Light hadron spectrum in 2+1 flavor QCD with Wilson quarks

CP-PACS and JLQCD Collaborations

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Collaborators of Nf=2+1 full QCD project (CP-PACS and JLQCD)

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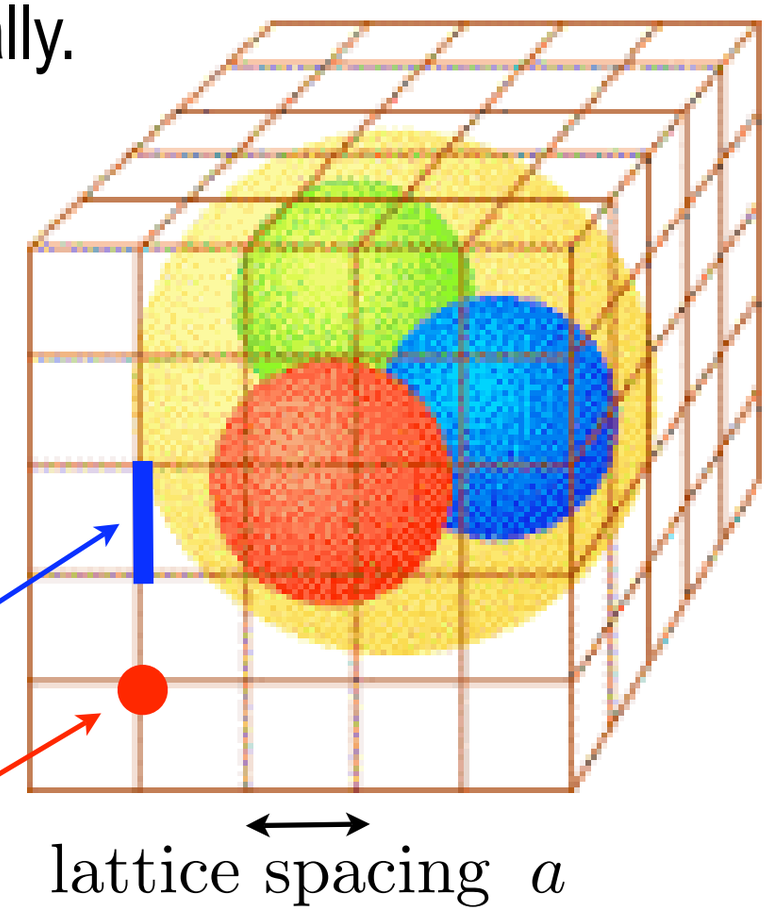
Lattice QCD

Mathematically fine definition of NP QCD

- Regularization is naturally introduced by lattice spacings.
- Finite number of d.o.f.
- Monte Carlo simulation is applicable.
- Hadronic observable can be obtained directly from “first principle” of QCD.
- Discretization error can be controlled theoretically.

$$\langle O[U_\mu, \bar{\psi}, \psi] \rangle$$
$$= \frac{1}{Z} \int \mathcal{D}U_\mu \mathcal{D}\psi \mathcal{D}\bar{\psi} O[U_\mu, \bar{\psi}, \psi]$$
$$\times \exp\left(-S_{gauge}[U_\mu] - S_{quark}[U_\mu, \bar{\psi}, \psi]\right)$$



Difficulties of the lattice QCD simulation

- large number of d.o.f.

  e.g. $\#$ of d.o.f. $\sim (32^3 \times 64) \times (8 \times 4 + 4 \times 3) \sim O(10^8)$

- Increase in numerical cost as quark mass is reduced

Systematic errors and approximation

- finite lattice spacings $\longrightarrow$ continuum extrapolation
- finite volume $\longrightarrow$ infinite volume extrapolation
- large quark mass $\longrightarrow$ chiral extrapolation
- quenched approximation (not systematic approximation)

$$\int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left(\int d^4x \bar{\psi} \not{D}[U_\mu]\psi\right) = \boxed{\det \not{D}[U_\mu]} = 1$$

  - ignore vacuum polarization of quarks (sea quark effect)
  - theoretical inconsistency

Importance of the dynamical fermion to simulate the real world

Strategies for dynamical symulations

- Wilson fermion
  - Chiral symmetry is violated at finite a .
  - Numerical cost is large at small quark masses.
- Twisted mass fermion
  - Chiral, flavor and parity symmetry is violated at finite a.
  - Numerical cost is less at small quark masses.
- Staggered fermion
  - Exact non-singlet U(1) chiral symmetry is preserved at finite a.
  - Numerical cost is very small.
  - There is theoretical ambiguity (fourth root trick, locality).
- GW fermion (Domain-Wall, Overlap)
  - Exact chiral symmetry is preserved at finite a .
  - Numerical cost is very large.

Nf=2+1 full QCD project

- **CP-PACS and JLQCD joint project**
- **O(a) improved Wilson quark formalism**
  - gauge : Iwasaki gauge action (Iwasaki,1985)
  - quark : non-perturbatively $O(a)$ improved Wilson action
    - c_{SW} is non-perturbatively determined using SF method.

(Nucl.Phys.Proc.Suppl.129(2004)444,
Phys.Rev.D73(2006)034501)

- **Dynamical up, down and strange quark**
  - No quenching effects
  - Up and down are degenerate.
    Strange has a distinct mass. (Nf=2+1)

Algorithm

HMC for up and down quarks

- $\delta\tau \longleftarrow P_{HMC} \simeq 85\%$

PHMC for strange quark

- Polynomial approximation: $D^{-1/2} \sim T_n[D], \quad P_{2n}[x] = T_n[x]T_n[x]^*$

$$\begin{aligned} \det[D] &= \det[DP_{2n}[D]]/\det[T_n[D]]^2 \\ &= W[D]\int D\phi D\phi^\dagger \exp[-|T_n[D]\phi|^2] \end{aligned}$$

  - Chebyshev polynomial for $P_{2n}[x]$
  - noise Metropolis test for the correction factor $W[D] = \det[DP_{2n}[D]]$

