Quarkonium Spectral Functions and Correlators from Anisotropic Lattice

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References

Work in collaboration with A. Jakovac (Budapest, Tech. U.), P. Petreczky (Brookhaven) and K. Petrov (Bohr Inst.).

- **hep-lat**/0603005
- **PoS LAT2005**:153,2006 (**hep-lat**/0509138)

Outline

Introduction

- Meson Correlators and Spectral Functions
- Reconstruction of the Spectral Function
- Simulation parameters, lattices

Test of MEM

- Model spectral functions: Dirac's delta and Breit-Wigner
- Stable state spectral functions
- Finite width spectral functions

Charmonium

- Zero Temperature
- Finite Temperature

Bottomonium

Point meson operator

$$J_H(t,x) = \bar{q}(t,x)\Gamma_H q(t,x),$$

where $\Gamma_H = 1, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \gamma_\mu\gamma_\nu$.

Meson states in different channels:

Γ	$^{2S+1}L_J$	J^{PC}	$c\bar{c}$ (n=1)	$c\bar{c}$ (n=2)	$b\bar{b}(n=1)$ (n=1)	$b\bar{b}(n=2)$ (n=2)
γ_5	$^1S_0$	0^{-+}	η_c	η_c'	η_b	η_b'
γ_s	$^3S_1$	1^{--}	J/ψ	ψ'	$\Upsilon(1S)$	$\Upsilon(2S)$
$\gamma_s\gamma_{s'}$	$^1P_1$	1^{+-}	h_c		h_b	
1	$^3P_0$	0^{++}	χ_{c0}		$\chi_{b0}(1P)$	$\chi_{b0}(2S)$
$\gamma_5\gamma_s$	$^3P_1$	1^{++}	χ_{c1}		$\chi_{b1}(1P)$	$\chi_{b1}(2P)$
		2^{++}	χ_{c2}		$\chi_{b2}(1P)$	$\chi_{b2}(2P)$

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		2^{++}	χ_{c2}		$\chi_{b2}(1P)$	$\chi_{b2}(2P)$

The spectral function

$$\sigma_H(p_0,\vec{p}) = \frac{1}{2\pi}(D_H^{>}(p_0,\vec{p}) - D_H^{<}(p_0,\vec{p})) = \frac{1}{\pi} Im D_H^R(p_0,\vec{p})$$

$$D_H^{>(<)}(p_0,\vec{p}) = \int \frac{d^4p}{(2\pi)^4} e^{ip.x} D_H^{>(<)}(x_0,\vec{x}) \quad (1)$$

$$D_H^{>}(x_0,\vec{x}) = \langle J_H(x_0,\vec{x}), J_H(0,\vec{0})\rangle$$

$$D_H^{<}(x_0,\vec{x}) = \langle J_H(0,\vec{0}), J_H(x_0,\vec{x})\rangle, x_0 > 0 \quad (2)$$

The Euclidean propagator

$$G_H(\tau,\vec{p}) = \int d^3x e^{i\vec{p}.\vec{x}} \langle T_\tau J_H(\tau,\vec{x}) J_H(0,\vec{0})\rangle$$

is related to the spectral function through the integral representation

$$G(\tau,\vec{p}) = \int_0^\infty d\omega \sigma K(\omega,\tau), \quad K(\omega,\tau) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}.$$

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Reconstruction of the Spectral Function

- $G(\tau,\vec{p}) = \int_0^\infty d\omega \sigma(\omega,T) K(\omega,\tau,T)$
- $O(10)$ data and $O(100)$ degrees of freedom to reconstruct.
- Bayesian technique: find $\sigma(\omega,T)$ that maximizes $P[\sigma|DH]$.
  - D data
  - H prior knowledge: $\sigma(\omega,T) > 0$

Maximum Entropy Method: Asakawa, Hatsuda, Nakahara, PRD 60 (99) 091503, Prog. Part. Nucl. Phys. 46 (01) 459

$$P[\sigma|DH] = \exp(-\frac{1}{2}\chi^2 + \alpha S) \tag{3}$$

Shannon-Janes entropy: $S = \int d\omega \left[\sigma(\omega) - m(\omega) - \sigma(\omega)\ln(\frac{\sigma(\omega)}{m(\omega)})\right]$, $m(\omega)$ - the default model, $m(\omega \gg \Lambda_{QCD}) = m_0\omega^2$ - perturbation theory.

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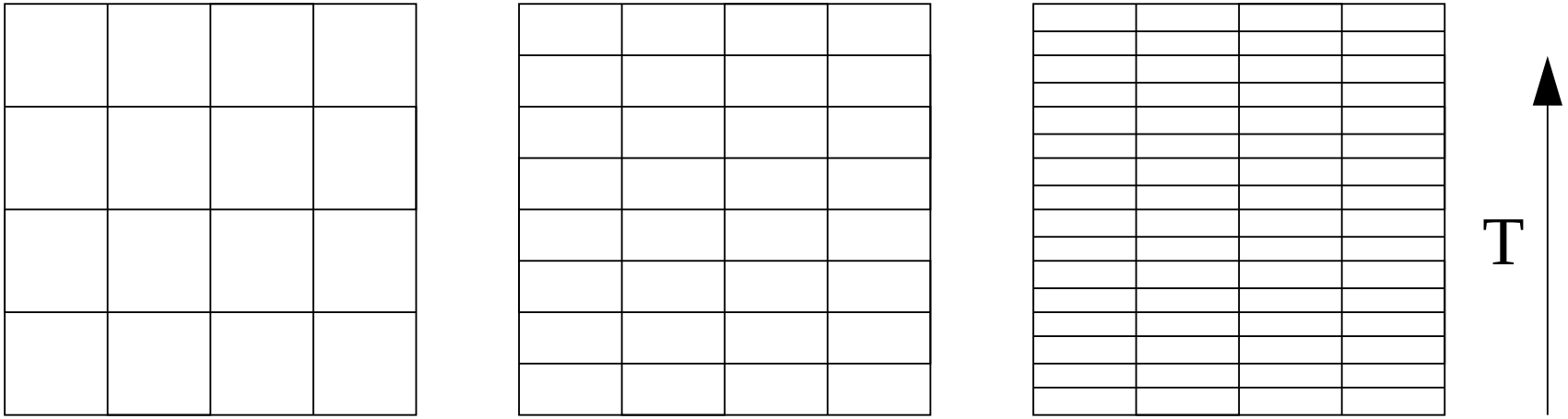
Anisotropic lattice $\xi = a_s/a_\tau = 2$ and 4.



Standard Wilson action in the gauge sector and the anisotropic clover improved action for heavy fermions. Quenched approximation. Sommer scale to fix the physical units.

β	5.7	5.9	6.1	6.1	6.5
$N_s^2 \times N_t$	$8^3 \times 64$	$16^3 \times 64$	$16^3 \times 64$	$16^3 \times 96$	$24^3 \times 160$
$(\xi,\ \xi_0)$	(2,1.655)	(2,1.691)	(2,1.718)	(4,3.211)	(4,3.3166)
r_0/a_s	2.414(8)	3.690(11)	5.207(29)	5.189(21)	8.96(4)
a_t^{-1}[Gev]	1.905	2.913	4.110	8.181	14.12
L_s [fm]	1.66	2.17	1.54	1.54	1.34
configs	2000	1560	930	500	160

Table: Simulation parameters for charmonium at zero temperature.

Model spectral functions

- Dirac's delta function

$$\hat{\sigma}(\omega) = \frac{3}{2\pi} R_1 \delta(\omega - m)$$

- Breit-Wigner

$$\hat{\sigma}(\omega) = \frac{3}{2\pi} R_1 \frac{1}{\pi} \frac{\gamma}{(\omega - m)^2 + \gamma^2} \frac{\omega^3}{m^3},$$

- + continuum

$$\sigma(\omega) = \hat{\sigma}(\omega) + \Theta(\omega - s_0) m_0 \omega^2$$

Model spectral function is integrated with the kernel to produce the propagator. The covariance matrix is taken from the lattice simulation ($\beta = 6.5$, $\xi = 4$, $N_t = 160$)

