Electromagnetic splittings of Hadrons, and calculations towards $g_\mu - 2$ light-by-light contribution.

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based on a collaborations with [RBC, RBC-UKQCD collaboration]

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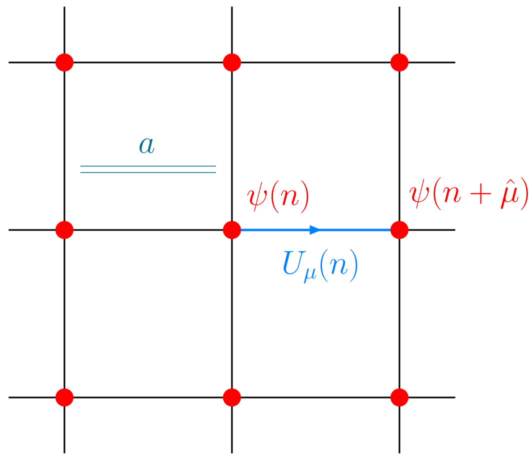
Lattice QCD

- Quantum Chromo Dynamics : strong coupling, needs non-perturbative calculation.

  $\Psi(x), A_\mu(x), x \in \mathcal{R}^4$: *continuous infinity*

  quantum divergences: needs regularization and renormalization



- Discretize Euclidean space-time
- lattice spacing $a \sim 0.1$ fm
  (UV cut-off $|p| \leq \pi/a$)

- $\psi(n)$: Quark field (Grassmann number)
- $U_\mu(n)$: Gluon field

- Accumulate samples of QCD vacuum, typically $\mathcal{O}(100) \sim \mathcal{O}(1,000)$ files of gluon configuration $U_\mu(n)$ on disk.

- Then measure physical observables on the vacuum ensemble.

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U_\mu \ \mathrm{Prob}[U_\mu] \times \mathcal{O}[U_\mu]$$

- A part of shorter fluctuation is taken care either perturbatively or by techniques called non-perturbative renormalization.

Taking Limits

- repeat the procedure for the three parameters,

  V : space-time Volume
  m_f : mass of quarks
  β : coupling among quarks and gluons

  A. Thermodynamic limit $V \to \infty$

  $\langle O \rangle |_V \sim const. \quad (V \gg V_c(m_f) \sim 1/m_{ps}^4)$,
  m_{ps} is the lightest hadron(pion) mass.

  B. Quark mass, $m_f \to m_q(phys) \sim 10$ MeV

  needs to extrapolate from $m_f > m_q(phys)$ because smaller m_f needs larger V and more computational power to solve the Dirac equation.
  chiral perturbation theory (ChPT) helps.

  C. Continuum limit, $a \to 0$ to eliminate discretization errors

  $\beta = 6/g^2 \to \infty$ à la asymptotic freedom $a\Lambda_L = \exp(-\frac{1}{2b_0 g^2}) \cdots$.
  $\langle O \rangle = O_{cont.} + c_n a^n + \cdots$.
  For DWF $n = 2$ not 1. (chiral symmetry).
  Each lattice action has different discretization error, useful for estimation of the discretization error by comparing among various lattice fermion/gauge actions.

  to get the final answer.

Use of chiral symmetry on lattice

- Spontaneous breaking of chiral symmetry, $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R + \bar{q}_R q_L \rangle \neq 0$.
  NG boson, $M_\pi^2 \propto m_q$. 99% origin of mass of constituent quark or proton.

- massless quark $q(x)$

$$S_f = \bar{q} \not{D} q = \bar{q}_L \not{D} q_L + \bar{q}_R \not{D} q_R$$

$$\text{symmetry:} \quad q_L \rightarrow e^{i\theta_L} q_L, q_R \rightarrow e^{i\theta_R} q_R$$

  Left and Right handed fermions are independently moving.

- Old lattice fermions ($e.g.$ Wilson fermion, staggered fermion) break chiral and flavor symmetries: Domain Wall Fermions (DWF) has exact flavor symmetry and a good chiral symmetry (Kaplan, Furman & Shamir, Blum & Soni)

- discretization error should be small.(lattice spacing, $a > 0$)
  No local operator with dimension five preserving chiral symmetry.
  $\mathcal{O}_5 = F_{\mu\nu}\bar{q}\sigma_{\mu\nu}q, \bar{q}D^2q$
  $\mathcal{L}_{lat} = Z\mathcal{L}_{cont.} + a^2 O_6 + \cdots$
  O(a) error is suppressed. Results on relatively coarse lattice (large a ,smaller computational cost) is much closer to the continuum limit: $a\Lambda_{QCD} \sim 0.1$

- unphysical operator mixing is prohibited by χ-sym.

Full QCD (including dynamical quarks)

- Entering Era of unquenched simulations ([Shinya's talk])

$$\text{Prob}[U_\mu] \propto \det \not{D} e^{-S_g},$$

$$\text{quench: } \det \not{D} \to 1 \text{ (or sometimes } \det \not{D}' \neq \det \not{D})$$

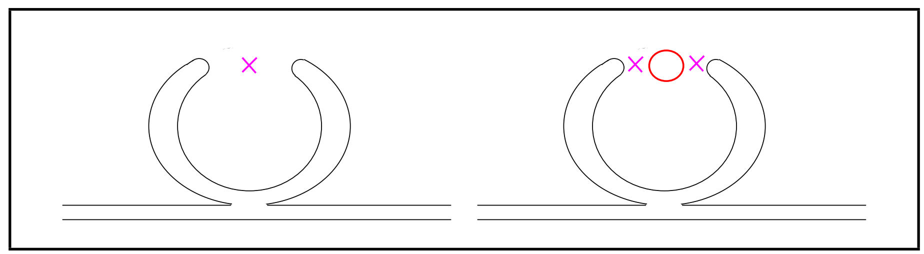
  Ignoring quark loops (sea quark loop) in QCD vacuum, and only using the external quarks (valence quarks) representing hadrons.

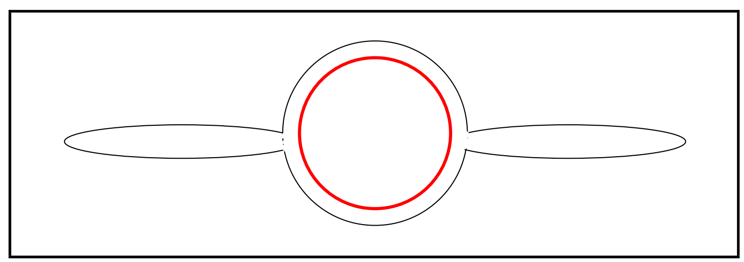
- This approximation causes the quenched pathologies.

  - Lack of Unitarity.
  - quenched chiral divergences (η' loops):

    $$M_\pi^2 = 2B_0 m_q [1 - 2\delta \ln(m_f)]$$

    $\delta \propto m_f$ in Full QCD.
  - can't decays.
    $e.g.\ \rho \to \pi\pi$:





- quark mass with less than $\sim \Lambda_{QCD}$ should play a significant role :$N_F = 2 + 1$.

Unitarity violation in Non-singlet scalar meson (a_0)

- Point to point propagator of non-singlet scalar meson, $C_{a_0}(t)$, was found to be negative in quenched QCD, which is a clear signal of the unitarity violation in quenched QCD.

- In the language of mesons (ChPT),

  $$a_0 \longrightarrow \eta' + \pi \longrightarrow a_0$$

  η' has double pole in (partially) quenched QCD.

