RBRC Symposium
June 22, 2006

Semi-leptonic decays on the lattice

- from CKM unitarity to Proton spin problem -

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Semi-leptonic decays

- Kaon semi-leptonic decay (K_{l3} decay)

$$K^+ \to \pi^0 + l^+ + \nu_l \qquad K^0 \to \pi^- + l^+ + \nu_l$$

- Hyperon beta decay (HBD)

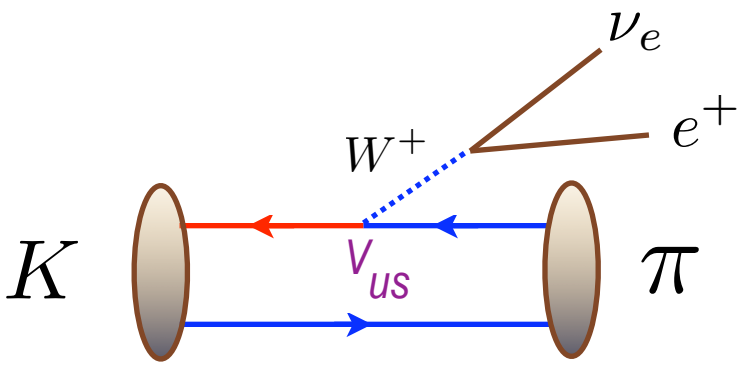
$$B' \to B + l^\pm + \nu_l(\bar{\nu}_l)$$

The octet baryons (p,n,Λ,Σ,Ξ) admit various β-type decays.

e.g. $n \to p + e^- + \bar{\nu}_e$ (neutron decay)

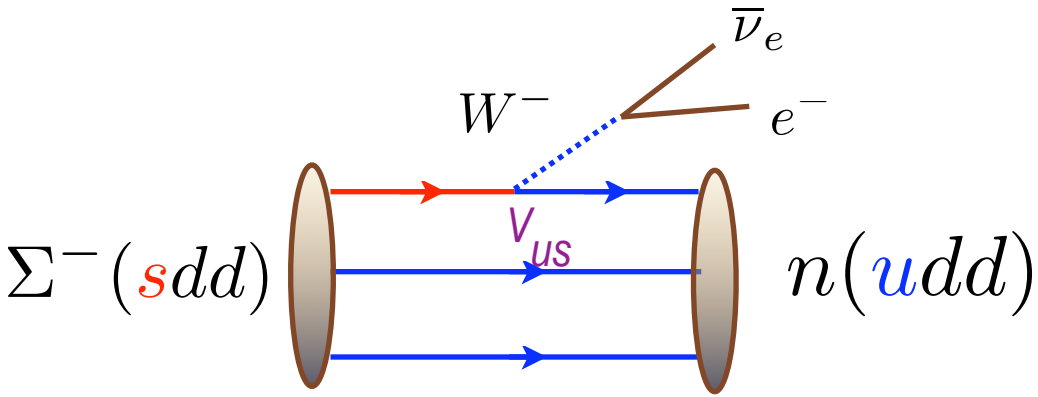
Semi-leptonic decays

- Kaon semi-leptonic decay (K_{l3} decay)



Determination of V_{us}

- Hyperon beta decay ($|\Delta s|=1$)



Determination of V_{us}

Vital input to analysis of strange quark spin fraction

$\Delta\Sigma(=\Delta u+\Delta d+\Delta s)_{\rm Expt.}=0.213\pm0.138$

Precise theoretical evaluation of $SU(3)_f$ breaking on form factors

Semi-leptonic decays

- Kaon semi-leptonic decay (K_{l3} decay)

$$\langle \pi(p)|\bar{s}\gamma_\mu u|K(p')\rangle = f_+(q^2)(p'+p)_\mu + f_-(q^2)(p'-p)_\mu$$

SU(3) symmetry: $f_+(0) = 1,\ f_-(q^2) = 0$ $\quad \Gamma_{\text{expt.}} \propto |f_+(0)|^2 |V_{us}|^2$

- Hyperon beta decay $B' \to B + l^\pm + \nu_l(\bar{\nu}_l)$

$$\langle B|V_\alpha - A_\alpha|B'\rangle = \bar{u}_B(p)[f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{M_{B'}}\sigma_{\alpha\beta}q_\beta + \frac{f_3(q^2)}{M_{B'}}q_\alpha$$

$$+g_1(q^2)\gamma_\alpha\gamma_5 + \frac{g_2(q^2)}{M_{B'}}\sigma_{\alpha\beta}\gamma_5 q_\beta + \frac{g_3(q^2)}{M_{B'}}q_\alpha\gamma_5]u_{B'}(p')$$

$$\frac{g_1(0)}{f_1(0)} = \frac{g_A}{g_V}$$

SU(3) symmetry: $f_3(q^2) = 0,\ g_2(q^2) = 0$ $\quad \Gamma_{\text{expt.}} \propto \left(|f_1(0)|^2 + 3|g_1(0)|^2\right)|V_{us}|^2$

Semi-leptonic decays

The simpler weak matrix elements:

✓ Vector and Axial symmetry play important roles

- SU(3) breaking effects ($|\Delta s|=1$ process)
  - CKM unitarity (Kl3, HBD)

✓ described by form factors

- struture of hadrons
  - Proton spin problem (HBD)

- Needs the precise theoretical calculation !

CKM unitarity

Cabibbo-Kobayashi-Maskawa matrix

weak eigen-states mass eigen-states

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$= \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Wolfenstein parameterization (λ, A, ρ, η)

Cabibbo-Kobayashi-Maskawa matrix

weak eigen-states mass eigen-states

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

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Wolfenstein parameterization (λ, A, ρ, η)

$$V_{us} = \lambda = \sin(\theta_{\mathrm{Cabibbo}})$$

an important ingredient

Status of $|V_{us}|$

- Can be determined from the various approachs
  - K_{l3} decays: 0.2200(26) [PDG2004]
  - hyperon β decays: 0.2250(27)
  - leptonic kaon & pion decays: 0.2238(30)
  - hadronic τ-decays: 0.2208(34)
  - CKM unitarity + $|V_{ud}|$: 0.2269(21)

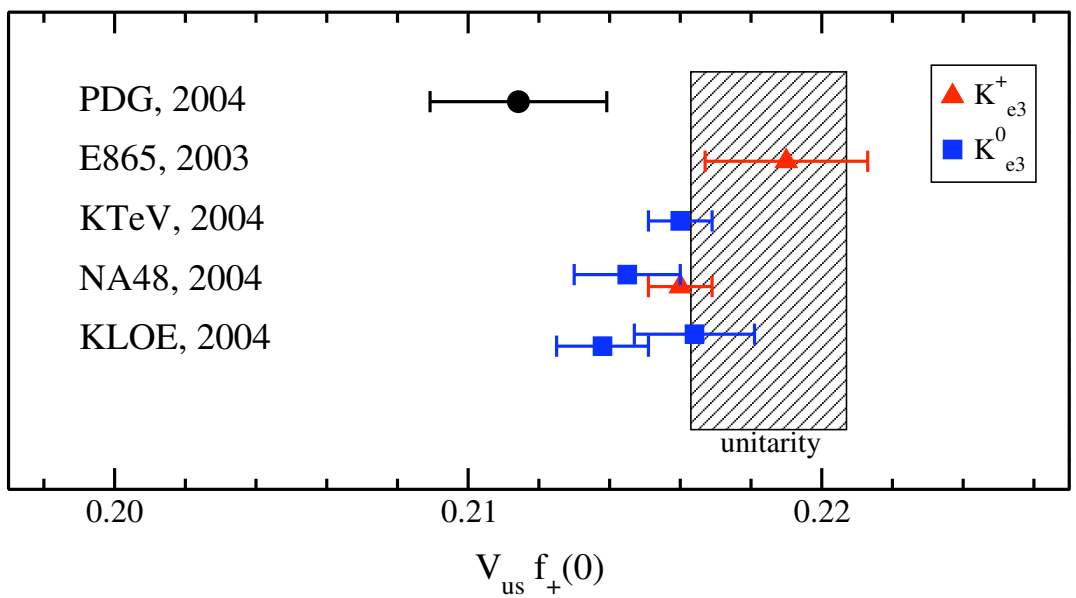
    ($|V_{ud}|$=0.9739(5), super-allowed nuclear β-decays, $|V_{ub}|$~O(10^{-3}))

"2-2.5 σ problem"

Status of $|V_{us}|$

- New experiments have been performed very recently.
  - K_{l3} decays: 0.2200(26) [PDG2004] old 70's experiments

$|V_{us}|\, f_+^{K^0\pi^-}(0)$ from Γ (2004, summer)



$$\Gamma_{\text{expt.}} \propto |f_+(0)|^2 |V_{us}|^2$$

  - new K_{l3} average: 0.2257(9)(0.961/$f_+(0)$) [Blucher-Marciano]
    - $f_+(0)$=0.961(8): the old quark model estimate (Leutwyler-Roos)

Kl3 decays

Kl3 decays

- Kl3 decays are described by 2 form factors

pure vector transition

$$\langle \pi(\mathbf{p}) | \bar{s}\gamma_\mu u | K(\mathbf{k}) \rangle = f_+(q^2)(k+p)_\mu + f_-(q^2)(k-p)_\mu$$

✓ SU(3) limit : $f_+(0) = 1, \; f_-(q^2) = 0$

$$f_+(0) = 1 + \text{“SU(3) breaking”}$$

$$f_-(q^2) = \text{“SU(3) breaking”}$$

Kl3 decays

- Kl3 decays are described by 2 form factors

pure vector transition

$$\langle \pi(\mathbf{p})|\bar{s}\gamma_\mu u|K(\mathbf{k})\rangle = f_+(q^2)(k+p)_\mu + f_-(q^2)(k-p)_\mu$$

✓ SU(3) limit : $f_+(0) = 1,\ f_-(q^2) = 0$

decay rate:

$$\Gamma = \frac{G_F^2}{192\pi^2} M_K^5 C^2 |f_+(0)|^2 |V_{us}|^2 I S_{\rm ew}(1+\delta_{\rm em})$$

Normalization

$C = \frac{1}{\sqrt{2}} \quad (K^+)$

$C = 1 \quad (K^0)$

Phase space integral

$I = I(f_+, (m_l/m_K)^2 f_0)$

Radiative corrections

Kl3 decays

- Kl3 decays are described by 2 form factors

pure vector transition

$$\langle \pi(\mathbf{p})|\bar{s}\gamma_\mu u|K(\mathbf{k})\rangle = f_+(q^2)(k+p)_\mu + f_-(q^2)(k-p)_\mu$$

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To determine $|V_{us}|$

Γ from experiment
$f_+(0)$ from theory

Theory: ChPT for $f_+(0)$

- ChPT expansion in terms of M_K, M_π

$$f_+(0) = 1 + \boxed{f_2 + f_4 + \cdots}\,(f_n = O(M_{K,\pi}^n))$$

SU(3) chiral limit

correction: quadratic in SU(3) breaking

"Ademollo-Gatto theorem"

$$\Delta f = f_+(0) - 1 \propto (m_s - m_l)^2 \sim (M_K^2 - M_\pi^2)^2$$

chiral-loop corrections are well constrained

Theory: ChPT for $f_+(0)$

- ChPT expansion in terms of M_K, M_π

$$f_+(0) = 1 + f_2 + f_4 + \cdots \quad (f_n = O(M_{K,\pi}^n))$$

$$f_2 = -0.023$$ [Gasser-Leutwyler]

$$f_2 = \frac{3}{2} H_{K\pi} + \frac{3}{2} H_{\eta K}$$

$$H_{12} = -\frac{1}{64\pi^2 f_\pi^2} \left[M_1^2 + M_2^2 + \frac{2M_1^2 M_2^2}{M_1^2 - M_2^2} \ln \frac{M_2^2}{M_1^2} \right]$$ no LECs

$$\to -\frac{(M_1^2 - M_2^2)^2}{64\pi^2 f_\pi^2 M_1^2} \quad (\text{if} \quad M_1 \approx M_2)$$

