Using AdS/CFT to explore the Strong Coupling Regime of Gauge Theories: I

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Contents

Based primarily on

- "Drag force in AdS/CFT," `hep-th/0605182`.
- "Dissipation from a heavy quark moving through N=4 super-Yang-Mills plasma," `hep-th/0605292`, with J. Friess and G. Michalogiorgakis.
- Work to appear with J. Friess, G. Michalogiorgakis, and S. Pufu.

Closely related works include:

- "Calculating the jet quenching parameter from AdS/CFT," `hep-ph/0605178`, by H. Liu, K. Rajagopal, U. Wiedemann.
- "Energy loss of a heavy quark moving through N=4 supersymmetric Yang-Mills plasma," `hep-th/0605158`, by C. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. Yaffe.
- "Heavy quark diffusion in strongly coupled N=4 Yang-Mills," `hep-ph/0605199`, by D. Teaney and J. Casalderrey-Solana.
- "Ampere's Law and Energy Loss in AdS/CFT Duality," `hep-ph/0606049`, by S.-J. Sin and I. Zahed.

1. Introduction

A recent string theory computation of drag force on a heavy quark moving through a thermal plasma of $\mathcal{N} = 4$ super-Yang-Mills theory gives

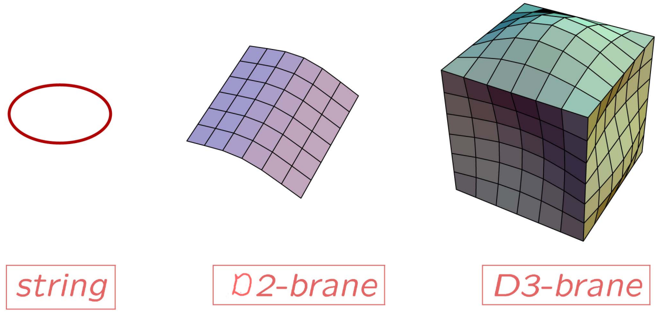
$$F \equiv \frac{dp}{dt} = -\frac{\pi\sqrt{g_{YM}^2 N}}{2} T^2 \frac{p}{m} \,. \tag{1}$$

My aims are

- To summarize relevant aspects of string theory (i.e. AdS/CFT).
- To caution you that string theorists have not yet solved QCD!
- To explain where (1) comes from.
- To describe further calculations that give evidence for a "wake" of gluons and their superpartners.
- To speculate about the possible relevance to jet-quenching at RHIC.

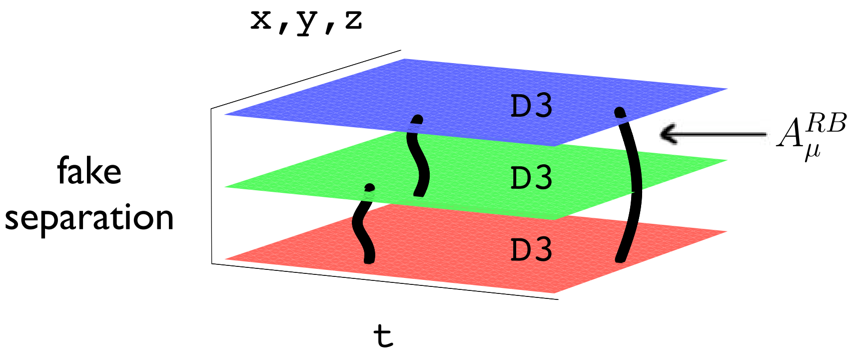
2. String theory and $\mathcal{N} = 4$ super-Yang-Mills

Strings can't exist without higher-dimensional objects: D-branes.

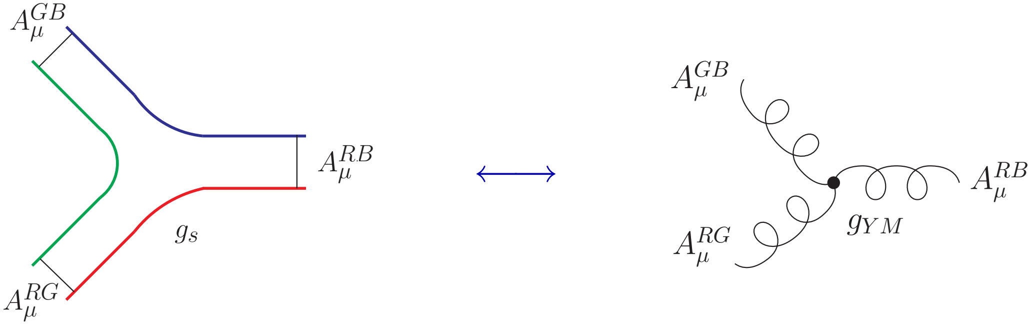


D-branes can be defined as locations where strings can end.