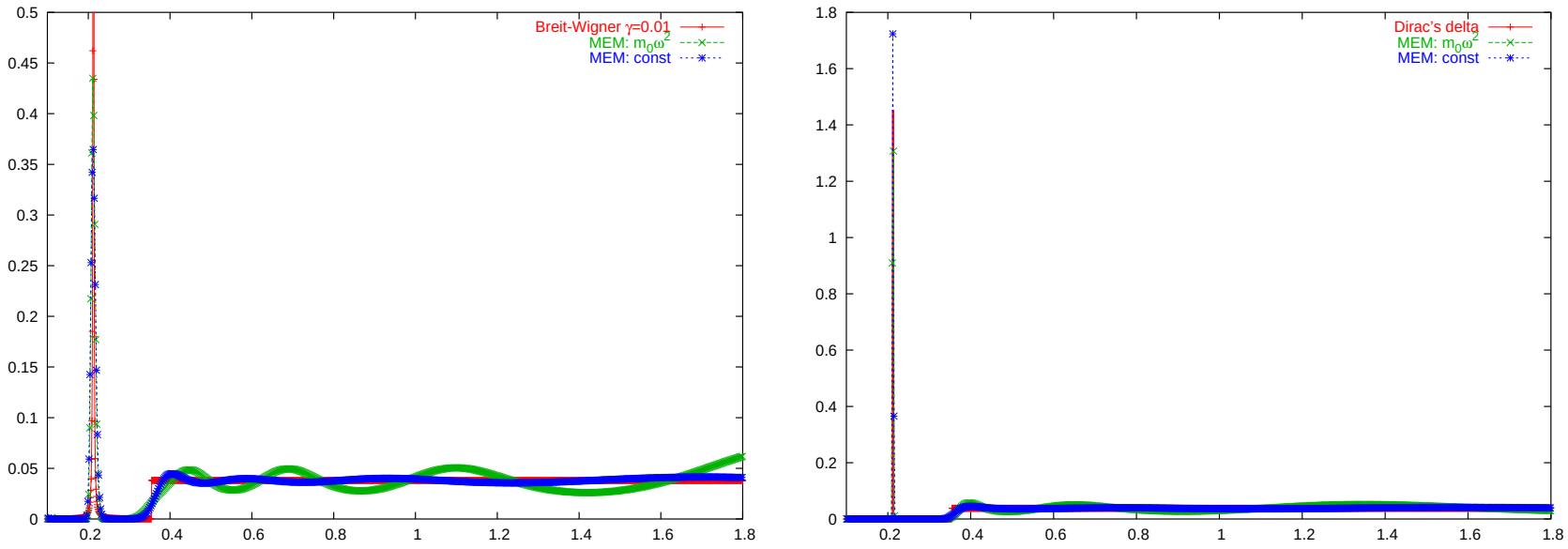


Figure: The $\gamma = 0.01$ Breit-Wigner (left) and Dirac's delta (right) spectral function reconstruction.

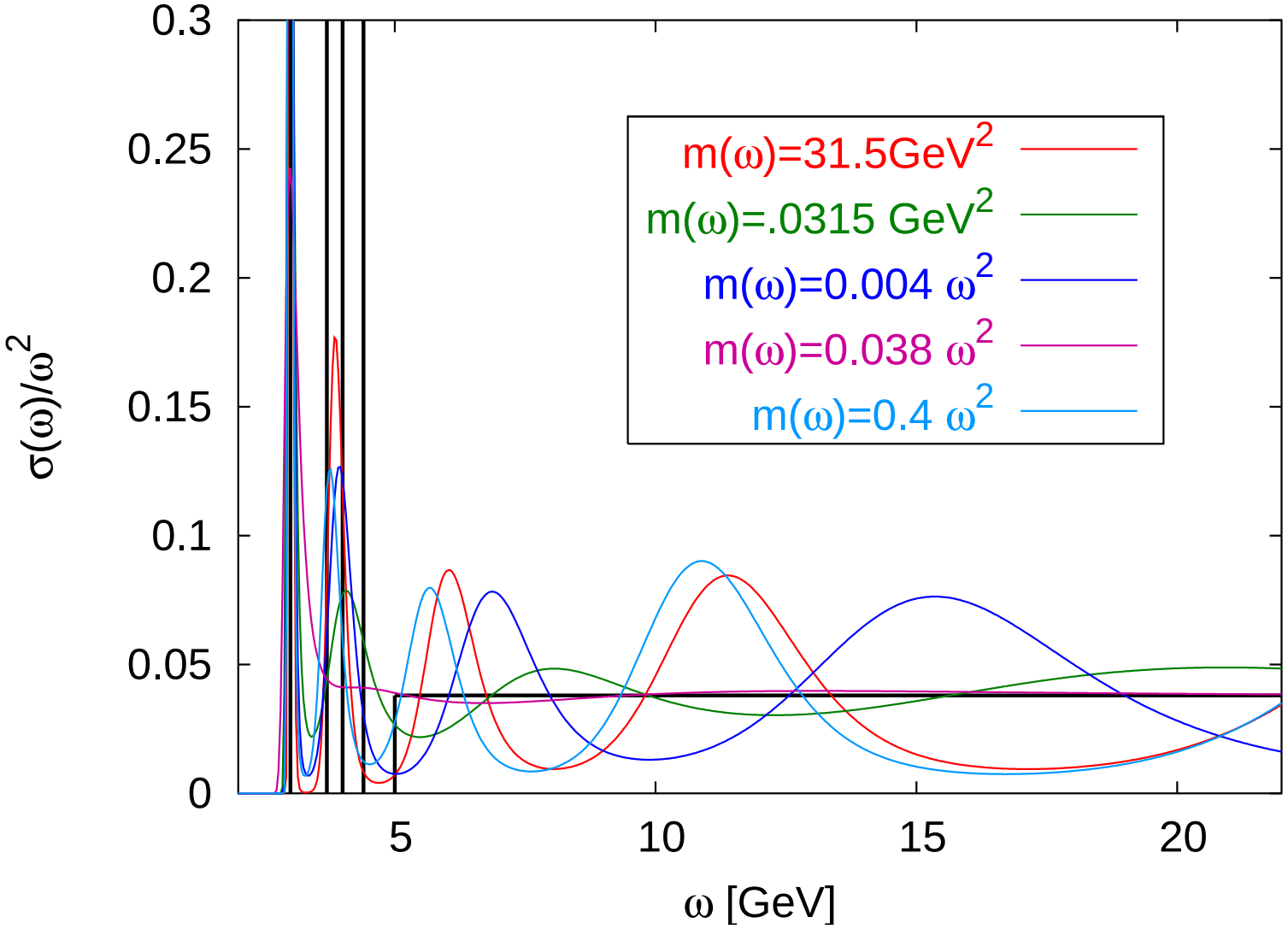


Figure: The Dirac's delta (4 state) spectral function reconstruction.

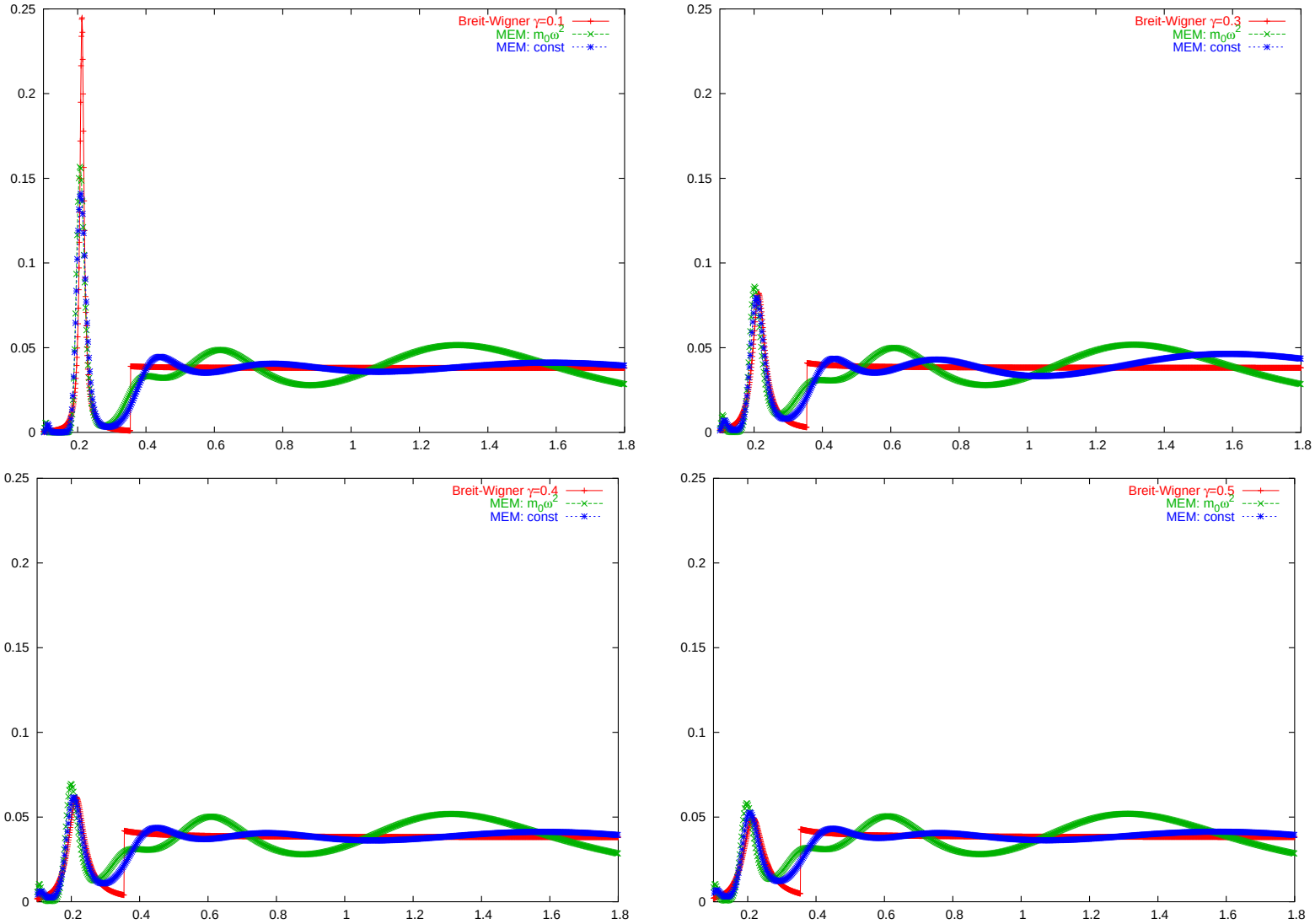


Figure: Reconstruction of $\gamma = 0.1, 0.3, 0.4, 0.5$ Breit-Wigner spectral functions.

