement transitions





- Onset transition at $\mu_o \approx 0.4 \approx m_\pi/2$
- μ-behaviour completely at odds with χPT predictions
- Approximately linear dependence of $n_B, \langle\psi\psi\rangle$ on J.
- Deconfinement transition at $\mu_d \approx 0.65$, virtually no j-dependence
- Quark and gluon energy densities increase from zero above μ_o

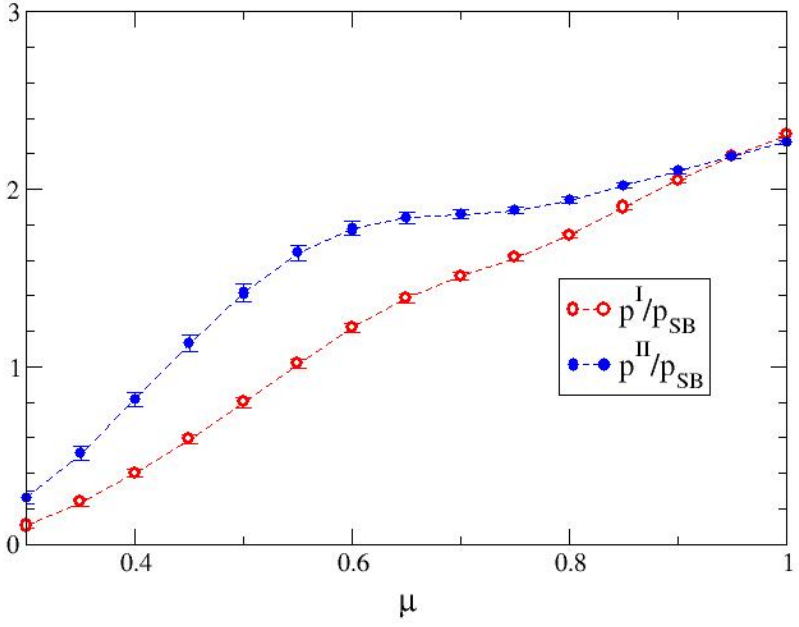
Thermodynamics results



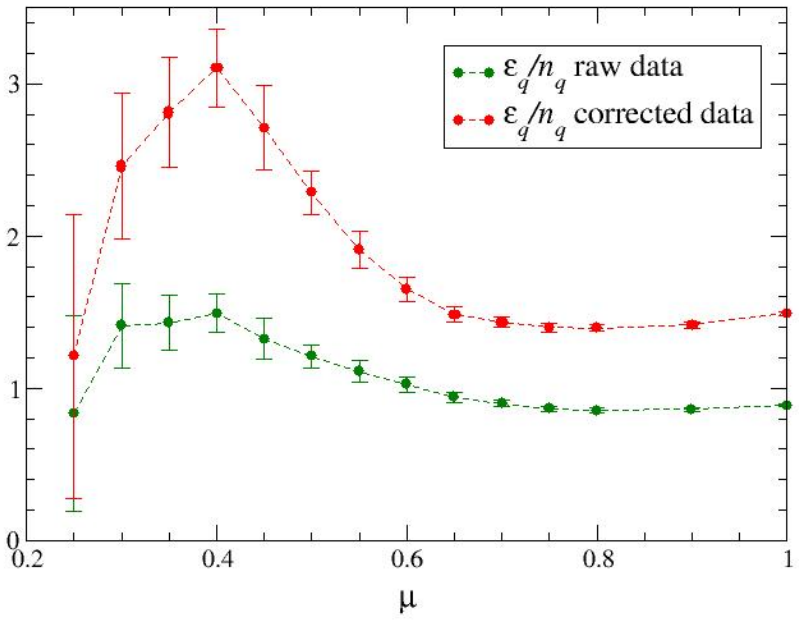
- Close to SB scaling for $\mu > \mu_d$
- $\varepsilon_q \sim 2\varepsilon_{SB} \rightarrow k_F > E_F \Longrightarrow$ binding energy?
- 30–40% of total energy from gluons!?