- $n = N_{poly} \longleftarrow P_{GMP} \simeq 90\%$

◆ $\delta\tau, \;\; N_{poly}$

$$\delta\tau, \;\; N_{poly} \;\; \longleftarrow P_{HMC} \simeq 85\%, \;\; P_{GMP} \simeq 90\%$$

κ_{ud}	κ_s	$d\tau$	N_{poly}	κ_{ud}	κ_s	$d\tau$	N_{poly}
0.13655		1/80	80	0.13655		1/90	110
0.13710		1/85	80	0.13710		1/100	110
0.13760	0.13710	1/100	100	0.13760	0.13760	1/110	120
0.13800		1/120	110	0.13800		1/120	130
0.13825		1/140	120	0.13825		1/150	150

$\beta = 1.83$

κ_{ud}	κ_s	$d\tau$	N_{poly}	κ_{ud}	κ_s	$d\tau$	N_{poly}
0.13580		1/125	110	0.13580		1/125	140
0.13610		1/125	110	0.13610		1/125	140
0.13640	0.13580	1/140	110	0.13640	0.13640	1/140	140
0.13680		1/160	110	0.13680		1/160	140
0.13700		1/180	110	0.13700		1/180	140

$\beta = 1.90$

κ_{ud}	κ_s	$d\tau$	N_{poly}	κ_{ud}	κ_s	$d\tau$	N_{poly}
0.13470		1/175	200	0.13470		1/175	250
0.13510		1/195	200	0.13510		1/195	250
0.13540	0.13510	1/225	200	0.13540	0.13540	1/225	250
0.13550		1/235	200	0.13550		1/235	250
0.13560		1/250	200	0.13560		1/250	250

$\beta = 2.05$

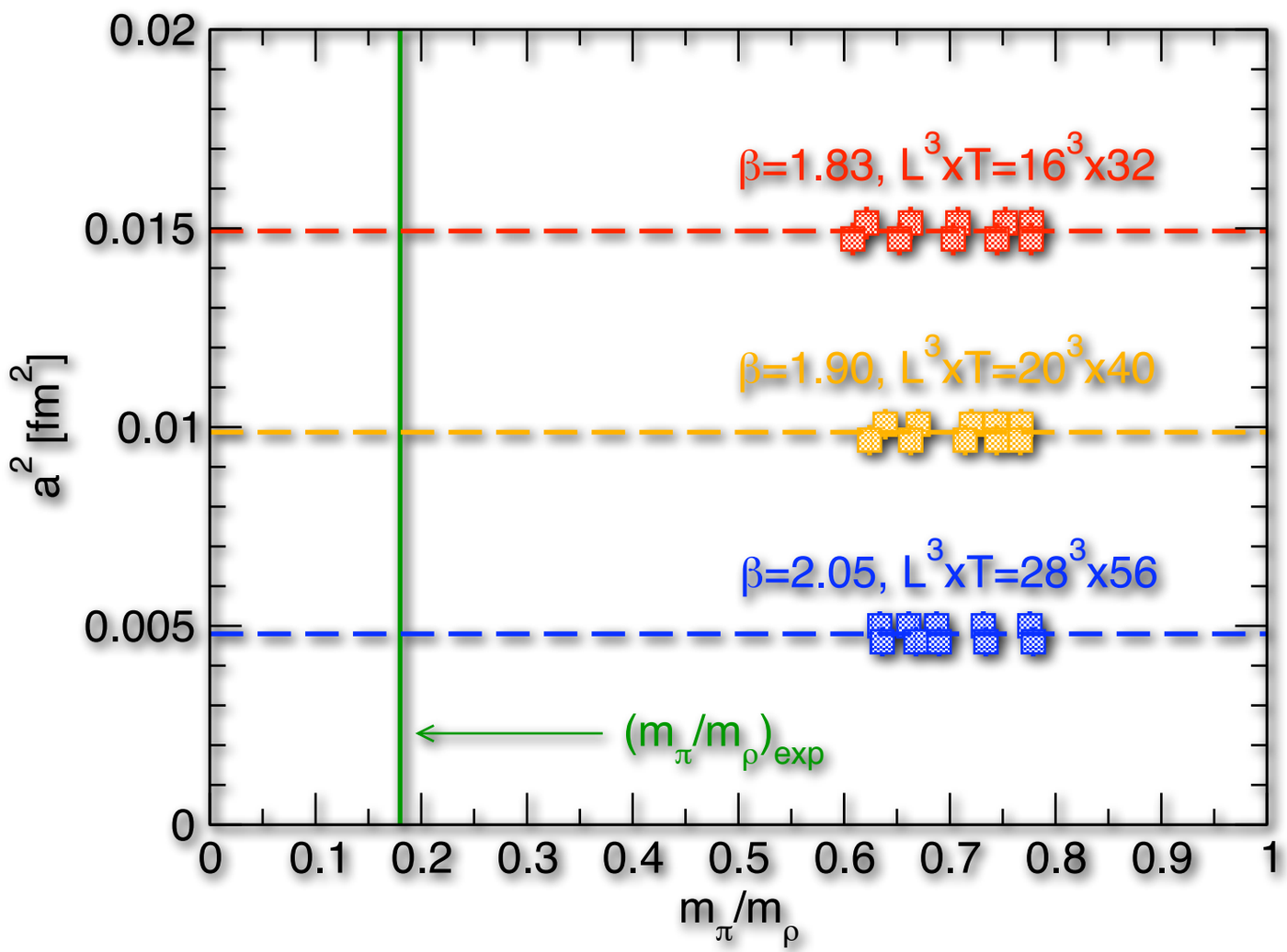
Simulation parameters

- **Lattice spacing, size, etc.**

$\beta = 2.05$	$\beta = 1.90$	$\beta = 1.83$
$(a \sim 0.070 \text{ fm})$	$(a \sim 0.100 \text{ fm})$	$(a \sim 0.122 \text{ fm})$
$c_{SW} = 1.628$	$c_{SW} = 1.715$	$c_{SW} = 1.761$
$L^3 \times T = 28^3 \times 56$	$L^3 \times T = 20^3 \times 40$	$L^3 \times T = 16^3 \times 32$
HMC traj.	HMC traj.	HMC traj.
$= 6000 - 6500$	$= 5000 - 9000$	$= 7000 - 8600$

fixed physical volume $\sim (2 \text{ fm})^3$

Quark mass parameters



5 ud quark parameters

$m_{PS}/m_V \sim 0.60 - 0.78$

(0.18 : experiment)