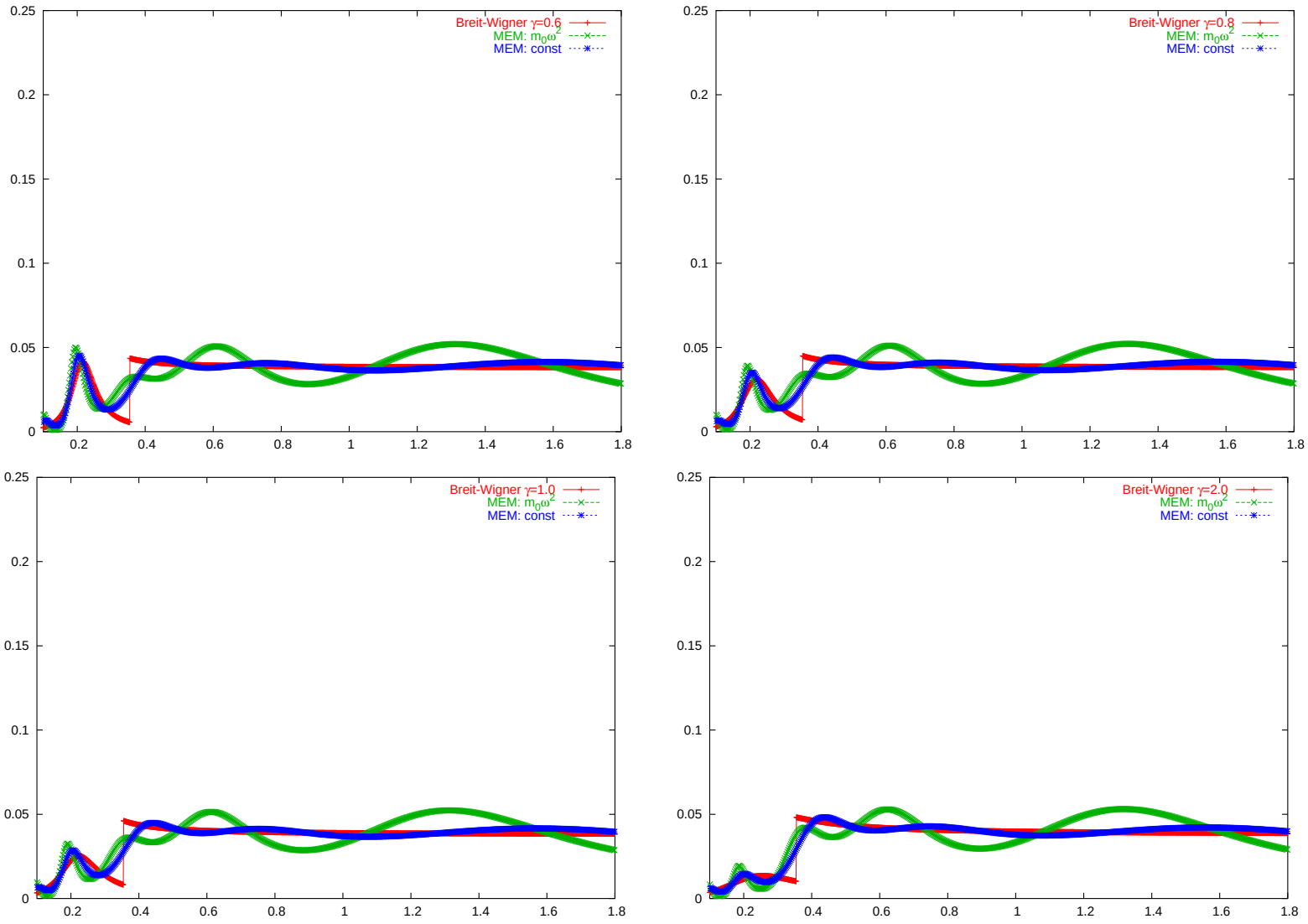


Figure: Reconstruction of $\gamma = 0.6, 0.8, 1.0, 2.0$ Breit-Wigner spectral functions.

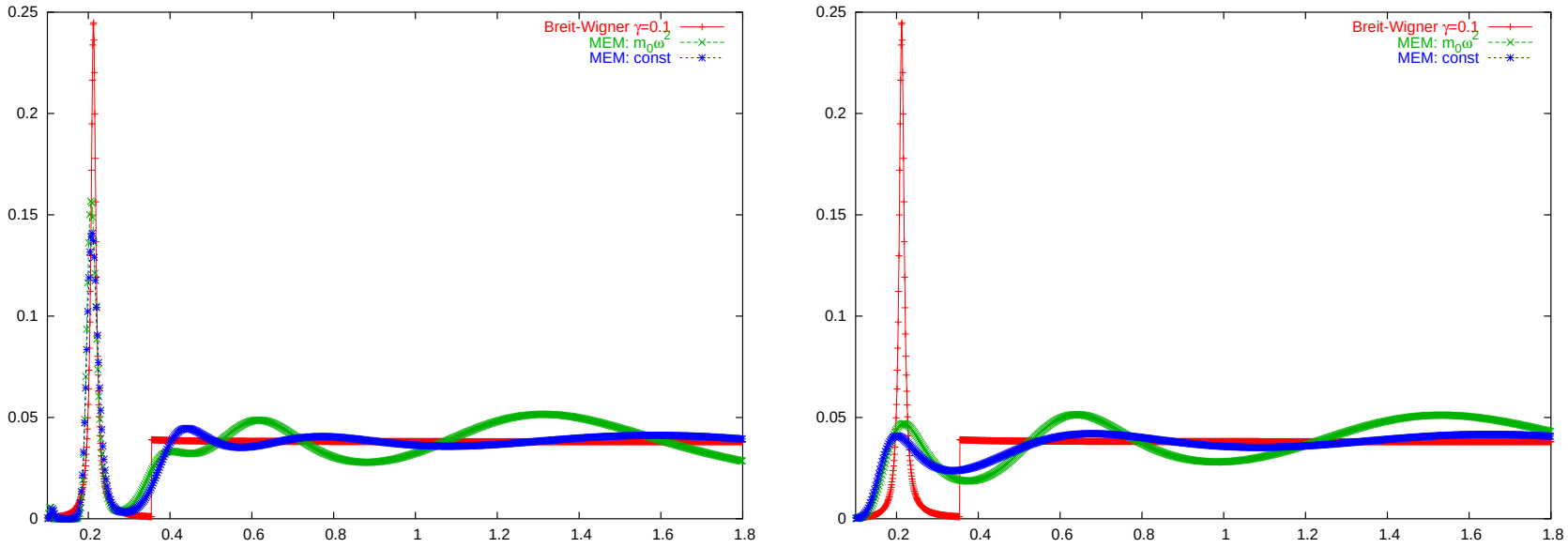


Figure: $\gamma = 0.1$ Breit-Wigner spectral function reconstruction with $N_t = 160$ (left) and $N_t = 32$ (right) covariance matrix.

Amplitude

γ	range	A_{in}	A^0_{out}	A^1_{out}	$\lvert A_{in} - A^0_{out}\rvert / A_{in}$
0.01	0.14-0.28	0.00530	0.00535	0.00533	0.009
0.1	0.14-0.28	0.00511	0.00507	0.00495	0.008
0.3	0.14-0.28	0.00443	0.00427	0.00410	0.035
0.4	0.14-0.28	0.00411	0.00394	0.00376	0.043
0.5	0.14-0.28	0.00382	0.00362	0.00346	0.053
0.6	0.14-0.28	0.00355	0.00335	0.00320	0.057
0.8	0.14-0.28	0.00308	0.00289	0.00275	0.063
1.0	0.14-0.28	0.00269	0.00252	0.00239	0.066
2.0	0.14-0.28	0.00158	0.00147	0.00137	0.073

Table: Amplitudes of the peaks for the input spectral function A_{in} and for the reconstructed spectral functions with *const* and $m_0\omega^2$ default models.