  This contribution was argued to give a negative contribution (also finite size effect), and predicted using Quenched ChPT in finite volume. (Bardeen *et. al*)

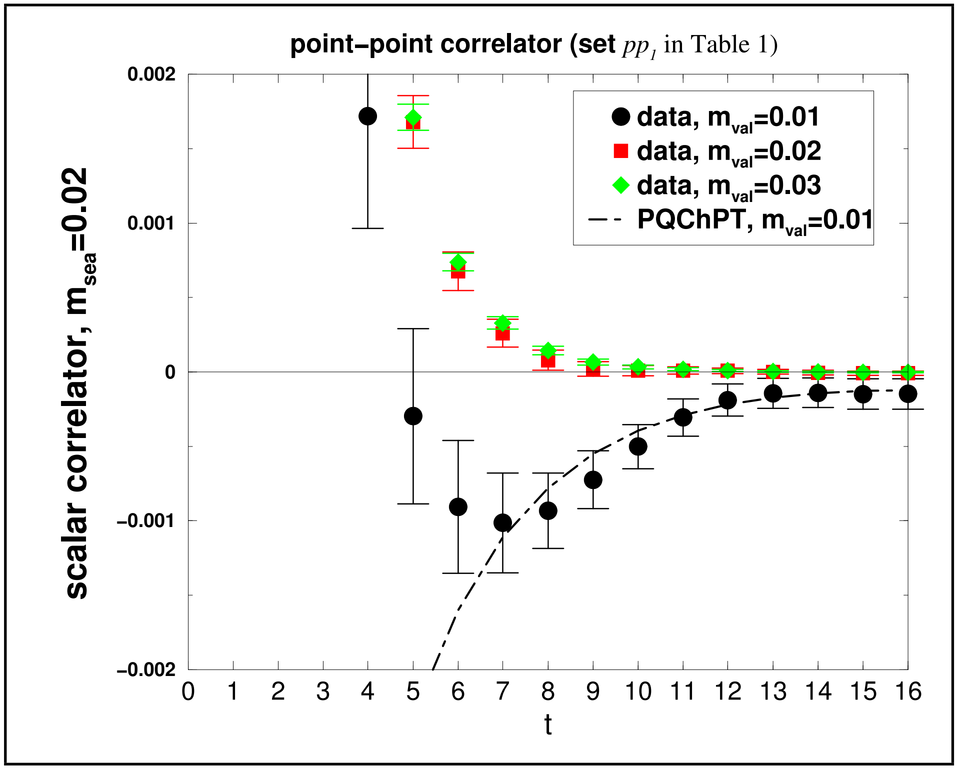
(S. Prelovsek, C. Dawson, T.I. K. Orginos, A. Soni (RBC))

- By fixing m_{sea} and changing m_{val} we found

$$C_{a_0}(t) < 0 \qquad (m_v < m_s)$$

$$C_{a_0}(t) > 0 \qquad (m_v \geq m_s)$$

- This behaviour could be understood by Partially Quenched ChPT also.

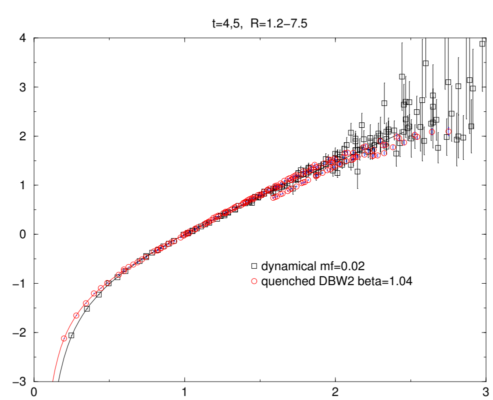


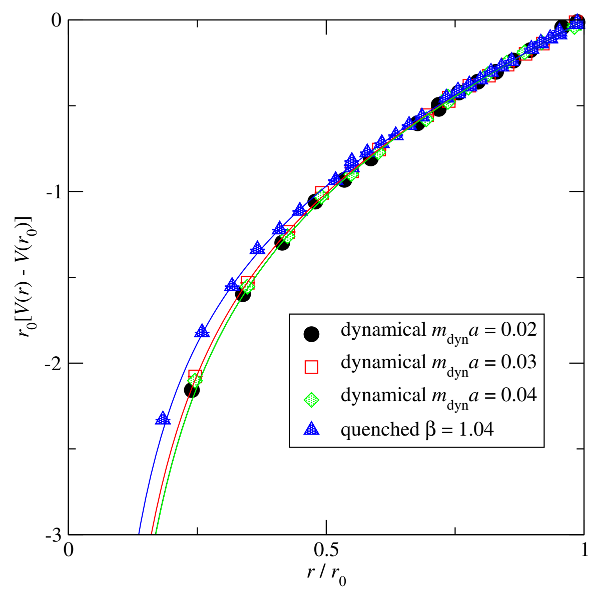
Dynamical quark effects

Quenching error (dynamical quark effect) is not a minor issue.

Other quantities very sensitive to dynamical quarks

- $I = 0$ $\pi\pi$ scattering length (M. Golterman, T.I, Y. Shamir)
- Nucleon-Nucleon potential
- Static quark potential (K. Hashimoto)
  In shorter distance, $r\Lambda_{QCD} \ll 1$, coupling is weaker for $N_F = 2$:
  $\alpha_S(r; N_F = 0) < \alpha_S(r; N_F = 2)$.
  asymptotic freedom $b_0 = (33 - 2N_F)/2$





Dynamical simulations 2006

Improvements ($\times 6 \sim$) in algorithms. ([Norman's talk])

Entering Era of Dynamical simulations. ([Shinya's talk])

various Lattice quarks

- DWF $N_F = 0$, ($\sim$ 2001 QCDSP) $N_F = 2$ (2001 $\sim$ 2005, QCDSP), $N_F = 2+1$ (QCDOC 2004$\sim$) ((BNL, RBRC) , Tsukuba, J-Lab) $\mathcal{O}(a\Lambda_{QCD})^2 + a\mathcal{O}(am_{res}) \sim \mathcal{O}(1\%)$, $m_q/m_s \lessapprox\sim 0.3 \rightarrow 0.2, 0.1$.

- staggered $N_F = 0, 2+1$: leading runner, many productions & publications academic question : $\sqrt[4]{\det \not{D}}$, universal RGT among non-local theories ? $\mathcal{O}(a\Lambda_{QCD})^2$

- 4D Wilson-types (RG-improved clover, twisted mass Wilson) : SAP ([Tomomi's talk])

- dynamical overlap, chirally improved fermions

Motivations

- The first principle calculations of isospin breaking effects due to electromagnetic (EM) and the up, down quark mass difference are necessary for accurate hadron spectrum, quark mass determination.