2 s quark parameters

$m_{PS}/m_V \sim 0.7$

(0.68 : 1-loop ChPT)

Gauge configuration generation

Earth Simulator @JAMSTEC

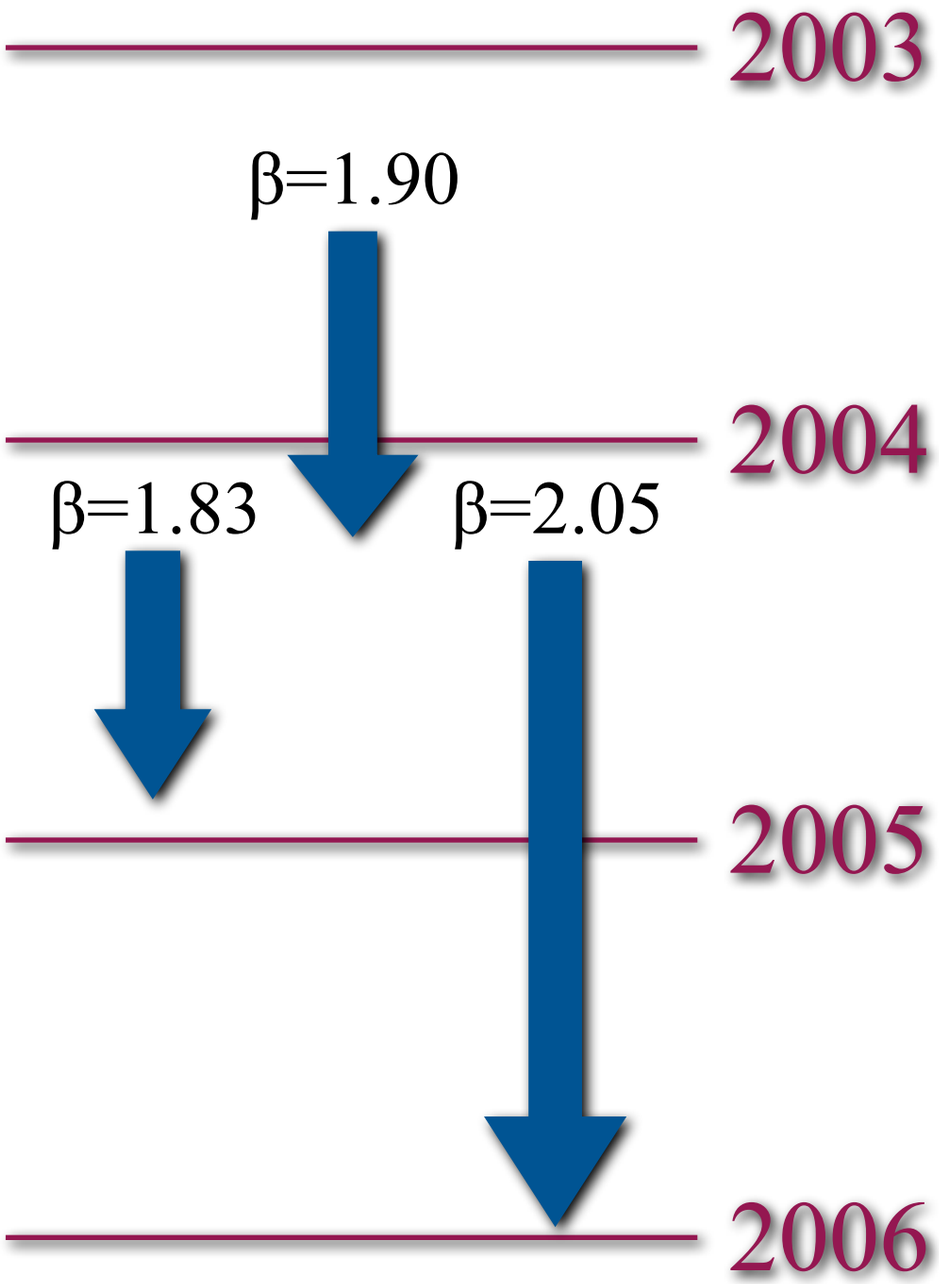
SR8000/F1 @KEK

CP-PACS @CCS, Univ. of Tsukuba

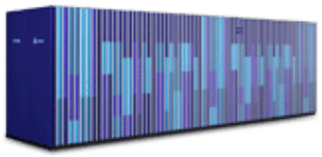
SR8000/G1 @CCS, Univ. of Tsukuba

VPP5000 @ACCC, Univ. of Tsukuba

Production runs



Computing facilities for Nf=2+1



machines	GF/node	total		for Nf=2+1 QCD		use
		# of node	GFlops	# of node	GFlops	
SR11000/J1 @ISSP, Univ. of Tokyo	60.8	88	5350	~2	~122	measurement
SR8000/G1 @CCS, Univ. of Tsukuba	14.4	12	173	12	173	production (β=1.83, 1.90), measurement
VPP5000 @ACCC, Univ. of Tsukuba	9.6	80	768	~24	~230	production (β=1.83, 1.90)
CP-PACS @CCS, Univ. of Tsukuba	0.3	2048	614	2048	614	production (β=1.83, 1.90)
SR8000/F1 @KEK	12	100	1200	~64	~768	production (β=1.90, 2.05), measurement
Earth Simulator @JAMSTEC	64	640	40960	~28	~1792	production (β=1.90, 2.05)

total ~3.7 TFlops for Nf=2+1

Measurement of the meson masses

Procedure

- We perform the measurements at only unitary points.

$$m_{valence} = m_{sea}$$

- Physical volume $\sim (2\ \mathrm{fm})^3$ is not large to calculate baryon masses.