Theory: ChPT for $f_+(0)$

- ChPT expansion in terms of M_K, M_π

$$f_+(0) = 1 + f_2 + f_4 + \cdots \quad (f_n = O(M_{K,\pi}^n))$$

$$f_2 = -0.023$$ [Gasser-Leutwyler]

f_4 contains LECs in $O(p^6)$ chiral lagrangian

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$$f_2 = -0.023$$ [Gasser-Leutwyler]

f_4 contains LECs in $O(p^6)$ chiral lagrangian

quark model estimation: [Leutwyler-Roos, 1984]

$$f_4 = -0.016(8) \qquad f_+(0) = 0.961(8)$$

Main theoretical uncertainty

First lattice calculation

- Becirevic et al., 2004
  - Quench (N_f=0)
  - Wilson gauge + NP clover fermion
  - L ~ 2.0 fm, a^{-1}~ 2.7 GeV
  - m_q ~ $m_s/2$ - m_s

$$f_+(0) = 0.960(5)_{\rm stat}(7)_{\rm syst}$$

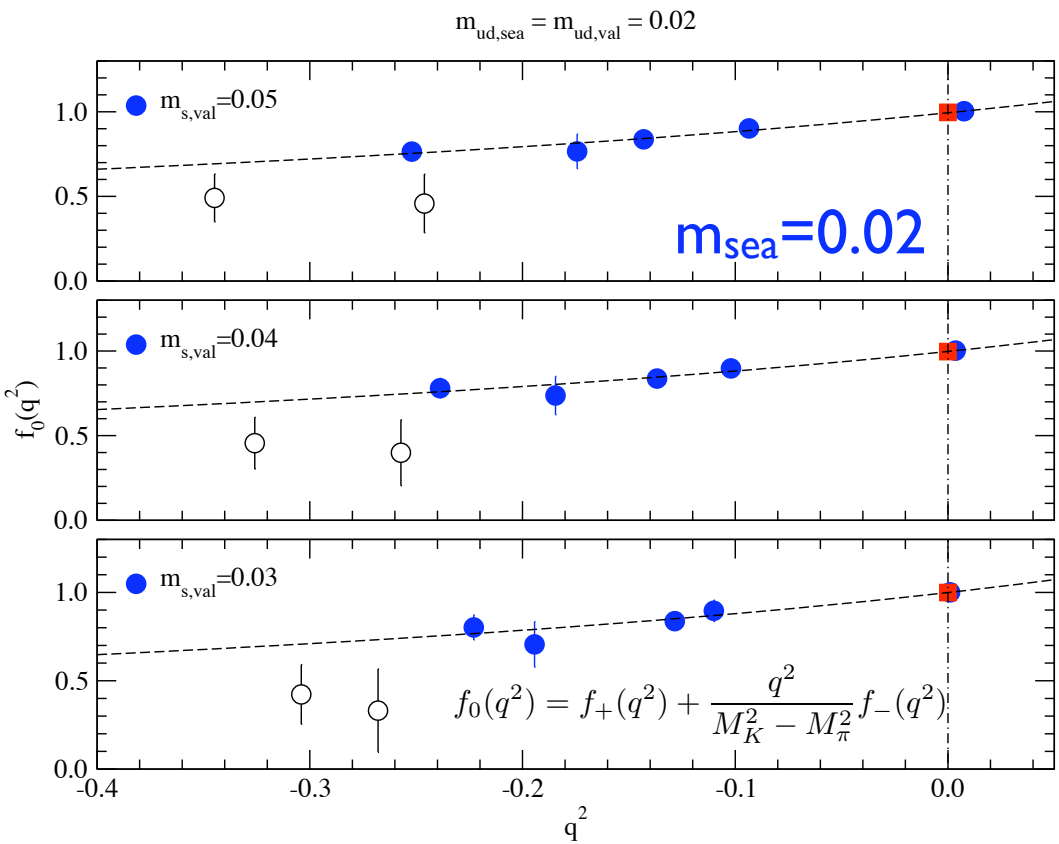
very close to Leutwyler-Roos estimate $f_+(0) = 0.961(8)$

Dynamical DWF calculation

- 2 flavors DWF + DBW2 gauge action (N_f=2)
- Lattice cutoff: a^{-1} ~ 1.7 GeV (β=0.80, c_1= –1.4069)
- Small residual quark mass: a few MeV (L_5=12, M_5=1.8)
- Box size: $V=16^3 \times 32$, L ~ 1.9 fm
- sea quark mass: m_{light}=0.02, 0.03, 0.04 (M_π=0.49, 0.61, 0.70 GeV)
- valence quark mass: $3*m_{strange} \in [0.02,0.03,0.04,0.05]$
- # of statistics: each 94 measurements
- f_π= 134.0(42), f_K=157.4(38) MeV, f_π / f_K =1.175(11)

RBC collaboration, Phys. Rev. D72, 114505 (05)

K_{l3} ($K \to \pi$) form factor



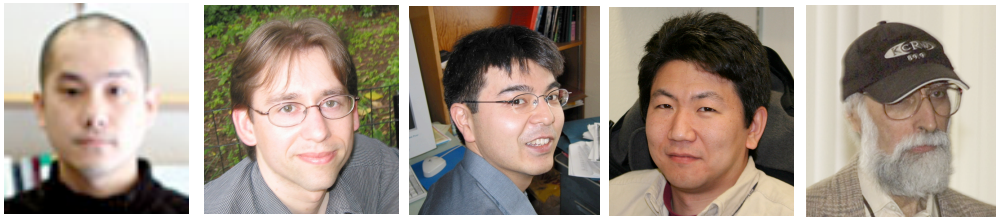
- DWF-DBW2 (Nf=2 dynamical)

  La~2.0 fm, a^{-1}=1.7 GeV

  m_{sea}=0.02 (QCDSP & QCDOC)

  m_{sea}=0.03, 0.04 (QCDOC)

T. Kaneko, C. Dawson, T. Izubuchi, S. Sasaki, A. Soni



$f_+(0)(=f_0(0))=0.968(9)(6)$

combine with the experimental data $|V_{us}f_+(0)|=0.2173(8)$

→ CKM matrix $|V_{us}| = 0.2245(8)_{expt}(26)_{theory}$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta \qquad \delta = 0.0013(16)$$

Other lattice calculations

- Becirevic et al. 2004: first lattice calc.
  - Nf=0, Wilson gauge + NP clover fermions
  - L~2.0 fm, a^{-1}~2.7GeV,
  - $f_+(0)=0.960(5)(7)$
- JLQCD, 2005
  - Nf=2, Wilson gauge + NP clover fermions
  - L~1.8 fm, a^{-1}~2.2 GeV
  - $f_+(0)=0.952(6)$
- Fermilab-MILC-HPQCD, 2004
  - Nf=2+1, impr. gauge + Asqtad (impr. Wilson for val. d-quark)
  - L=2.6 fm, a^{-1}~1.6 GeV
  - $f_+(0)=0.962(6)(9)$ using an experimental input for q^2-extrapolation

Baryon semileptonic decays

Baryon semi-leptonic decay

The octet baryons (p,n,Λ,Σ,Ξ) admit various β-type decays.

$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

- neutron beta decay

$$n \to p + e^- + \bar{\nu}_e$$

CVC (conserved vector current) relation:

$$\langle p|V_\mu^+ - A_\mu^+|n\rangle = 2\langle p|V_\mu^3 - A_\mu^3|p\rangle$$

Weak matrix element Iso-vector nucleon matrix element

way to access the nucleon structure

• Contents

✳ Nucleon axial charge

Finite size effect
$O(a^2)$ correction

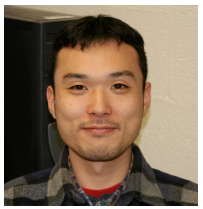
S.S., K. Orginos, S. Ohta, T. Blum, Phys. Rev. D68 (03) 054509

✳ Hyperon beta decay

SU(3) breaking effect

✓ An exploratory study in quench (Nf=0) DWF calculation

S.S., T.Yamazaki



Nucleon axial charge

Nucleon axial charge g_A

- Well measured quantity in experiment

  Iso-symmetry gives rise to the relation

$$\langle p|V_\mu^+ - A_\mu^+|n\rangle = 2\langle p|V_\mu^3 - A_\mu^3|p\rangle$$

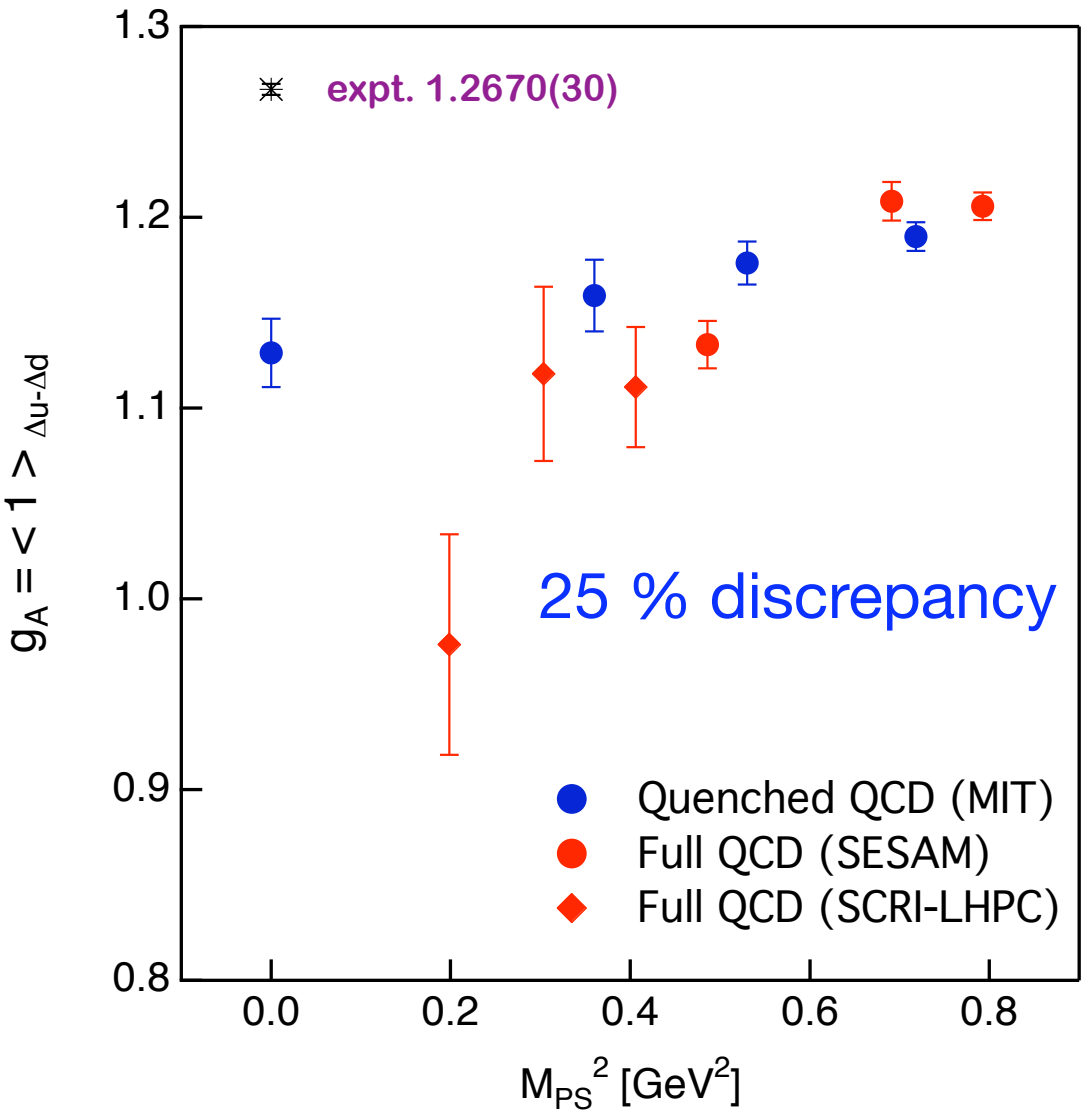
  from neutron beta decay; g_A/g_V= 1.2670(35).

- The simplest nucleon matrix elements

  - lowest moment (no covariant derivative)
  - zero momentum transfer
  - no disconnected diagram

a benchmark calculation = a "gold plated" test

But . . .



$$n \to p + e^- + \bar{\nu}_e$$

$$g_A = \lim_{q^2 \to 0} g_1(q^2)$$

$$\left(\frac{g_A}{g_V}\right)_{\text{expt.}} = 1.2670(30)$$

long-standing problem

Lattice calculation of g_A (before 2002)

type	group	fermion	lattice	β	volume	configs	$m_\pi L$	g_A
quench	KEK [1]	Wilson	16^3x20	5.7	(2.2 fm)^3	260	≥ 5.9	0.985(25)
	Kentuchy [2]	Wilson	16^3x24	6.0	(1.5 fm)^3	24	≥ 5.8	1.20(10)
	DESY [3]	Wilson	16^3x32	6.0	(1.5 fm)^3	1000	≥ 4.8	1.074(90)
	LHPC-SESAM [7]	Wilson	16^3x32	6.0	(1.5 fm)^3	200	≥ 4.8	1.129(98)
			16^3x32	6.0	(1.5 fm)^3	O(500)		
	QCDSF [4]	Wilson	24^3x48	6.2	(1.6 fm)^3	O(300)		1.14(3)
			32^3x48	6.4	(1.6 fm)^3	O(100)		
			16^3x32	6.0	(1.5 fm)^3	O(500)		
	QCDSF-UKQCD [5]	Clover	24^3x48	6.2	(1.6 fm)^3	O(300)		1.135(34)
			32^3x48	6.4	(1.6 fm)^3	O(100)		
full	LHPC-SESAM [7]	Wilson	16^3x32	5.5	(1.7 fm)^3	100	≥ 4.2	0.914(106)
	SESAM [6]	Wilson	16^3x32	5.6	(1.5 fm)^3	200	≥ 4.5	0.907(20)

1. M. Fukugita et al., Phys. Rev. Lett. 75 (1995) 2092.
2. K.F. Liu et al., Phys. Rev. D49 (1994) 4755.
3. M. Göckeler et al., Phys. Rev. D53 (1996) 2317.
4. S. Capitani et al., Nucl. Phys. B (Proc. Suppl.) 79 (1999) 548.
5. R. Horsley et al., Nucl. Phys. B (Proc. Suppl.) 94 (2001) 307.
6. S. Güsken et al., Phys. Rev. D59 (1999) 114502.
7. D. Dolgov et al., Phys. ReV.D66 (2002)034506.