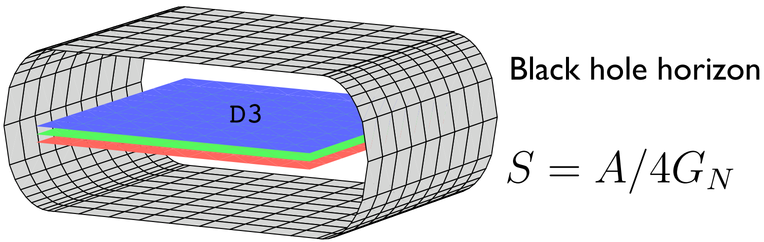
- One one hand, strings ending on D3-branes act as gluons for interesting four-dimensional gauge theories.



Merging interactions of strings mimics standard 3-vertex of gauge theory:



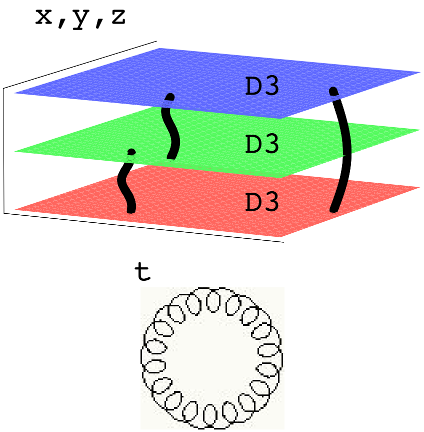
• On the other hand, many coincident D3-branes back-react on spacetime to produce a black hole horizon.

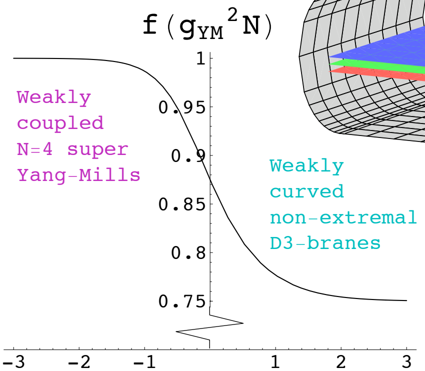


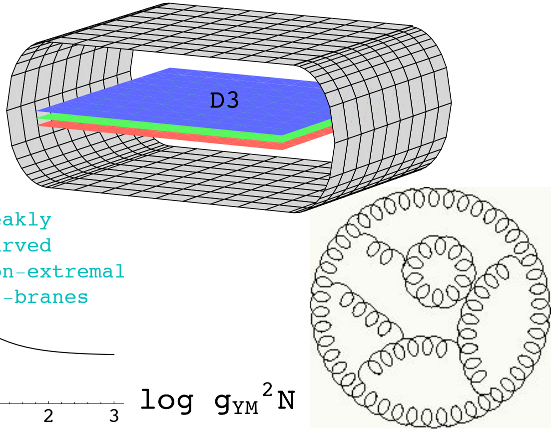
The temperature, entropy, shear viscosity, etc. of this horizon are supposed to match those of $\mathcal{N} = 4$ super-Yang-Mills.

Free energy (as calculated in hep-th/9602135):

$$-F/V_3T^4 = \frac{\pi^2}{6} N^2 f(g_{YM}^2 N) \tag{2}$$



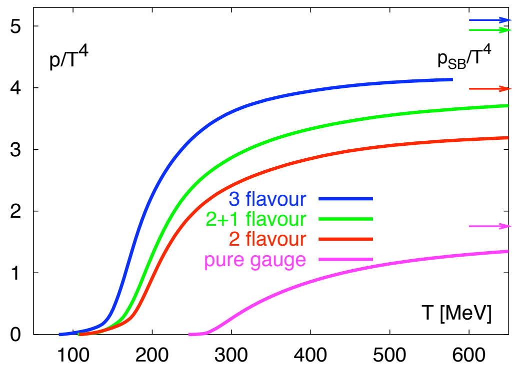




The punch-line:

$$F_{\text{strong coupling}} = \frac{3}{4} F_{\text{weak coupling}} \,.$$

This seems roughly in line with lattice simulations of QCD, for example in Karsch's hep-lat/0106019:



A computation of Policastro, Son, and Starinets (hep-th/0104066) shows that

$$\frac{\eta}{s} = \frac{1}{4\pi} \tag{3}$$

for near-extremal D3-branes. The smallness of this ratio seems to fit to elliptic flow measurements at RHIC better than other first-principles calculations.