Charmonium: $T = 0$

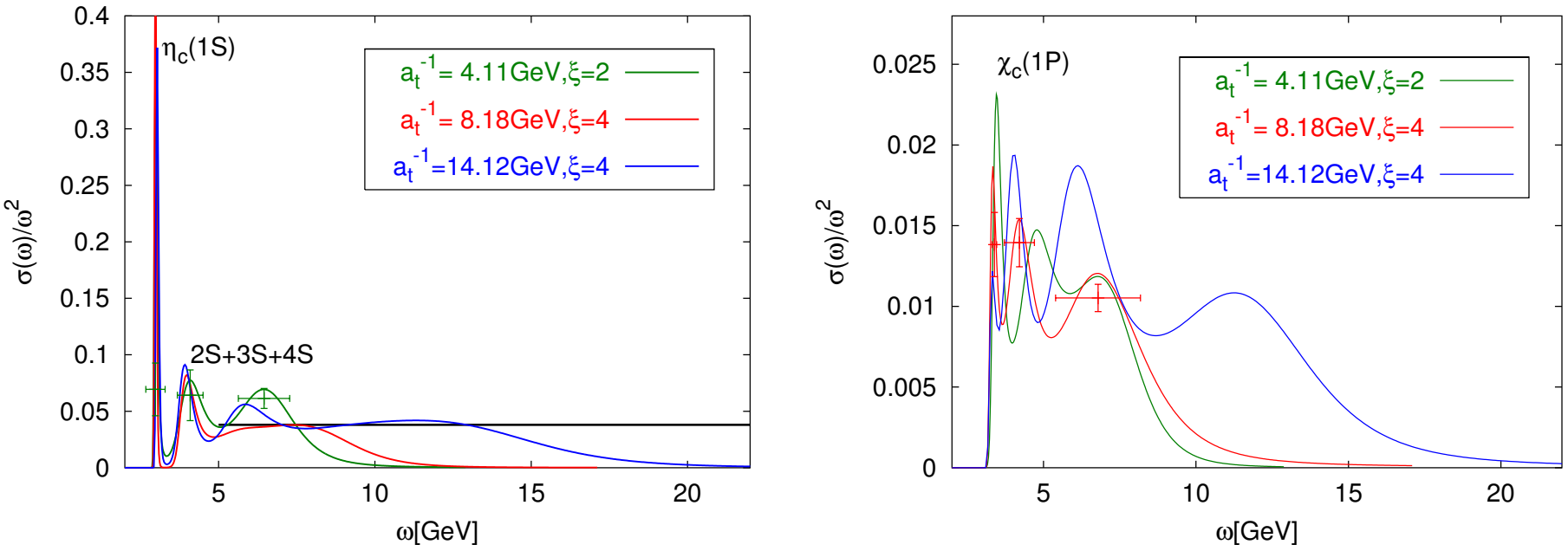


Figure: Charmonium spectral function in the pseudo-scalar channel (left) and the scalar channel (right) at different lattice spacings and zero temperature.

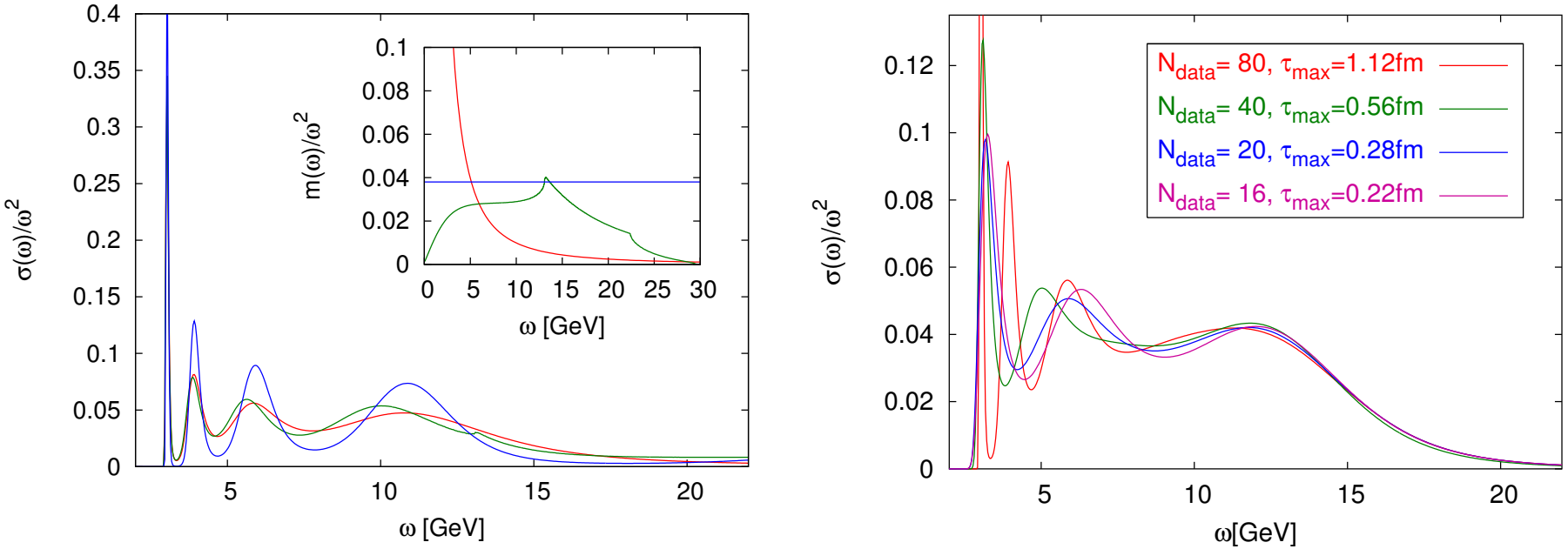


Figure: Charmonium spectral function dependence on the default model (left) and on the maximum time extend (right). Pseudo-scalar channel at $a_t^{-1} = 14.12 GeV$ and zero temperature.

Charmonium: $T > 0$

$$G_{recon}(\tau, T) = \int_0^\infty d\omega \sigma(\omega, T = 0) K(\tau, \omega, T)$$

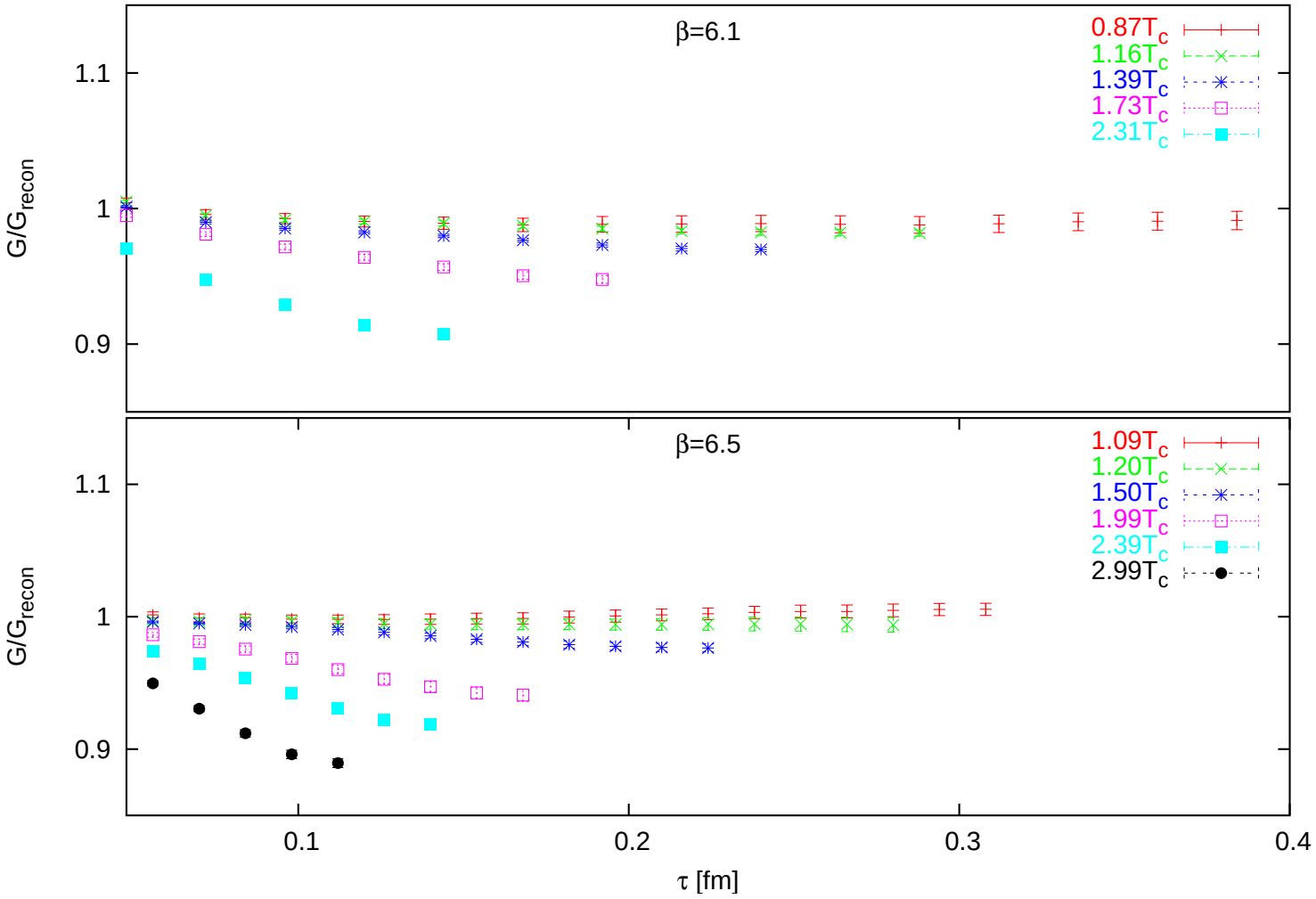


Figure: The ratio $G(\tau, T)/G_{recon}(\tau, T)$ of charmonium for the pseudo-scalar channel at $a_t^{-2} = 8.18$ and 14.11GeV at different temperatures.