Bulk thermodynamics (II)

Pressure from integral method
Correcting for leading lattice artefacts $\rightarrow p^I, p^{II}$

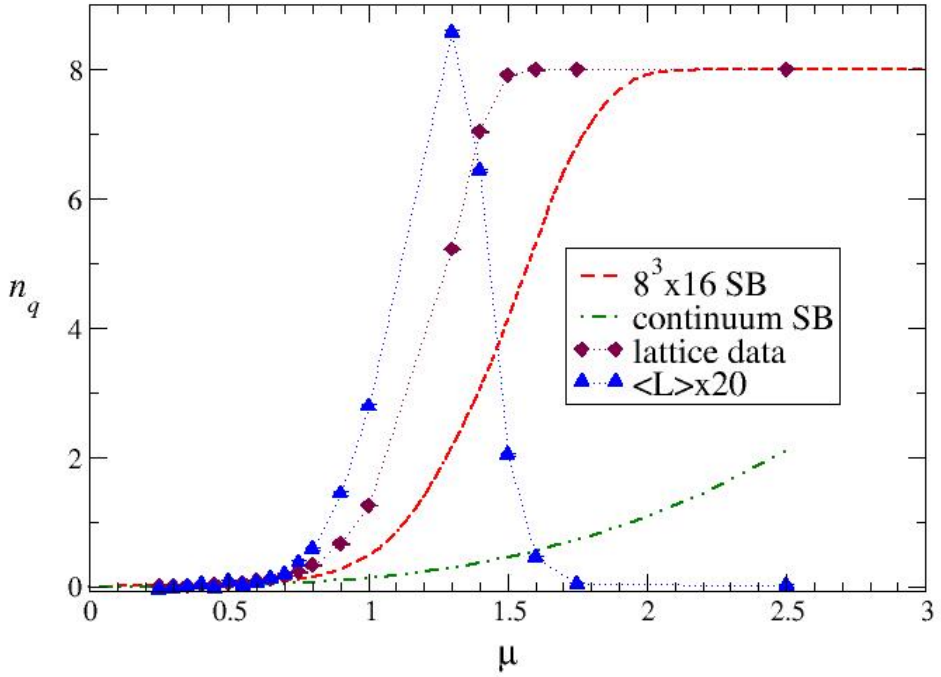


2-colour stars?



- Shallow minimum in ε_q/n_q — saturation?
- 2-colour stars would be quark stars?
- Lattice artefacts do not change qualitative picture
- Non-zero pressure at minimum $\rightarrow$ hadron gas atmosphere?

Saturation



- Lattice saturation at $\mu > 1.3$, well separated from μ_d
- Saturated phase similar to quenched, but nonzero ε_g

Gluon and quark propagators

- Details of phase diagram depend critically on the effective quark mass in the medium.
- Gap equation with effective or one-gluon interaction used to determine superconducting gap $\rightarrow$ more realistic input?
- Unscreened magnetic gluon plays crucial role.

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$$S^{-1}(\vec{p}, \tilde{\omega}) = i \ \vec{\not{p}} A(\vec{p}^2, \tilde{\omega}^2) + i\gamma_4 \tilde{\omega} C(\vec{p}^2, \tilde{\omega}^2) + B(\vec{p}^2, \tilde{\omega}^2)$$

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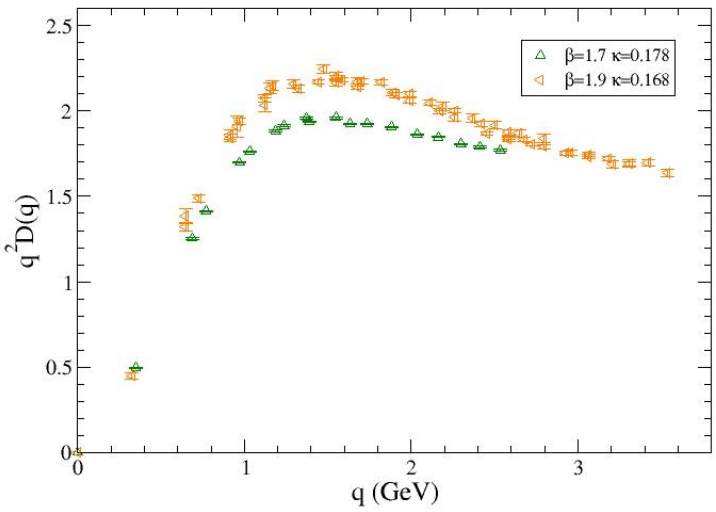
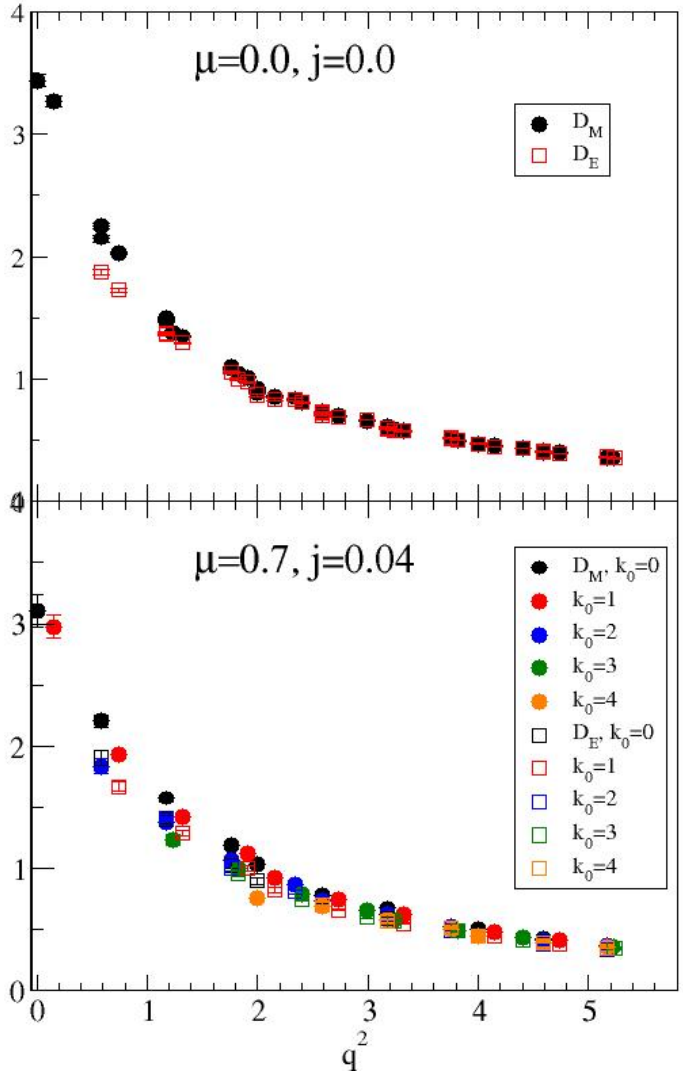
where $\tilde{\omega} \equiv p_t - i\mu$.

