Numerical Studies of Thermalization

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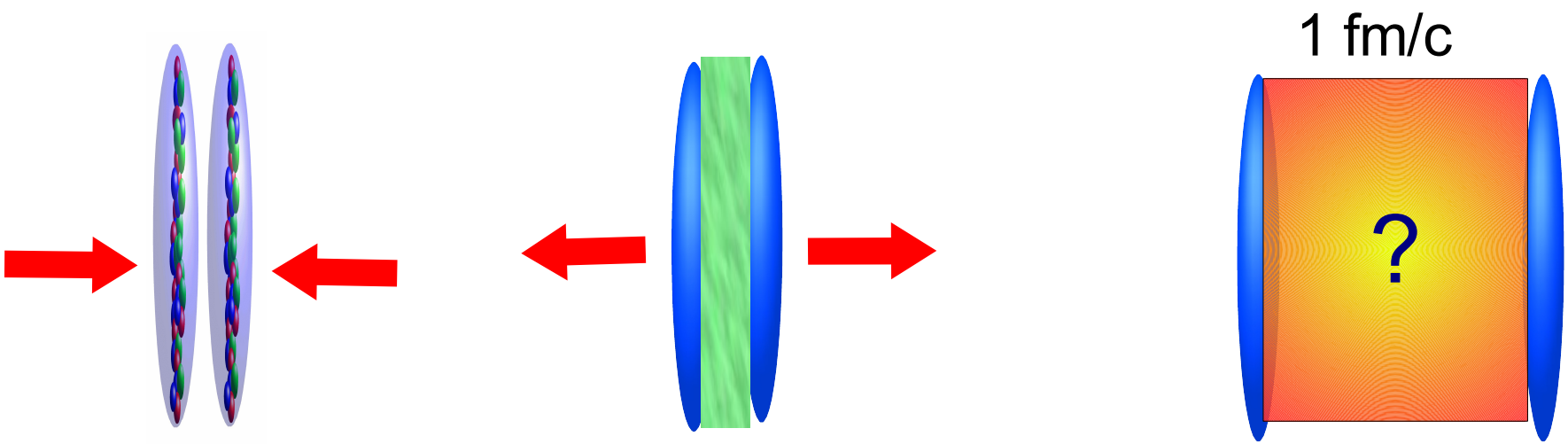
- Introduction.
- Vlasov simulation for gluons in 1D.
- Vlasov simulation for gluons in 3D.
- Summary and outlook.

Adrian Dumitru and Y. N, Phys. Lett. B621,89 (2005).
Adrian Dumitru, Y.N. , Mike Strickland, hep-ph0604149.

RHIC Physics in the Context of the Standard Model: June 18-23th 2006

Long-term Goal (motivation)

- Understanding the pre-equilibrium dynamics in high energy heavy ion collisions.
- Do we have thermal deconfined matter (QGP) at RHIC/LHC?
- If so, How? When? Which T?



Non-equilibrium dynamics within kinetic theory

Traditional plasma physics

L.M.Lifshitz and L.P.Pitaevskii, Physical kinetics

The fourth state of matter

- Plasmas consist of moving charged particles (electrons and ions).
- Plasmas have unique physical properties ,distinct from solids, liquids and gases.

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\nabla_x f + g(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})\cdot\nabla_p f = C[f]$$

$$\dot{\boldsymbol{E}} = \nabla\times\boldsymbol{B} - \boldsymbol{J}$$

$$\dot{\boldsymbol{B}} = -\nabla\times\boldsymbol{E}$$

$$\boldsymbol{J} = g\int\frac{d^3p}{(2\pi)^3}\boldsymbol{v}f$$

$f(\boldsymbol{x},\boldsymbol{p})$ One-particle distribution function

- Particle' motion is determined by the Lorentz force
- Self-consistent solution of the field
- Collision among particles
- Self-interaction of the field if Yang-Mills

Non-equilibrium dynamics in QCD

- Real time evolution of the Classical Yang-Mills (MV model)
- Vlasov approach (self-consistent mean field, no collision)
- Boltzmann approach (collision term only)

Bottom-up thermalization?

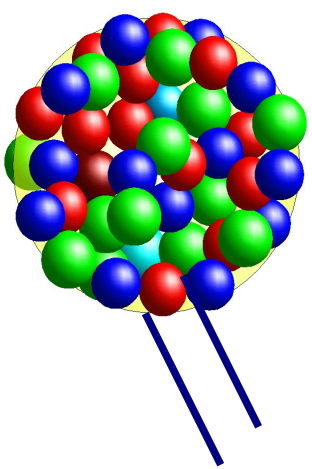
time

$\tau = 0$

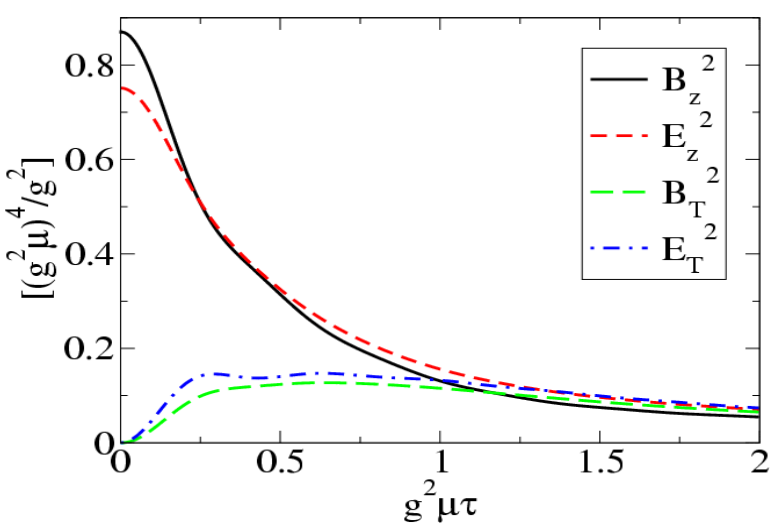
$\tau \approx 1/Q_s$

QCD Plasma instabilities?

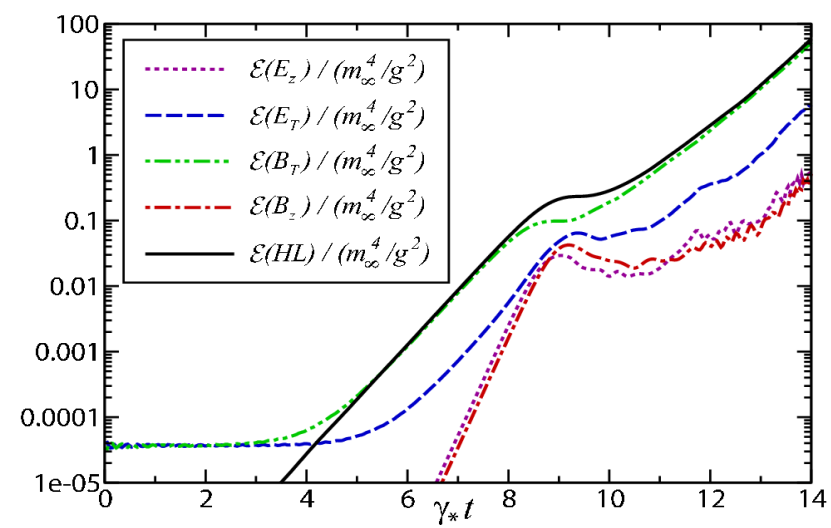
$\tau \approx few\ fm/c$



$r \sim \frac{1}{Q_s}$



T.Lappi, L. McLerran, N.P.A772



A. Rebhan, P. Romatschke and M. Strickland

Simulation of the Classical Yang-Mills field

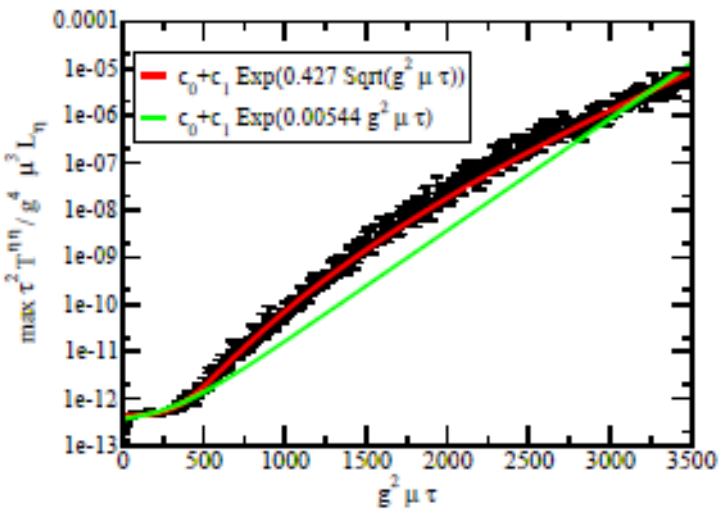
$$\dot{A}_i = -E_i \qquad \dot{E}_i = -D_j F_{ji}$$

- classical YM simulation of gluon production

  Krasnitz, Venugopalan, Y.N., Lappi

- Evolution of soft rapidity fluctuations

  (Romatschke & Venugop. 06)



- But: Dynamics of the hard modes (dominate pressure, viscosity etc) ???