  $\longrightarrow$ We mainly focus on the meson sector.

- FSE is comparable to or slightly larger than statistical error of the meson spectrum at physical points.

  $\longrightarrow$ It does not change conclusions.

- Measurements are performed at every 10 HMC trajectories.
- 2 source points
- Smearing: Doubly (exponentialy) smeared source - point sink

Statistics for the measurement

β=1.83

κ_{ud}	κ_s	traj.	m_π/m_ρ	m_{η_s}/m_ϕ	κ_{ud}	κ_s	traj.	m_π/m_ρ	m_{η_s}/m_ϕ
0.13655		7000	0.7772(13)	0.7521(15)	0.13655		7000	0.7769(14)	0.7235(19)
0.13710		7000	0.7524(21)	0.7524(21)	0.13710		8600	0.7447(14)	0.7128(16)
0.13760	0.13710	7000	0.7076(18)	0.7414(17)	0.13760	0.13760	8000	0.7033(18)	0.7033(18)
0.13800		8000	0.6628(22)	0.7365(16)	0.13800		8100	0.6524(23)	0.6941(20)
0.13825		8000	0.6212(24)	0.7343(15)	0.13825		8100	0.6083(32)	0.6884(21)

β=1.90

κ_{ud}	κ_s	traj.	m_π/m_ρ	m_{η_s}/m_ϕ	κ_{ud}	κ_s	traj.	m_π/m_ρ	m_{η_s}/m_ϕ
0.13580		5000	0.7673(15)	0.7673(15)	0.13580		5200	0.7667(16)	0.7210(21)
0.13610		6000	0.7435(18)	0.7647(17)	0.13610		8000	0.7443(15)	0.7182(17)
0.13640	0.13580	7600	0.7204(19)	0.7687(15)	0.13640	0.13640	9000	0.7145(16)	0.7145(16)
0.13680		8000	0.6701(27)	0.7673(17)	0.13680		9200	0.6630(21)	0.7127(17)
0.13700		8000	0.6389(21)	0.7693(15)	0.13700		8000	0.6241(28)	0.7101(20)

β=2.05

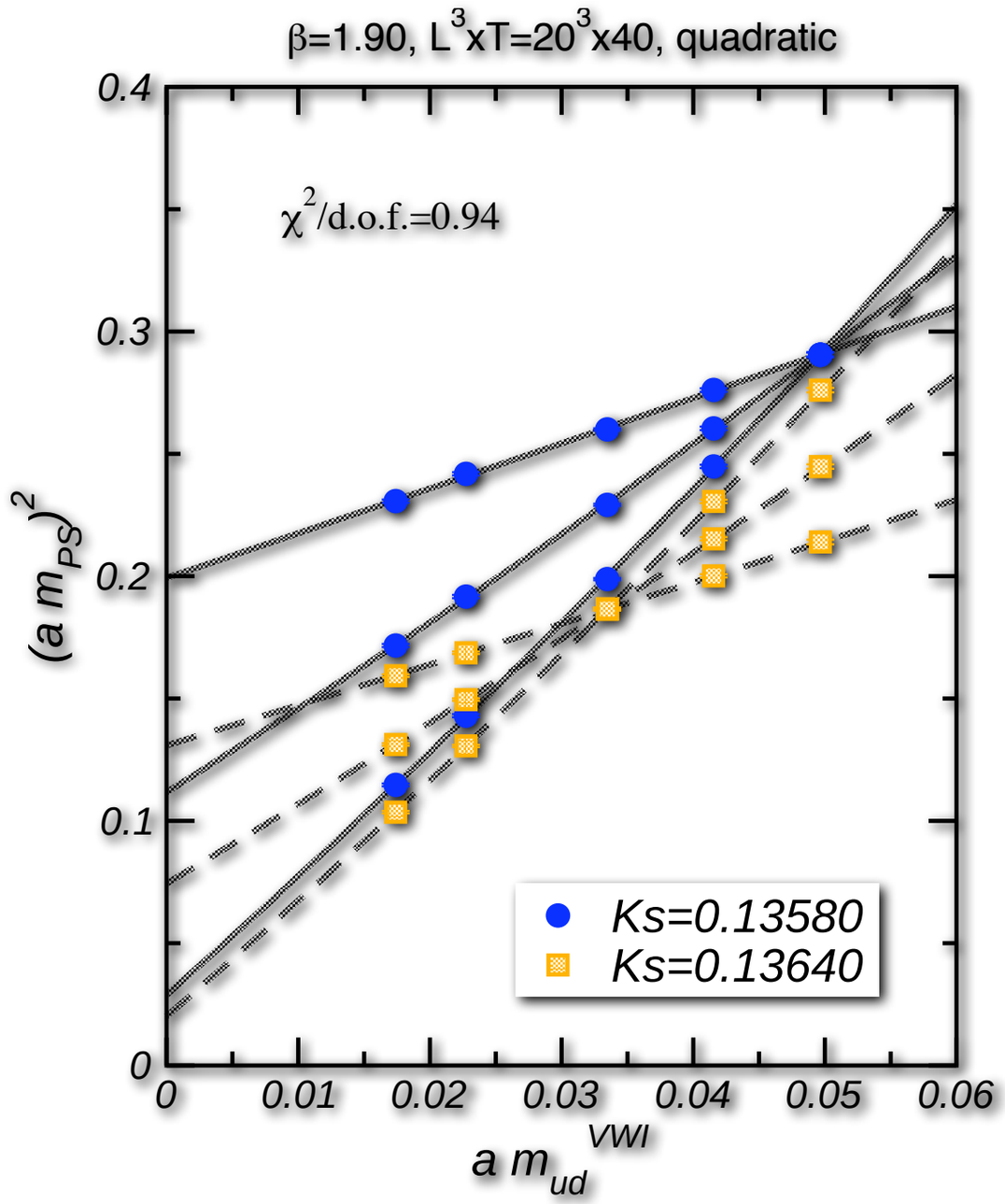
κ_{ud}	κ_s	traj.	m_π/m_ρ	m_{η_s}/m_ϕ	κ_{ud}	κ_s	traj.	m_π/m_ρ	m_{η_s}/m_ϕ
0.13470		6000	0.7757(26)	0.7274(29)	0.13470		6000	0.7790(23)	0.6821(32)
0.13510		6000	0.7316(23)	0.7316(23)	0.13510		6000	0.7341(29)	0.6820(39)
0.13540	0.13510	6000	0.6874(30)	0.7395(23)	0.13540	0.13540	6000	0.6899(34)	0.6899(34)
0.13550		6500	0.6611(34)	0.7361(25)	0.13550		6500	0.6679(45)	0.6899(43)
0.13560		6500	0.6337(38)	0.7377(28)	0.13560		6500	0.6360(47)	0.6852(46)