Anomalous propagation:
$\langle \Psi(y) \overline{\Psi}(x) \rangle$ contains off-diagonal elements $\Delta(y, x)$

Gluon propagator results (Landau gauge)

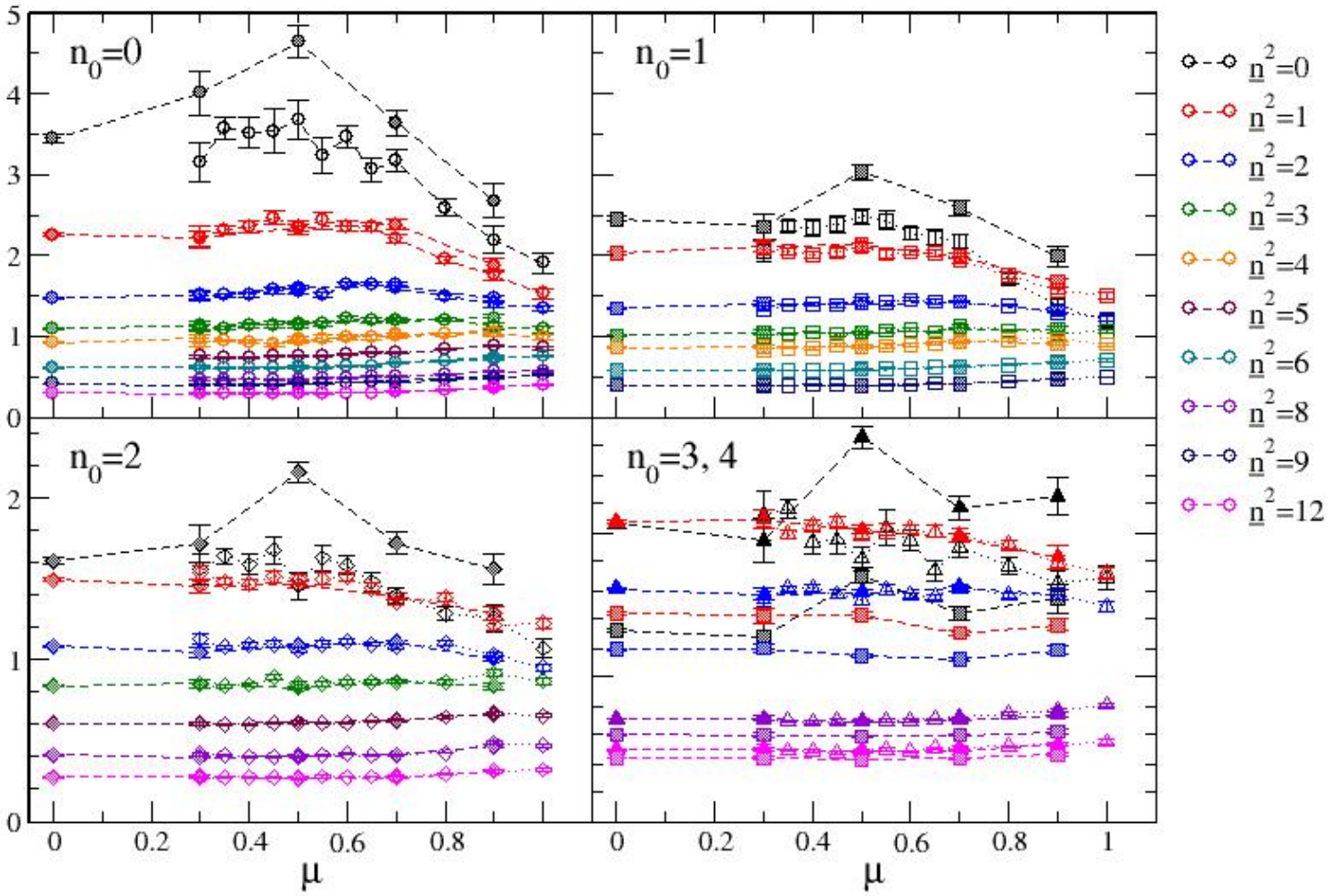




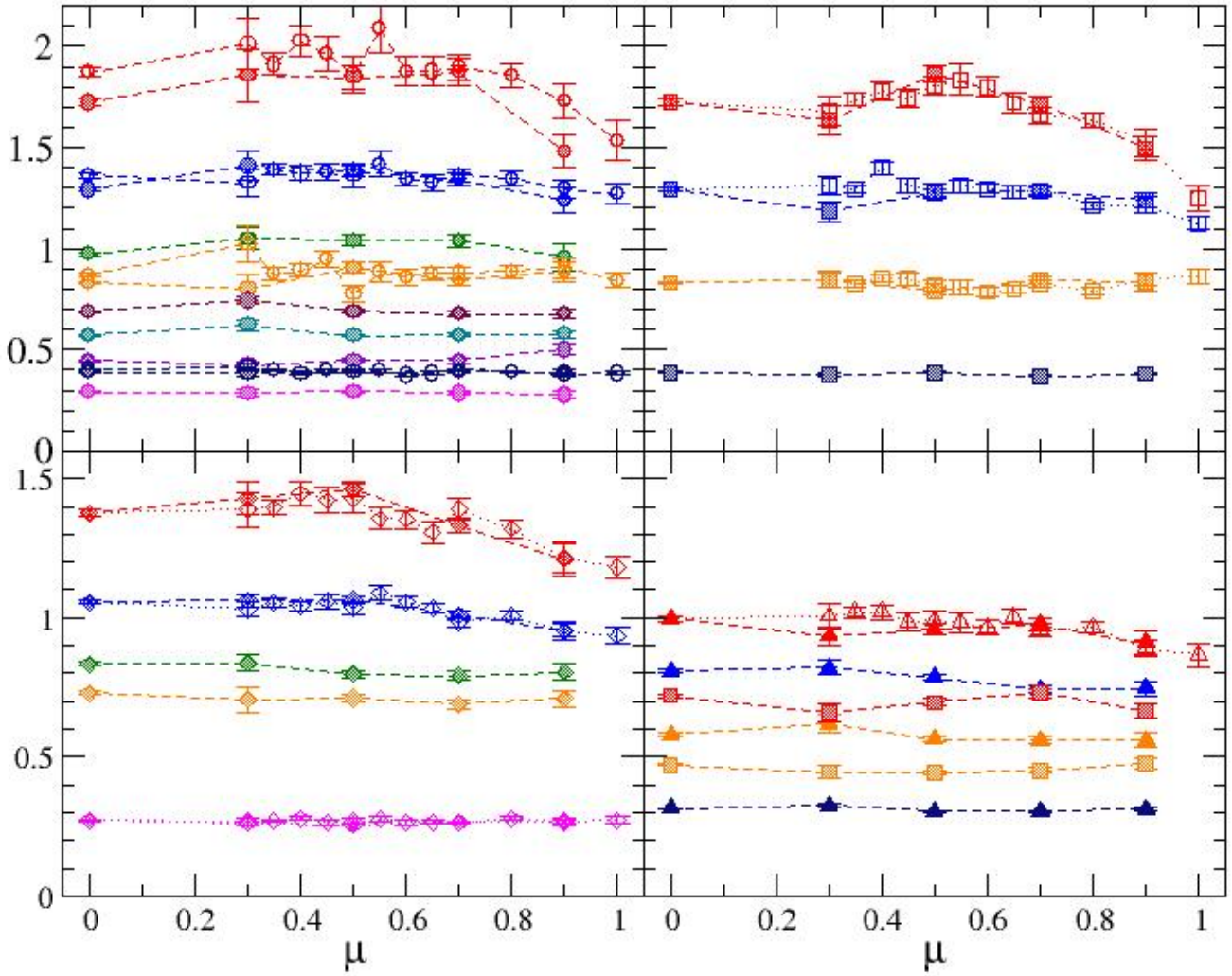
Some finite volume and lattice spacing effects and at $\mu = 0$

