Thermodynamics at $\mu = 0$ on the lattice

Y. Aoki

U. Wuppertal

RHIC Physics in the Context of the Standard Model

Staggered thermodynamics

- Equation of State : YA, Z. Fodor, S.D. Katz, K.K. Szabó
  JHEP 0601:089,2006 [hep-lat/0510084]
- Order of the transition : YA, G. Endrődi, Z. Fodor, S.D. Katz, K.K. Szabó
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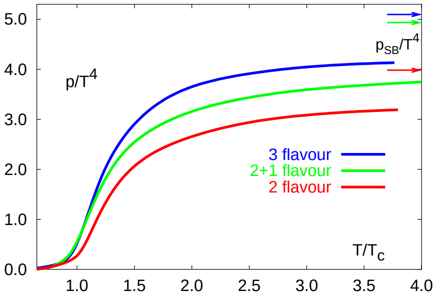
Systematic errors of the EoS



Bielefeld 2000.

- finite MD step size $\Delta\tau$
- *a* dependence is not investigated
- *ma* fixed while changing *a* ($T = 1/(N_t a)$)

Solution

- Use exact algorithm: RHMC
- Do several *a* with improved action
- Tune *ma* to reproduce the $T = 0$ experimental spectrum: LCP

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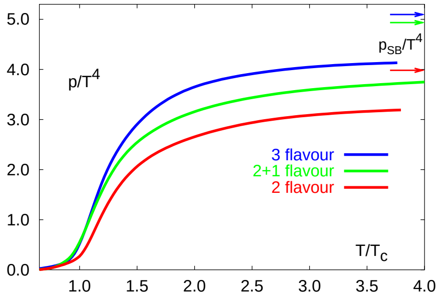

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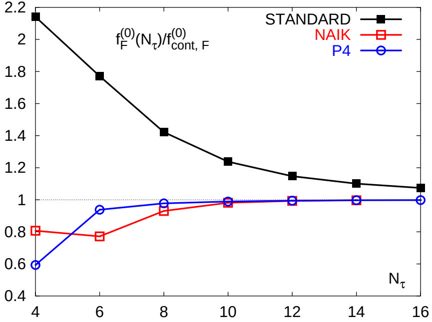
Discritization error

- Rorational symmetry breaking
- Taste symmetry breaking

Improving discritization error

- Rotational symmetry breaking
  - Symanzik (tree level) gauge action $\rightarrow$ improves gauge sector
  - Single (stout) link action $\rightarrow$ does not improve f at $T = \infty$
    - ★ Large deviation from continuum SB
    - ★ $a \rightarrow 0$ behavior is very good



Heller, Karsch, Sturm

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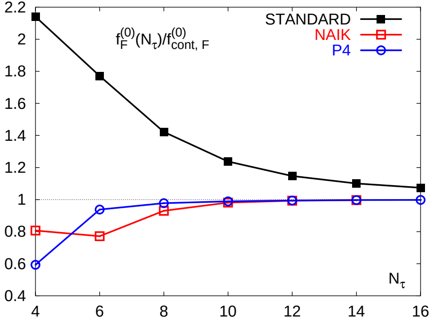

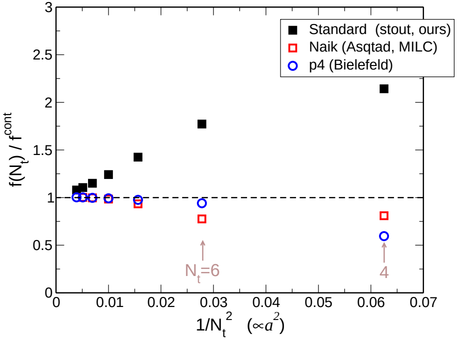

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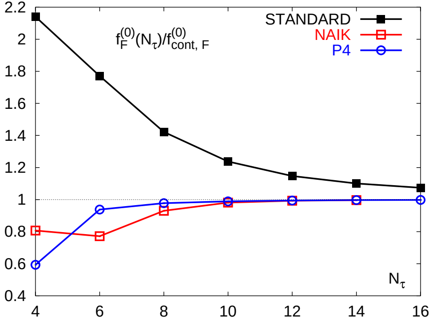

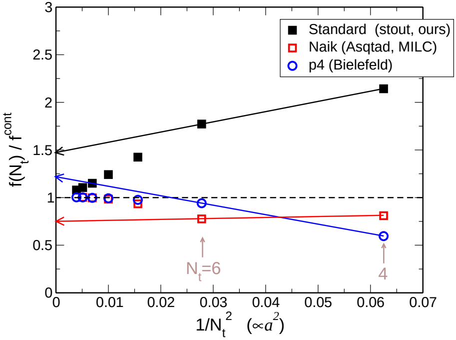

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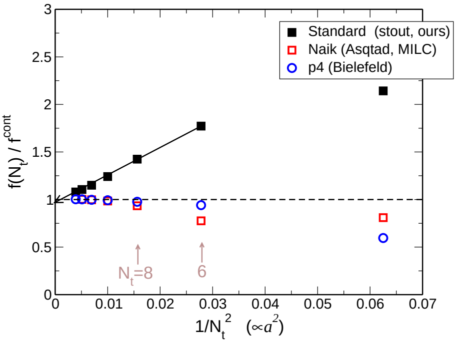