Low value of g_A in lattice QCD

- Possible systematic errors
  - ✓ Quenching

    $g_A^{Full} \lesssim g_A^{Quench}$ at a~0.1 fm ~ 5-10 % ↘

  - ✓ Finite lattice spacing

    $(g_A)_{at\ a \to 0} > (g_A)_{at\ a \sim 0.1\ fm}$ ~ 5 % ↗

  - ✓ Determination of Z_A

    $Z_A^{Non\text{-}pert} < Z_A^{Pert}$ (Clover) ~ 10 % ↘

  - ✓ Finite volume

    No estimation ?

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6. S. Güsken et al., Phys. Rev. D59 (1999) 114502.
7. D. Dolgov et al., Phys. ReV.D66 (2002)034506.

DWF calculation of g_A

- Big advantage in dealing with the axial symmetry
  - g_A are supposed to respect the axial WT identity
  - Empirically known as Goldberger-Treiman relation: $M_N g_A = f_\pi g_{\pi N}$

✓ Excellent chiral properties of DWF (continuum-like)

✳ lighter pion mass

✳ Especially, a significant relation $Z_V = Z_A$ for local lattice currents

T. Blum et al, PRD66 (02) 014504

- up to $O(a^2)$

A ratio g_A^{latt} / g_V^{latt} directly yields the renormalized value of g_A

➡ Just require to calculate the ratio of three-point functions!

CVC relation

- The vector weak current is the iso-spin rotation of the electromagnetic current, $j_\mu^{\rm em} = \frac{2}{3}V_\mu^u - \frac{1}{3}V_\mu^d + \cdots$

$$[I_+, j_\mu^{\rm em}] = -\bar{u}\gamma_\mu d$$

$$\langle p|\bar{u}\gamma_\mu d|n\rangle = -\langle p|[I_+, j_\mu^{\rm em}]|n\rangle$$

$$= \langle p|j_\mu^{\rm em}|p\rangle - \langle n|j_\mu^{\rm em}|n\rangle$$

$$g_V = \lim_{q^2\to 0}\langle p|\bar{u}\gamma_\mu d|n\rangle = \lim_{q^2\to 0}\langle p|j_\mu^{\rm em}|p\rangle = 1$$

way to calculate the renormalization factor $Z_V = g_V^{\rm ren}/g_V^{\rm lat} = 1/g_V^{\rm lat}$

CVC relation (cont'd)

- Further consideration of iso-spin symmetry provides

$$\lim_{q^2\to 0}\langle p|j_\mu^{\rm em}|p\rangle = \lim_{q^2\to 0}\langle p|V_\mu^d|p\rangle = \lim_{q^2\to 0}\langle p|V_\mu^u - V_\mu^d|p\rangle = 1$$

in the continuum

- For the local lattice currents in the chiral limit,

$$\mathcal{V}_\mu^f = Z_V V_\mu^f + \mathcal{O}(a^2)$$

$$\partial_\mu V_\mu^f \neq 0$$
$$\partial_\mu \mathcal{V}_\mu^f = 0$$

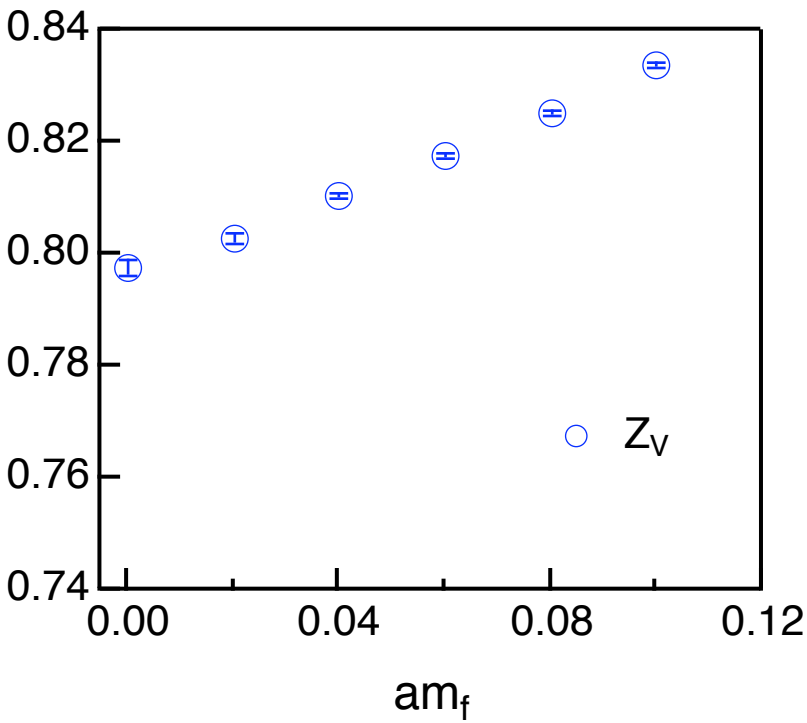
➡ Three different determination of Z_V can expose an $O(a^2)$ lattice artifact.

Check of the relation $Z_A = Z_V$

- Nf0: DWF-DBW2 at β=0.87 (a^{-1}=1.3GeV)
  - $16^3 \times 32 \times 16$ with M_5=1.8

coarse lattice

Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509

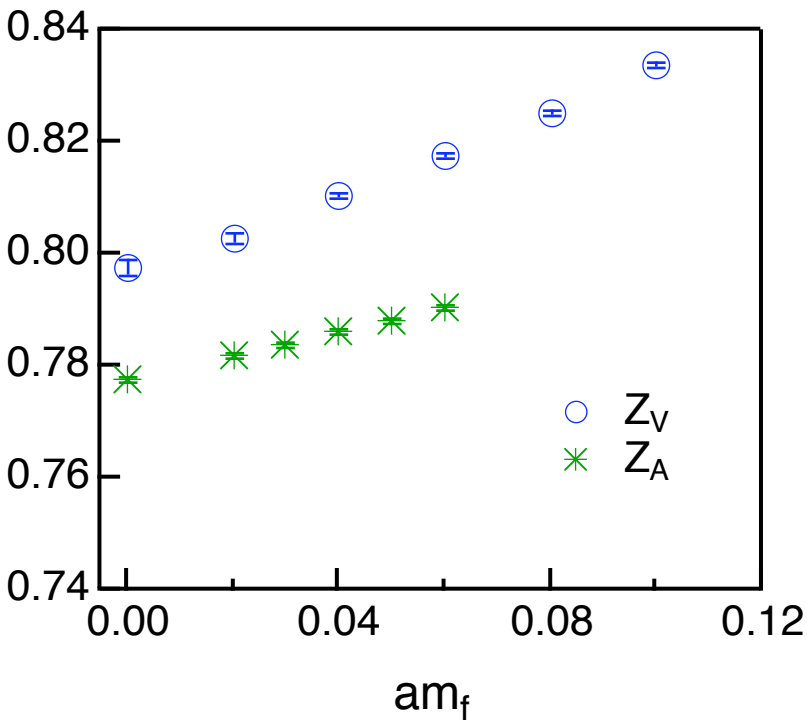


$$Z_V = \frac{1}{g_V^{\rm latt}} \propto \frac{\langle N(t')\bar{N}(0)\rangle}{\langle N(t')V_4(t)\bar{N}(0)\rangle}$$

Check of the relation Z_A=Z_V

- Nf0: DWF-DBW2 at β=0.87 (a^{-1}=1.3GeV)
  coarse lattice
  - 16^3 x 32 x 16 with M_5=1.8

Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509



$$Z_V = \frac{1}{g_V^{\rm latt}} \propto \frac{\langle N(t')\bar{N}(0)\rangle}{\langle N(t')V_4(t)\bar{N}(0)\rangle}$$

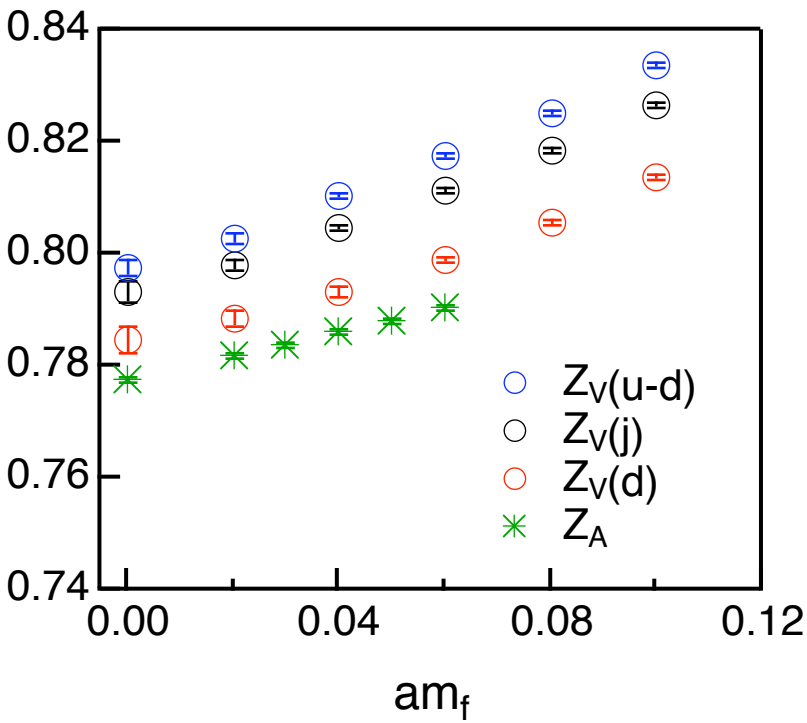
$$Z_A \propto \frac{\langle \mathcal{A}_0^{\rm con}(t)\pi(0)\rangle}{\langle A_0(t)\pi(0)\rangle}$$

Y. Aoki et al., PRD69 (04) 074504

Check of the relation $Z_A = Z_V$

- Nf0: DWF-DBW2 at β=0.87 (a^{-1}=1.3GeV)
  coarse lattice
  - $16^3 \times 32 \times 16$ with M_5=1.8

Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509



$$Z_V = \frac{1}{g_V^{\rm latt}} \propto \frac{\langle N(t')\bar{N}(0)\rangle}{\langle N(t')V_4(t)\bar{N}(0)\rangle}$$

Violation of

$$\lim_{q^2\to 0}\langle p|j_\mu^{\rm em}|p\rangle = \lim_{q^2\to 0}\langle p|V_\mu^d|p\rangle = \lim_{q^2\to 0}\langle p|V_\mu^u - V_\mu^d|p\rangle$$

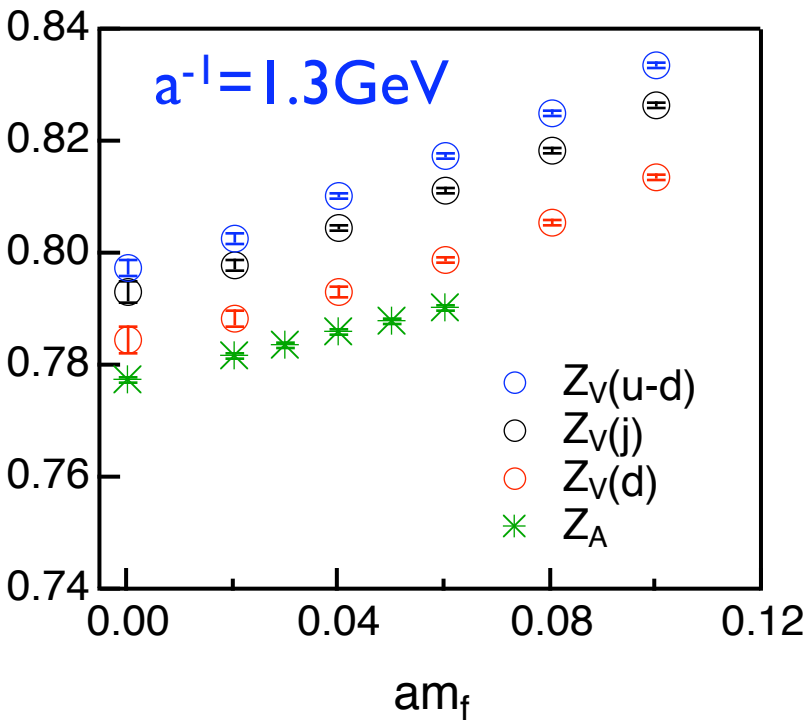
implies an $O(a^2)$ lattice artifact

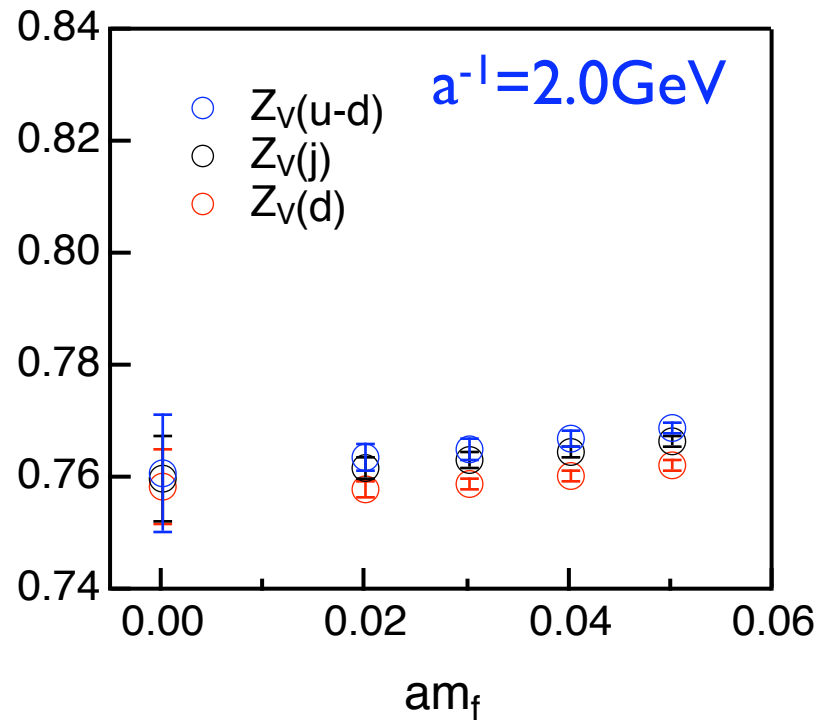
Check of the relation $Z_A=Z_V$

- Nf0: DWF-DBW2 at β=0.87 (a^{-1}=1.3GeV)
  - $16^3 \times 32 \times 16$ with M_5=1.8