- EM splittings are measured very accurately :

$$m_{\pi^\pm} - m_{\pi^0} = 4.5936(5)\text{MeV}, \quad m_N - m_P = 1.2933317(5)\text{MeV}$$

- From $\Gamma(\pi^+ \to \mu^+\nu_\mu, \mu^+\nu_\mu\gamma)$ + V_{ud}(exp)

$$f_{\pi^+} = 130.7 \pm 0.1 \pm 0.36\text{MeV} \qquad \text{PDG 2004}$$

  the last error is due to the uncertainty in the part of $\mathcal{O}(\alpha)$ radiative corrections that depends on the hadronic structure of the π meson.

$$\Gamma(PS^+ \to \mu^+\nu_\mu, \mu^+\nu_\mu\gamma) \propto [1 + C_{\text{PS}}\ \alpha]_{\text{had. struc.}}$$
$$C_\pi \sim 0 \pm 0.24, \quad C_\pi - C_K = 3.0 \pm 1.5$$

  c.f. Marciano 2004 : V_{us} from f_π/f_K (MILC) + $\Gamma(\pi_{l2})/\Gamma(K_{l2})$.

Motivations...

- A practice for $g_\mu - 2$ light-by-light calculation.

$$a_\mu^{\text{exp}} = \frac{g_\mu - 2}{2} = 116,592,080(60) \times 10^{-11}$$
$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{had}} + a_\mu^{\text{EW}}, \qquad a_\mu^{\text{new}} \sim \mathcal{O}((m_\mu/M_{new})^2)$$
$$a_\mu^{\text{had}} = a_\mu^{\text{had,LO}} + a_\mu^{\text{had,HO}} + a_\mu^{\text{had,LBL}} \tag{1}$$

$$\begin{aligned}
a_\mu^{\text{exp}} &= 116,592, \quad 080(60) \times 10^{-11} \\
a_\mu^{\text{QED}} &= 116,584, \quad 706(3) \times 10^{-11} + \mathcal{O}(\alpha^4) \\
a_\mu^{\text{had,LO}} &= 6, \quad 963(62)(36) \times 10^{-11} \\
a_\mu^{\text{had,HO}} &= - \quad 100(6) \times 10^{-11} \\
a_\mu^{\text{had,LBL}} &= 134(25) \times 10^{-11} (\text{before}: 86(35) \times 10^{-11}) \\
a_\mu^{\text{EW}} &= 154(1)(2) \times 10^{-11}
\end{aligned}$$

- $a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (220 \pm 100) \times 10^{-11}$ (e^+e^-, A. Vainshtein *et. al.* 2004)

QCD + QED simulations

(T. Blum, T. Doi, M. Hayakawa, T.I., N. Yamada)

- In most of lattice QCD simulations, up and down quarks are treated to have equal mass and effects of electromagnetism (EM) is ignored (Isospin symmetry).
- More realistic first principle calculation is desirable for accurate hadron spectrum and quark mass determination.

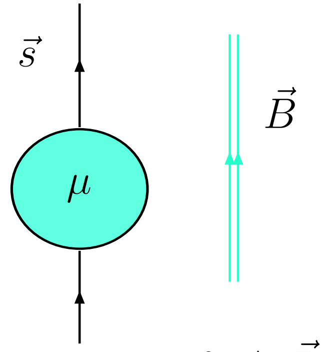
$$m_{up} \neq m_{down}, \qquad Q_{up} = 2/3e, \;\; Q_{down} = -1/3e.$$

- Hadron mass differences due to isospin breaking are measured very accurately in experiments:

$$m_{\pi^\pm} - m_{\pi^0} = 4.5936(5)\text{MeV},$$
$$m_N - m_P = 1.2933317(5)\text{MeV}$$

QCD + QED simulations

- muon anomalous magnetic moment $g_\mu - 2$ (BNL-E821) .
  g_μ gyromagnetic ratio: muon (spin 1/2)'s coupling to magnetic field



$E = -g_\mu \frac{e}{2m_\mu} \vec{s} \cdot \vec{B}$

$$a_\mu^{\text{exp}} = \frac{g_\mu - 2}{2} = 116,592,080(60) \times 10^{-11}$$

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{Had}} + a_\mu^{\text{EW}},$$

$$a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (220 \pm 100) \times 10^{-11}$$

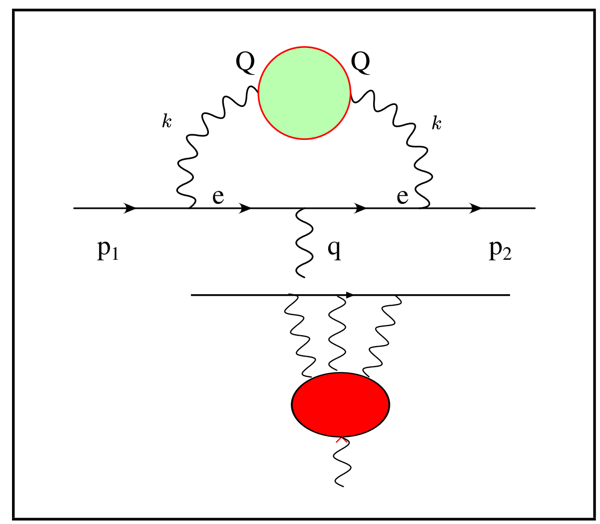
- Hadronic contributions dominates theory error.

$$a_\mu^{\text{Had}} = a_\mu^{\text{Had,LO}} + a_\mu^{\text{Had,HO}} + a_\mu^{\text{Had,LBL}}$$

$$a_\mu^{\text{had,LBL}} = 134(25) \times 10^{-11}$$

$$(\text{before} : 86(35) \times 10^{-11})$$

$$a_\mu^{\text{new}} \sim \mathcal{O}((m_\mu / M_{new})^2)$$



$a_\mu^{\text{Had,HO}}$ was explored by T. Blum in PRL 91, 2003,
C. Aubin & T. Blum new analysis using SChPT.

EM splittings

- Axial WT identity with EM $(N_F = 2, m_u = m_d)$,

$$\mathcal{L}_{EM} = -iA_\mu(x)\bar{q}Q\gamma_\mu q(x), \;\; Q = \frac{e}{3}\left(T^3 + \frac{1}{2}\right),$$

$$\partial_\mu \mathcal{A}_\mu^a(x) = 2mJ_5^a(x) - ieA_\mu(x) f_{3ab}\mathcal{A}_\mu^b$$

neutral current, $\mathcal{A}_\mu^3(x)$, is conserved: π^0 is still a NG boson.

- ChPT with EM at $\mathcal{O}(p^4, p^2e^2)$:

$$M_{\pi^\pm}^2 = 2mB_0 + 2e^2\frac{C}{f_0^2} \qquad +\mathcal{O}(m^2\log m, m^2) + I_0 e^2 m\log m + K_0 e^2 m$$

$$M_{\pi^0}^2 = 2mB_0 \qquad\qquad +\mathcal{O}(m^2\log m, m^2) + I_\pm e^2 m\log m + K_\pm e^2 m$$

Dashen's theorem :
The difference of squared pion mass is independent of quark mass upto $\mathcal{O}(e^2m)$,

$$\Delta M_\pi^2 \equiv M_{\pi^\pm}^2 - M_{\pi^0}^2 = 2e^2\frac{C}{f_0^2} + (I_\pm - I_0)e^2 m\log m + (K_\pm - K_0)e^2 m$$

C is a new low energy constant. $I_\pm, I_0$ is known in terms of C.