Mean field only: Vlasov equation

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla_x f + g(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \cdot \nabla_p f = 0$$

$$\dot{\boldsymbol{E}} = \nabla \times \boldsymbol{B} - \boldsymbol{J} \qquad \boldsymbol{J} = g \int \frac{d^3 p}{(2\pi)^3} \boldsymbol{v} f$$

$$\dot{\boldsymbol{B}} = -\nabla \times \boldsymbol{E}$$

$f(\boldsymbol{x}, \boldsymbol{p})$ One-particle distribution function

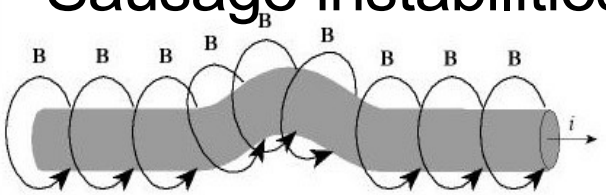
- Linearized Vlasov (hard-loop approx.):
  A.Rebhan, P.Romatschke M.Strickland,P.Arnold,G.D.Moore.
- Full solution by particle in cell simulation:
  linear regime : G.D.Moore, C.R.Hu B.Muller
  application to the instabilities: A.Dumitru, Y.N. M.Strickland.

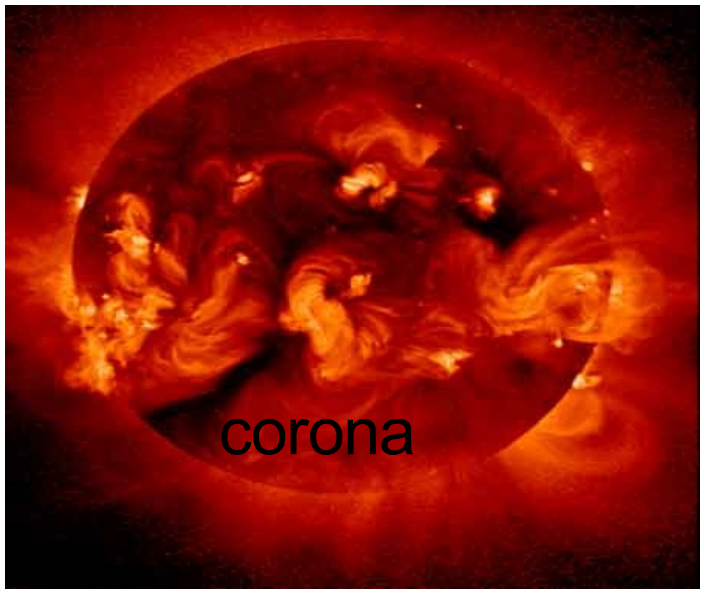
Plasma instabilities

Exponential growth of wave solutions: $A \sim e^{\omega t}$

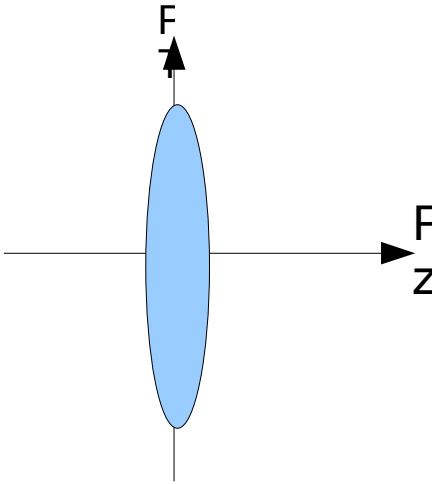
- **Macroscopic instabilities:**
  coordinate space nonuniformity

Sausage instabilities





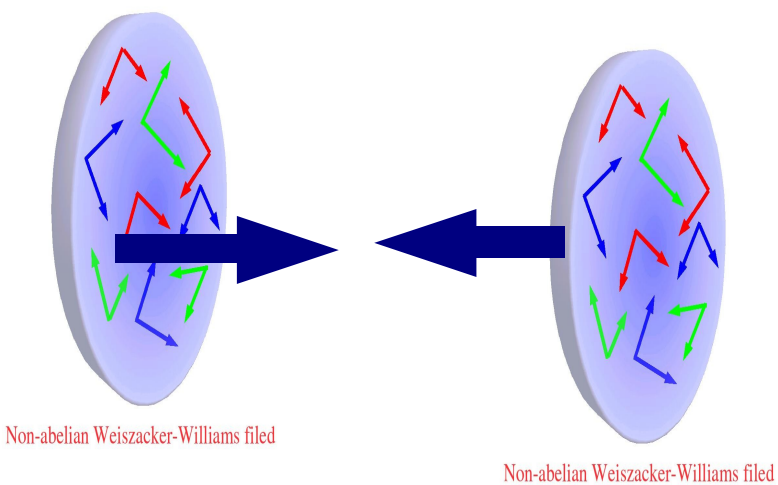
- **Microscopic instabilities:**
  momentum space anisotropy

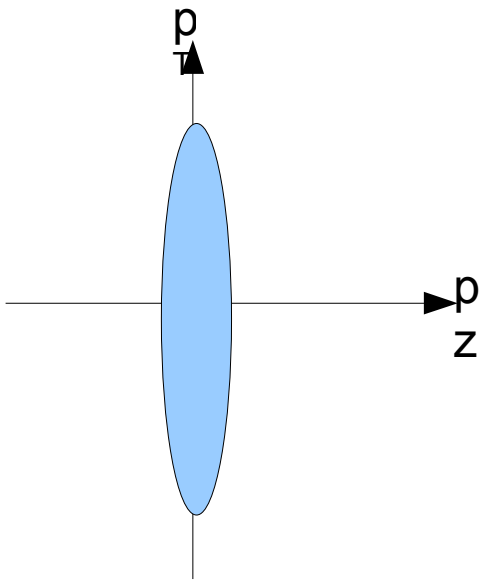


A Picture in $\alpha_s \ll 1$ limit

"Bottom-up", Phys. Lett. B502 (2001)51

- Gluons are produced at $\tau \sim 1/Qs$,
  $f \sim 1/\alpha_s$ typical momentum $p \sim Qs$
- Due to longitudinal expansion, typical longitudinal momentum of gluons becomes
  $p_z \sim 1/\tau$, $p_x \sim p_y \sim Q_s$ at $\tau >> 1/Qs$





Non-Abelian Plasma Instabilities

- In Abelian plasma: instabilities stop when the effect of the soft field on the hard particle trajectory becomes non-perturabative , i.e.