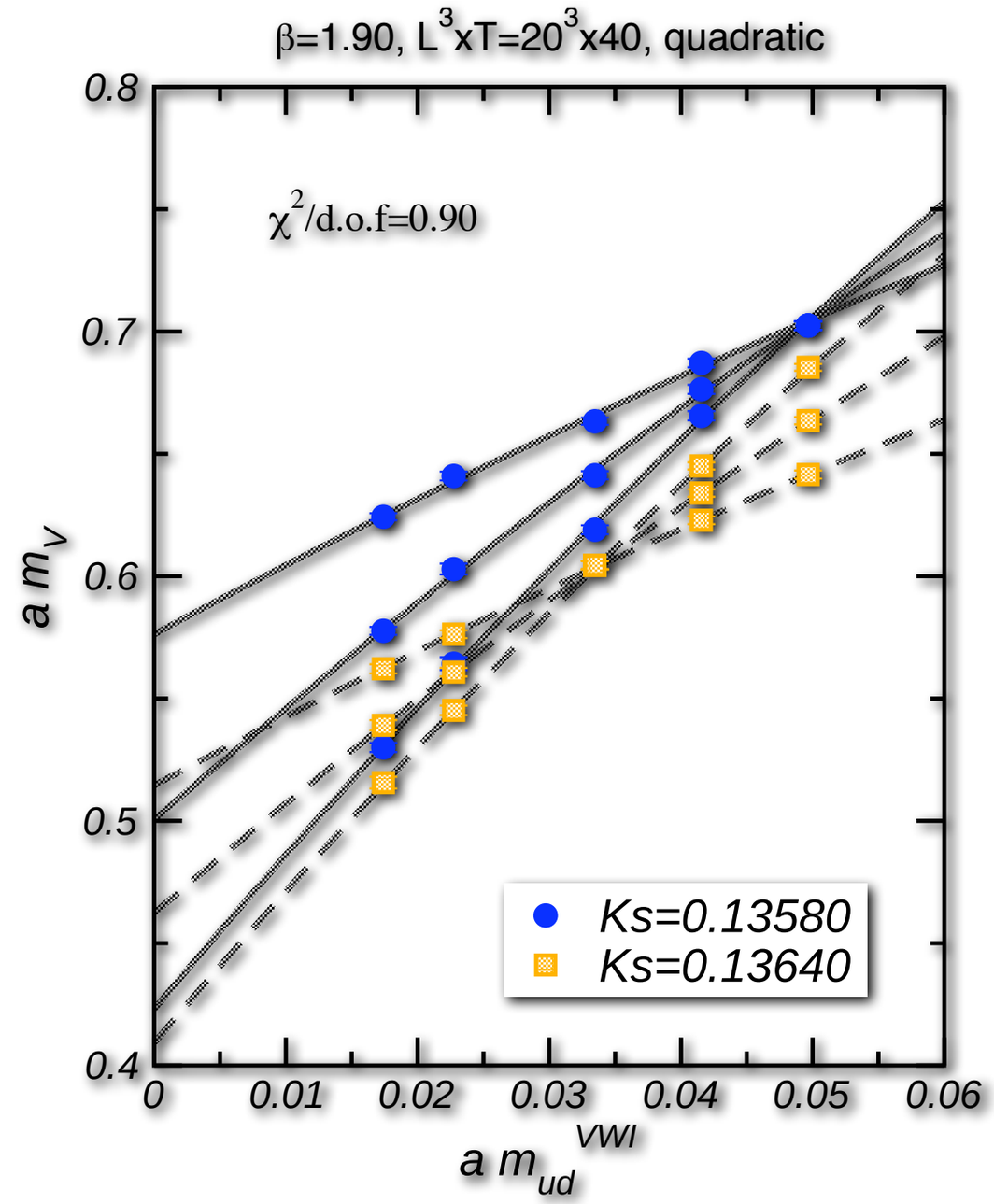
jackknife bin size = 100 HMC traj.

Chiral extrapolation

Fitting procedure

- Polynomial fit functions in quark masses are used.
  - include up to quadratic terms
  - interchanging symmetry among 3 sea quarks
    among 2 valence quarks
- Chiral fits are made to
  - light-light (LL), light-strange (LS) and strange-strange (SS) meson simultaneously.
  - ignoring correlations among LL, LS and SS
- # of parameters = 16





# of data points for each fit = 26

Fitting form from Wilson ChPT (WChPT)

- chiral log behavior $m_\pi^2 \ln m_\pi^2$
- Wilson ChPT (WChPT) (Sharpe et al. '98, Lee et al. '99)
  - include explicit chiral symmetry breaking effect of Wilson quark
  - Nf=2+1 version of WChPT has already been obtained (LO, NLO).