AdS/CFT subsumes these calculations and provides a complete map (in principle) between string theory in $AdS_5 \times S^5$ (AdS) and $\mathcal{N} = 4$ super-Yang-Mills theory on $\mathbf{R}^{3,1}$ (CFT).

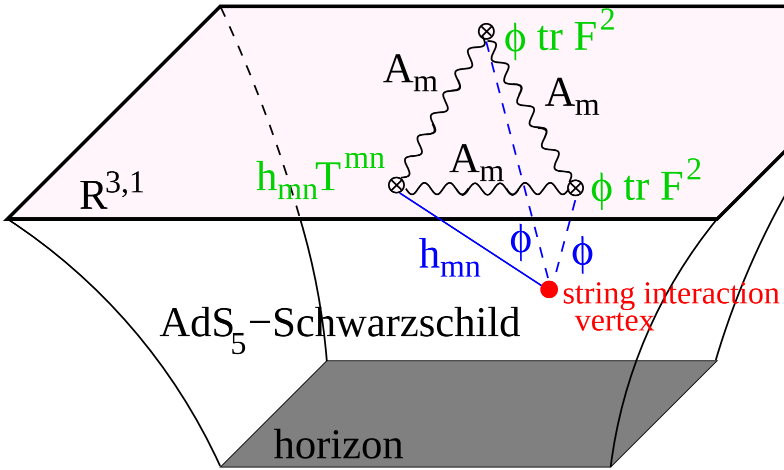
The basic statement:

$$\begin{aligned} I_{\text{string theory}} \equiv \frac{1}{2\kappa_5^2} \int_{AdS_5} d^5x \sqrt{-G} \left[R + \frac{12}{L^2} - \frac{1}{2}(\partial\phi)^2 + \ldots \right] \\ - \frac{1}{2\pi\alpha'} \int_\Sigma d^2\sigma \sqrt{-g}\, e^{\phi/2} \end{aligned} \tag{4}$$

$$W_{\mathcal{N}=4} \equiv \left\langle \int_{\mathbf{R}^{3,1}} d^4x \left[\phi\big|_{\text{bdy}} \operatorname{tr} F^2 + h_{mn}\big|_{\text{bdy}} T^{mn} \right] \right\rangle_{\text{connected}}$$

$$\boxed{W_{\mathcal{N}=4}\left[\phi\big|_{\text{bdy}},\, h_{mn}\big|_{\text{bdy}} \right] = -I_{\text{string theory}}\,[\text{on shell}]} \tag{5}$$

- You prescribe boundary values for string theory fields like the dilaton ϕ.
- Then you solve classical equations of string theory subject to these boundary conditions.
- Then evaluate $I_{\text{string theory}}$ on shell. (Pay due heed to total derivative terms).
- The result is a connected correlator of the gauge theory!
- Corrections arise in inverse powers of $g_{YM}^2 N = L^4/\alpha'^2$ and $N = 2\pi L^{3/2}/\kappa_5$ due to α' corrections and quantum effects in string theory.
- Here's how to calculate $\langle T^{mn}(x_1)\,\mathrm{tr}\,F^2(x_2)\,\mathrm{tr}\,F^2(x_3)\rangle$:



3. String theorists have not yet solved QCD!

$\mathcal{N}=4$ super-Yang-Mills misses several essential features of QCD:

- No confinement. A CFT's coupling doesn't run. But you can dial $g_{YM}^2 N$ and N.
- No chiral condensate. No chiral fermions! Instead, there's a large global symmetry, $SO(6)$.
- All fundamental matter fields are in adjoint representation: A_μ, four Majorana fermions λ_i, six real scalars X_I. NO FUNDAMENTAL QUARKS!
- External quarks can be added: strings ending on $\mathbf{R}^{3,1}$.