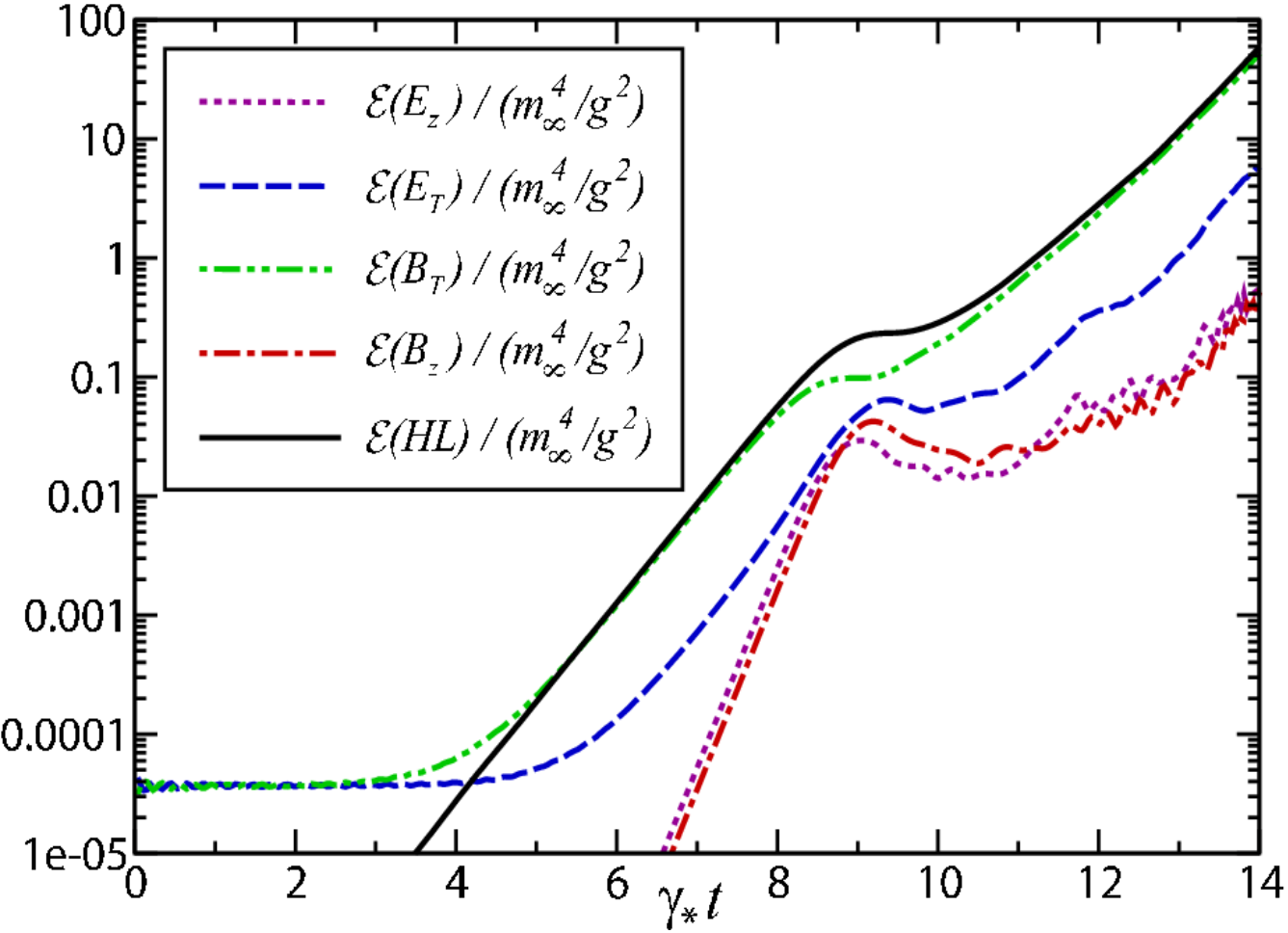
$$gA \approx p_{hard}$$

- In Non-Abelian plasma, what is the role of non-Abelian interactions between the soft modes?

$$gA \approx k_{soft}$$

1+1D SU(2) hard loop simulations

A. Rebhan, P. Romatschke and M. Strickland, Phys. Rev.Lett 94,102303(2005),hep-ph/0412016.



Linear Vlasov $f \simeq f_0 + \delta f$

First confirmation of instabilities in SU(2) gauge theory predicted by

Arnold, Lenaghan, Moore
Romatschke, Strickland

Initial Conditions:

- random fields
- squeezed particle momentum distr.

Purpose of our Study

solving full Vlasov equation for gluons,
i.e. dynamics of fields and particles.

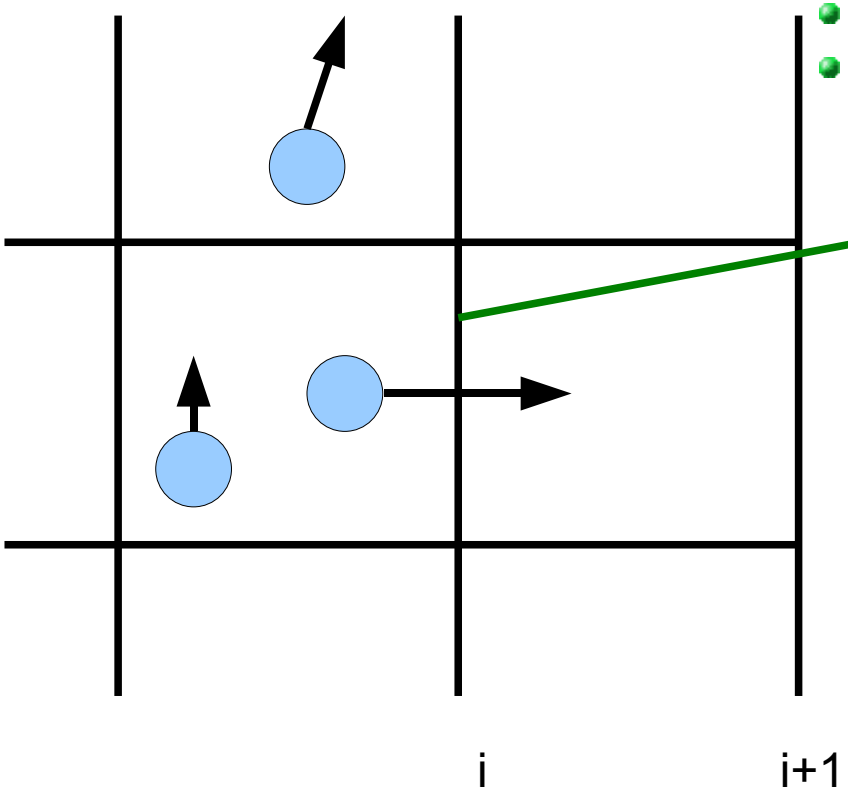
- non-linear effects: saturation of instabilities.
- study isotropization of the system (particles and field) by interactions between particles and field.
- weak and strong initial field.

Particle in cell simulation for colored particles (CPIC)

$$\frac{d\,\boldsymbol{x}_i}{dt}=\boldsymbol{v}_i,\quad \frac{d\,\boldsymbol{p}_i}{dt}=gQ_i^a(\boldsymbol{E}_i^a+v_i\times\boldsymbol{B}_i^a),\quad \frac{d\,\boldsymbol{Q}_i}{dt}=igv_i^\mu[A_\mu,Q_i].$$

$$D_\mu F^{\mu\nu}=J^\nu=g\sum Q\,v^\mu\,\delta(x-x_i(t))$$

NGP method for non-Abelian gauge theories



- Density: Count the number of particles within a cell
- A current is generated only when a particle cross a cell

$$J(i)=Q\,\delta(t-t_{cross})/N_{test}$$

- Color charge must be parallel transported:

$$Q(i+1)=U_x(i)^\dagger Q(i)U_x(i)$$

to satisfy the lattice covariant continuity equation,

$$\dot{\rho}(i)=\sum_x U_x(i-x)^\dagger J_x(i-x)U_x(i-x)-J_x(i)$$

in oder to conserve Gauss law.

C. R. Hu and B. Muller, Phys. Lett. B409 (1997)377
G. D. Moore, C.R. Hu and B. Muller, Phys. Rev. D58 (1998)045001.

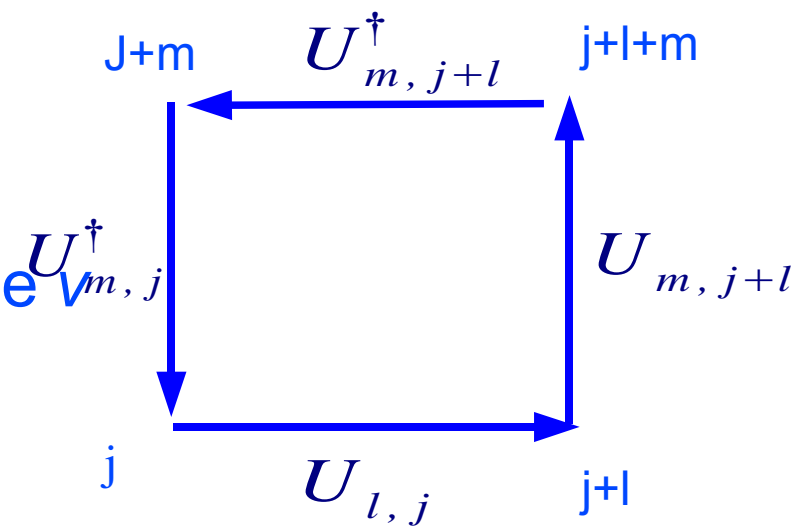
Yang-Mills Hamiltonian on the lattice

Numerical implementation of real time simulation in classical YM is well established. Kogut-Susskind Hamiltonian:

$$H_{YM} = \frac{1}{2}\sum_{l} E_l^2 + \frac{1}{2}\sum_{\square}\left(N_C - \Re\, Tr\, U_{\square}\right)$$

Link $\quad U_{j,i} = \exp[-iga\, A_j(i)]$

Plaquette $U_{\square} \equiv U_{l,j}\, U_{m,j+l}\, U^{\dagger}_{l,j+m}\, U^{\dagger}_{m,j}$