coarse lattice

Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509





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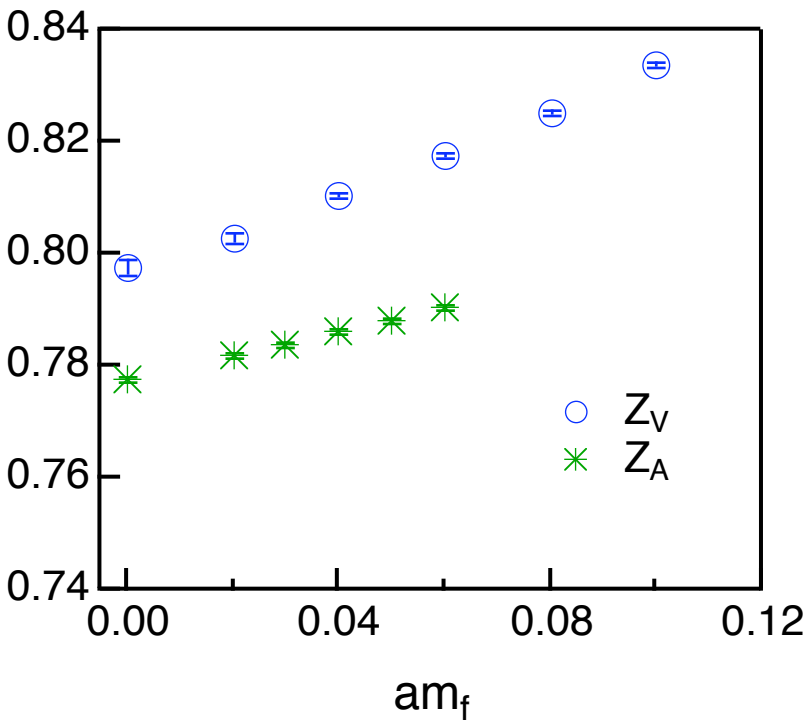
Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509



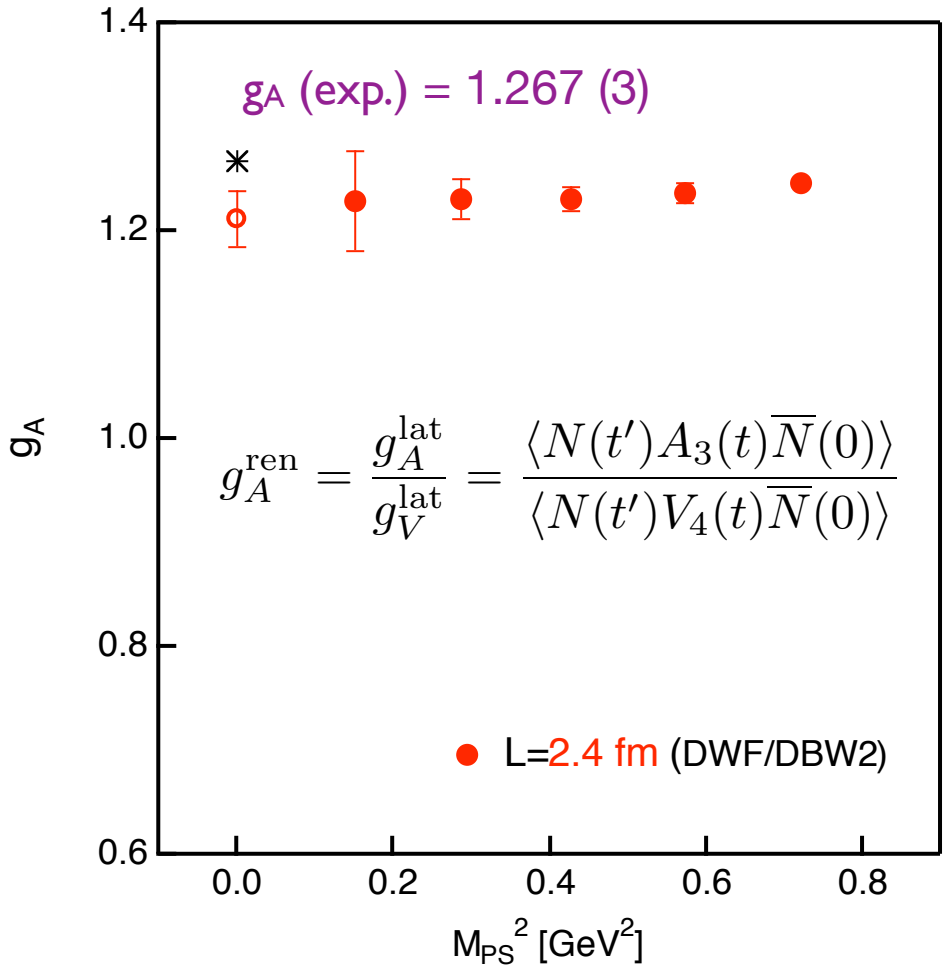
In the chiral limit

$$Z_V = 0.796(3)$$

$$Z_A = 0.7776(5)$$

2-3 % systematic error stemming from determination of Z-factor mainly due to O(a^2) corrections

Quench DWF calculation of g_A



Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509

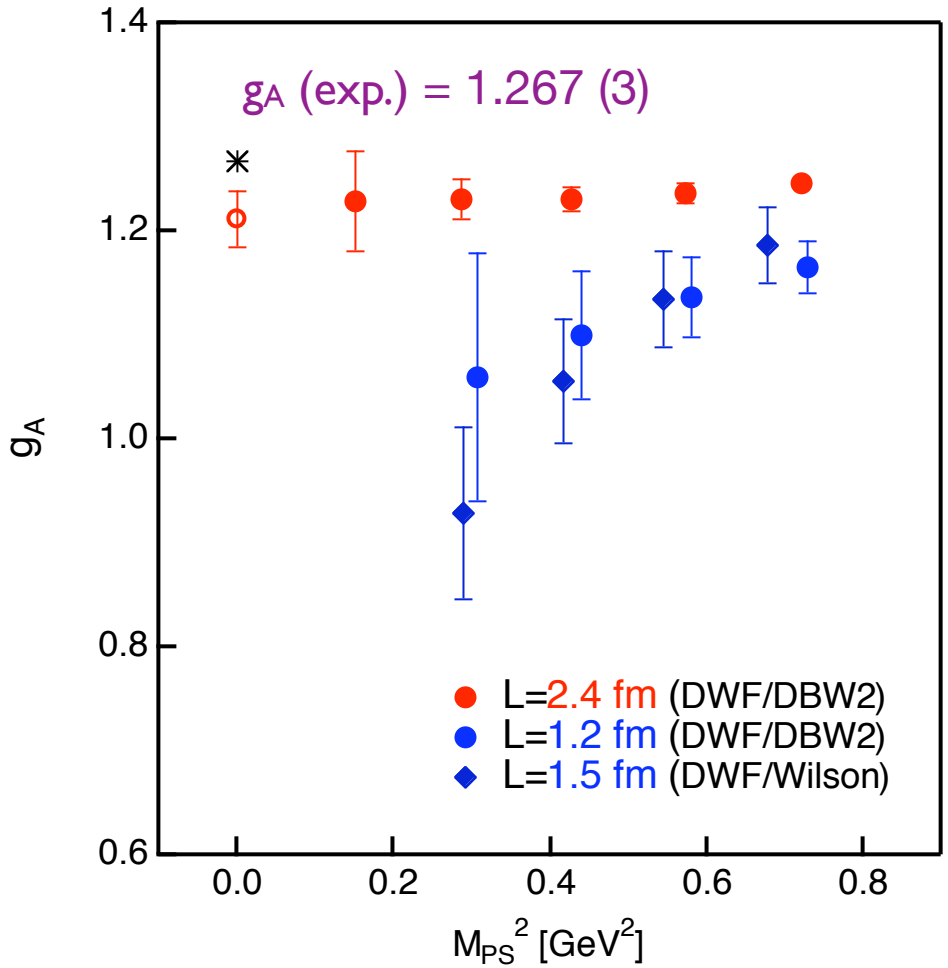
- ✓ the lightest pion mass, $M_\pi \sim 0.39$ GeV
- ✓ relatively large volume, $V \sim (2.4\ \text{fm})^3$
- ✓ large statistics, 416 configs

✳ mild quark mass dependence

➡ Linear extrapolation yields

$$g_A = 1.212\ (27)$$

Quench DWF calculation of g_A



Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509

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- ✓ relatively large volume, $V \sim (2.4\ \text{fm})^3$
- ✓ large statistics, 416 configs

✳ mild quark mass dependence

✳ clear finite volume dependence

- a 20 % increase from 1.2 fm to 2.4 fm

resolve the long-standing problem!

- Nf2: DWF-DBW2 at beta=0.80 (a^{-1}=1.7GeV)
  - 16^3x32x12 (L=1.9 fm): O(5000) MC trajectories
  - m_{sea}=0.02, 0.03, 0.04 (M_π=0.49, 0.61, 0.70 GeV)
  - m_{res}=0.00137(5)
  - f_π= 134.0(42), f_K=157.4(38) MeV
  - $B_K^{\overline{MS}}$(2 GeV)=0.495(18)

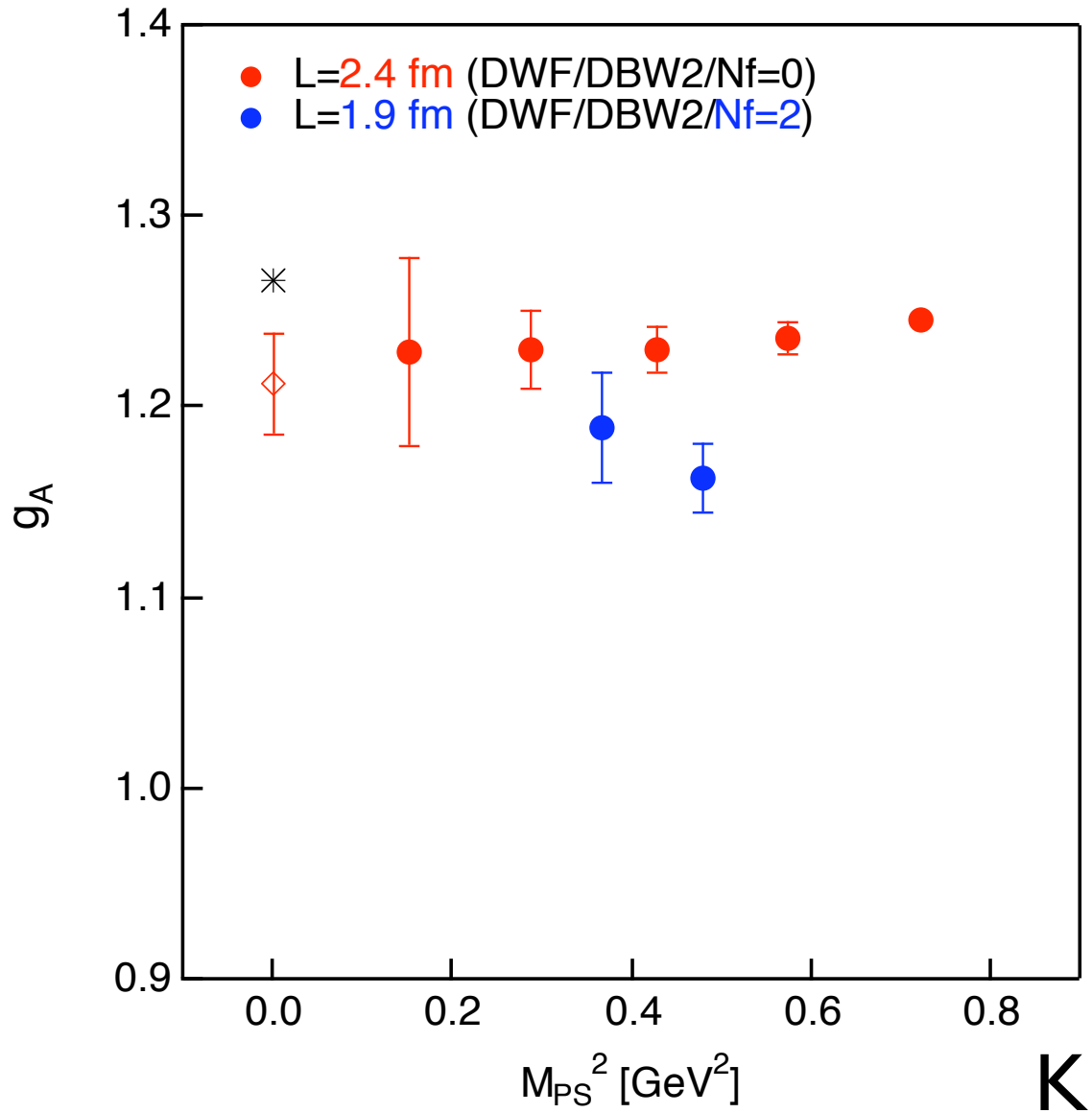
RBC collaboration, Phys. Rev. D72, 114505 (05)