In-medium modifications, incl. violations of Lorentz symmetry, visible in magnetic gluon at $\mu = 0.7$

Magnetic gluon

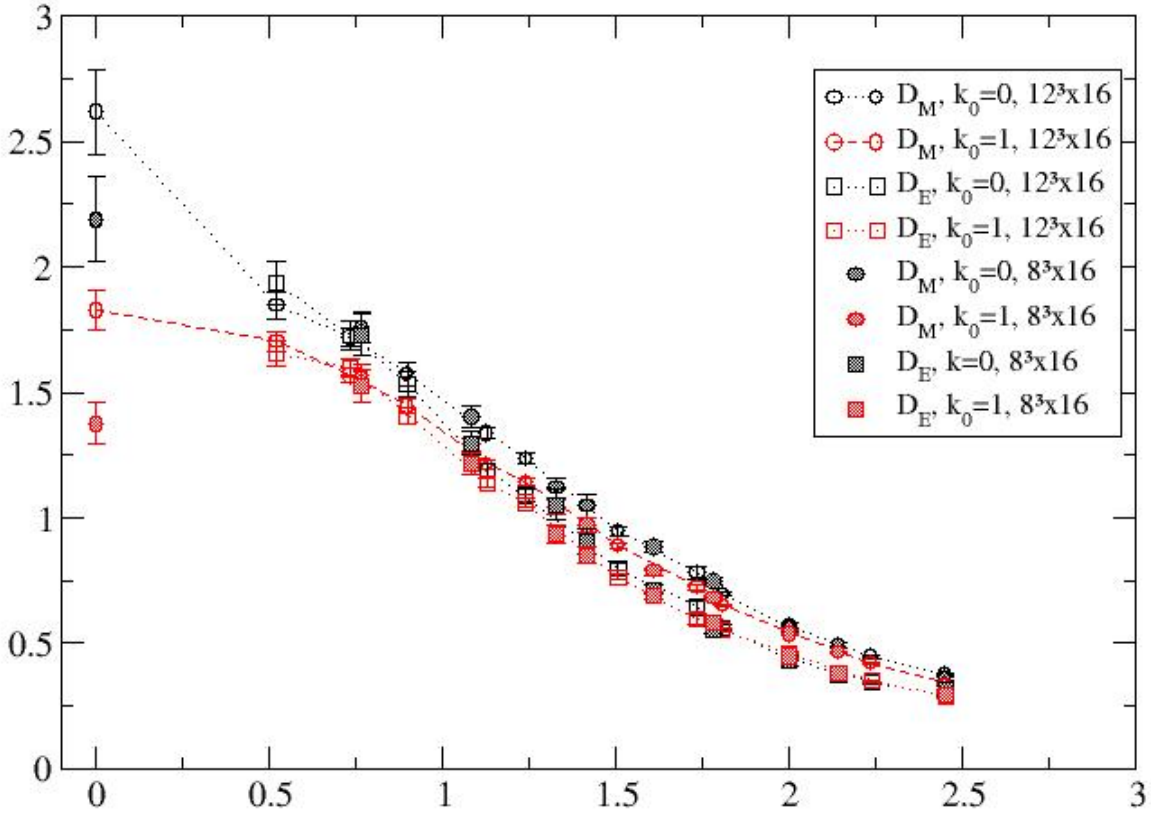


Electric gluon

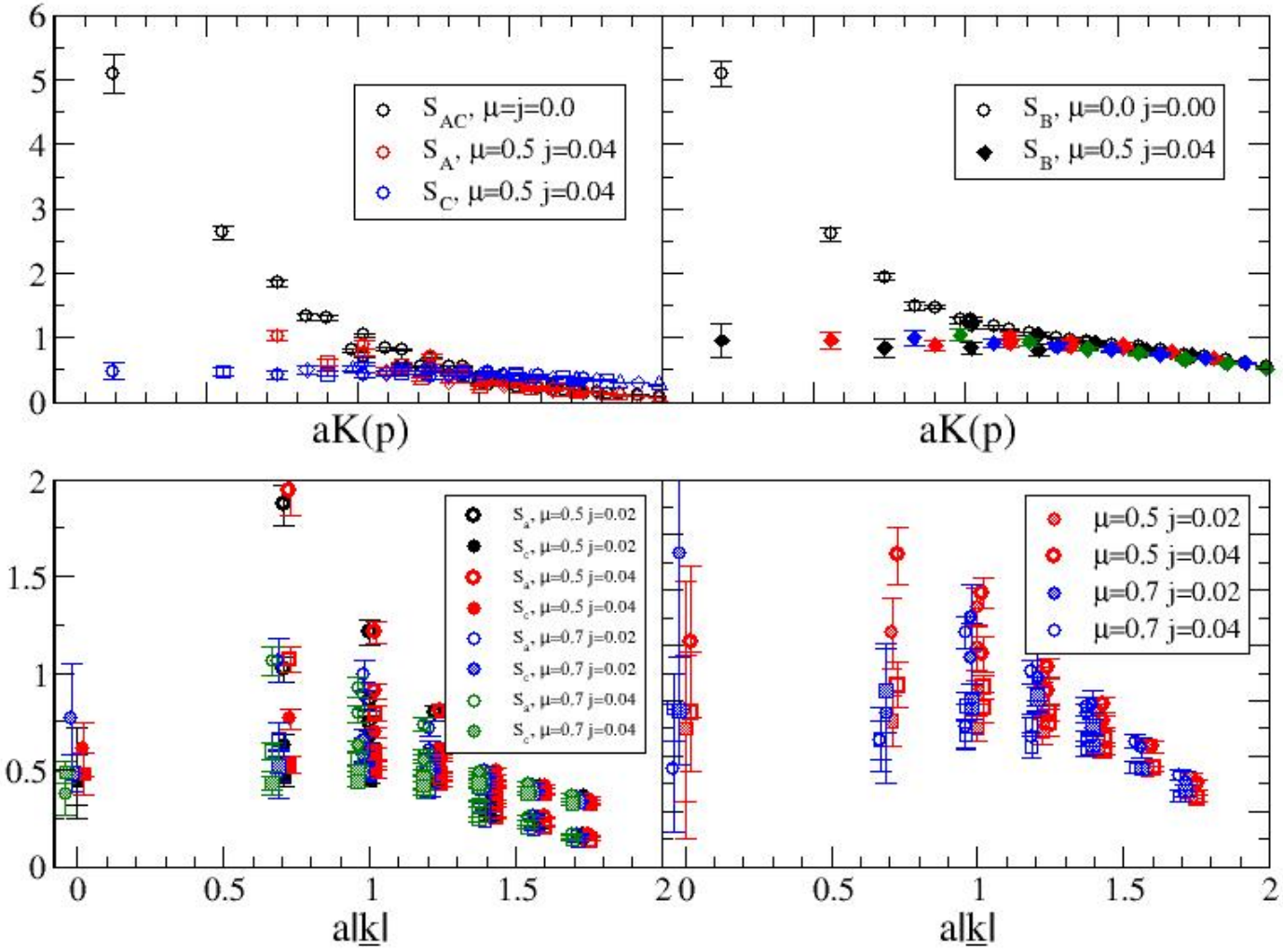