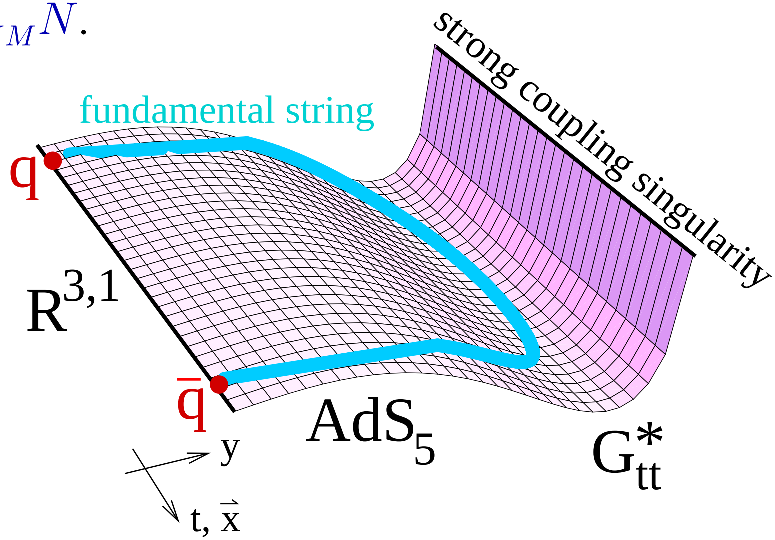
Variants on the D3-brane construction exist which exhibit confinement, but usually there are two funny things about them:

1. There's lots of extra matter near the confinement scale, often with an unbroken global symmetry. It's a bit like having a bunch of copies of the s quark.

2. The QCD string tension is much bigger than the mass gap.

$$\tau_{\text{QCD string}}/m_{\text{gap}}^2 \sim \sqrt{g_{YM}^2 N} \tag{6}$$

Roughly, this arises because $m_{\text{gap}}^2 \sim G_{tt}^*/L^2$, but $\tau_{QCD} \sim G_{tt}^*/2\pi\alpha'$, and $L^2/\alpha' = \sqrt{g_{YM}^2 N}$.

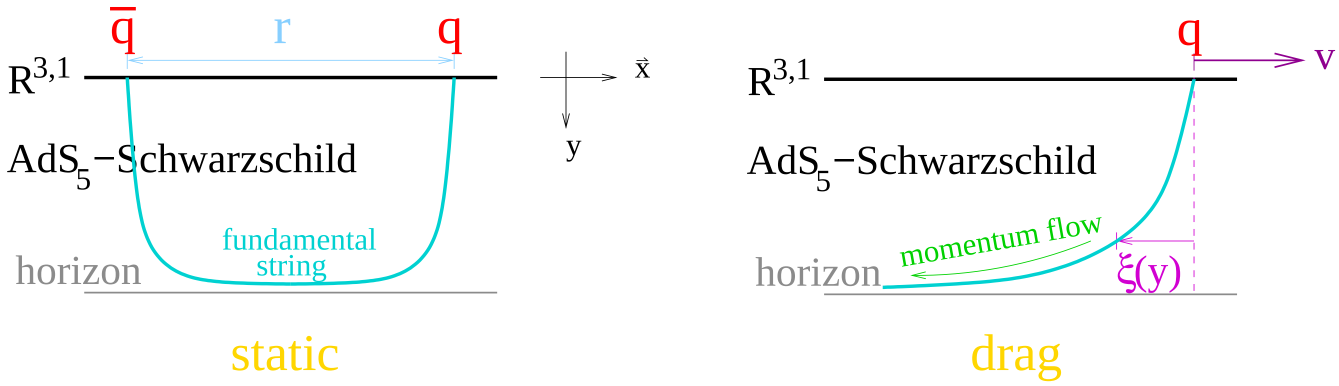


"In most circumstances, replacing QCD by $\mathcal{N} = 4$ super-Yang-Mills can be charitably described as an uncontrolled approximation." (`hep-th/0605182`)

But maybe when confinement and chiral condensate go away (e.g. at $T \sim 300\,\text{GeV}$) the "approximation" is better.

4. A drag force computation

$G_{tt} \to 0$ at the horizon of AdS-Schwarzschild, so the *static force* between quarks goes to zero as separation increases. But *drag force* on a moving quark is finite.