    (Phys.Rev.D73(2006)014511, Phys.Rev.D73(2006)095401

- for π and ρ (O(a) improved theory)

$$m_\pi^2 = \tilde{m}_\pi^2\left[1+\frac{L_\pi}{2}-\frac{L_\eta}{6}\right]-\frac{1}{f^2}(E_{xx}x^2+E_{yy}^\pi y^2+2Exy)$$

$$\boxed{+C(5L_\pi+4L_K+L_\eta)-\frac{1}{f^2}(D_xX+2D_yy)}$$

lattice artifacts

$$m_\rho = m_O+\lambda_x x+2\lambda_y y-\frac{2}{3\pi f^2}\left[(g_1^2+\frac{2}{3}g_2^2)\tilde{m}_\pi^3+2g_2^2\tilde{m}_K^3+\frac{2}{3}g_2^2\tilde{m}_\eta^3\right]$$

no lattice artifacts in vector meson masses

$$L_\psi = \frac{\tilde{m}_\psi^2}{16\pi^2 f^2}\ln\tilde{m}_\psi^2, \quad \tilde{m}_\pi^2 = 2Bm_{ud},\ \tilde{m}_K^2 = B(m_{ud}+m_s),\ \tilde{m}_\eta^2 = 2Bm_s$$

$$x = \frac{2B}{3}(2m_{ud}+m_s), \quad y = \frac{B}{3}(m_{ud}-m_s)$$

# of fit parameters = 18

Fitting is very difficult due to highly non-linear form (in progress).

Our quark mass at simulation point is not so small.

Physical point and lattice spacing

- Inputs to fix the quark masses and the lattice spacing

K-input ⟵ $m_\pi,\ m_\rho,\ m_K$

ϕ-input ⟵ $m_\pi,\ m_\rho,\ m_\phi$

to set s quark mass

- lattice spacings ⟵ m_ρ

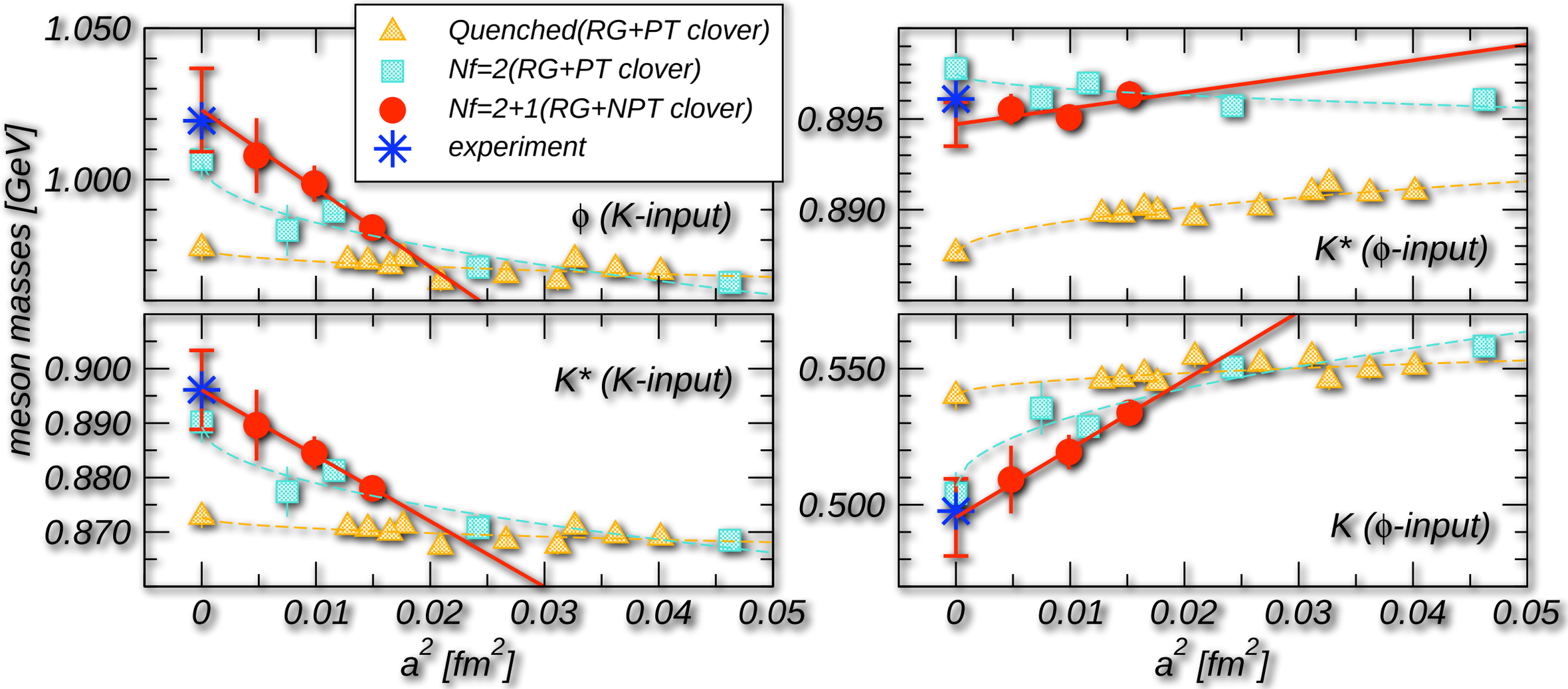
	K-input		ϕ-input	
β	a^{-1}[GeV]	a[fm]	a^{-1}[GeV]	a[fm]
1.83	1.612(22)	0.1222(17)	1.598(26)	0.1233(20)
1.90	1.983(38)	0.0993(19)	1.980(37)	0.0995(19)
2.05	2.84(11)	0.0693(26)	2.84(10)	0.0695(26)

K-input and ϕ-input give consistent results.

Light meson spectrum

Results

- In the continuum, meson spectrum is consistent with experiment.
- The statistical error in the continuum limit is large.



Scaling violation is large?