Equations of motion for dynamical variable v

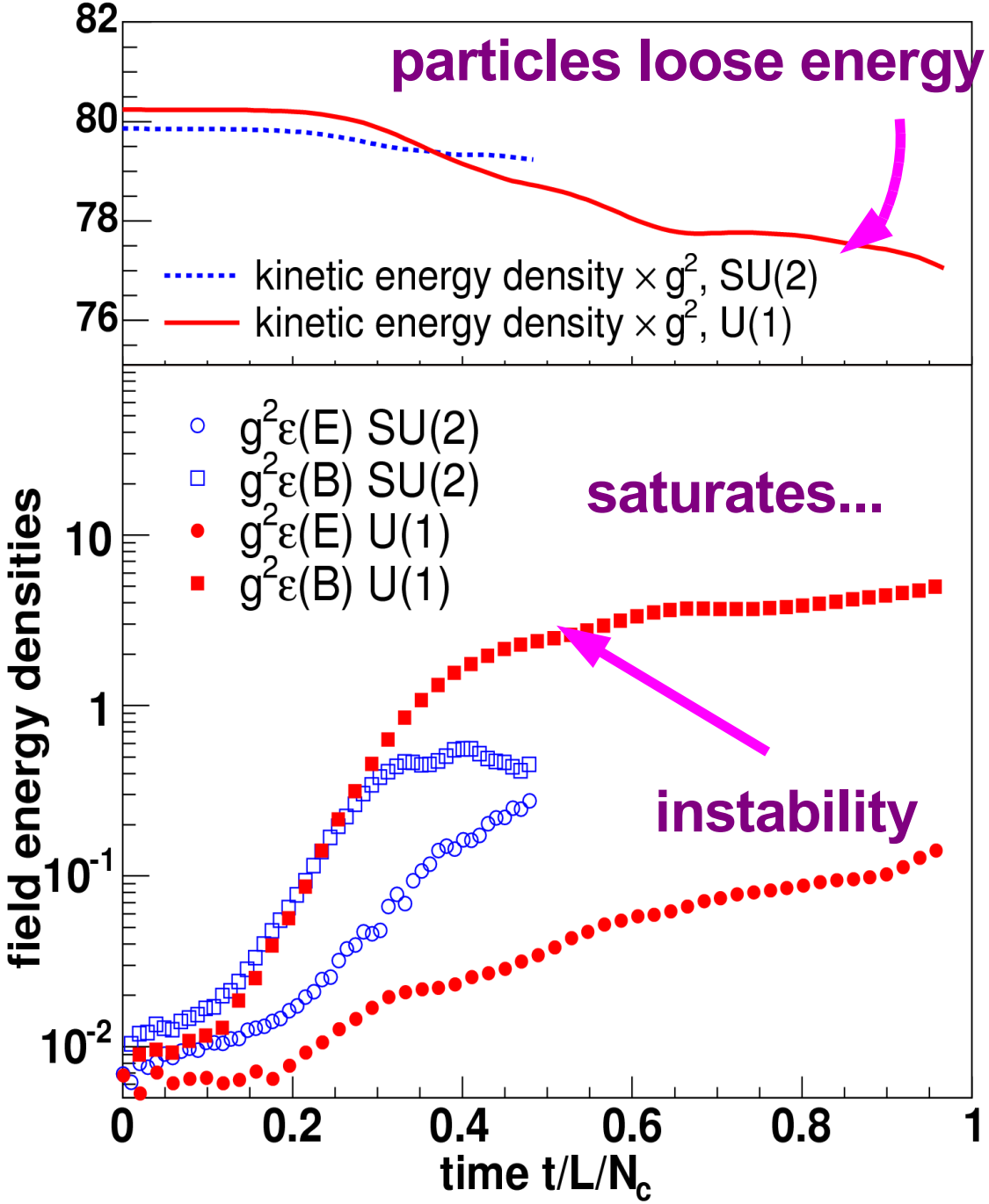
$$\frac{dv}{d\tau} = \{H_L, v\}$$

Initial conditions for simulations

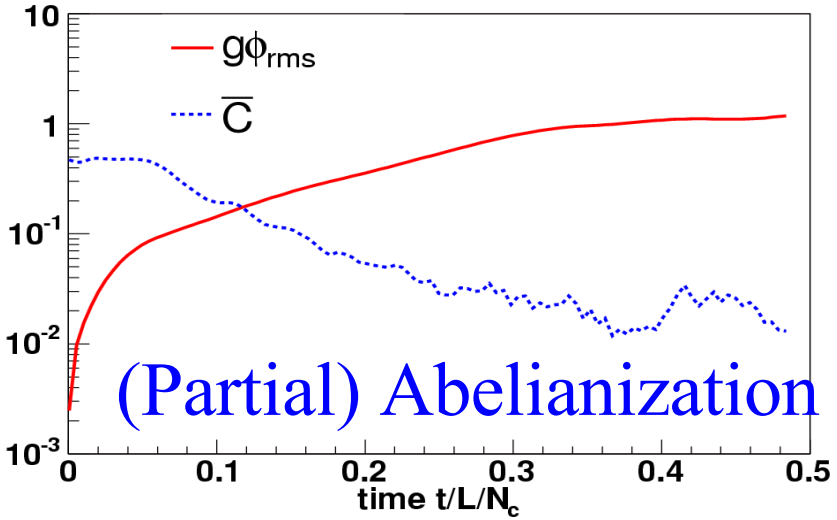
- 1D-3V simulation:
  Fields depend only on x-direction (1D), but particle motion is in 3D.
- 3D-3V simulation: full 3D.
- Initial field configuration: random white noise for the field.
- Anisotropic initial particle distribution:

$$f(\boldsymbol{p}) \sim \delta(p_z)\, \mathrm{e}^{-\sqrt{p_x^2+p_y^2}/p_{hard}}$$

Weak initial fields (1D+3V)



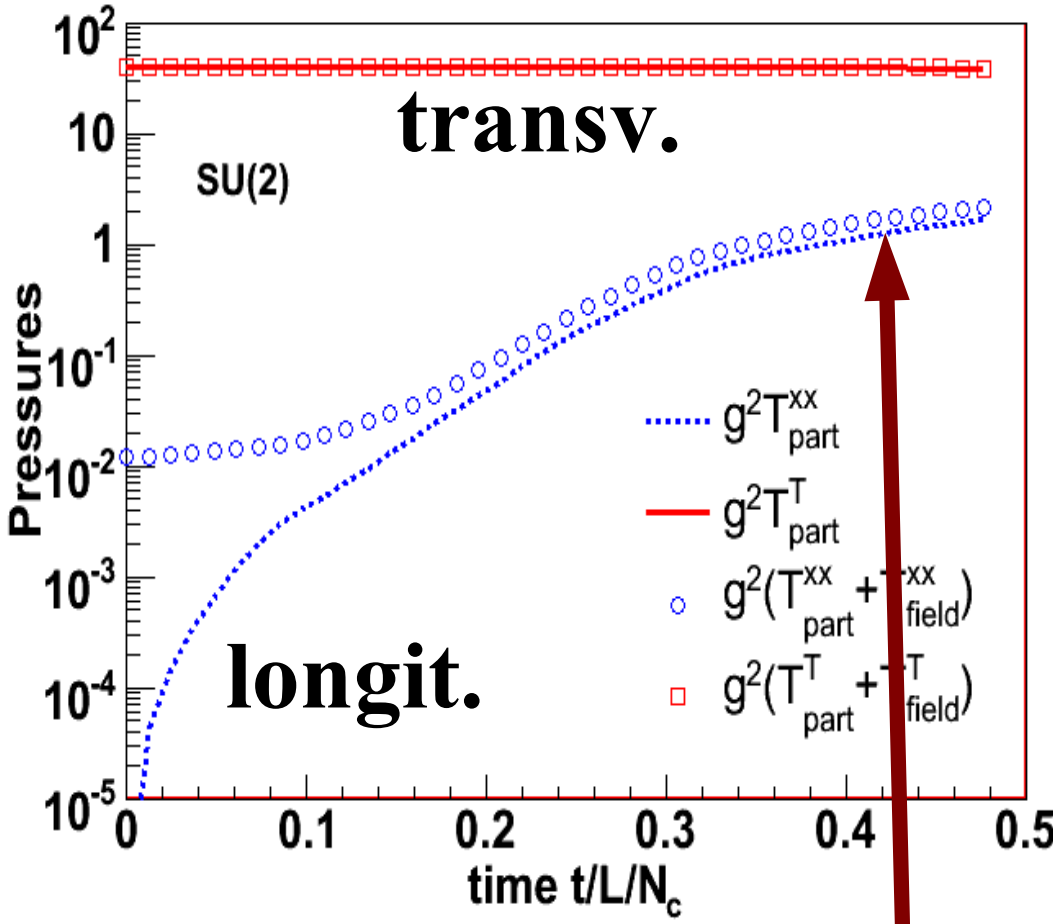
- Confirms existence of instabilities.
- Particles ARE affected
- Saturation well before fields reach particle scale.