EM splittings on lattice

- The correlator for neutral meson is calculated using the interpolation field of the third component of isospin:

$$C_{X^0}(t) = \frac{1}{2}\left[\left\langle J_X^{uu}(t) J_X^{uu\dagger}(0)\right\rangle_{conn} + \left\langle J_X^{dd}(t) J_X^{dd\dagger}(0)\right\rangle_{conn}\right]$$

- Quark mass renormalization due to EM has dependence to renormalization prescription. The counter term for quark mass due to EM:

$$\frac{3e^2}{16\pi^2} Q^2 m_q \log(\mu^2 a^2) \sim 10^{-3} m_q \quad \text{for} \quad \mu \to 2 \times \mu \quad ,$$

  thus the ambiguity of quark mass renormalization is tiny. $\sim$ 0.01 MeV for light quarks and $\sim$ 0.1 MeV for strange quark.

- Isospin breaking due to quark mass, $m_u - m_d$, is higher order effect in π, $\mathcal{O}\left((m_u - m_d)^2, (m_u - m_d)e^2\right)$.
  This is not the case for Kaons and Nucleons.

EM splittings on lattice

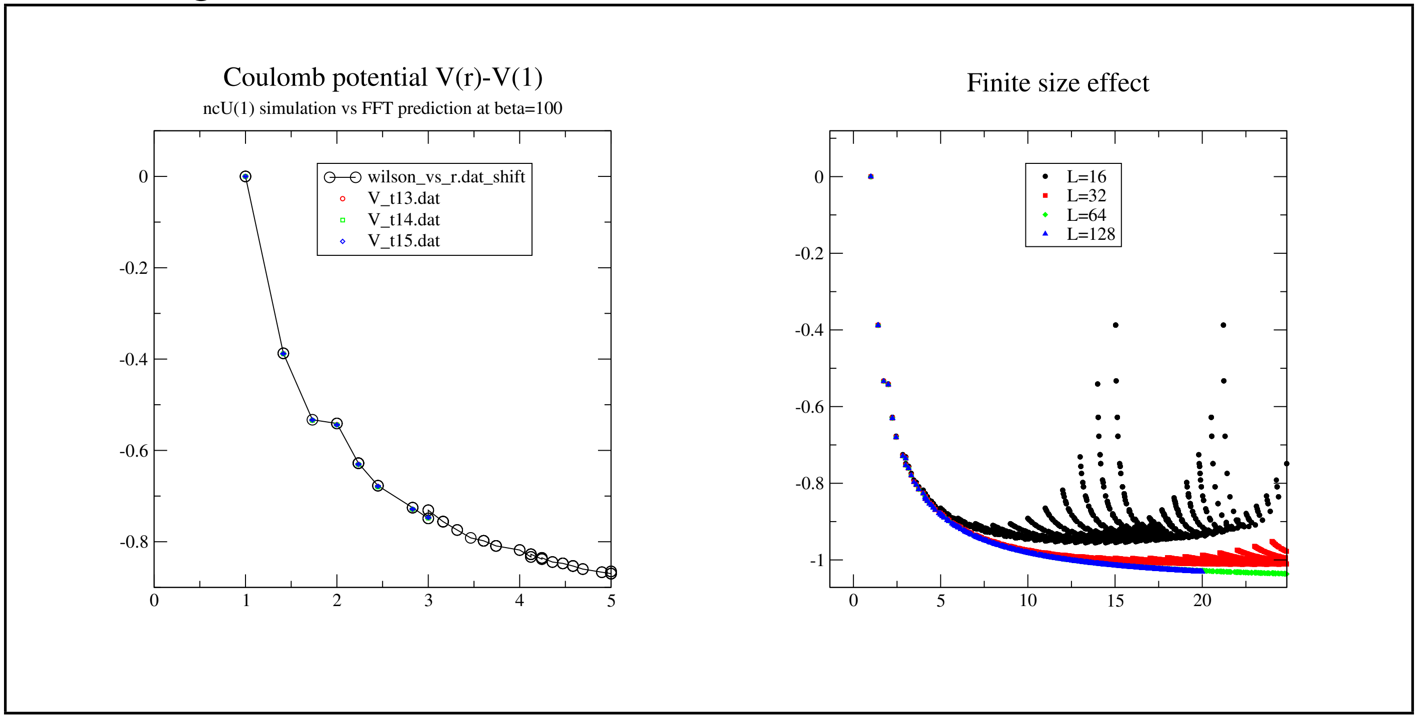
- In 1996, Duncan, Eichten, Thacker carried out SU(3)×U(1) simulation to do the EM splittings for the hadron spectroscopy using quenched Wilson fermion on $a^{-1} \sim 1.15$ GeV, $12^3 \times 24$ lattice.
- Using $N_F = 2$ Dynamical DWF ensemble (RBC) would have advantages such as better scaling and smaller quenching errors.
- Especially smaller systematic errors due to the the quark massless limits, $m_f \to -m_{res}(Q_i)$, has smaller Q_i dependence than that of Wilson fermion, $K \to K_c(Q_i)$.
- Generate Coulomb gauge fixed (quenched) non-compact U(1) gauge action with $\beta_{QED} = 1$. $U_\mu^{EM} = \exp[-iA_\mu(x)]$.
- Quark propagator, $S_{q_i}(x)$ with EM charge $Q_i = q_i e$ with Coulomb gauge fixed wall source

$$D\left[(U_\mu^{EM})^{Q_i} \times U_\mu^{SU(3)}\right] S_{q_i}(x) = b_{src}, \qquad (i = \text{up,down})$$

$$q_{\text{up}} = 2/3, q_{\text{down}} = -1/3$$

photon field on lattice

- non-compact $U(1)$ gauge generated using FFT.
- static lepton potential on $16^3 \times 32$ lattice ($\beta_{QED} = 100$, 4,000 confs) vs lattice Coulomb potential.
- L=16 has significant finite volume effect for $ra > 6 \sim 1.5r_0 \sim 0.75$ fm. We hope it's less problematic for the calculation EM splitting of hadron due to confinement. It would be worth considering for generation of U(1) on a larger lattice and cutting it off.



simulation parameters

- $N_F = 2$ Dynamical DWF configuration for QCD
- $a^{-1} = 1.691(53)$ GeV.
- degenerate quark mass at dynamical quark mass points,
  $m_{val} = m_{sea} = (0.02), 0.03, 0.04 \sim$ 50%, 75%, 100% of m_{strange}.
- $16^3 \times 32$ or (1.9 fm)^3.
- $Ls = 12$, $m_{res}a = 0.0013$ or a few MeV.
- EM charge: $e = 1.0, 0.6, 0.3028 = \sqrt{4\pi/137}$
- $\sim$ 94$\longrightarrow$ 190 configurations for each m
- one or two QED configuration per a QCD configuration.
- All 16 mesonic connected correlators + Neutron, Proton.