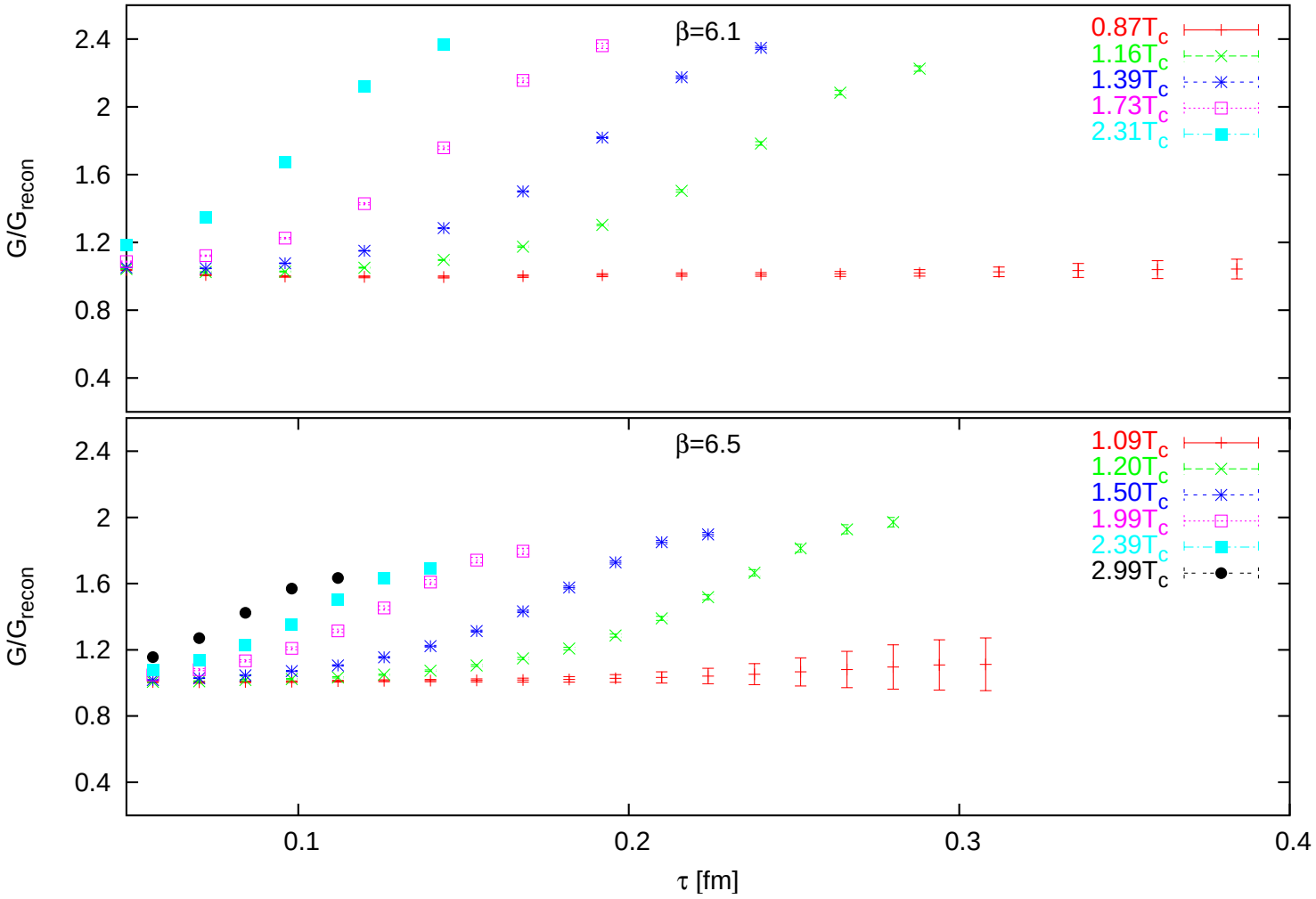


Figure: The ratio $G(\tau, T)/G_{recon}(\tau, T)$ of charmonium for the scalar channel at $a_t^{-2} = 8.18$ and 14.11GeV at different temperatures.

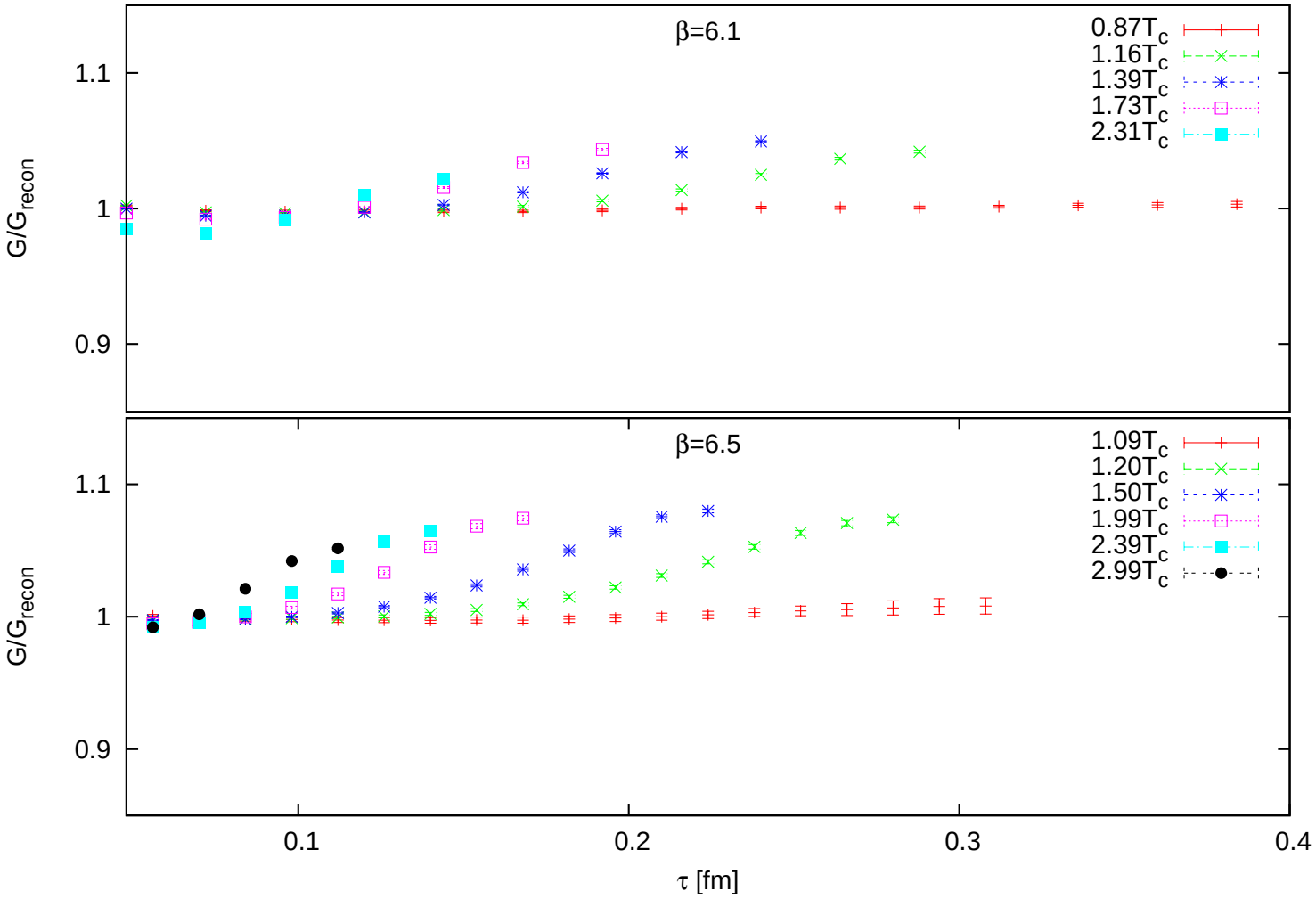


Figure: The ratio $G(\tau, T)/G_{recon}(\tau, T)$ of charmonium for the vector channel at $a_t^{-2} = 8.18$ and 14.11GeV at different temperatures.