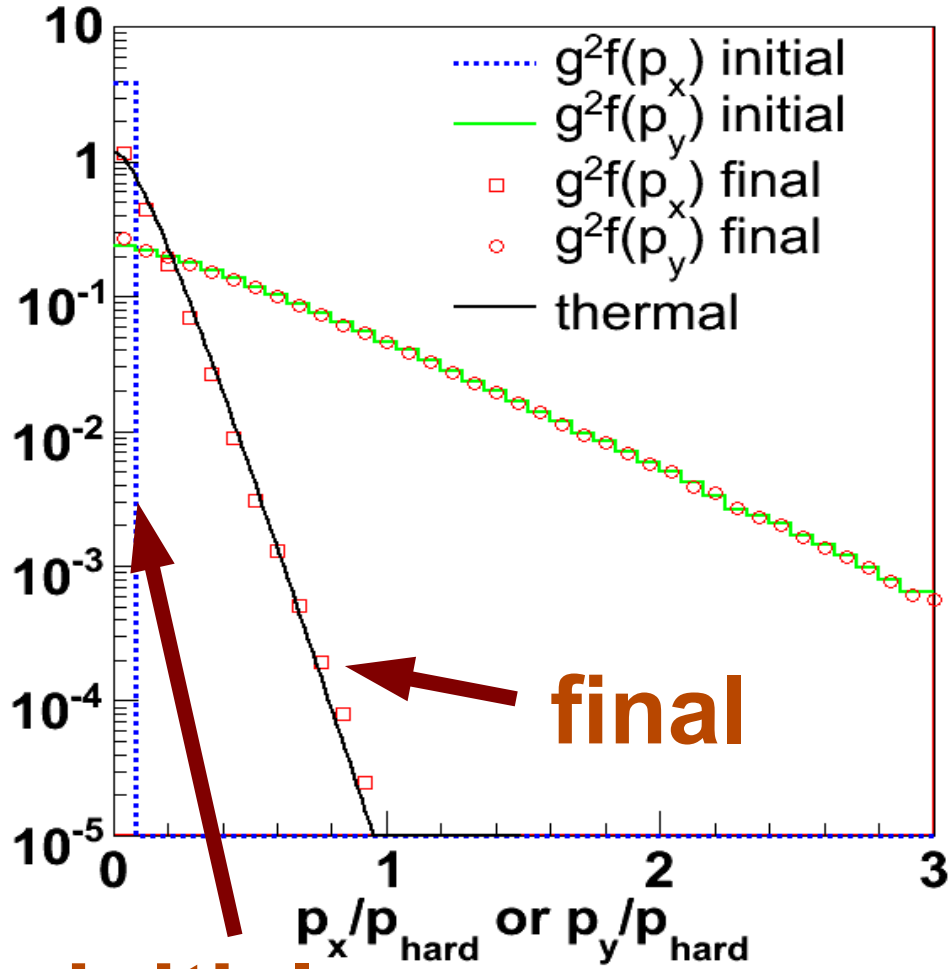


$$\overline{C} = \int_0^L \frac{dx}{L} \frac{\sqrt{Tr\ \ (i[A_y, A_z])^2}}{Tr\ A_y^2 + A_z^2}$$

$$\Phi_{rms} = \sqrt{\int_0^L \frac{dx}{L} (A_y^a A_y^a + A_z^a A_z^a)}$$

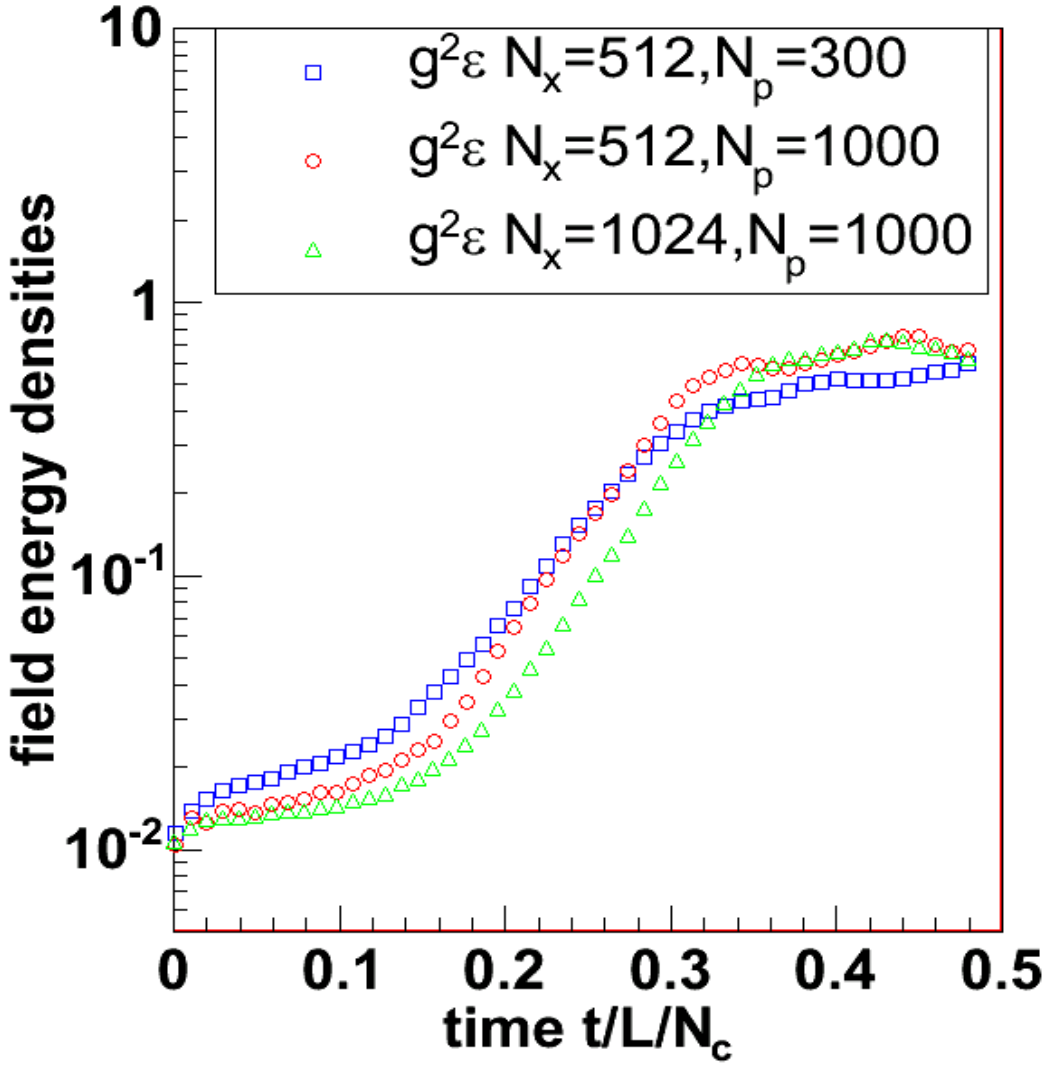
(Partial) Isotropization





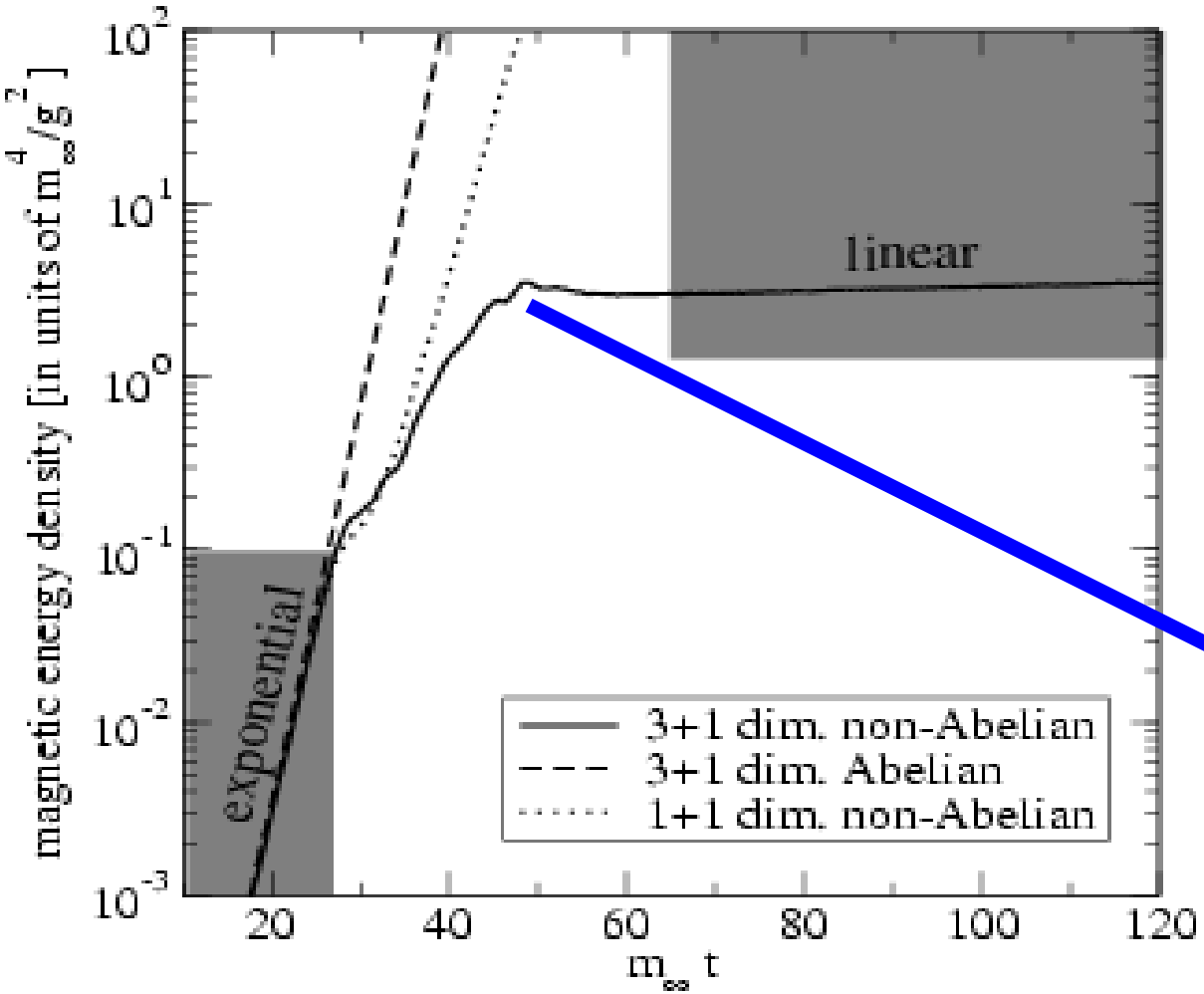
exponential growth of long. Pressure due to deflection of particles