- Nf2: DWF-DBW2 at beta=0.80 (a^{-1}=1.7GeV)
  - 16^3x32x12 (L=1.9 fm): 220 statistics
  - m_{sea}=0.03, 0.04
  - m_{sea}=0.02 (underway)
  - Nucleon structure function

Blum, Lin, Ohta, Orginos, Sasaki

K. Orginos's talk

Nf=2 DWF calculation of g_A



K. Orginos's talk

Hyperon beta decay

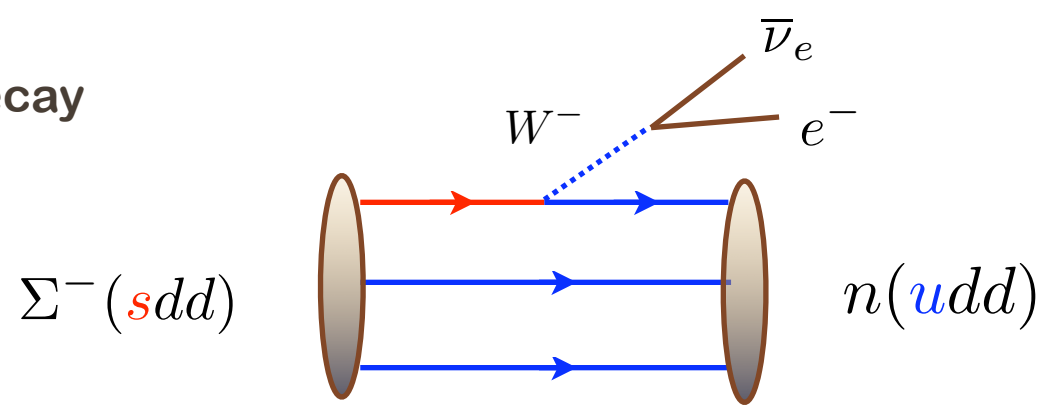
Hyperon beta decays

The octet baryons (p,n,Λ,Σ,Ξ) admit various β-type decays.

$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

- hyperon beta decay
  - ✓ *Alternative way to determine $|V_{us}|$ other than K_{l3} decays*

Δs=1 decay



the weak mixing element (CKM) V_{us}

Hyperon beta decays

The octet baryons (p,n,Λ,Σ,Ξ) admit various β-type decays.

$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

- hyperon beta decay
  - ✓ *Vector and Axial-vector transition unlike Kl3 decays.*

$$\langle B|V_\alpha - A_\alpha|B'\rangle = \bar{u}_B(p)[f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{M_{B'}}\sigma_{\alpha\beta}q_\beta + \frac{f_3(q^2)}{M_{B'}}q_\alpha$$
$$+g_1(q^2)\gamma_\alpha\gamma_5 + \frac{g_2(q^2)}{M_{B'}}\sigma_{\alpha\beta}\gamma_5 q_\beta + \frac{g_3(q^2)}{M_{B'}}q_\alpha\gamma_5]u_{B'}(p')$$

These decays are described by 6 form factors

Hyperon beta decays

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$$g_V = \lim_{q^2\to 0} f_1(q^2) \qquad g_A = \lim_{q^2\to 0} g_1(q^2)$$ forward limit

Hyperon beta decays

The octet baryons (p,n,Λ,Σ,Ξ) admit various β-type decays.

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$$\lim_{q^2\to 0} f_2{}^{n\to p}(q^2) = \frac{M_n}{M_p}\frac{\mu_p - \mu_n}{2}$$ Weak magnetism

Hyperon beta decays

The octet baryons (p,n,Λ,Σ,Ξ) admit various β-type decays.

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1st class: $f_1(q^2),\ \ f_2(q^2),\ \ g_1(q^2),\ \ g_3(q^2)$

2nd class: $f_3(q^2),\ \ g_2(q^2)$

2nd-class current

$$\langle B|V_\alpha - A_\alpha|B'\rangle = \bar{u}_B(p)[f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{M_{B'}}\sigma_{\alpha\beta}q_\beta + \frac{f_3(q^2)}{M_{B'}}q_\alpha$$

$$+g_1(q^2)\gamma_\alpha\gamma_5 + \frac{g_2(q^2)}{M_{B'}}\sigma_{\alpha\beta}\gamma_5 q_\beta + \frac{g_3(q^2)}{M_{B'}}q_\alpha\gamma_5]u_{B'}(p')$$

- Time reversal invariance requires all 6 form factors to be real
- With respect to transformation under (extended) G-parity,

1st class $\quad Gf_{1,2}(q^2)G^{-1} = +f_{1,2}(q^2) \qquad Gg_{1,3}(q^2)G^{-1} = -g_{1,3}(q^2)$

2nd class $\quad Gf_3(q^2)G^{-1} = -f_3(q^2) \qquad Gg_2(q^2)G^{-1} = +g_2(q^2)$

- (extended) G-parity invariance requires $\qquad G = Ce^{-i\pi T_{2,5,7}}$

$$f_3(q^2) = 0 \qquad g_2(q^2) = 0$$

neutron beta decay

2nd-class current

$$\langle B|V_\alpha - A_\alpha|B'\rangle = \bar{u}_B(p)[f_1(q^2)\gamma_\alpha + \frac{f_2(q^2)}{M_{B'}}\sigma_{\alpha\beta}q_\beta + \frac{f_3(q^2)}{M_{B'}}q_\alpha$$

$$+g_1(q^2)\gamma_\alpha\gamma_5 + \frac{g_2(q^2)}{M_{B'}}\sigma_{\alpha\beta}\gamma_5 q_\beta + \frac{g_3(q^2)}{M_{B'}}q_\alpha\gamma_5]u_{B'}(p')$$

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- With respect to transformation under (extended) G-parity,

1st class $\quad Gf_{1,2}(q^2)G^{-1} = +f_{1,2}(q^2) \qquad Gg_{1,3}(q^2)G^{-1} = -g_{1,3}(q^2)$

2nd class $\quad Gf_3(q^2)G^{-1} = -f_3(q^2) \qquad Gg_2(q^2)G^{-1} = +g_2(q^2)$

- ~~(extended) G parity invariance~~ requires $\qquad G = Ce^{-i\pi T_{2,5,7}}$

$f_3(q^2) \neq 0 \qquad g_2(q^2) \neq 0 \qquad$ SU(3) breaking

$|V_{us}|$

Hyperon beta decays (cont'd)

The octet baryons (p,n,Λ,Σ,Ξ) admit various β-type decays.

$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

- hyperon beta decay

$|\Delta s| = 1$ decay rate:

$$\Gamma \approx \frac{G_F^2}{60\pi^3} M_{B'}^5 \delta^5 |V_{us}|^2 R$$

$$R = \left(1 - \frac{3}{2}\delta\right) f_1(0)^2 + \left(3 - \frac{9}{2}\delta\right) g_1(0)^2 - 4\delta g_1(0) g_2(0) + O(\delta^2)$$

$$\delta = \frac{M_{B'} - M_B}{M_{B'}}$$

Hyperon beta decays (cont'd)

The octet baryons (p,n,Λ,Σ,Ξ) admit various β-type decays.

$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

- hyperon beta decay

$|\Delta s| = 1$ decay rate:

neglecting higher orders of δ

assuming $g_2(0) = 0$

$$\Gamma \approx \frac{G_F^2}{60\pi^3} M_{B'}^5 \delta^5 |V_{us}|^2 \left(1 - \frac{3}{2}\delta\right) |f_1(0)|^2 \left[1 + 3\left|\frac{g_1(0)}{f_1(0)}\right|^2\right]$$

① measure semileptonic decay width

② measure $g_1(0)/f_1(0)$

③ $f_1(0)$ from theory → $f_1(0)_{SU(3)}$ value

Theoretical uncertainties of $f_1(0)$

		$\Lambda \to p$	$\Sigma^- \to n$	$\Xi^- \to \Lambda$	$\Xi^0 \to \Sigma^+$
$\tilde{f}_1 = \lvert f_1/f_1^{\mathrm{SU}(3)}\rvert$	DHK'87 (quark model)	0.987	0.987	0.987	0.987
	Sch'95 (quark model)	0.976	0.975	0.976	0.976
	Kr'90 (HBChPT)	0.943	0.987	0.957	0.943
	AL'93 (HBChPT)	1.024	1.100	1.059	1.011
	V'06 (HBChPT)	1.027	1.041	1.043	1.009
	FMJM'98 (large N_c)	1.02(2)	1.04(2)	1.10(4)	1.12(5)
$\lvert\tilde{f}_1 V_{us}\rvert$		0.2221(33)	0.2274(49)	0.2367(97)	0.209(27)

HBD (ave.): $|V_{us}|$ = 0.2250(27) with f_1=1

- quark model: $|f_1(0)| < |f_1(0)_{\mathrm{SU}(3)}| \Longrightarrow V_{us} \nearrow$
- HBChPT, large N_c: $|f_1(0)| > |f_1(0)_{\mathrm{SU}(3)}| \Longrightarrow V_{us} \searrow$

cf. CKM unitarity prediction: $|V_{us}|_{\mathrm{unitary}}$ = 0.2271(21)

Proton spin problem

Hyperon beta decays (cont'd)

The octet baryons (p,n,Λ,Σ,Ξ) admit various β-type decays.

$$B' \to B + l^{\pm} + \nu_l(\bar{\nu}_l)$$

- hyperon beta decay

✓ *Vital information about strange quark spin fraction of proton*

$$\Delta\Sigma(= \Delta u + \Delta d + \Delta s)_{\rm Expt.} = 0.213 \pm 0.138$$

$$\left.\begin{array}{l} (g_A/g_V)_{np} = \Delta u - \Delta d \\ (g_A/g_V)_{\Lambda p} = (2\Delta u - \Delta d - \Delta s)/3 \\ (g_A/g_V)_{\Xi\Sigma} = (\Delta u + \Delta d - 2\Delta s)/3 \\ (g_A/g_V)_{\Sigma n} = \Delta d - \Delta s \end{array}\right\} \text{Assumption : SU(3) symmetry}$$

Jaffe's talk

Hyperon beta decays (cont'd)

The octet baryons (p,n,Λ,Σ,Ξ) admit various β-type decays.

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- hyperon beta decay

✓ *Vital information about strange quark spin fraction of proton*

$$\Delta\Sigma(= \Delta u + \Delta d + \Delta s)_{\rm Expt.} = 0.213 \pm 0.138$$

$$\Delta s = -0.124 \pm 0.046$$

The hidden uncertainty of Δs coming from unknown SU(3) breaking in hyperon beta decays.

Jaffe's talk

What's to be measured ?

- For proton spin problem
  - ➡ 1st order of SU(3) breaking effect on $g_1(0)/f_1(0)$
  - ➡ nonzero 2nd-class form factors $f_3(q^2)$, $g_2(q^2)$
- For precise determination of $|V_{us}|$
  - ➡ 2nd order of SU(3) breaking effect on $f_1(0)$

Hyperon beta decays (Expt.)

$B' \to Bl\nu$	$f_1^{SU(3)}$	g_1/f_1 (Exp.)	$(g_1/f_1)^{SU(3)}$
$n \to p$	1	1.2670 ± 0.0030	$F+D$
$\Lambda \to p$	$-\frac{\sqrt{6}}{2}$	0.718 ± 0.015	$F+\frac{1}{3}D$
$\Xi^- \to \Lambda$	$\frac{\sqrt{6}}{2}$	0.25 ± 0.05	$F-\frac{1}{3}D$
$\Sigma^- \to n$	-1	-0.340 ± 0.017	$F-D$
$\Xi^0 \to \Sigma^+$	1	1.32 ± 0.21	$F+D$
$\Xi^- \to \Xi^0$	-1	N/A	$F-D$

$g_V = \lim_{q^2 \to 0} f_1(q^2) \quad g_A = \lim_{q^2 \to 0} g_1(q^2)$

Hyperon beta decays (Expt.)