Analysis methods

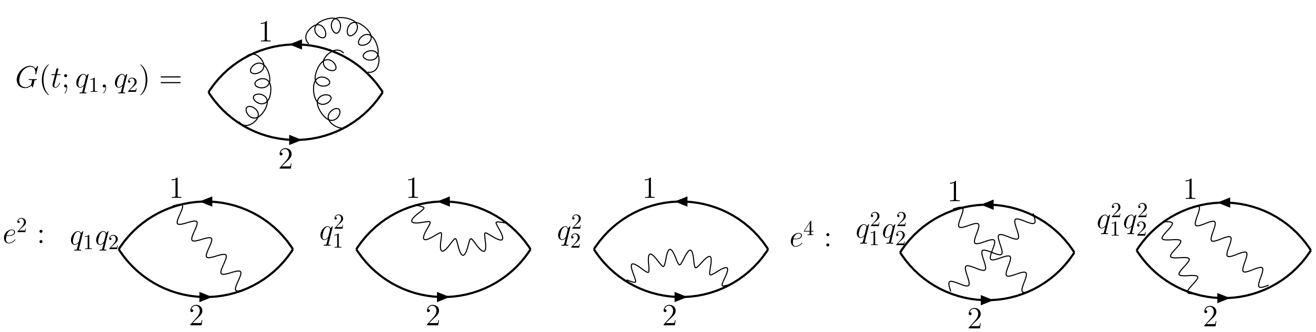
- Analysis method I :
  Fit correlator for each charge combination separately,
  then calculate the mass splittings under jackknife.

$$X = \pi, \rho, N : \Delta M_X = M_{X^\pm} - M_{X^0},$$

- Analysis method II :
  Subtract charged correlator by neutral correlator,
  and fit it by a linear function in t:

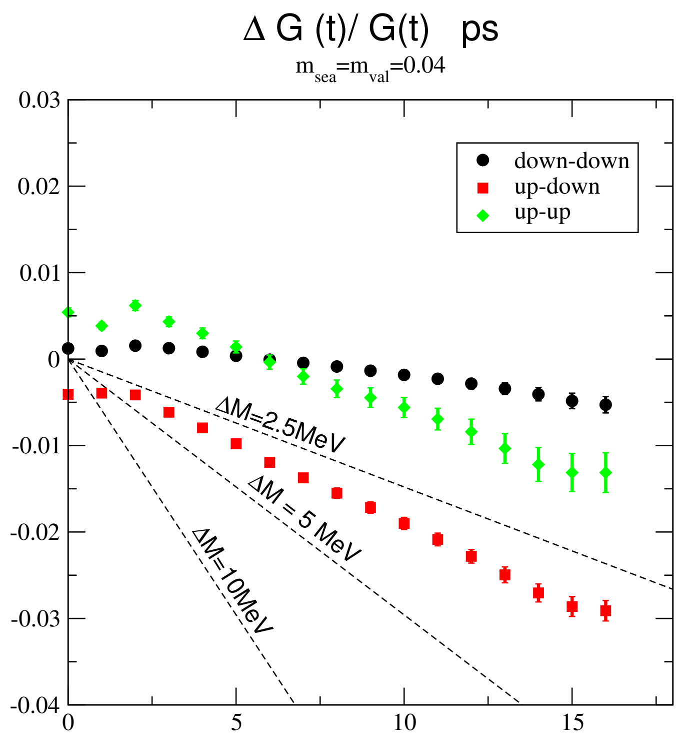
$$C_X(t) = A(e^2) e^{-M_X(e^2) t}$$

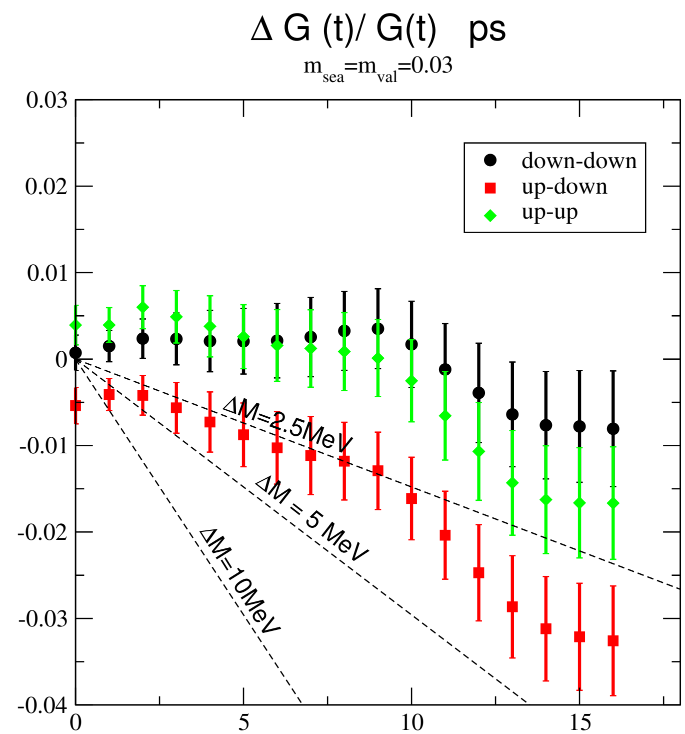
$$\frac{C_{X^\pm}(t) - C_{X^0}(t)}{C_{X^0}(t)} = \Delta M_X \times t + Const$$



results

- $G(t) = \langle J_5(0) J_5(t) \rangle$ at $m = 0.04$ and 0.03.





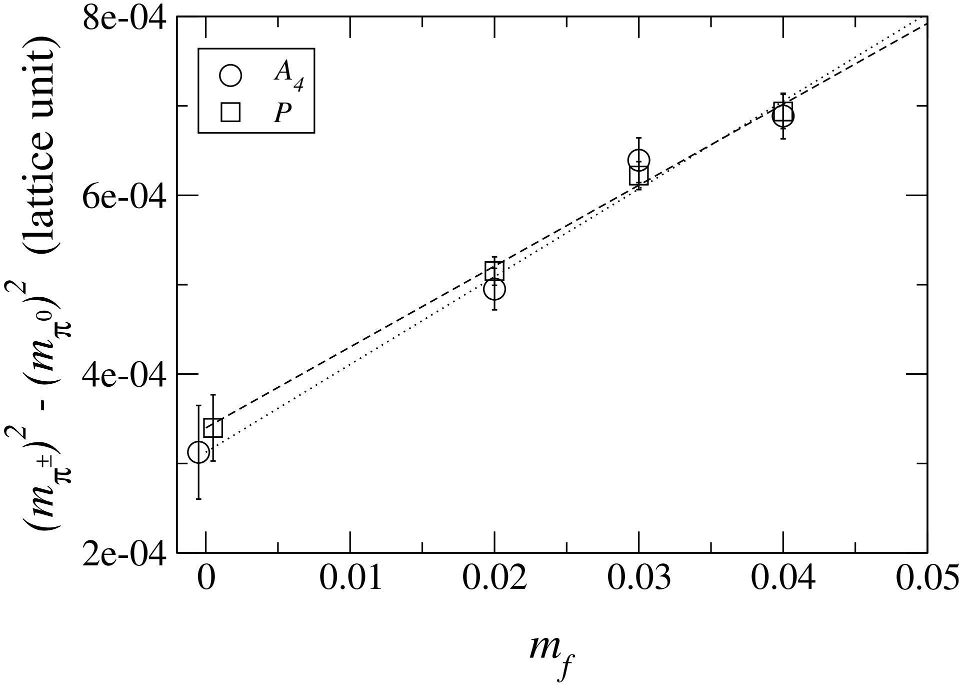
- larger e^2 doesn't help to reduce error for J_5: relative error stays almost same.

- Fluctuations due to SU(3) are comparable to that from U(1): by double the QED statistics: ΔM_π reduces by $\sim$ 4, 10, (30) % for $A_4, J_5, (N)$ resp. at $m = 0.04$.

$$\frac{\sigma_{QCD}^2 + 0.5\sigma_{QED}^2}{\sigma_{QCD}^2 + \sigma_{QED}^2} = (0.9)^2 \Longrightarrow \sigma_{QED}/\sigma_{QCD} \sim 0.85$$

ΔM_π^2

- Mass splitting of Π_0 and $\Pi_\pm$ with fixed quark mass.
- Method I, Method II are consistent within statistical error.