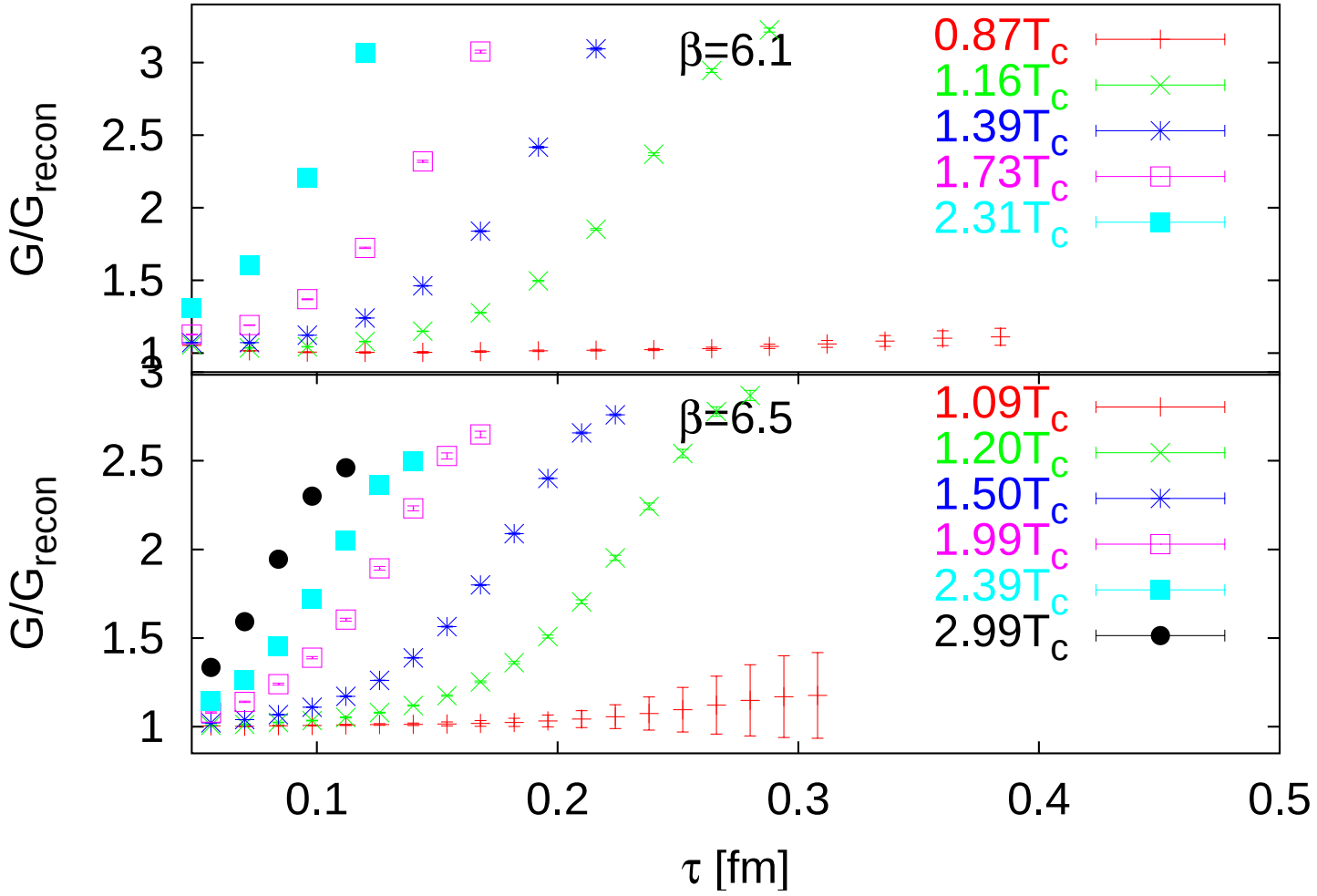


Figure: The ratio $G(\tau, T)/G_{recon}(\tau, T)$ of charmonium for the axial vector channel at $a_t^{-2} = 8.18$ and 14.11GeV at different temperatures.

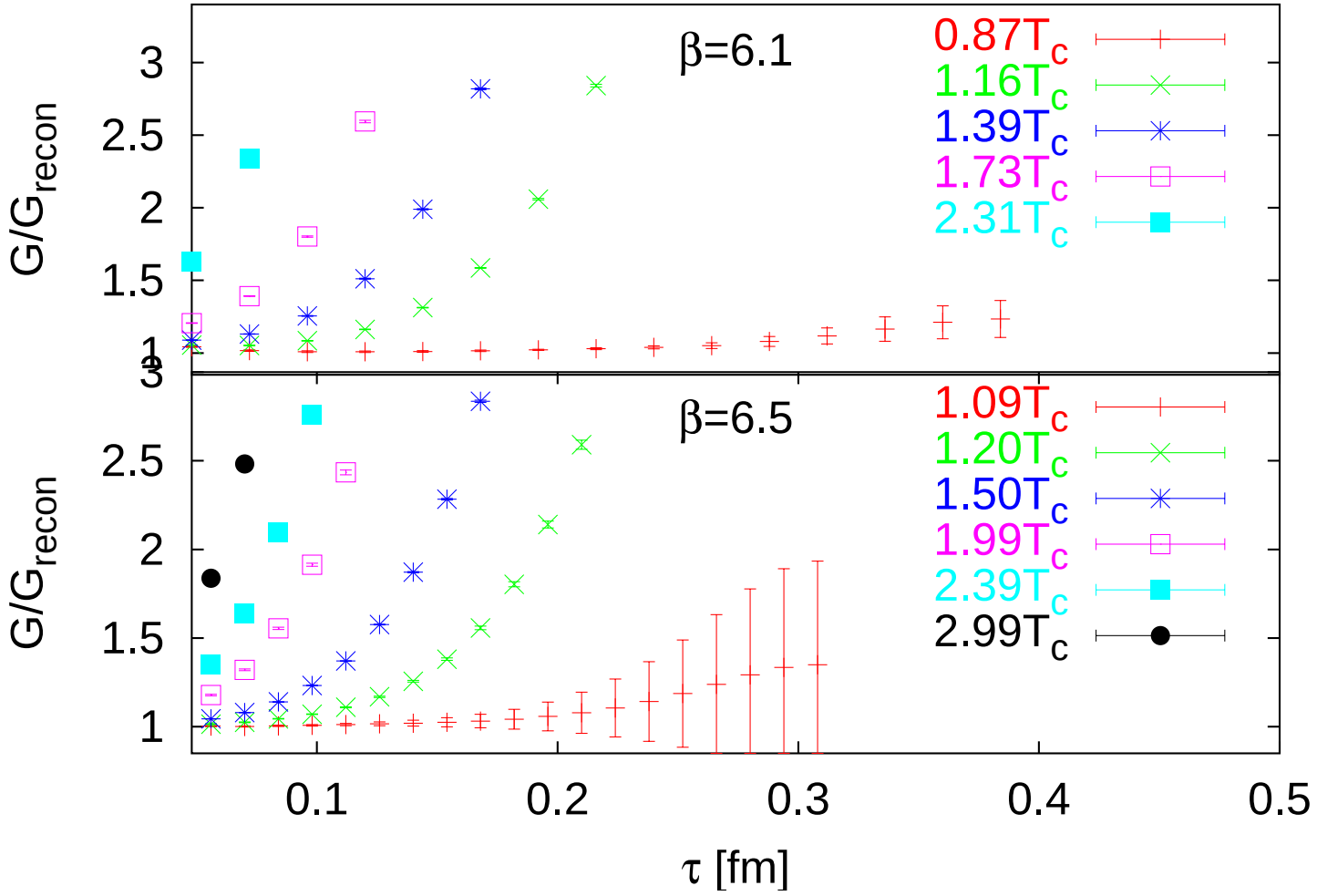


Figure: The ratio $G(\tau, T)/G_{recon}(\tau, T)$ of charmonium for the tensor channel at $a_t^{-2} = 8.18$ and 14.11GeV at different temperatures.

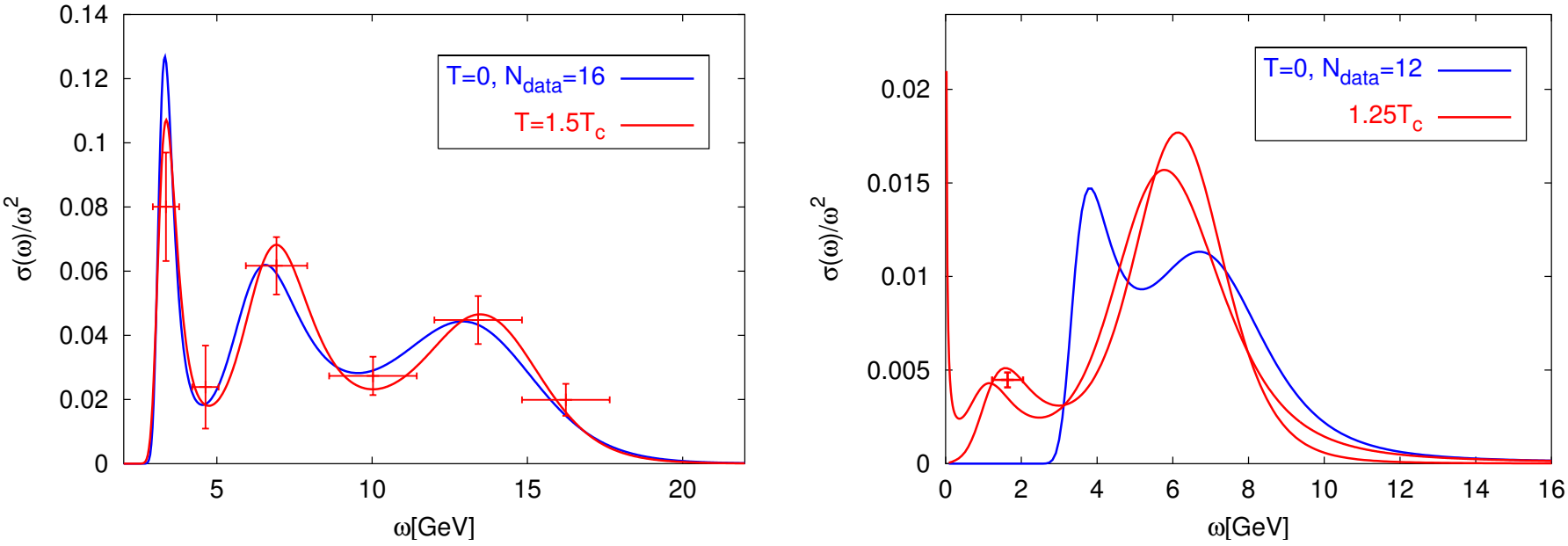


Figure: Charmonium spectral function in the pseudo-scalar channel at $a_t^{-2} = 14.11$GeV (left) and the scalar channel (right) at $a_t^{-2} = 8.18$GeV at zero and above deconfinement temperatures. For finite temperature scalar channel two different default models are shown.