	$B' \to Bl\nu$	$f_1^{SU(3)}$	g_1/f_1 (Exp.)	$(g_1/f_1)^{SU(3)}$
✔	$n \to p$	$\boxed{1}$	1.2670 ± 0.0030	$\boxed{F+D}$
	$\Lambda \to p$	$-\frac{\sqrt{6}}{2}$	0.718 ± 0.015	$F + \frac{1}{3}D$
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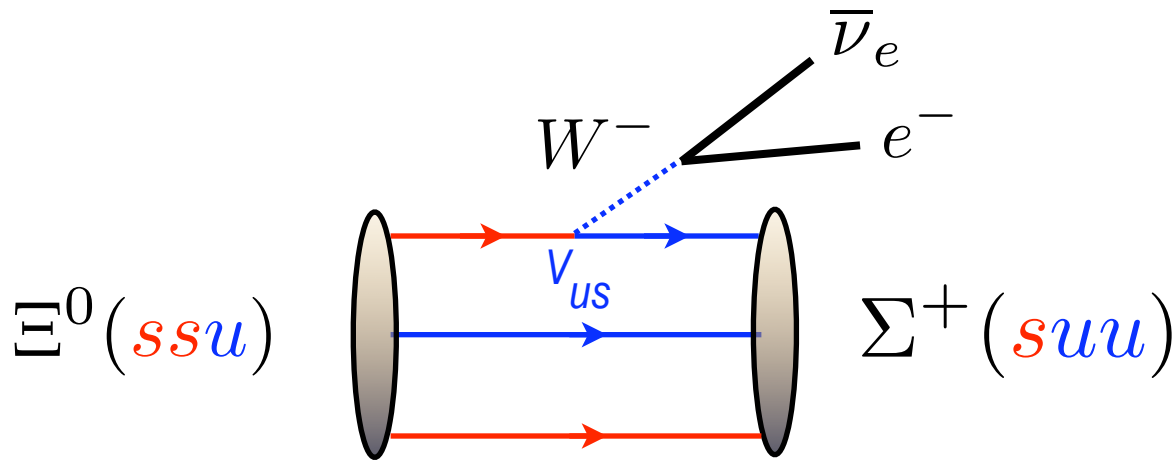
✔	$\Xi^0 \to \Sigma^+$	$\boxed{1}$	1.32 ± 0.21	$\boxed{F+D}$
	$\Xi^- \to \Xi^0$	-1	N/A	$F - D$

$g_V = \lim_{q^2 \to 0} f_1(q^2) \quad g_A = \lim_{q^2 \to 0} g_1(q^2)$

$\Xi^0 \to \Sigma^+$ is the direct analogue of $n \to p$ under $d \leftrightarrow s$

Hyperon beta decay ($\Xi^0 \to \Sigma^+$)

$\Xi^0 \to \Sigma^+$ is the direct analogue of $n \to p$ under $d \leftrightarrow s$



highly sensitive to SU(3) breaking

Center of mass correction approach (Ratcliffe)

$(g_A/g_V)_{np} > (g_A/g_V)_{\Xi\Sigma}$ 8-10%

$1/N_c$ expansion approach (Flores-Mendieta-Jenkins-Manohar)

$(g_A/g_V)_{np} > (g_A/g_V)_{\Xi\Sigma}$ 20-30%

Summary on $\Xi^0 \to \Sigma^+$ (expt.)

- First and Single experiment at KTeV@FNAL
  - $g_1 / f_1 = 1.17 \pm 0.28(\text{stat}) \pm 0.05(\text{syst})$
  - $g_2 / f_1 = -1.7 \pm 2.0(\text{stat}) \pm 0.5(\text{syst})$
    - ➡ no evidence for a nonzero value of the g_2 form factor (2nd-class)

✓ Assumming $g_2 / f_1 = 0$ $\qquad$ $n \to p$: $g_1 / f_1 = 1.2670(35)$

- $g_1 / f_1 = 1.32 \pm 0.21(\text{stat}) \pm 0.05(\text{syst})$
  - ➡ no indication of flavor SU(3) breaking effects $\dfrac{(g_A/g_V)_{\Xi\Sigma}}{(g_A/g_V)_{np}} \sim 1.04$

- Nf0: DWF-DBW2 at beta=0.87 (a^{-1}=1.3GeV)
  - 16^3x32x16 (L=2.4 fm): 377 statistics
  - m_l=0.04, 0.05, 0.06 (M_π=0.53, 0.60, 0.65 GeV)
  - fixed "strange" quark masses at m_s=0.08 (0.10)
  - neutron beta decay is well reproduced
    - g_A/g_V = 1.212 (27) Sasaki-Orginos-Ohta-Blum, PRD68 (03) 054509

Exploratory study

- ✳ Explore SU(3) breaking effect on $g_1(0)/f_1(0)$
- ✳ Extract the 2nd-class form factors (g_2 and f_3)
  - ✓ SU(3) breaking = existence of non-zero g_2 and f_3
- ✳ Measure 2nd order of SU(3) breaking on $f_1(0)$
  - ✓ theoretical uncertainties in evaluation of $|V_{us}|$

SU(3) breaking effect on g_1 / f_1 (g_A /g_V)

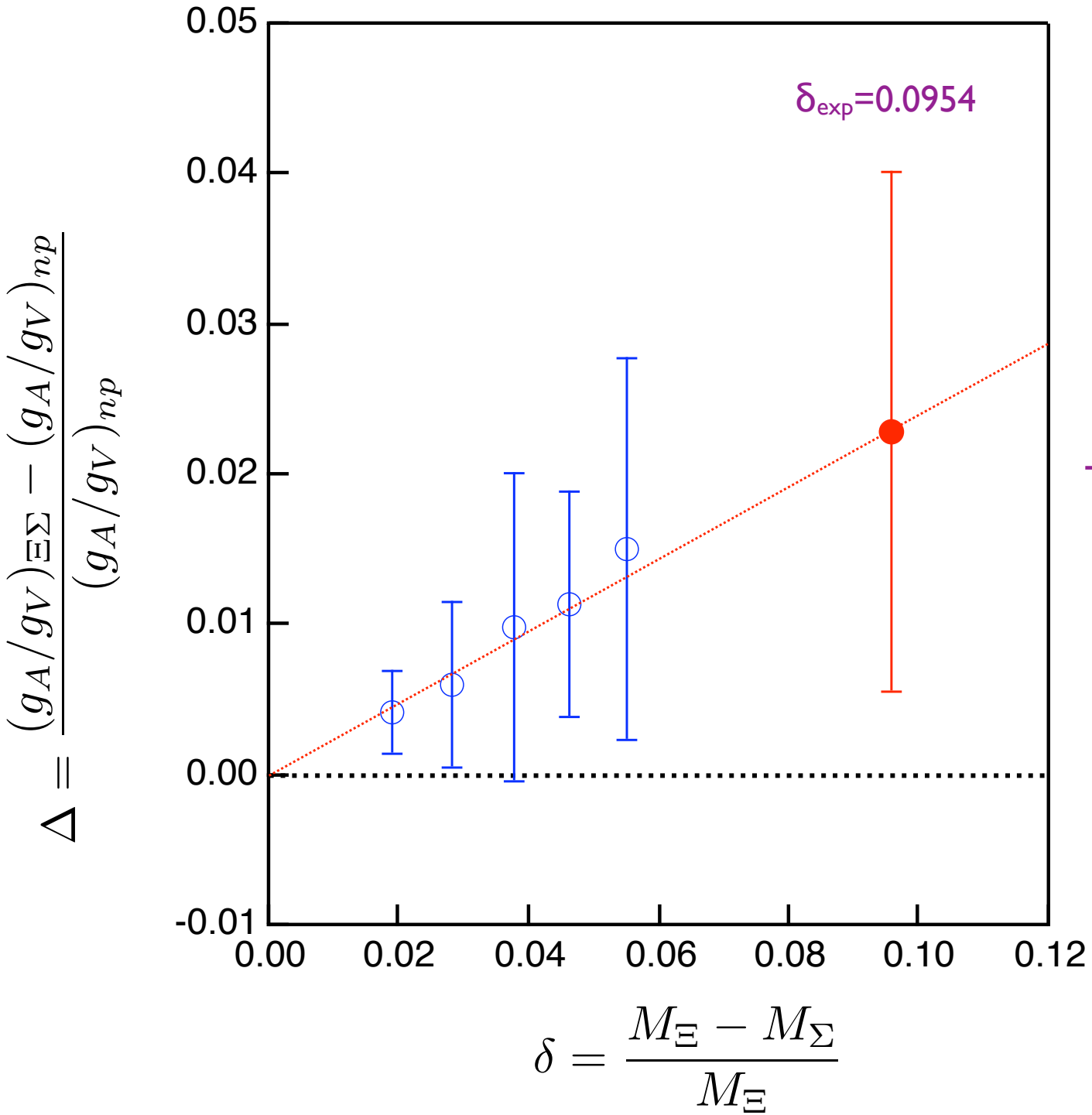
- Consider the following double ratio at the rest frame (p,k=0)

$$\frac{\langle \Sigma(t') A_3(t) \overline{\Xi}(0) \rangle}{\langle \Sigma(t') V_4(t) \overline{\Xi}(0) \rangle} \frac{\langle p(t') V_4(t) \overline{n}(0) \rangle}{\langle p(t') A_3(t) \overline{n}(0) \rangle} = \frac{g_1(q^2_{\max}) - \delta g_2(q^2_{\max})}{f_1(q^2_{\max}) + \delta f_3(q^2_{\max})} \left(\frac{f_1(0)}{g_1(0)} \right)_{\mathrm{SU}(3)}$$

$$= \frac{g_1(0)/f_1(0)}{(g_1(0)/f_1(0))_{\mathrm{SU}(3)}} + \mathcal{O}(\delta^2)$$

1st order symmetry breaking

where $\delta = \frac{M_\Xi - M_\Sigma}{M_\Xi}$ $\quad q^2_{\max} = -(M_\Xi - M_\Sigma)^2$



$$\Delta = 0.023(17)$$

Theoretical expectation

$$\Delta \propto \delta$$

1st order symmetry breaking

5 data points

$(m_l, m_s) =$ (0.04, 0.08)
(0.05, 0.08)
(0.06, 0.08)
(0.04, 0.10)
(0.05, 0.10)

SU(3) breaking effect on g_1 / f_1

✓ Preliminary result (quenched lattice QCD)

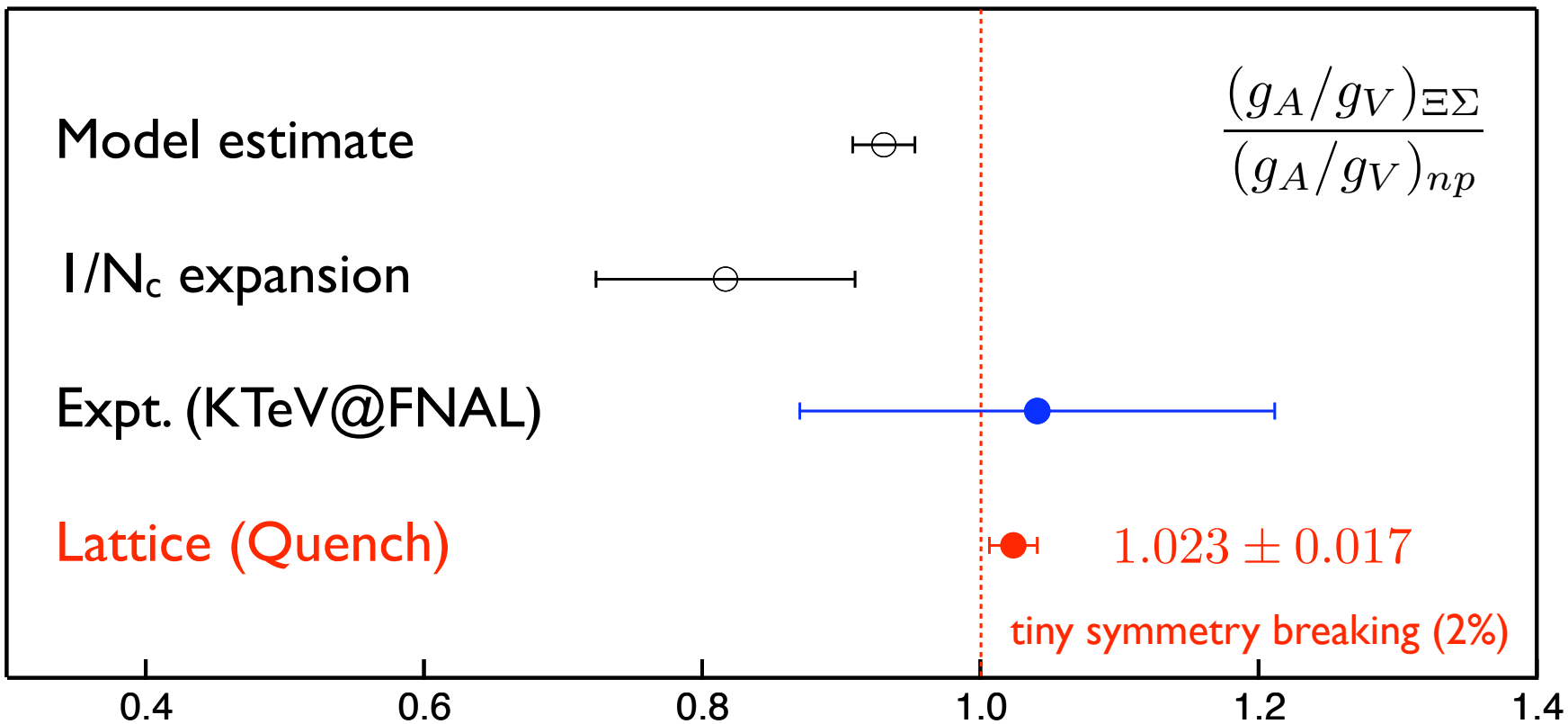
$$\frac{(g_A/g_V)_{\Xi\Sigma}}{(g_A/g_V)_{np}} = 1.023 \pm 0.017$$

2% (tiny symmetry breaking)

✳ KTeV@FNAL

$$\frac{(g_A/g_V)_{\Xi\Sigma}}{(g_A/g_V)_{np}} = 1.04 \pm 0.17(\mathrm{stat}) \pm 0.04(\mathrm{syst})$$

SU(3) breaking effect on g_1 / f_1 (g_A/g_V)



* Center-of-mass correction approach (with a bag model)
  Ratcliffe, Phys. Rev. D59 (1999) 014038
* 1/Nc expansion approach
  R. Flores-Mendieta, E. Jenkins, A.V. Manohar, Phys. Rev. D58 (1998) 094028