- preliminary results

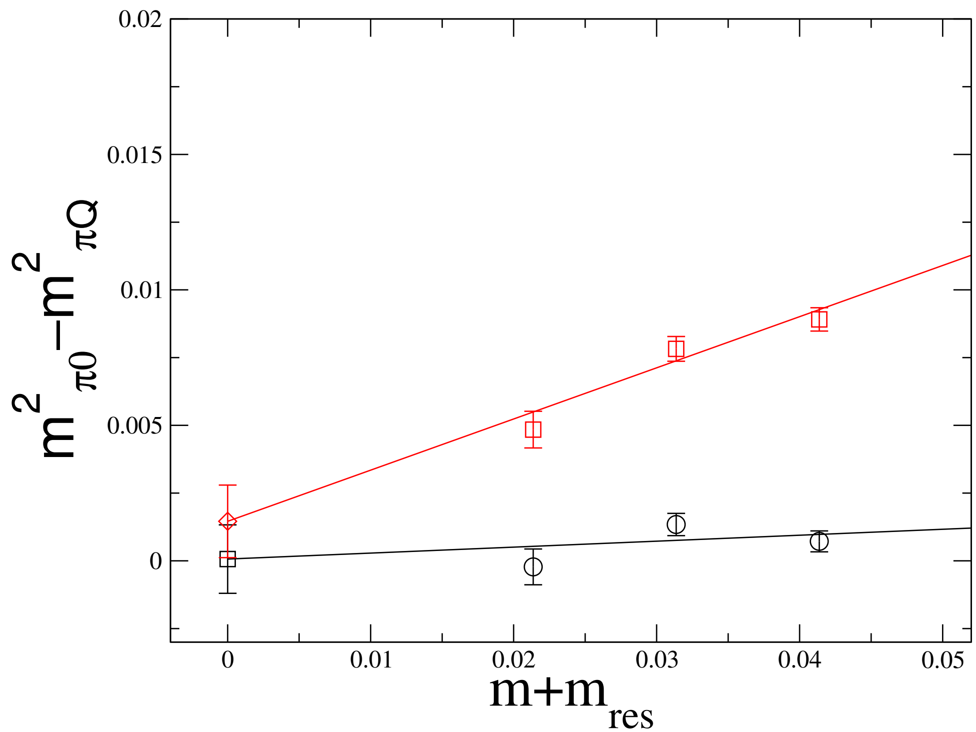
$$m_\pi^\pm - m_\pi^0 = 3.5(6)\text{MeV}(4.43\text{exp}) m_d - m_u = 2.6(3)\text{MeV}$$

- ρ and Nucleon EM splitting is milder.

Dashen's theorem

- $M_{\pi,Q}$: pure QCD pion ($e^2 = 0$)

$$M^2_{\pi^0} - M^2_{\pi Q} = I e^2 m \log m + K e^2 m$$



- Extraction of quark masses, $m_{up}, m_{down}, m_{strange}$, from experimental values of $m_{\pi^\pm}, m^0_\pi, m_{K^\pm}, m_{K^0}$.

non-perturbative technique with perturbative one

- In Lattice QCD calculations, perturbative treatment is often used.
- If the expansion paramter is enough small why not rely on perturbative methods if it extends the reach to a new region ! Perturbation is nice, intuitive, particle picture....
-

$$[\text{answer}] = [\text{pertubative vlue}] * [\text{non-pertubative value}]$$

'$*$' could be a simple operation (multiplication) or a compilcated operation (integrals):

answer	pertubative	non-perturbative	*
$\Gamma(\pi \to e\bar{\nu}_e)$	$G_F V_{ud} m_\mu$	F_π	mult.
ϵ_K $(K_0 - \overline{K_0})$	$\propto$ $G_F^2 M_W^2$ [$V_{qq'} F$'s]	B_K	lin. comb.
$O(\alpha^2) g - 2$	kernel $f(q^2)$	$\Pi_{\mu\nu} = \langle V_\mu V_\nu \rangle (q)$	fit & integ.
EM splits	Photon prop. $G_{\mu\nu}(q)$	$\langle H \| V_\mu(q) V_\nu(-q) \| H \rangle$	M.C. integral

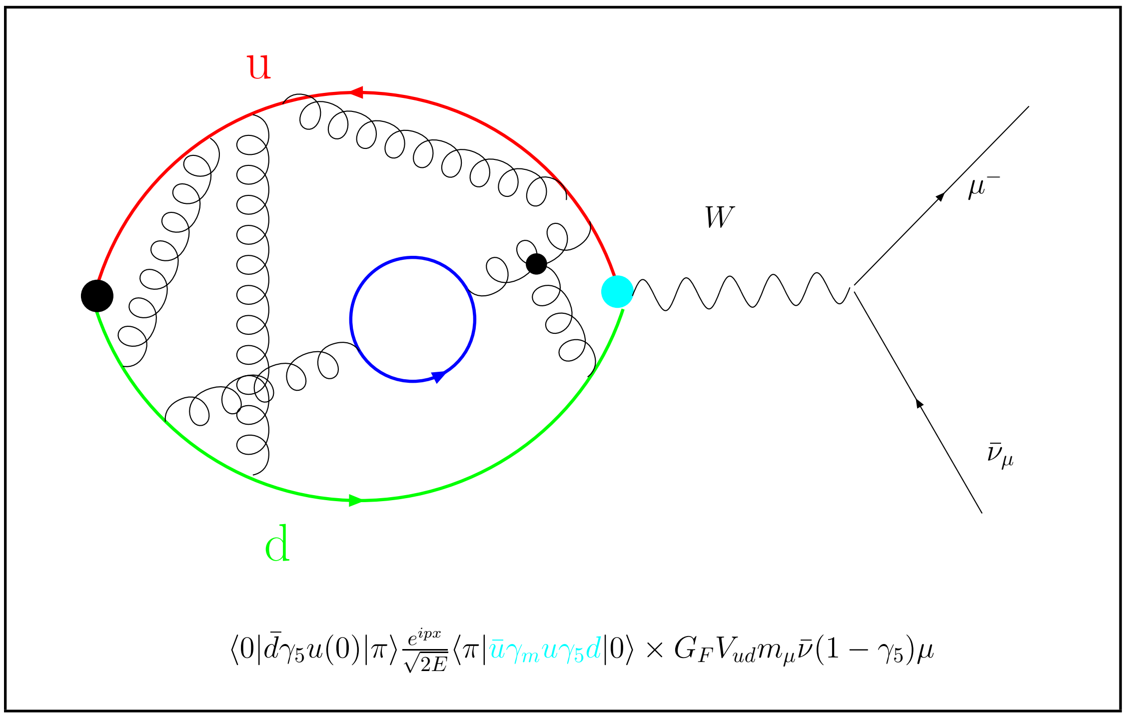
(K_{l3} : (Shoichi's talk) , SF (NEDM!) : (Kostas' talk) , B_K : (Jun's talk))