No sensitivity to UV (in 1D !)



Hard loop 3D simulations

P.Arnold, G.D.Moore and L.G.Yaffe, Phys. Rev. D72,054003(2005), hep-ph/0505212.
A. Rebhan, P. Romatschke and M. Strickland, JHEP 09,041(2005),hep-ph/0505261.

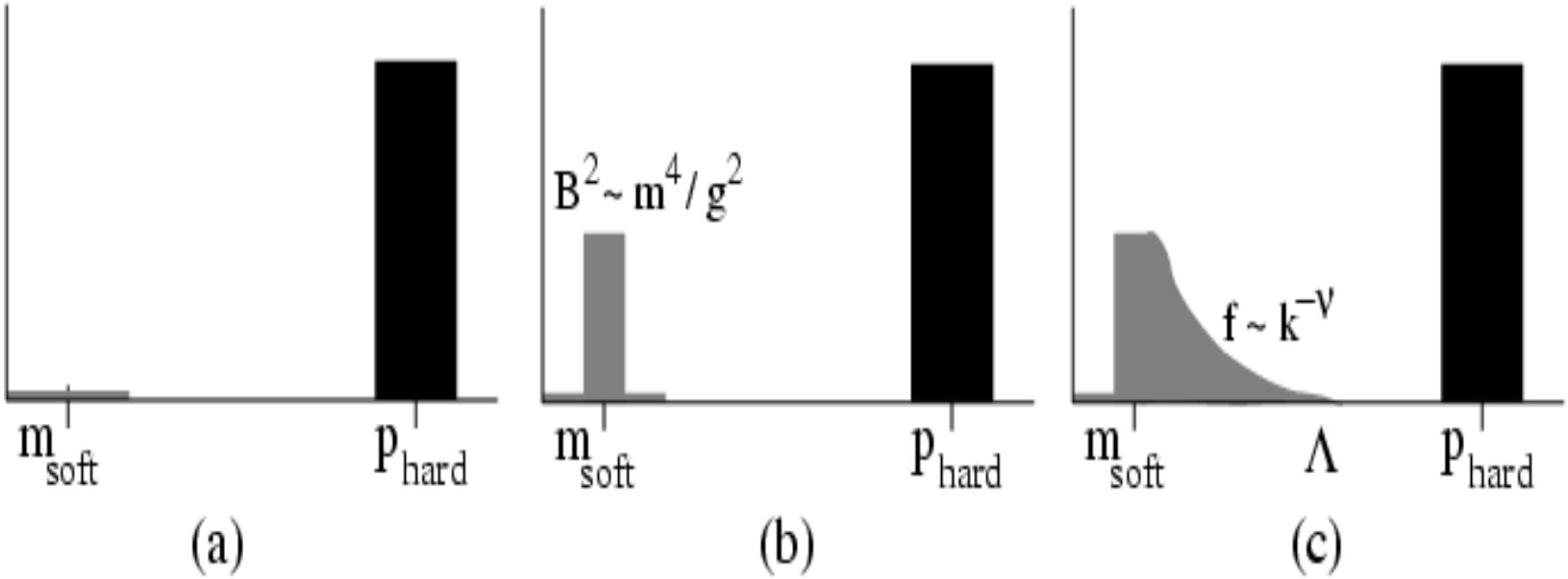


Instability grows linearly at late times in 3D, when self-interaction of the non-Abelian field becomes non-perturbative.

Abelianization not seen!

$$gA \approx k_{soft}$$

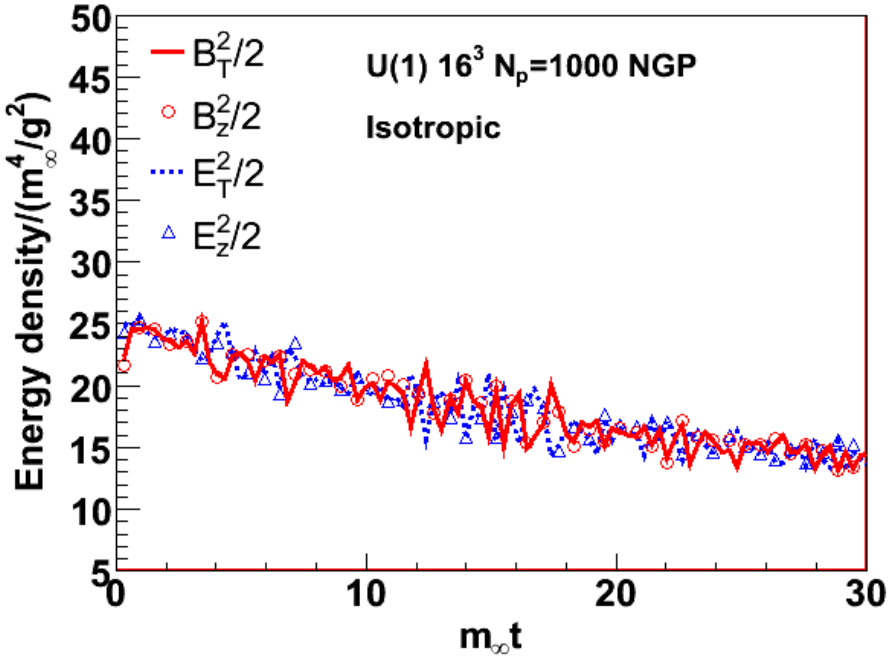
Kolmogorov Cascade of energy

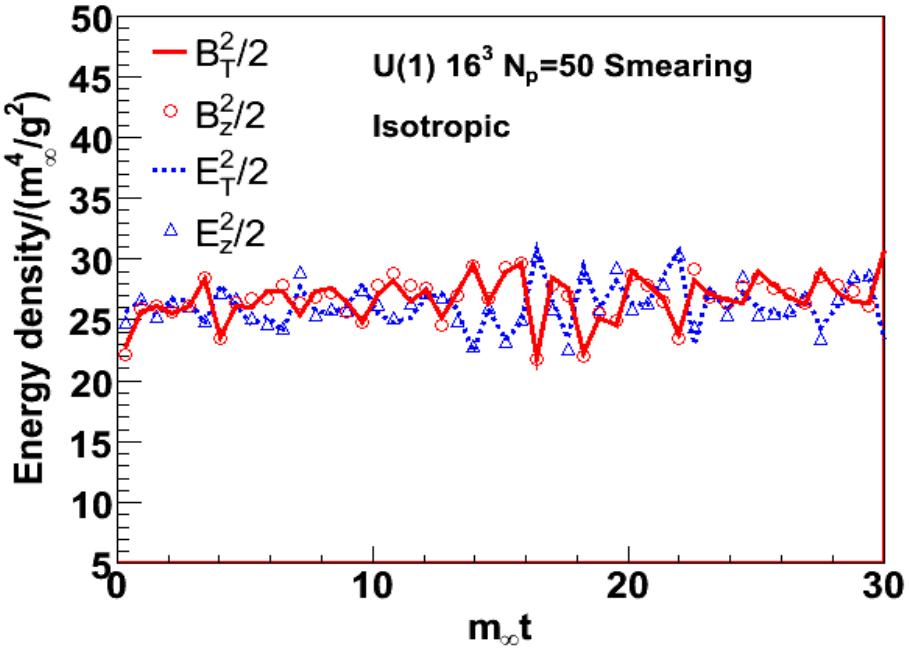


- Λ <<< phard for moderate anisotropy:
  cascade happens in the IR,
  UV safe (Arnold, Moore: PRD73)

- How about strong anisotropy ?