We need to know the shape of the trailing string and the momentum flow down it. We assume a "co-moving" ansatz:

$$x^1(t,y) = vt + \xi(y) \tag{7}$$

The AdS_5-Schwarzschild background is

$$ds^2 = \frac{L^2}{z_H^2 y^2}\left(-h dt^2 + d\vec{x}^2 + z_H^2 \frac{dy^2}{h}\right) \qquad \boxed{h \equiv 1 - y^4} \qquad z_H = \frac{1}{\pi T} \tag{8}$$

Classical string EOM's are precisely the conservation of worldsheet current of space-time energy-momentum.

$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma\, e^{\phi/2}\sqrt{-\det g_{\alpha\beta}} \qquad g_{\alpha\beta} \equiv G_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$$
$$\nabla_\alpha P^\alpha{}_\mu = 0 \qquad P^\alpha{}_\mu \equiv -\frac{1}{2\pi\alpha'} G_{\mu\nu}\partial^\alpha X^\nu \tag{9}$$

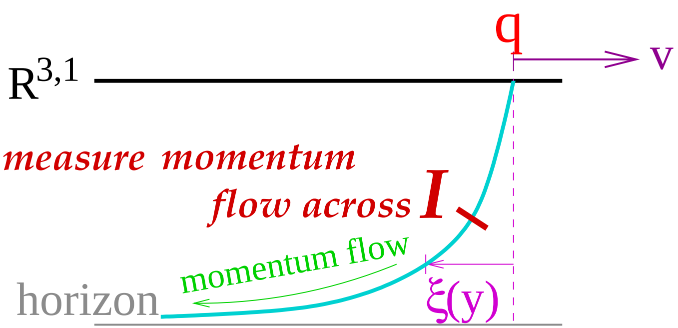
Differential equation for ξ follows from a "reduced" lagrangian:

$$\mathcal{L} = -\frac{1}{y^2}\sqrt{1-\frac{v^2}{h}+\frac{h}{z_H^2}\xi'^2}$$
$$\frac{d}{dy}\frac{\partial\mathcal{L}}{\partial\xi'} = 0 \qquad \Longrightarrow \qquad \xi = -\frac{z_H v}{4i}\left(\log\frac{1-iy}{1+iy} + i\log\frac{1+y}{1-y}\right) \tag{10}$$

Momentum "drains down the string:"

$$\Delta P_1 = -\int_{\mathcal{I}} dt\,\sqrt{-g}\,P^y{}_{x^1} = \frac{dp_1}{dt}\Delta t\,.$$

dp_1/dt is precisely the drag force!



$$F = \frac{dp}{dt} = -\frac{\pi\sqrt{g_{YM}^2 N}}{2}\frac{v}{\sqrt{1-v^2}} = -\frac{p}{t_0} \qquad t_0 = \frac{2}{\pi\sqrt{g_{YM}^2 N}}\frac{m}{T^2} \tag{11}$$

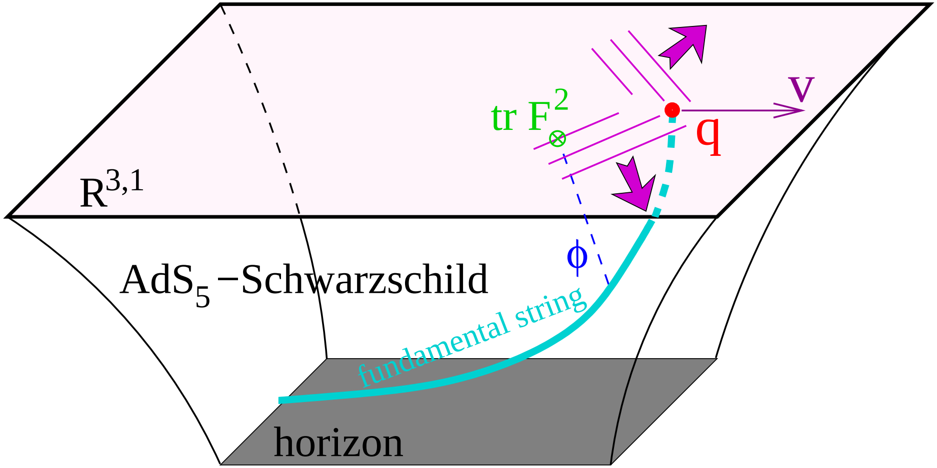
- The $\sqrt{g_{YM}^2 N}$ scaling comes from $F \propto L^2/\alpha' = \sqrt{g_{YM}^2 N}$.
- The quark is fundamentally charged, but non-dynamical, i.e. infinitely massive. So (11) is slightly formal: p and m are infinite but p/m is finite.
- Comparisons with RHIC physics should work best when the quark mass is well above the QCD scale. For example,