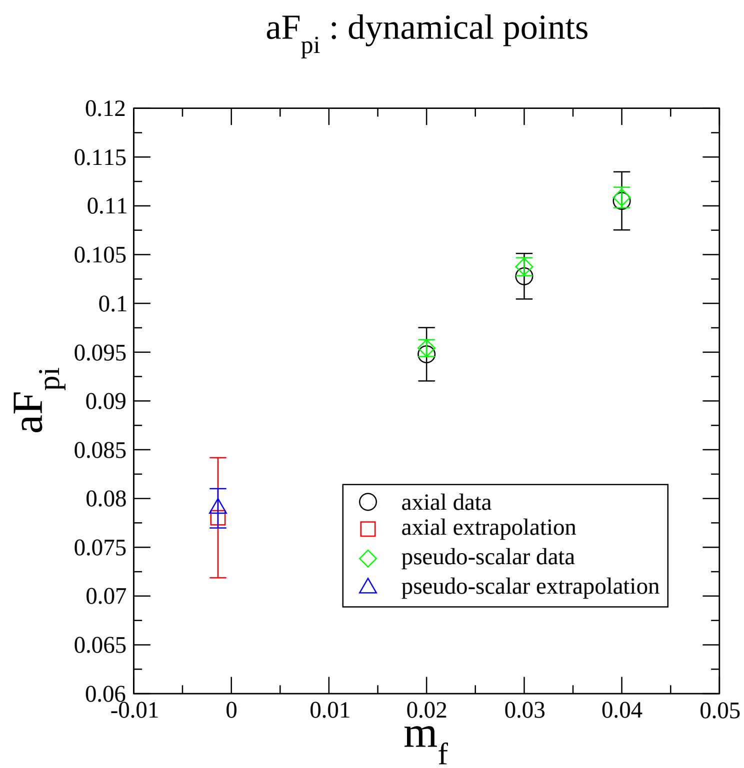
Decay Constant

$\sqrt{2}\langle 0|A_\mu(0)|\pi(q)\rangle = i f_\pi q_\mu$(non-perturbative part)

$G_F V_{ud} m_\mu \bar{\nu}(1-\gamma_5)\mu$(perturbative part)

$\Longrightarrow \Gamma(\pi \to e\bar{\nu}_e)$





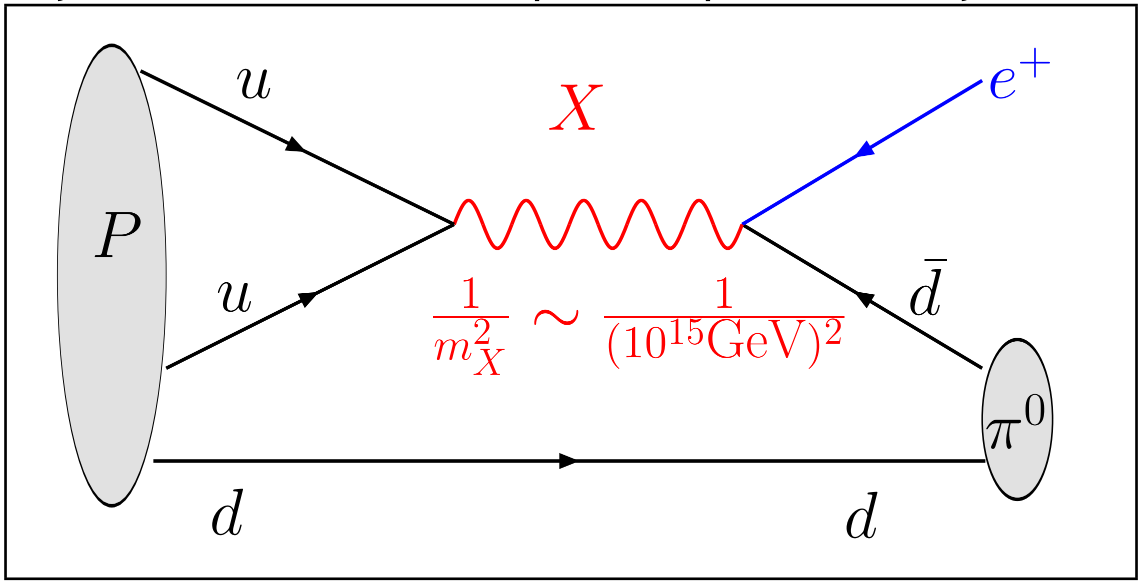
- linear fit to pseudo-scalar data gives

$$F_\pi = 142(6) MeV$$

(quenched: 129.0(7.3) MeV)

Nucleon decay matrix elements

- Beyond-Standard-Models predict proton decay



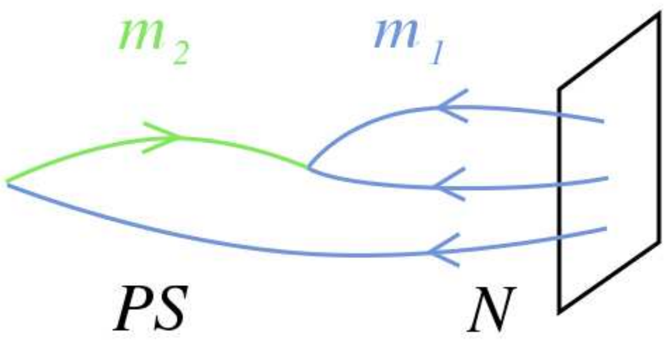
(Y. Aoki)

- One of the largest uncertainties for life time is Hadron matrix elements, a factor 10 difference among model calculations.

$$\Gamma_P \propto \left| \bar{v}_e \left\langle \pi | \epsilon_{abc}(u^{aT} C P_{R/L} d^b) P_L u^c | P \right\rangle \right|^2$$

- Experimental bound on mean life time of proton:

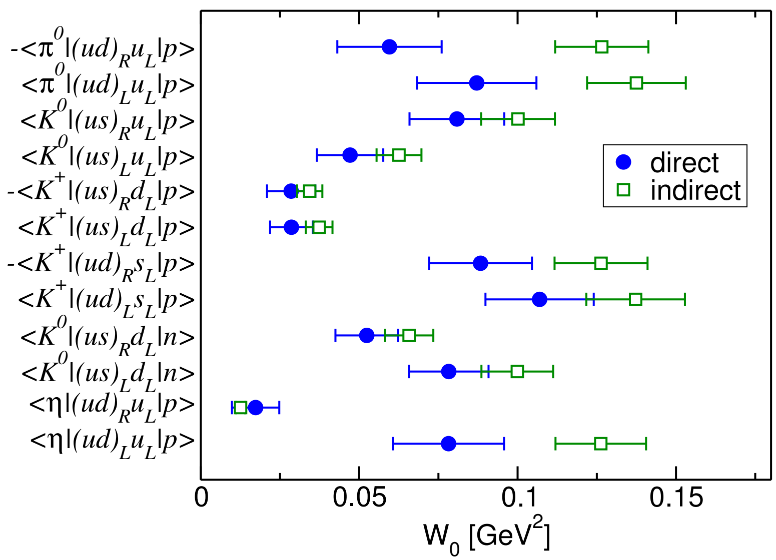
$$\tau_p > 10^{31} \sim 10^{33} \quad \text{years}$$