Three point functions

$B_1 \to B_2$

$$\langle \mathcal{B}_2(t')O(t)\overline{\mathcal{B}_1}(0)\rangle \propto \int dp \int dk\; e^{-E_2(p)(t'-t)}\langle \mathcal{B}_2|p\rangle\langle p|O|k\rangle\langle k|\overline{\mathcal{B}_1}\rangle e^{-E_1(k)t}$$

$$= G_{\mathcal{O}}(q) \times f(t, t', E_1(k), E_2(p))$$ where $q = p - k$

Vector $\vec{k} \neq \vec{0},\ \vec{p} = \vec{0}$

$$\mathrm{Tr}\mathcal{P}^t G_{V_t}(q) = M_2(E_1 + M_1)\left(f_1(q^2) - \frac{E_1 - M_1}{M_1} f_2(q^2) + \frac{E_1 - M_2}{M_1} f_3(q^2)\right)$$

$$\mathrm{Tr}\mathcal{P}^t G_{V_x}(q) = iq_x M_2\left(f_1(q^2) - \frac{E_1 - M_2}{M_1} f_2(q^2) + \frac{E_1 + M_1}{M_1} f_3(q^2)\right)$$

$$\mathrm{Tr}\mathcal{P}^{xy} G_{V_x}(q) = iq_y M_2\left(f_1(q^2) + \frac{M_1 + M_2}{M_1} f_2(q^2)\right)$$

Projection operator: $\mathcal{P}^{xy} = \frac{1}{4}\frac{1+\gamma_t}{2}\gamma_y\gamma_x$ $\quad \mathcal{P}^t = \frac{1}{4}\frac{1+\gamma_t}{2}\gamma_t$

Three point functions

$B_1 \to B_2$

$$\langle \mathcal{B}_2(t')O(t)\overline{\mathcal{B}_1}(0)\rangle \propto \int dp \int dk\; e^{-E_2(p)(t'-t)}\langle \mathcal{B}_2|p\rangle\langle p|O|k\rangle\langle k|\overline{\mathcal{B}_1}\rangle e^{-E_1(k)t}$$

$$= G_{\mathcal{O}}(q) \times f(t, t', E_1(k), E_2(p))$$ where $q = p - k$

Axial-vector $\vec{k} \neq \vec{0},\ \vec{p} = \vec{0}$

$$\mathrm{Tr}\mathcal{P}^z G_{A_z}(q) = M_2(E_1 + M_1)\left(g_1(q^2) - \frac{q_z^2 + (E_1 + M_1)(M_1 - M_2)}{M_1(E_1 + M_1)} g_2(q^2) + \frac{q_z^2}{M_1(E_1 + M_1)} g_3(q^2)\right)$$

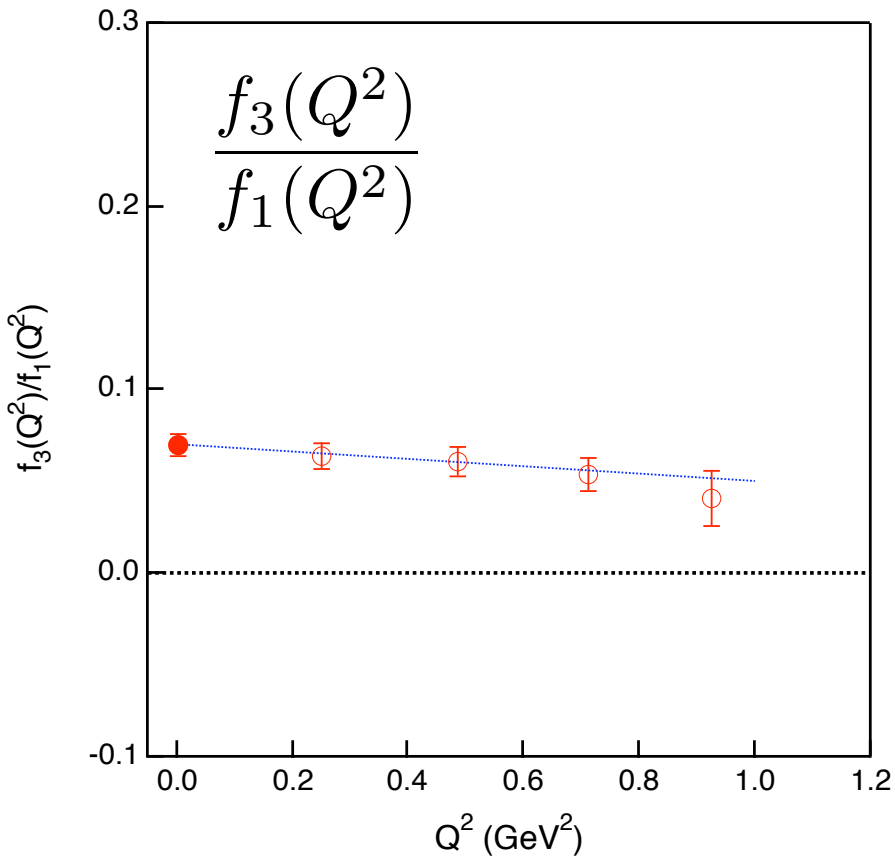
$$\mathrm{Tr}\mathcal{P}^z G_{A_x}(q) = \frac{q_x q_z M_2}{M_1}\left(-g_2(q^2) + g_3(q^2)\right)$$

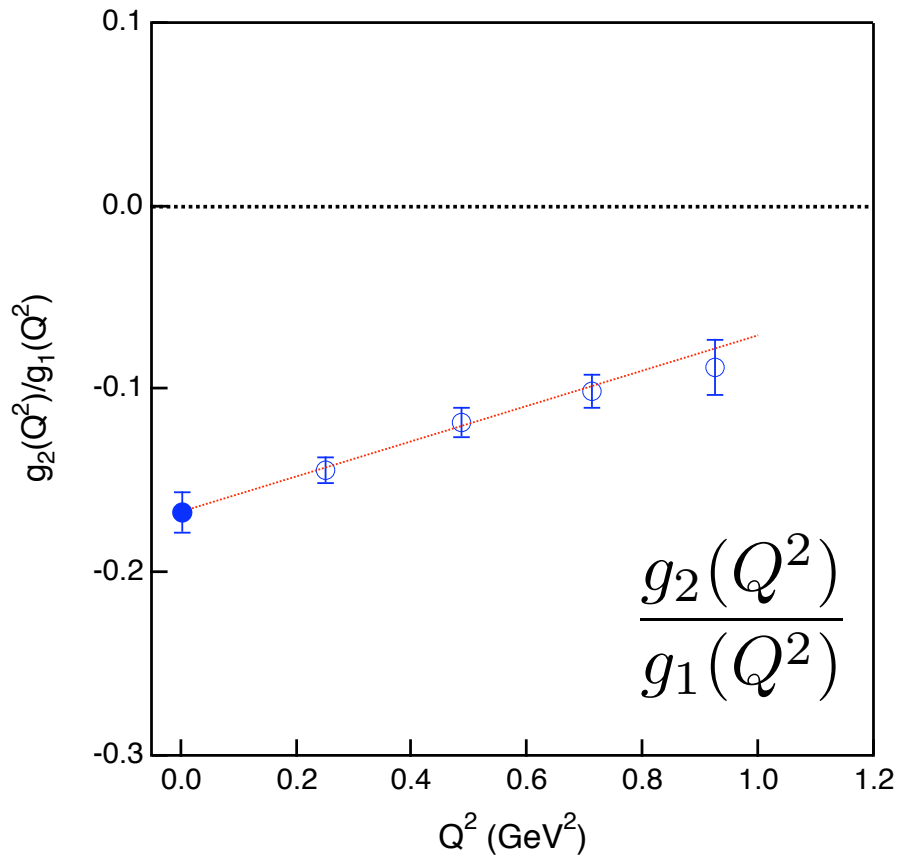
$$\mathrm{Tr}\mathcal{P}^z G_{A_t}(q) = q_z M_2\left(g_1(q^2) - \frac{E_1 + M_1}{M_1} g_2(q^2) + \frac{E_1 - M_2}{M_1} g_3(q^2)\right)$$

Projection operator: $\mathcal{P}^z = \frac{1}{4}\frac{1 + \gamma_t}{2}\gamma_z$

2nd-class form factors

m_l=0.05, m_s=0.08

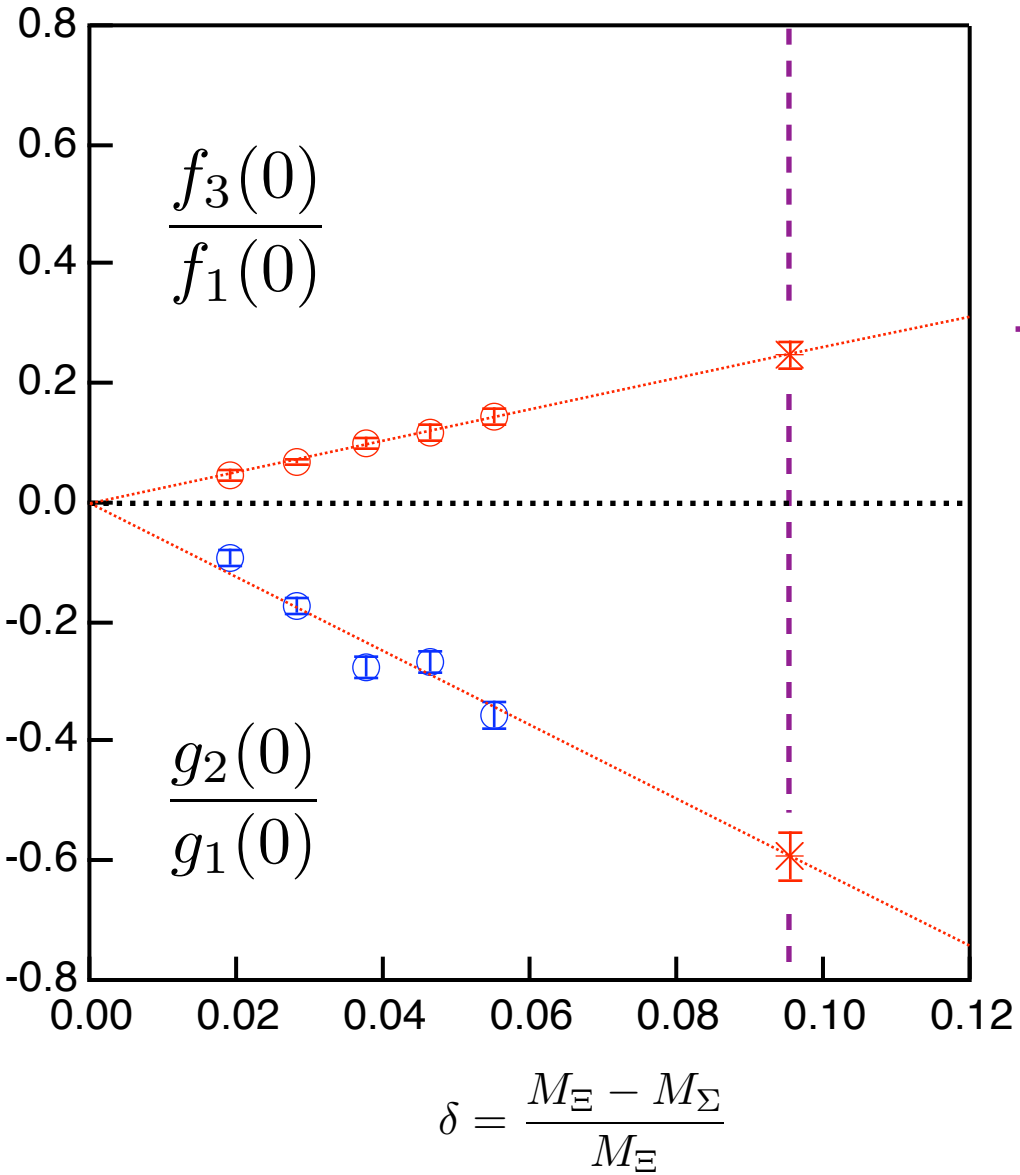




SU(3) breaking effects

$$\delta = \frac{M_\Xi - M_\Sigma}{M_\Xi} = 0.0281(9)$$

2nd-class form factors



δ=0.0945 (exp.)