$$\begin{aligned} &\text{bottom:} \qquad t_0 \approx 2\,\text{fm}/c\,\frac{m/m_b}{\sqrt{g_{YM}^2 N/10}\,(T/300\,\text{MeV})^2} \\ &\text{charm:} \qquad t_0 \approx 0.6\,\text{fm}/c\,\frac{m/m_c}{\sqrt{g_{YM}^2 N/10}\,(T/300\,\text{MeV})^2} \end{aligned} \tag{12}$$

- ϵ/T^4 for $\mathcal{N}=4$ $SU(3)$ super-Yang-Mills is about twice that of QCD with 3 flavors. Should we multiply t_0 by a "fudge factor" of 2 to correct for this? At any rate, t_0 for charm is pretty small.

5. Wake generated by the moving quark

The string "casts a shadow" on the boundary, and that shadow is the wake of the moving quark.



We can learn about color singlet VEV's by further string theory computations.

- Easiest is the dilaton, which relates to $\langle\, \mathrm{tr}\, F^2 + (\text{superpartners}) \rangle$ on the boundary. Worked out in `hep-th/0605292`.
- Graviton is harder but tells us about $\langle T_{mn} \rangle$, in particular the Poynting vector. Work in progress.

The dilaton equation of motion is

$$\Box\phi = J \equiv \frac{\kappa_5^2}{2\pi\alpha'}\int_\Sigma d^2\sigma\,\frac{\sqrt{-g}}{\sqrt{-G}}\delta^5(x^\mu - X^\mu(\sigma)) \tag{13}$$

We assume a co-moving ansatz and pass to Fourier space:

$$\begin{aligned}\phi &= \phi(x^1 - vt, x^2, x^3, y)\\ &= \sqrt{1-v^2}\frac{\kappa_5^2}{2\pi\alpha' L}\int\frac{d^3K}{(2\pi)^3}e^{i\left[K_1(x^1-vt)+K_2x^2+K_3x^3\right]/z_H}\phi_K(y)\end{aligned} \tag{14}$$

where I use dimensionless wave-numbers $\vec{K}$ in place of ordinary ones $\vec{k}$:

$$\vec{K} = z_H\vec{k} = \vec{k}/\pi T \qquad K_\perp \equiv \sqrt{K_2^2+K_3^2} \tag{15}$$

(13) and (14) lead to

$$\left[y^3\partial_y\frac{h}{y^3}\partial_y - \left(1-\frac{v^2}{h}\right)K_1^2 - K_\perp^2\right]\phi_K = y\left(\frac{1-iy}{1+iy}\right)^{vK_1/4}\left(\frac{1+y}{1-y}\right)^{ivK_1/4} \tag{16}$$

A series solution near the boundary gives

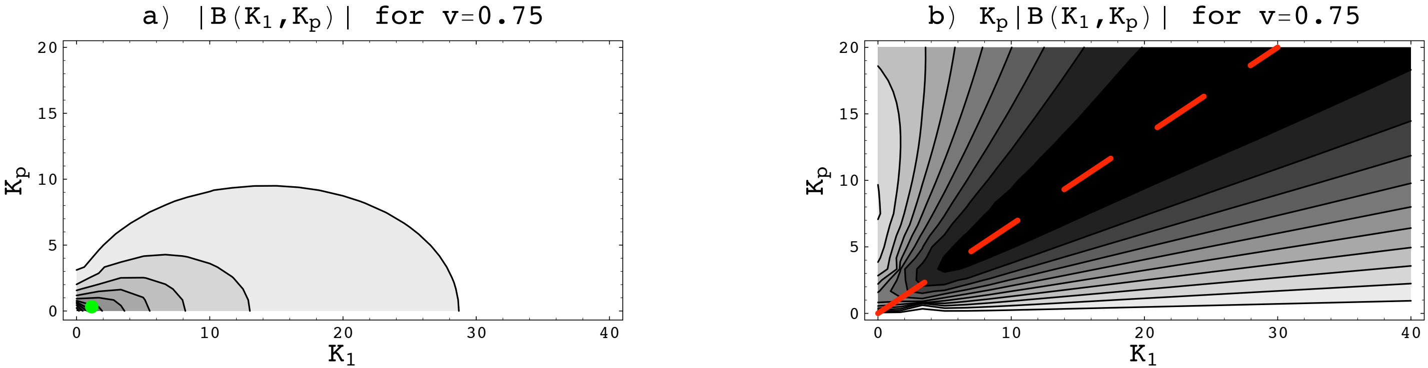
$$\phi_K = -\frac{y^3}{3}\left[1+O(y^2)\right] + A_K\left[1+O(y^2)\right] + B_K y^4\left[1+O(y^2)\right] \tag{17}$$