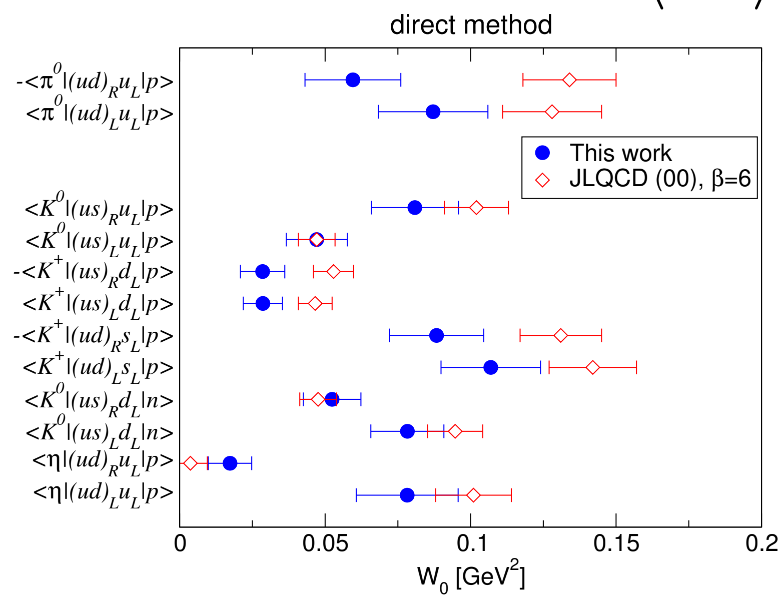


- Disentangles relevant piece from π's (PS) momentum dependence.

- Take a ratio to the 2pt functions

$$R_{\vec{p}} = \frac{\left\langle \pi(\vec{p})\ \mathcal{O}^{udu}_{R/L,L}(-\vec{p})\ \ P \right\rangle}{\left\langle \pi(\vec{p})\pi(\vec{-p}) \right\rangle \left\langle P\overline{P} \right\rangle}$$





- Non-perturbative renormalization at $\mu = 2$ GeV $\overline{\text{MS}}$ for $N_F = 0$ and 2.
- LO ChPT (Indirect) approximation gives systematically larger values for $P \to \pi$.
- Consistent results with the conservative choice in phenomenology.
- DWF scales better than Wilson fermion (JLQCD).

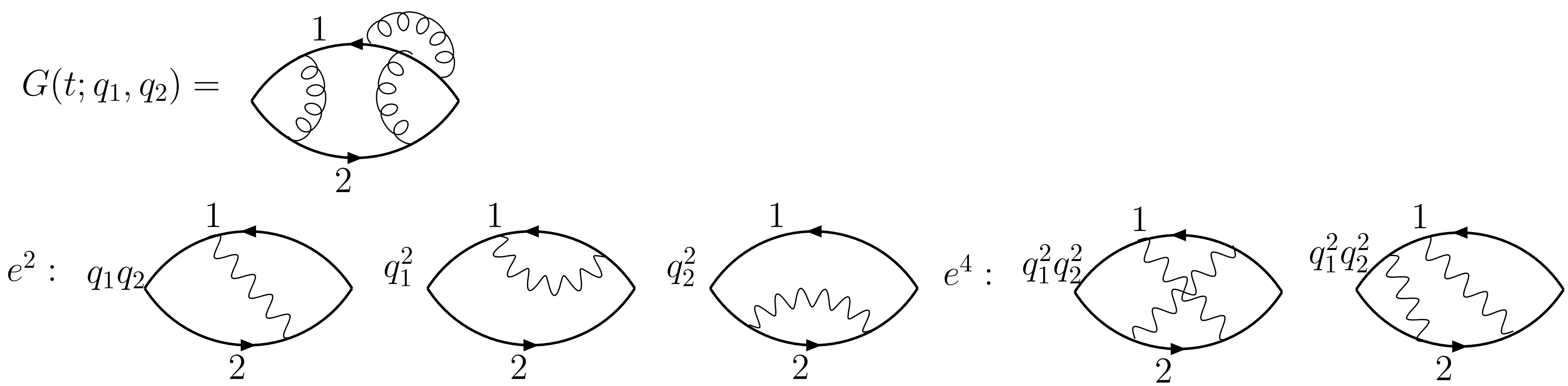
Direct calculation of the each QED diagram

- Each of the perturbative pieces could be obtained by a *direct calculation of QED diagram* treating QCD non-perturbatively. This method may have the following advantages:
  - Could obtain the contribution from each QED diagram separately, and/or order by order in α.
  - The charge, e and q_i, could be input later than simulation.
  - Would help or avoid the subtractions, for example in the light-by-light calculation.

$$\partial_\chi D^{-1}(\chi) = -D^{-1}\partial_\chi D D^{-1}$$

$$\chi = m, (m_u - m_d), e, \mu_q$$

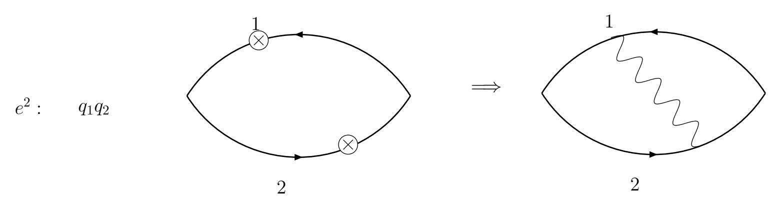
- An example for meson EM splittings :



Direct calculation ...

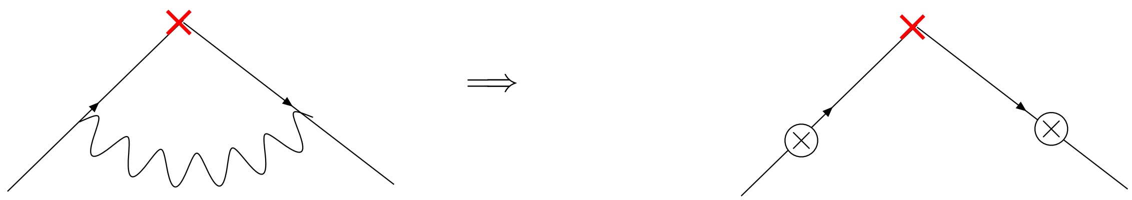
- Prepare a $U_{SU(3)\mu}$, and an A_μ.
- 1. Solve
$$D\left[U_{SU(3)}\right] S_q^{(1)}(x) = b,$$
- 2. multiply $-iA_\mu(x)K_\mu$ or $-ieq_iA_\mu(x)K_\mu$ to the first solution vector, $S_q^{(1)}(x)$, where K_μ is the kernel of the conserved vector current: $V_\mu^{con}(x) = \bar{\psi}K_\mu\psi$.
- 3. Sequentially solve
$$D\left[U_{SU(3)}\right] S_q^{(2)}(x) = -iA_\mu(x)\gamma_\mu S_q^{(1)}(x),$$
- (4. Repeat 2, 3. $(n-1)$ times for $S_q^{(n)}$)
- Then Tr$[S_q^{(2)}\Gamma\gamma_5[S_q^{(2)}]^\dagger\gamma_5\Gamma]$ is the $e^2q_1q_2$ diagram of the $G(t;q_1,q_2)$ in the background fields,

$$\frac{1}{e^2}\frac{\partial^2}{\partial q_1 q_2}G(t;q_1,q_2) =$$



Future prospects

- EM splittings using non-perturbative QED.
- Nucleon mass splittings in progress. (important !)
- EM splittings using the direct calculation of the QED diagrams.
- $\mathcal{O}(\alpha)$ contribution to $g_\mu - 2$ (pure QED).



- Auxially small uniform E/M field on lattice.
- $\mathcal{O}(\alpha^3)$ contribution (light-by-light) to $g_\mu - 2$.