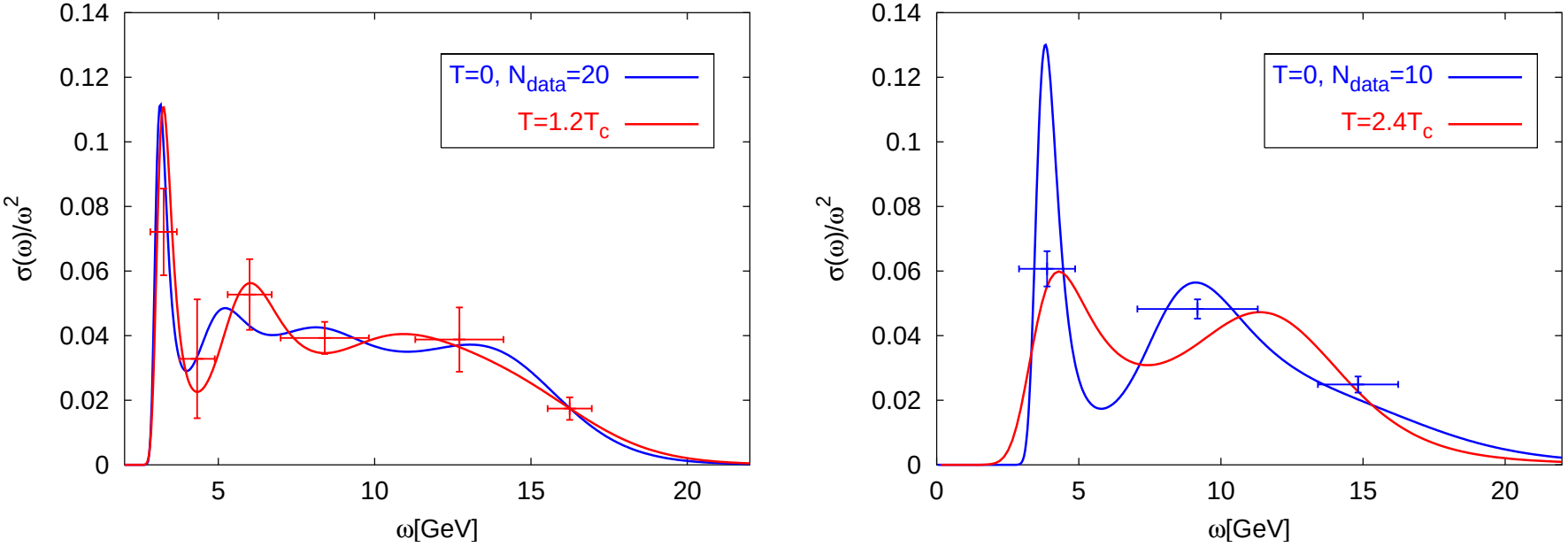


Figure: Charmonium spectral functions in the pseudoscalar channel at finite temperature.

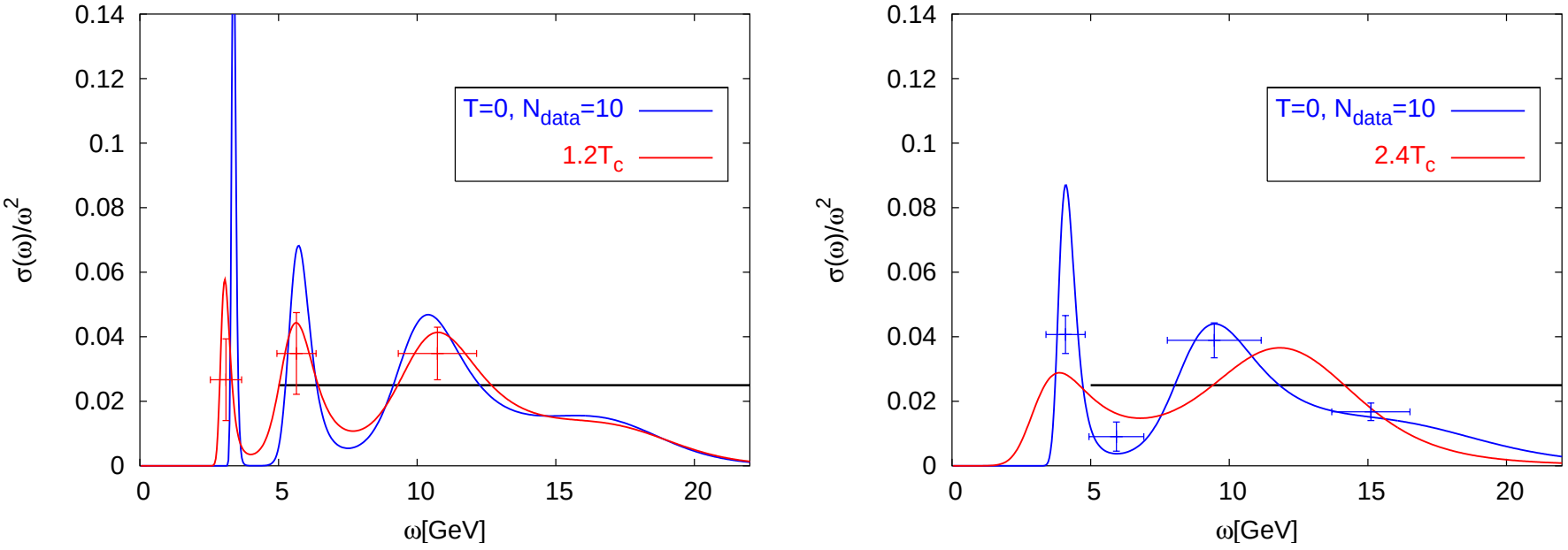
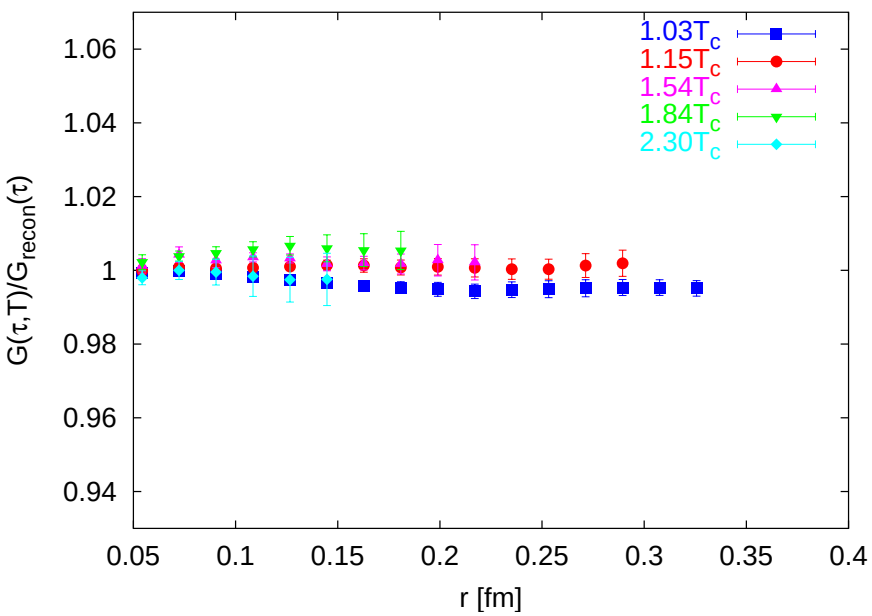


Figure: Charmonium spectral functions in the vector channel at finite temperature.