The $W = -I$ recipe of AdS/CFT, suitably elaborated, tells us that $A_K = 0$ and that B_K is proportional to the K-th Fourier mode of $\langle \mathrm{tr}\, F^2 \rangle$.

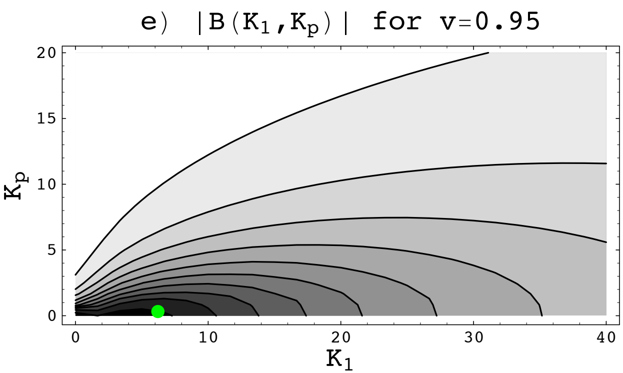
B_K is "the answer," but to get at it unambiguously we have to set one more boundary condition: the dilaton field is purely in-falling at the horizon.

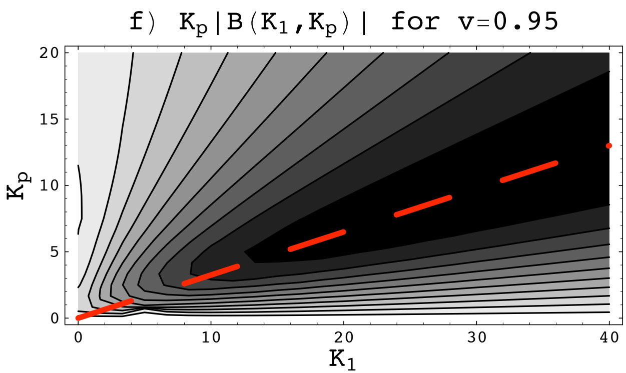
$$\phi_K = C_K^+ \underset{\textit{out-falling}}{(1-y)^{ivK_1/4}} + C_K^- \underset{\textit{in-falling}}{(1-y)^{-ivK_1/4}} + \text{subleading} \tag{18}$$

So $C_K^+ = 0$.



Same quantity in both plots, but phase space factor of $K_\perp$ enters Fourier transform to x-space, so right hand plot is more meaningful.





The striking directionality supports the idea of a wake of gluons (and superpartners). But let's think about energy scales. Say

$$T = \frac{1}{\pi}\,\mathrm{GeV} = 318\,\mathrm{MeV}\,. \tag{19}$$

Then the K_1 and $K_\perp$ axes are in GeV. The green dot is at

$$\frac{1+v^2}{1-v^2} T = 6.2\,\mathrm{GeV} \qquad \text{for } v = 0.95, \tag{20}$$

which is a typical recoil energy of a *free* massless gluon from a heavy quark.