Volume dependence

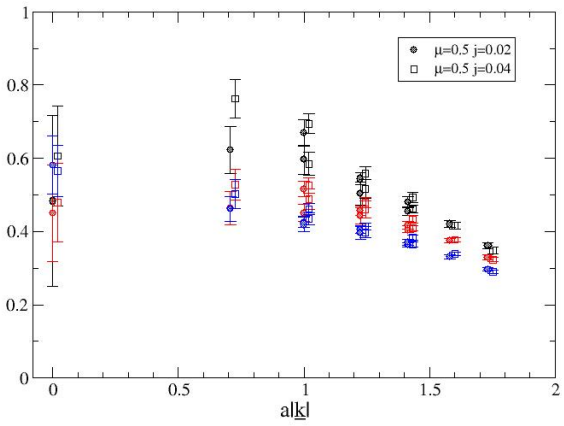
$[\mu = 0.9, j = 0.04]$

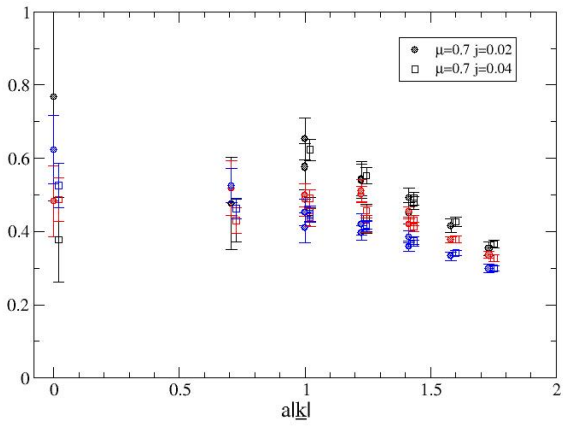


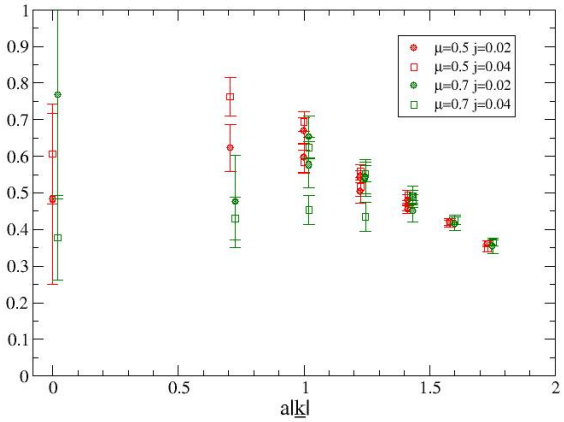
Quark propagator (Preliminary!)