Theoretical expectation

$$f_3(q^2), g_2(q^2) \propto \delta$$

1st order symmetry breaking

5 data points

(m_l, m_s) = (0.04, 0.08)
(0.05, 0.08)
(0.06, 0.08)
(0.04, 0.10)
(0.05, 0.10)

2nd-class form factors

✓ Lattice results (quenched lattice QCD)

$$\frac{g_2(0)}{f_1(0)} = -0.757 \pm 0.057 \qquad \frac{f_3(0)}{f_1(0)} = +0.250 \pm 0.022$$

✳ KTeV@FNAL

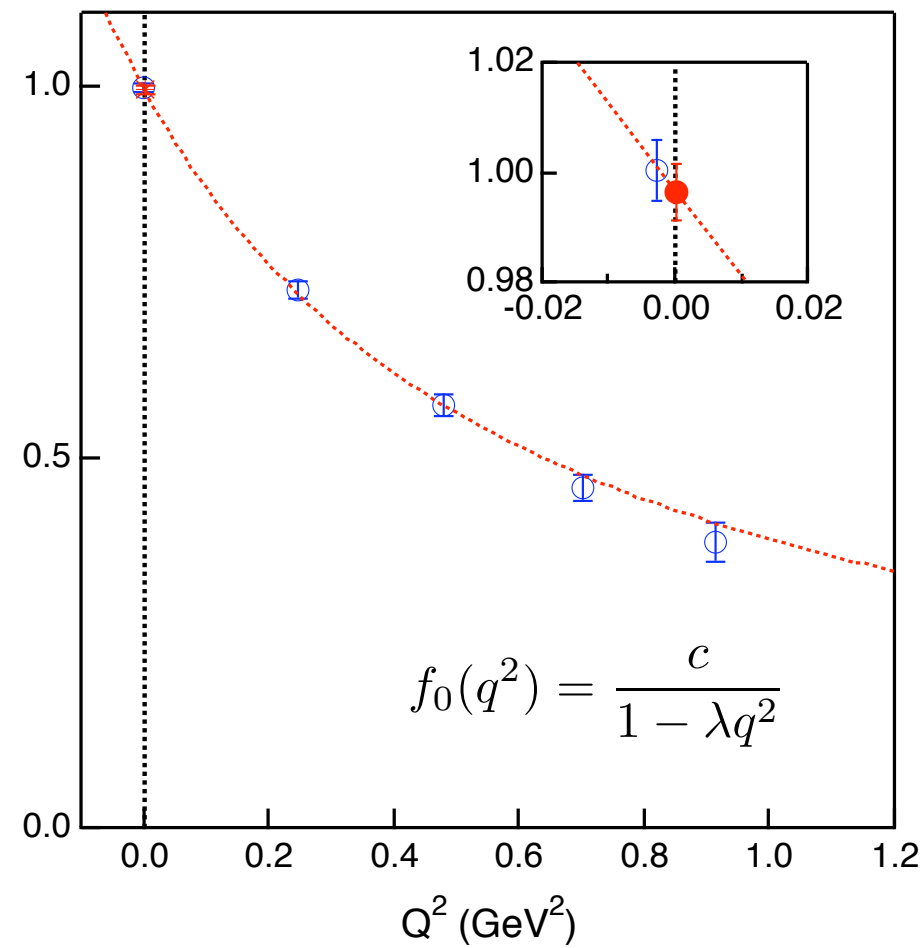
$$\frac{g_2(0)}{f_1(0)} = -1.7 \pm 2.0 \pm 0.5 \qquad \frac{f_3(0)}{f_1(0)} = N/A$$

Vector form factor $f_0(q^2)$

- Renormalization
  - $Z_V^{su} = \sqrt{Z_V^{uu} Z_V^{ss}}$
  - utilize CVC relation
- Calculate

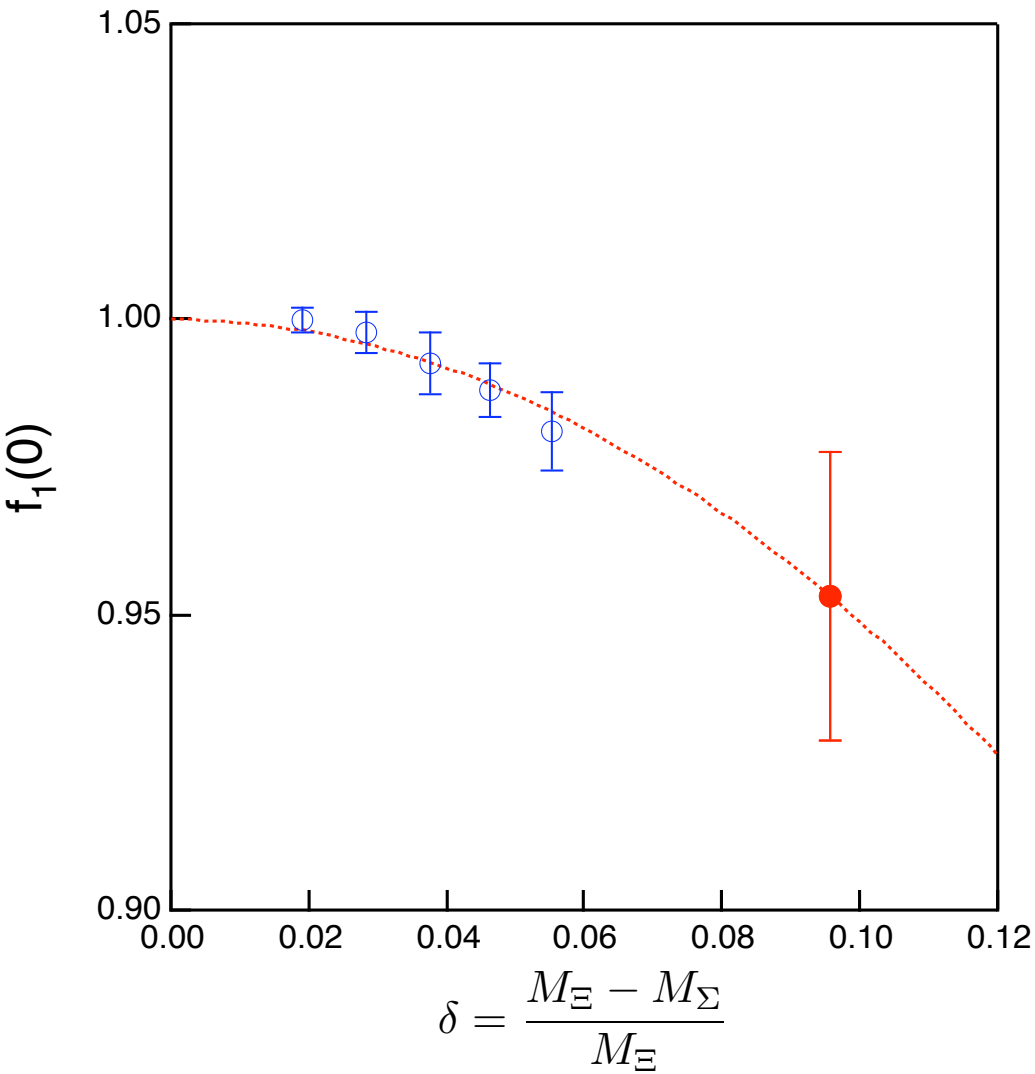
$$f_0(q^2) = f_1(q^2) + \delta f_3(q^2)$$

- Interpolate it to $q^2=0$
  - polar form
- Combined with $f_3(0)$



$$f_1(0) = f_0(0)\left(1 + \delta \frac{f_3(0)}{f_1(0)}\right)^{-1} \qquad 1 + \delta \frac{f_3(0)}{f_1(0)} = 1.024(2)$$

SU(3) breaking effect on $f_1(0)$



Ademollo-Gatto theorem

$$|1 - f_1(0)| \propto \delta^2$$

2nd order symmetry breaking

$$f_1(0) = 0.953(24)$$

Combined with

$$|f_1(0) V_{us}| = 0.209(27) \text{ (KTeV)}$$

$$|V_{us}|_{\Xi^0 \to \Sigma^+} = 0.219(27)_{\text{expt}}(5)_{\text{theory}}$$

Summary of $|V_{us}|$

✓ Kl3 decay: $f_+(0) = 0.960(5)_{\rm stat}(7)_{\rm syst}$ 2 flavor DWF

✓ Hyperon decay: $f_1(0) = 0.953(24)_{\rm stat}$ quench DWF

$$|V_{us}|_{K_{l3}} = 0.2245(8)_{\rm expt}(26)_{\rm theory}$$

$$|V_{us}|_{\Xi^0 \to \Sigma^+} = 0.219(27)_{\rm expt}(5)_{\rm theory}$$

$$|V_{us}|_{\rm unitarity} = 0.2274(21)$$

agreement with the CKM unitarity

Summary of SU(3) breaking effects

- $f_1(0)$ has about 5% negative breaking effects
- $f_2(0)$ has about 20% positive breaking effects
- $f_3(0)$ is large positive: about 1/4 of $f_1(0)$

  1st-order breaking (20-25%), 2nd-order breaking (~5%)

- $g_2(0)$ is large negative: about 3/5 of $g_1(0)$
- $g_1(0)/f_1(0)$ is suprisingly tiny breaking around 2%
  - This doesn't conflict with Cabibbo-model fits for HBD
  - However, this doesn't mean that an estimation of Δs is reliable.

Summary/Outlook

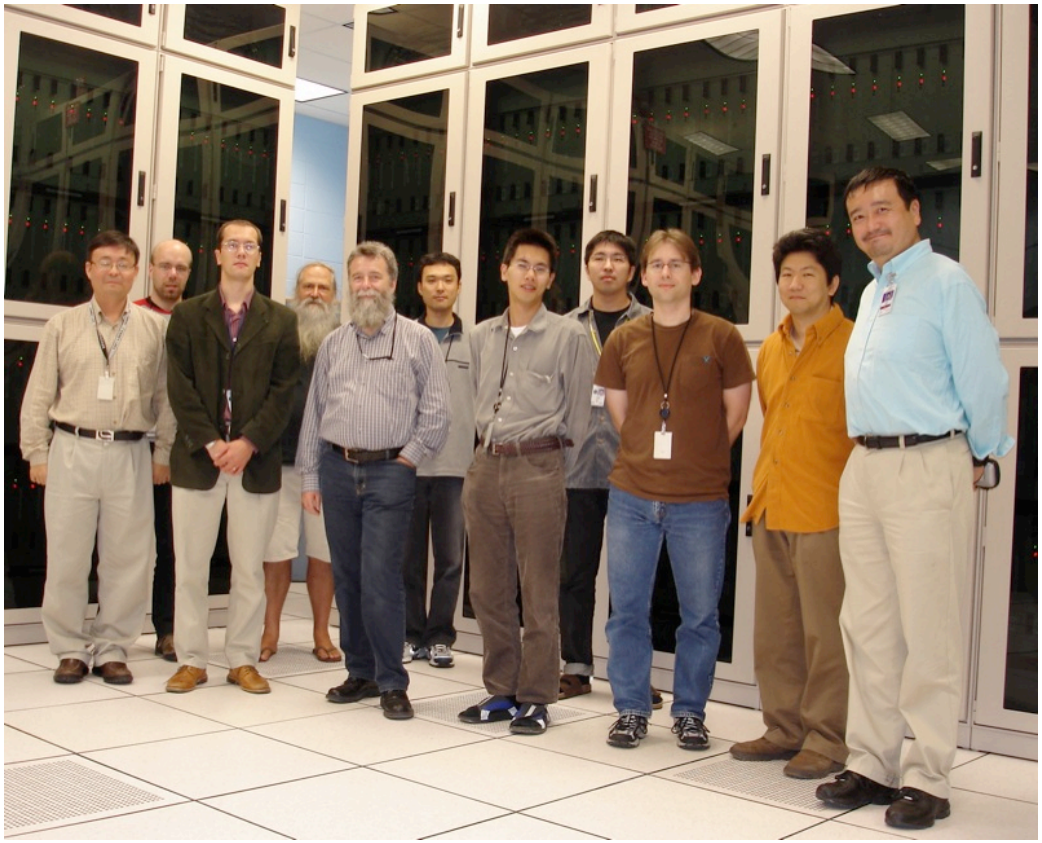
✳ The computation of weak matrix elements in lattice QCD is now progressing with steadily increasing accuracy by utilizing domain wall fermions (DWF).

- DWF has a big advantage in dealing with the axial symmetry
- It is easy to determine Z-factors for V(A) local currents

✓ Lattice studies of Kl3 and HBD are highly required for

- precise determination of $|V_{us}|$ and stringent test of the CKM unitarity
- exhibiting the pattern of SU(3) violation
- judging the validity of nucleon spin decomposition through HBD data

Generating 2+1 flavor DWF configurations



QCDOC with RBRC & BNL Lattice theorists

Large scale production run

- DWF + Iwasaki gauge action
- Lattice cutoff: $a^{-1} \sim 1.6$ GeV ($\beta = 2.13$, $c_1 = -0.331$)
- Box size: $V = 24^3 \times 64 \times 16$

  $L \sim 2.9$ fm
- $m_{light} = 1/4, 1/2, 3/4$ of $m_{strange}$

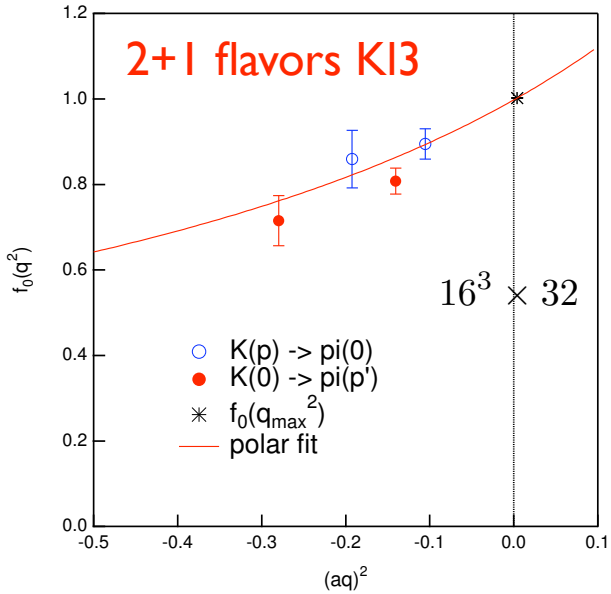
  $M_\pi \sim 350, 500, 750$ MeV

in collaboration with Columbia, UKQCD

Summary/Outlook

Our goal:

2+1 flavors DWF calculation (RBC+UKQCD) is promising for theoretical research on the SU(3) breaking effects in semi-leptonic decays.



in collaboration with UKQCD