Full 3D simulations (Isotropic case)



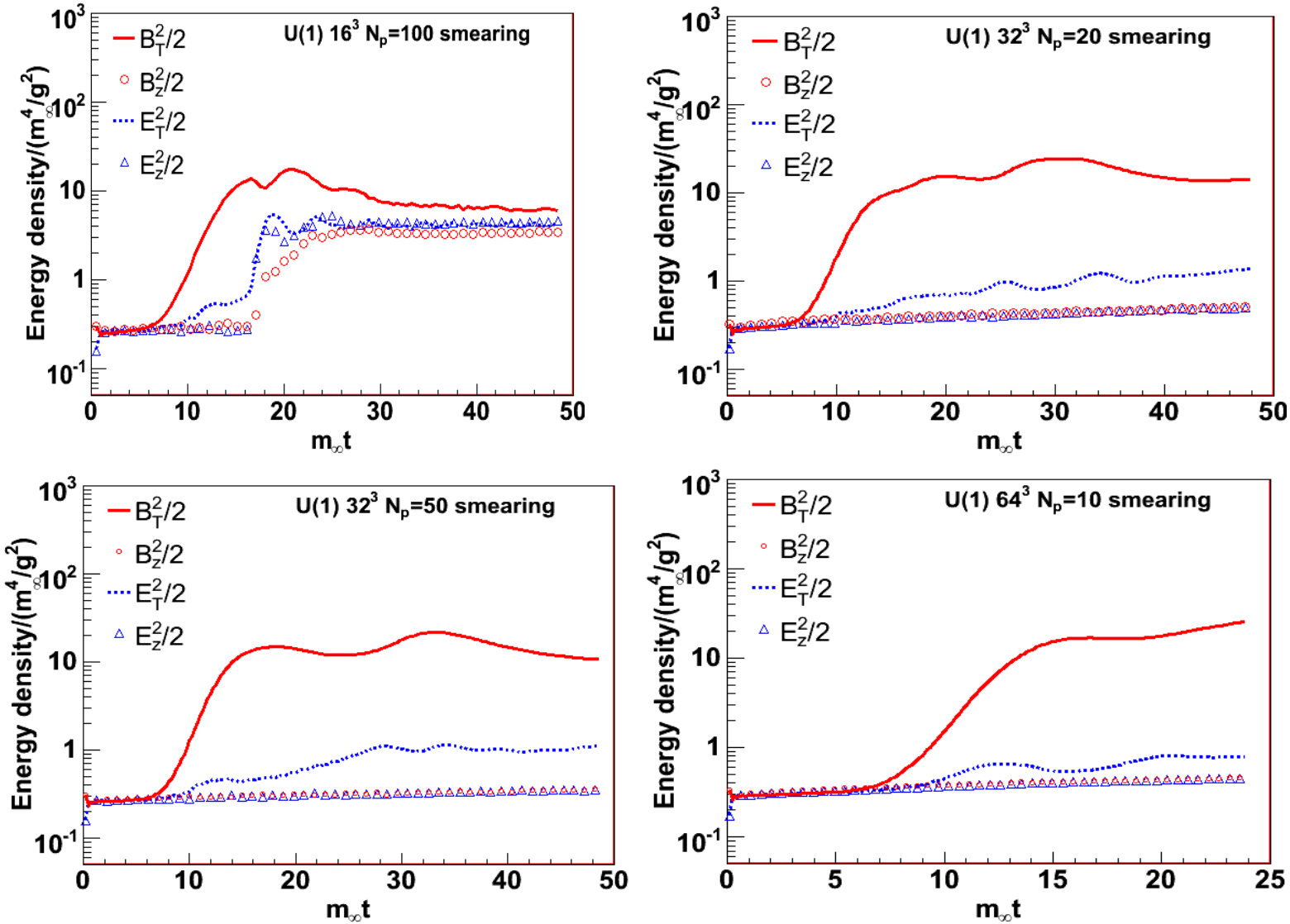


Point particle method does not work in 3D : too noisy and unphysical damping.

Smearing is needed to achieve reasonable simulations!

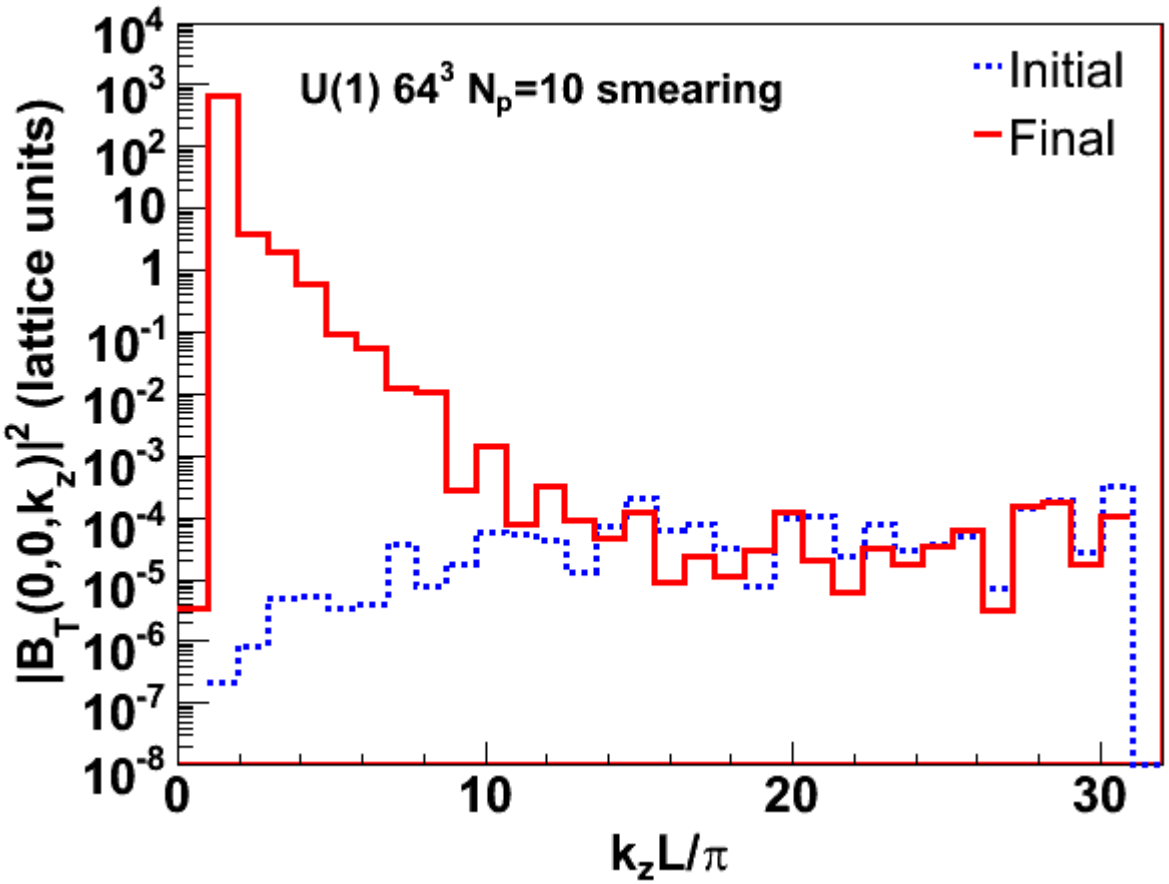
$$m_\infty^2 = g^2 N_c \int \frac{d^3 p}{(2\pi)^3} \frac{f(p)}{|p|}$$

U(1) results for 3D+3V: good!