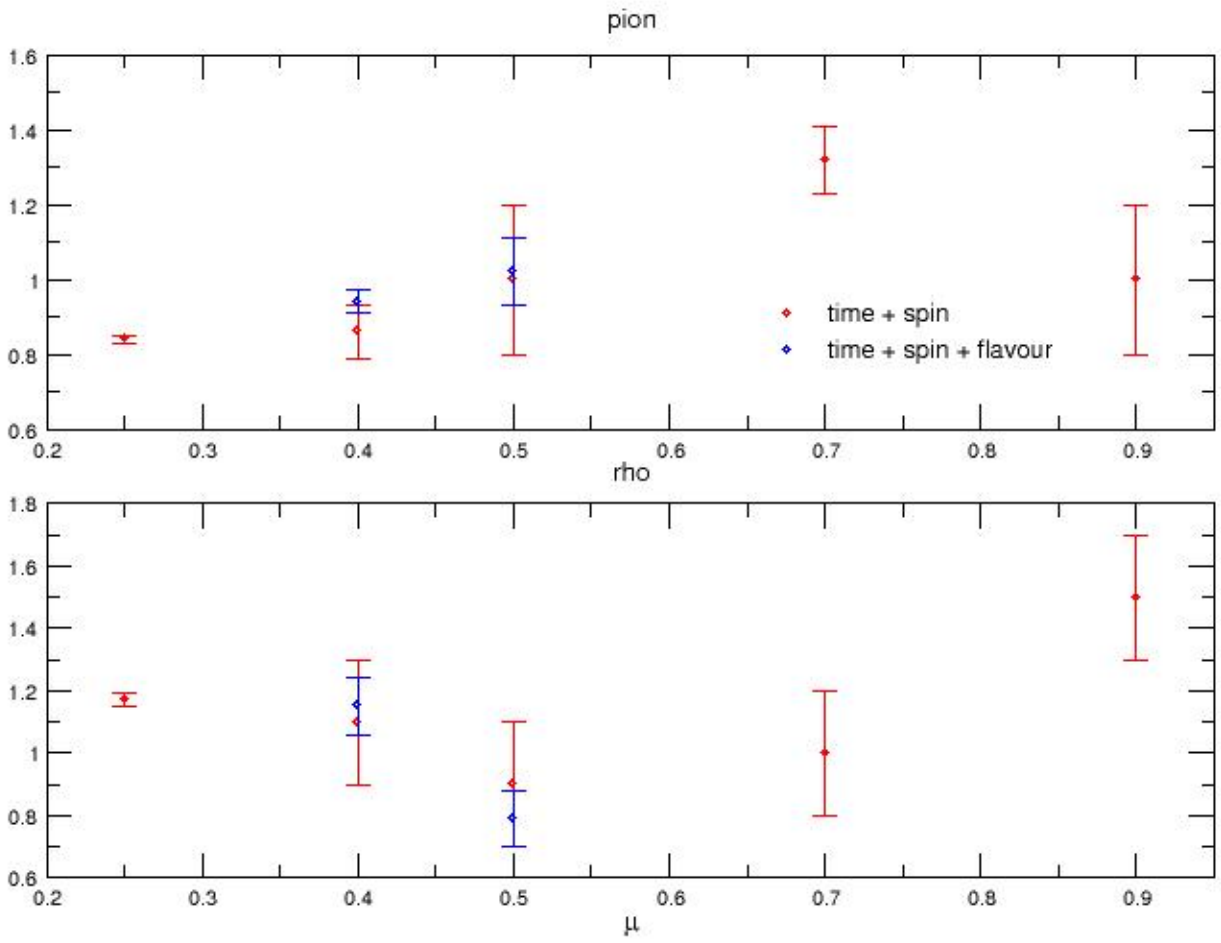
Anomalous propagation







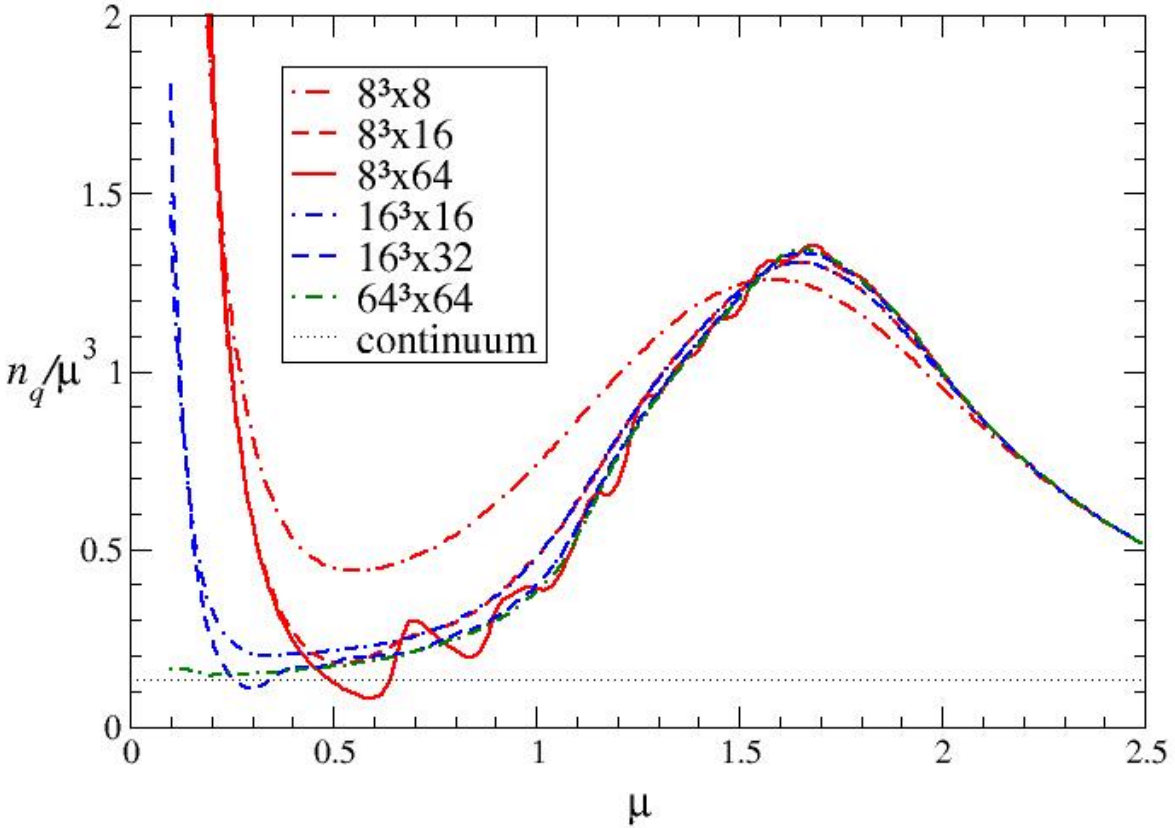
Hadron spectrum (Preliminary)



Summary

- Two separate phase transitions:
  - → Onset/Diquark condensation at $\mu = \mu_o \approx m_\pi/2$
  - → Deconfinement at $\mu = \mu_d \approx 0.65$
- Stefan–Boltzmann scaling of thermodynamic quantities at high μ
  - → indicating formation of a Fermi surface?
  - → binding energy from factor 2 mismatch?
- Gluons account for a substantial part of the energy density
  - → resulting solely from background quark charge density
- Screening of both magnetic and electric gluon propagator after deconfinement
  - → screening of static magnetic gluon is purely nonperturbative!
- Strong modifications of quark propagator in BEC phase
- Rho meson becomes lighter in the medium?

Outlook

- Simulations on finer (and larger) lattices underway
- Hadron spectrum and quark propagator study in progress
- Nonperturbative Karsch coefficients
  $\longrightarrow$ renormalised energy densities?
- Study quark mass dependence?
- Future work: Topology, eigenvalue spectrum, higher T, ...

Free lattice fermions