There is a striking amount of structure at large momentum! And all this is *after* we subtracted away a near-field contribution to B_K corresponding to the Coulombic field of the quark: in the quark's rest frame,

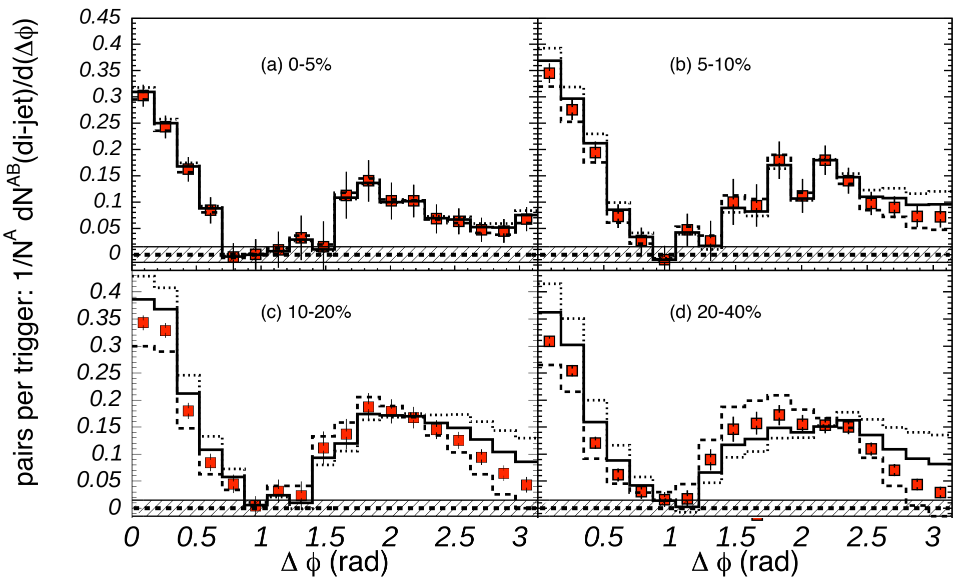
$$\left\langle \operatorname{tr} F^2 + \ldots \right\rangle_{\text{near field}} = \frac{1}{16\pi^2} \frac{\sqrt{g_{YM}^2 N}}{|\vec{x}|^4} \qquad \Longrightarrow \qquad B_K^{\text{near field}} = \frac{\pi}{16} |\vec{K}| \tag{21}$$

6. What does it all mean?

- A full description of the wake in AdS/CFT involves high-momentum modes. Directionality is far more pronounced for large $\vec{K}$ than for small $\vec{K}$.
- So maybe strongly coupled plasmas enhance fragmentation into a few energetic decay products.
- The stress tensor would give a more complete picture:

$$\langle T_{mn} \rangle = \int \frac{d^3K}{(2\pi)^3} e^{i\left[K_1(x^1 - vt) + K_2 x^2 + K_3 x^3\right]/z_H} T_{mn}^K \qquad F_{\text{drag}} \propto v \lim_{\vec{K} \to 0} K_1 T_K^{01} \tag{22}$$

  Structure of T_{mn}^K away from IR limit may help confirm wake picture and/or significance of high-energy modes.
- Energy scales of $10\text{-}30\ \text{GeV}$ are quite high for parton decay products. E.g. Di-hadron correlators probe few-GeV particles.



From `hep-ex/0507004`: trigger particle has $2.5\,\mathrm{GeV}/c < p_T < 4.0\mathrm{GeV}/c$, associated particle has $1.0\,\mathrm{GeV}/c < p_T < 2.5\mathrm{GeV}/c$. No heavy quark tagging.

- Considerations of recoil, kinematic limits, and secondary decays might all be important to relating $\mathcal{N} = 4$ calculation to RHIC—if indeed this is possible at all.

Also: hard processes in QCD see the weakening coupling that RG predicts, but there's no RG running in the $\mathcal{N} = 4$ computation.

7. Conclusions

- AdS/CFT gives us a lot of computational power over $\mathcal{N} = 4$ super-Yang-Mills at large N and large $g_{YM}^2 N$. Maybe this is useful to RHIC physicists.
- A drag force $F \sim \sqrt{g_{YM}^2 N}\, T^2$ comes out of a trailing string picture.
- The "shadow" on $\mathbf{R}^{3,1}$ of the trailing string is the QCD string stretching out and widening in a wake around and behind the quark.
- This wake involves high-momentum fields, at least for external quarks in $\mathcal{N} = 4$ super-Yang-Mills.
- More theoretical information soon, in the form of $\langle T_{mn} \rangle$. Possibly $\langle (\operatorname{tr} F^2)^2 \rangle$.
- A question for the experimentalists: Is energy dissipated through a wake of coherent low-energy fields, or are high-energy particles involved in an important way?