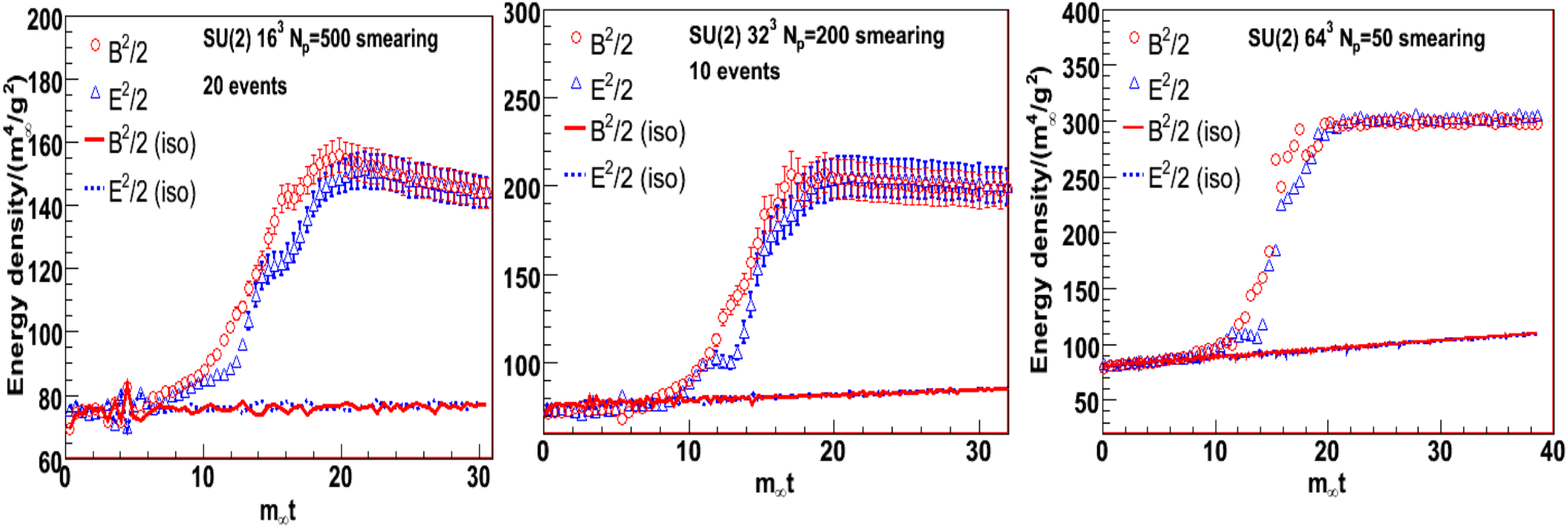
Exponential growth of magnetic field.
independent of lattice spacing.

Fourier transform of the transverse magnetic field in U(1)



No UV problem in U(1). Soft modes unstable, hard modes stable.

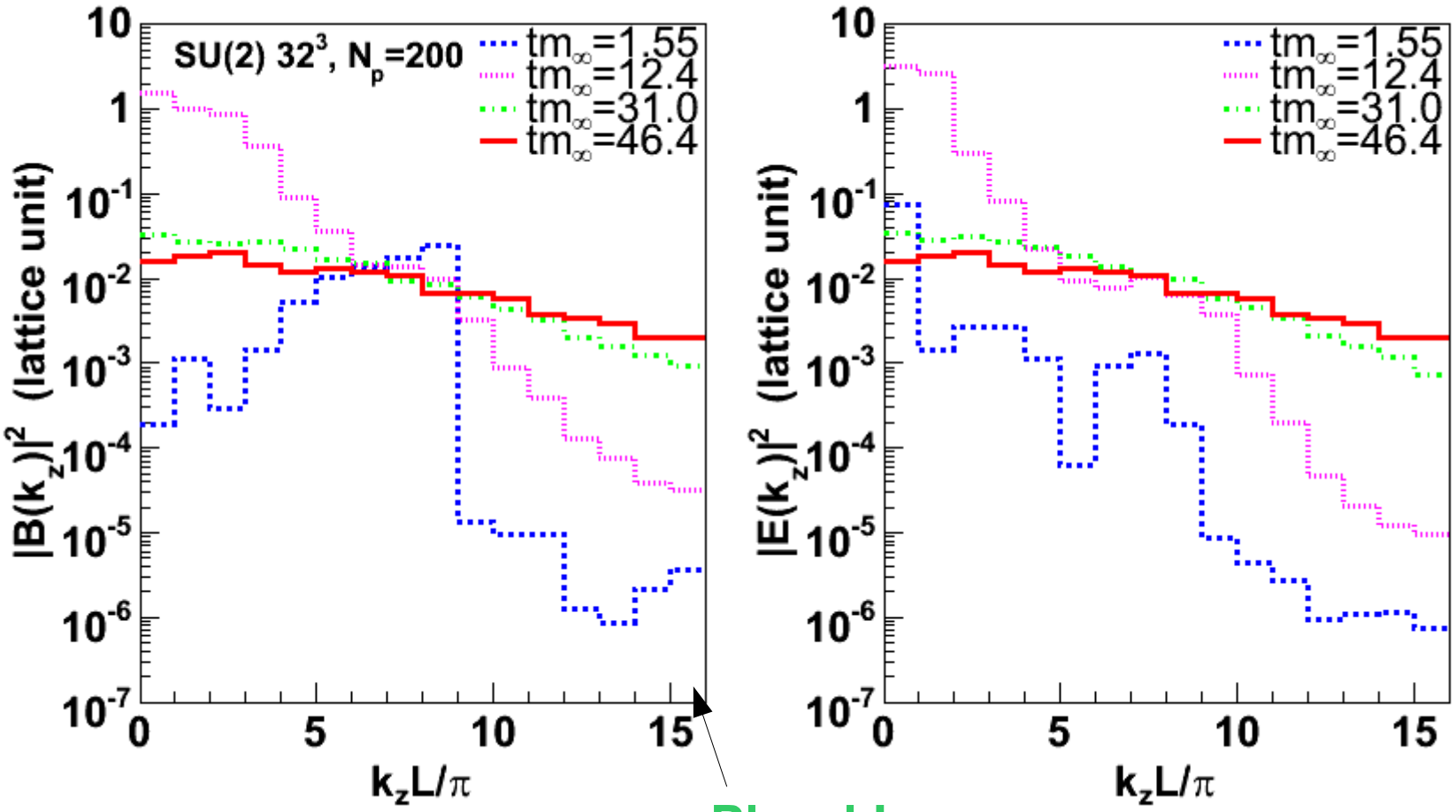
SU(2) for 3D+3V



We have developed a particle in cell simulation method by introducing current smearing in SU(2) case.

Field strength grow with inverse lattice spacing!

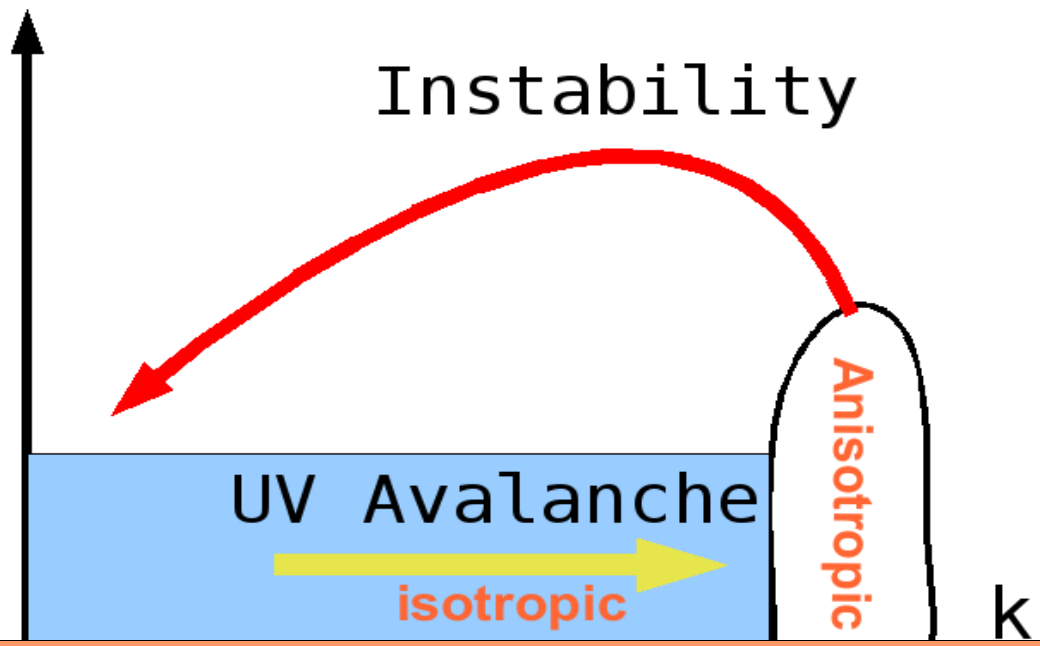
Time evolution of the Fourier transformed fields in SU(2) 3D+3V



Phard !

Soft fields rapidly avalanche to the UV

Collective process driving thermalization at RHIC/LHC ?



- Proper definition of classical dynamics of soft+hard modes in gauge theories beyond the hard-loop approx. is lacking !

Summary and Outlook

- Particle-field (Vlasov-YM) simulations in QCD.
- 3D results are totally different from 1D results. UV avalanche in 3D.
- Collective effects due to color field play impor-tant role for the pre-equilibrium dynamics.

- Include expansion of the system.
- Interplay between Collision term. and color mean-field.
- Proper definition of particle + field dynamics beyond HL in gauge theories.