Light quark masses

Renormalization

$$m_q^{LAT}(a^{-1}) \longrightarrow m_q^{\overline{MS}}(\mu = 2\ \mathrm{GeV})$$

- tad-pole improved 1-loop matching
- 4-loop running to 2 GeV

Results

$$m_{ud}^{\overline{MS}} = 3.50(14)(^{+26}_{-15})\ \mathrm{MeV}$$

$$m_s^{\overline{MS}} = 91.8(3.9)(^{+6.8}_{-4.1})\ \mathrm{MeV}$$

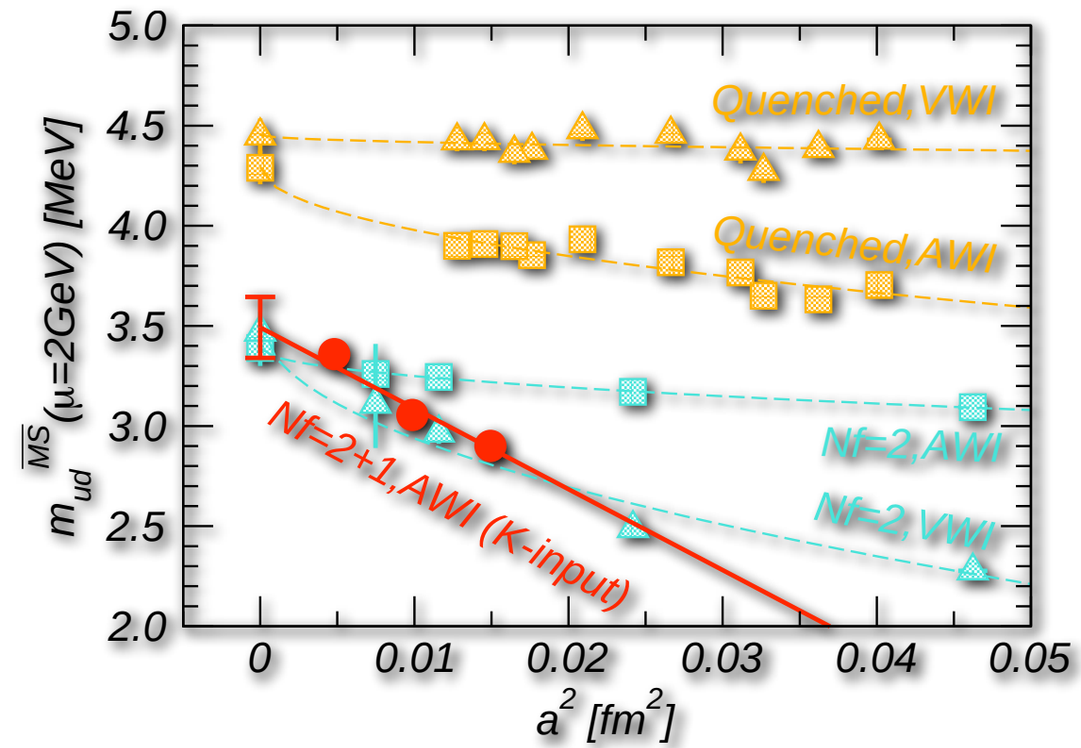
(AWI, combined K with ϕ-input)

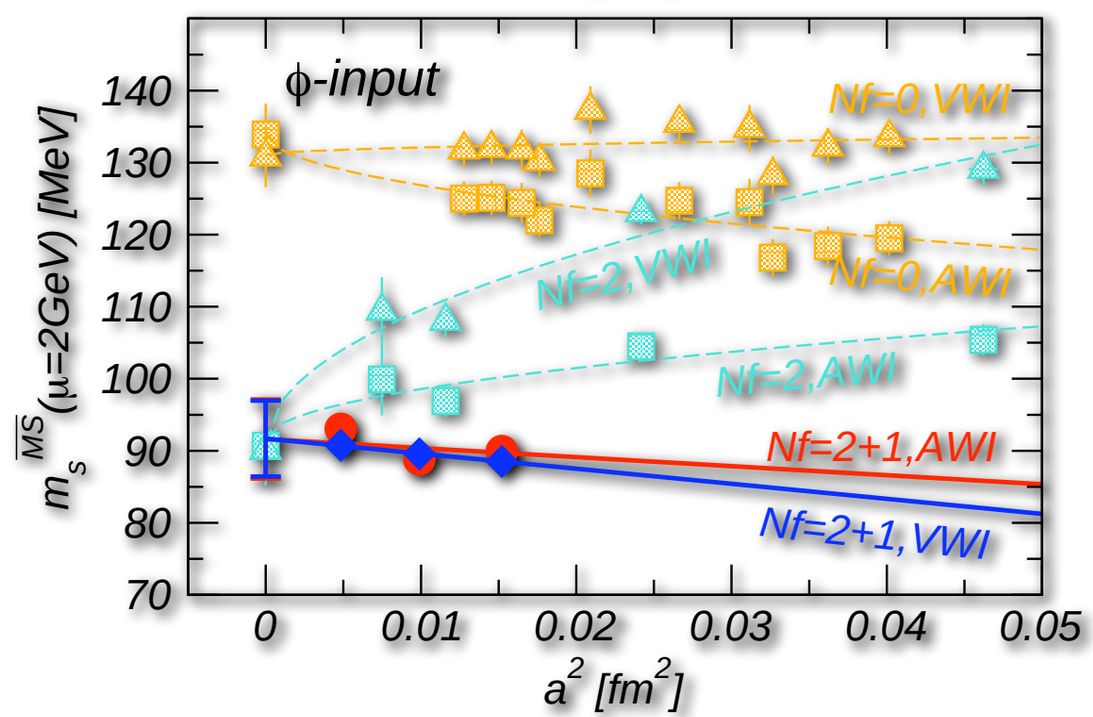
consistent with Nf=2

MILC staggered (2-loop matching)

$$m_{ud}^{\overline{MS}} = 3.2(0)(2)(2)(0)\ \mathrm{MeV}$$

$$m_s^{\overline{MS}} = 87(0)(4)(4)(0)\ \mathrm{MeV}$$





(Nf=2, 0 : RG+clover(PT))