Bottomonium



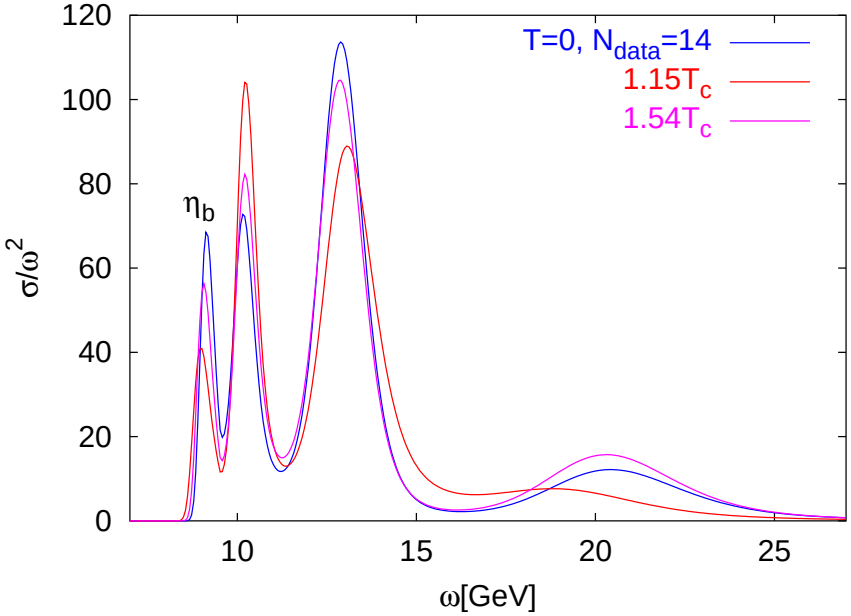
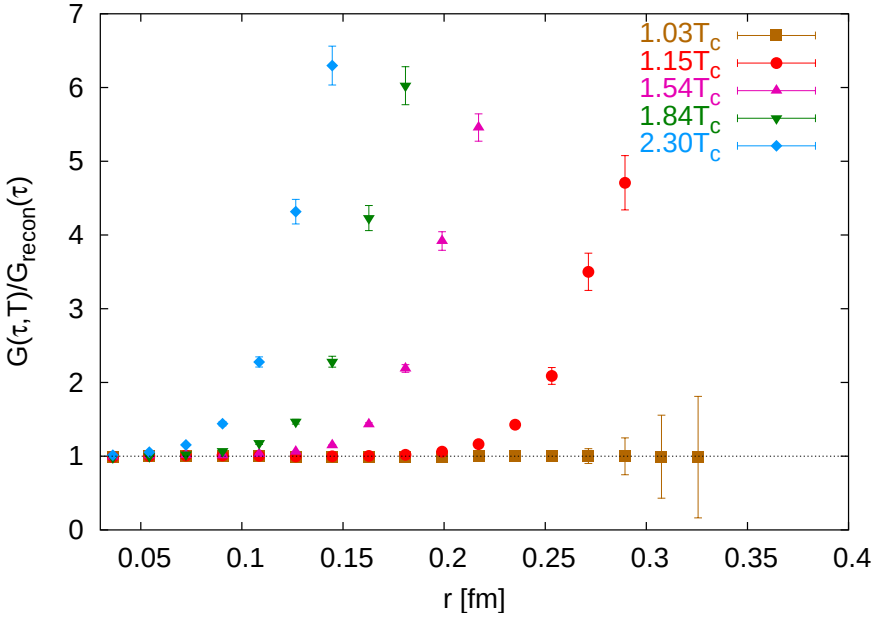


Figure: Bottomonia correlators (left) and spectral functions (right) in pseudo-scalar channel for different temperatures.



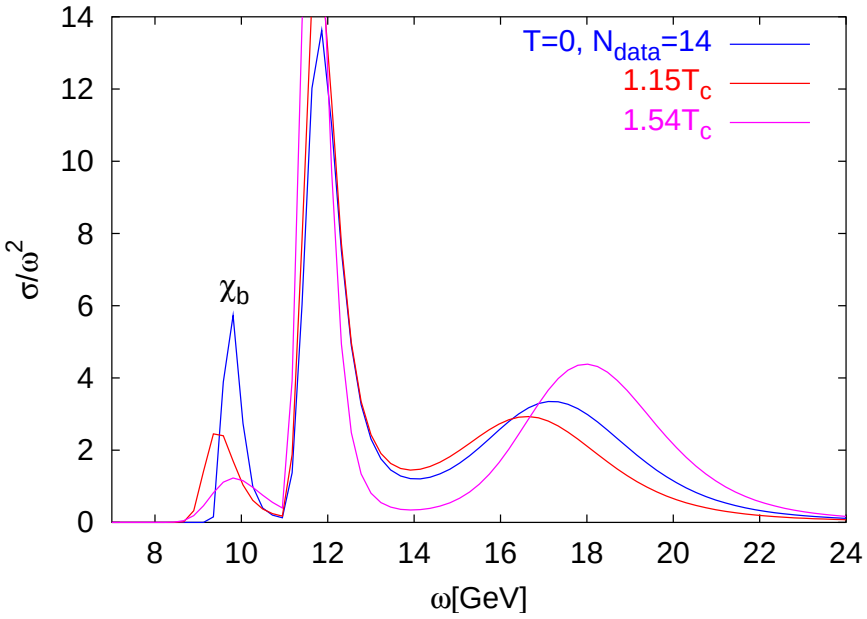


Figure: Bottomonia correlators (left) spectral functions (right) in scalar channel for different temperatures.

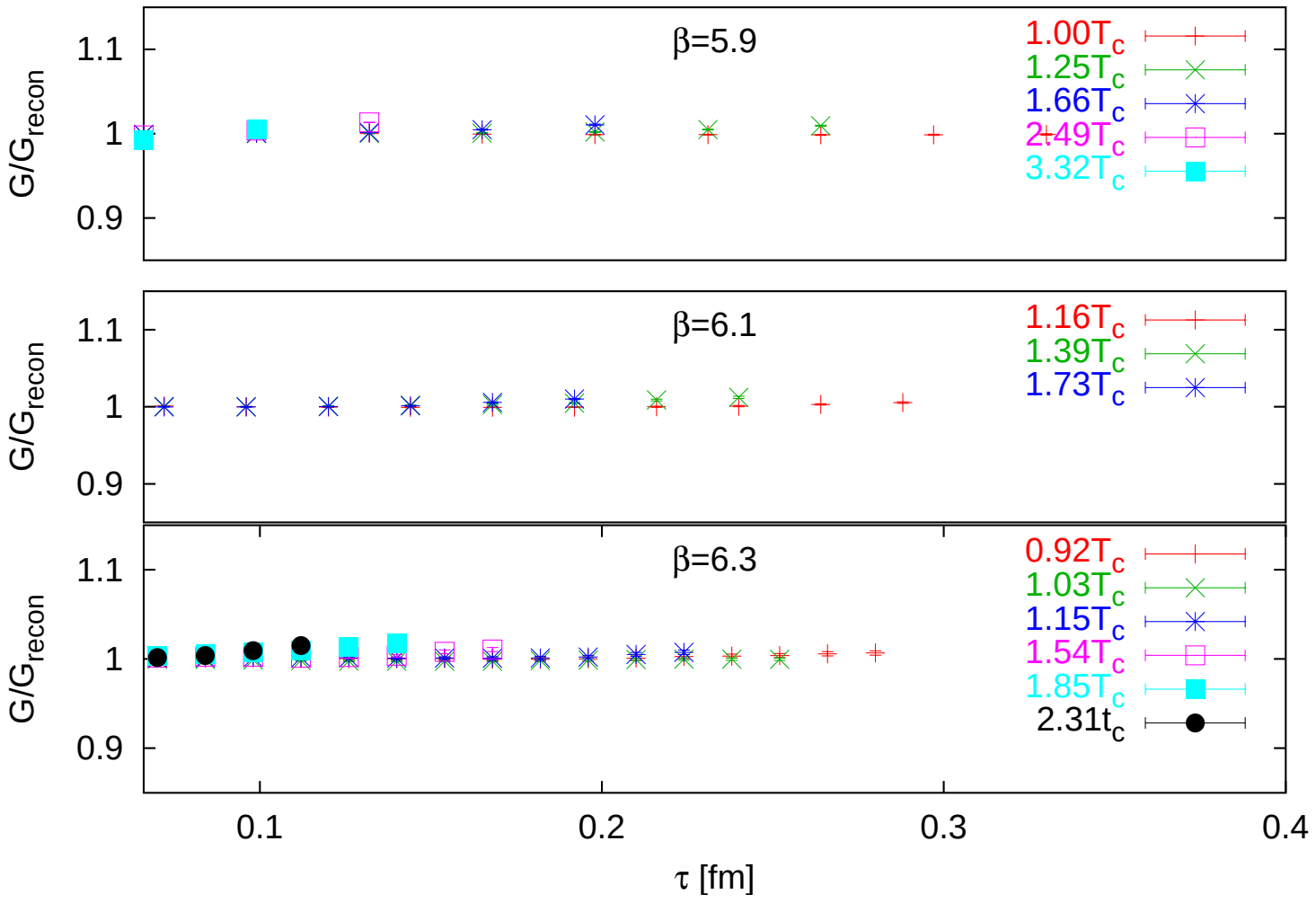


Figure: Bottomonia correlators for vector channel for different temperatures and lattices.

Summary

- MEM can resolve finite width bound states.
- The $1S$ (η_c, J/ψ) charmonium states exist as a resonance in the deconfined phase at $T \simeq 1.5\,T_c$.
- $1P$ (ξ_{c0}, ξ_{c1}) charmonium states dissolve at $1.1\,T_c$.
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- The emerging studies of heavy quarkonium properties with dynamical fermions produce consistent results with the quenched approximation.

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