PS meson decay constants

PS decay constant f_π, f_K

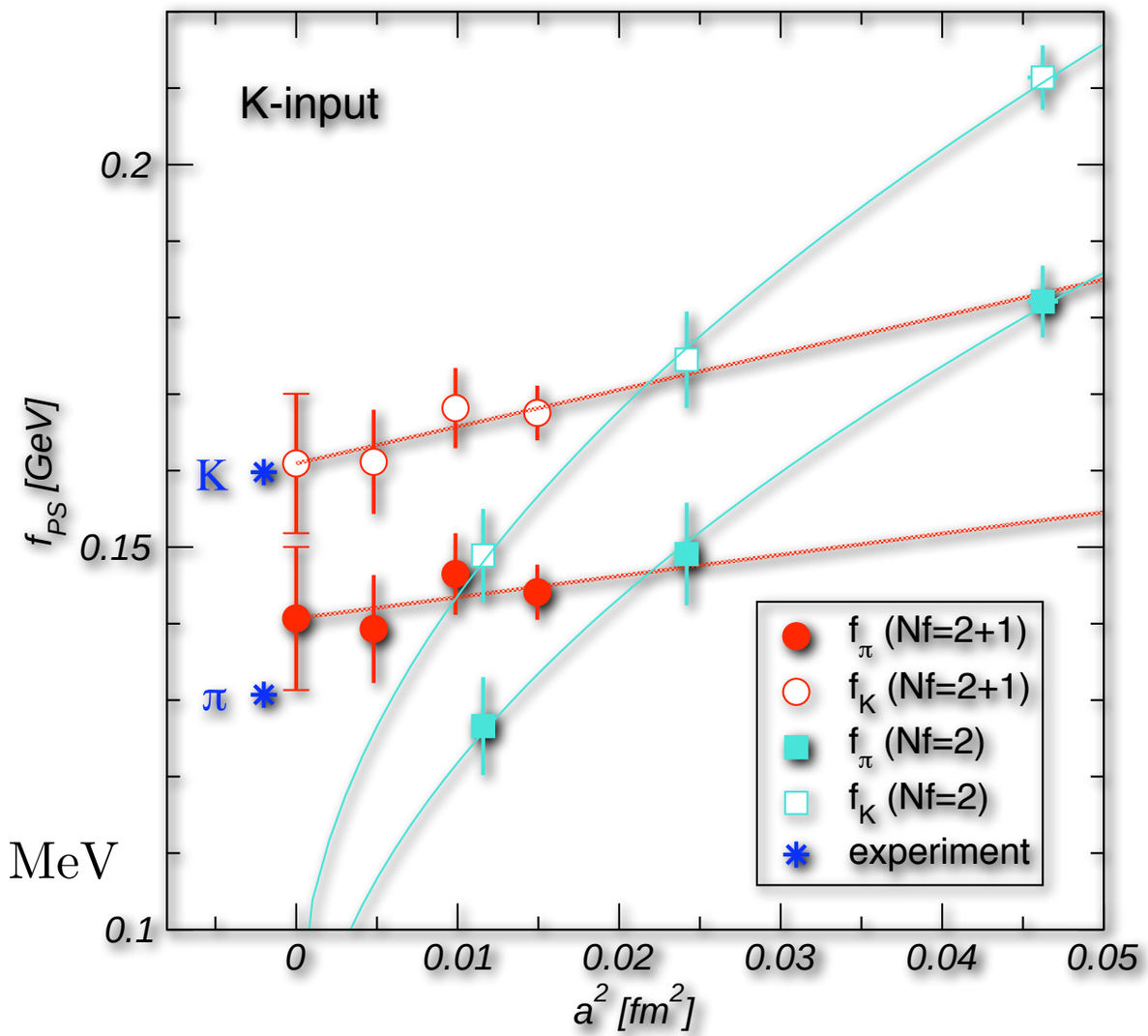
$$\langle 0|A_4|\pi\rangle = f_\pi m_\pi$$

Renormalization

- tad-pole improved 1-loop renormalization factor

Results

$$f_\pi = 140.7(9.3)\ \mathrm{MeV}$$
$$f_K = 160.9(9.1)\ \mathrm{MeV}$$
$$f_K/f_\pi = 1.142(17)$$
(K-input)



experiment

$f_\pi = 130.7(0.4)$ MeV, $f_K = 159.8(1.5)$ MeV

$f_K/f_\pi = 1.223(12)$

Summary and outlook

CP-PACS/JLQCD Nf=2+1 project

- Iwasaki gauge + NP clover (Wilson quark formalism)
- Gauge configuration generation has already been completed.

Analysis of spectrum and quark masses

- Encouraging results in the continuum limit are obtained.
- Assuming that the scaling is a^2 ,
  - Meson spectrum is consistent with experiment.
  - Quark masses in the continuum :

$$m_{ud}^{\overline{MS}} = 3.50(14)(^{+26}_{-15})\ \mathrm{MeV}$$

$$m_{s}^{\overline{MS}} = 91.8(3.9)(^{+6.8}_{-4.1})\ \mathrm{MeV}$$

Required task

- Non-perturbative determination of renormalization factor

Calculations in progress

- η' meson mass (Current status will be presented at LATTICE 2006.)
- Heavy meson quantities using a relativistic heavy quark action

(Aoki,Kuramashi,Tominaga,2001)

  - charmonium & bottomonium : hyperfine, fine and 1P-1S splitting
  - heavy-light decay constants : $f_D, \ f_{D_S}, \ f_B, \ f_{B_S}$
  - $m_c^{\overline{MS}}, \ m_b^{\overline{MS}}$
  - $B_B, \ B_{B_S}$
  - form factor in semileptonic decay

Next direction - lighter quark mass -

PACS-CS collaboration

- members : S. Aoki, T.I, N. Ishizuka, K. Kanaya, Y. Kuramashi, K. Sasaki, Y. Taniguchi, A. Ukawa and T. Yoshie @ Univ. of Tsukuba
  N. Tsutsui @ KEK
  K.-I. Ishikawa and M. Okawa @ Hiroshima Univ.
- clover quark with domain decomposed HMC (Luscher, 2004)
- We wish go down to light quark mass :

$$m_\pi/m_\rho \approx 0.4 \;\; (m_{ud}/m_s \approx 0.2) \quad \text{or less} \; \ldots$$

- PACS-CS @ CCS, Univ. of Tsukuba

  to be installed soon !! (July 1st, 2006)

  - cluster, 16x16x10=2560 nodes,
  - 14.3 Tflops peak

By this simulation we hope that the ambiguity in the chiral extrapolation will be removed !