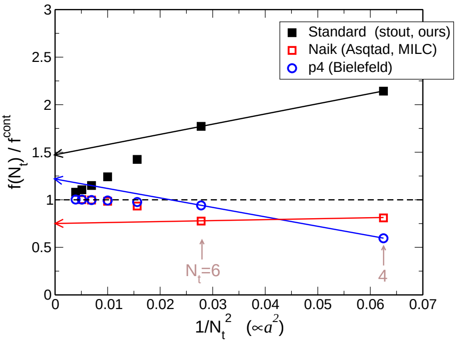

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Improving discritization error

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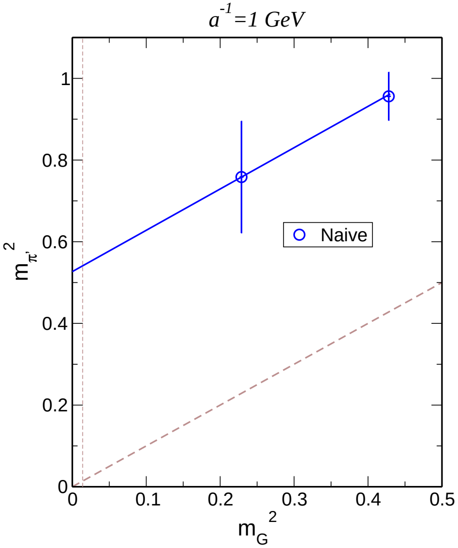

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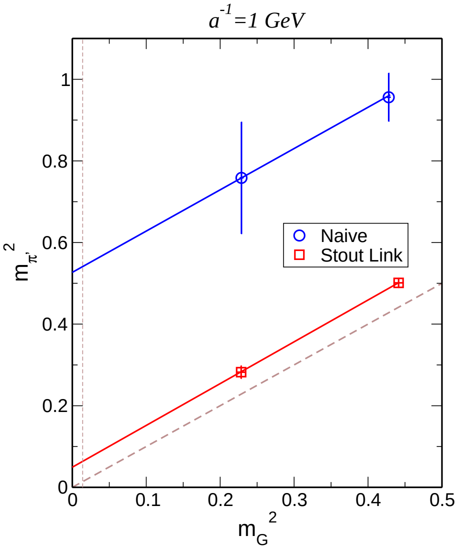
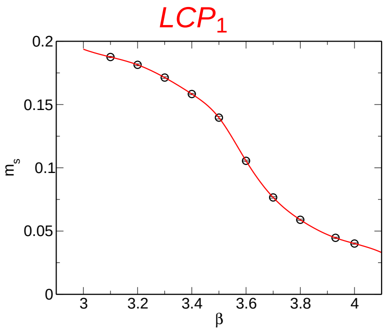


LCP: to be always on the physical point
The line of constant physics

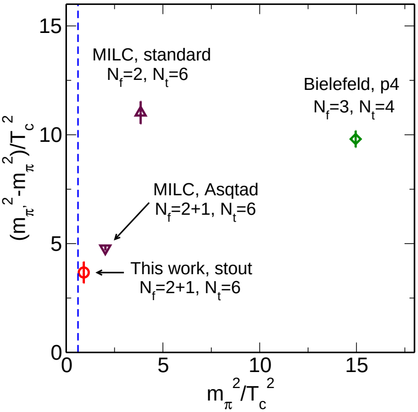
- $m_{ud}(\beta)$, $m_s(\beta)$.
- Tune m_s using $N_f = 3$ degenerate simulations and LO ChPT

$$\left.\frac{m_{PS}^2}{m_V^2}\right|^{N_f=3}(m = m_s) = \frac{m_{\eta_s}^2}{m_\phi^2} = \frac{2m_K^2 - m_\pi^2}{m_\phi^2}.$$

- obtain $m_s(\beta)$
- $m_{ud} = m_s/25$
- should be checked in $N_f = 2+1$ simulation.



How close to the physical and continuum limit ?



$$\Delta'_\pi = \frac{m_{\pi'}^2 - m_\pi^2}{T_c^2} = (m_{\pi'}^2 - m_\pi^2) N_t^2$$

($T = 0$ masses, measured at $\beta_c(N_t)$)

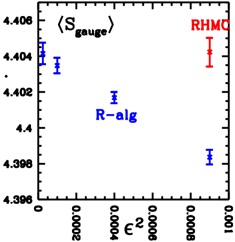
EoS procedure

integral along LCP

$$\left.\frac{p}{T^4}\right|_{(\beta_0,m_0)}^{(\beta,m)} = -\left.\frac{f}{T^4}\right|_{(\beta_0,m_0)}^{(\beta,m)}$$

$$= -N_t^4 \int_{(\beta_0,m_0)}^{(\beta,m)} d(\beta, m_{ud}, m_s) \begin{pmatrix} \langle -S_g/\beta \rangle \\ \langle \bar\psi\psi_{ud} \rangle \\ \langle \bar\psi\psi_s \rangle \end{pmatrix}$$

$$\left.\frac{p}{T^4}\right|^{(\beta,m)} = \left.\frac{p}{T^4}\right|_{(\beta_0,m_0)}^{(\beta,m)} (T \neq 0) - \left.\frac{p}{T^4}\right|_{(\beta_0,m_0)}^{(\beta,m)} (T = 0).$$



RHMC algorithm does not require the extrapolation in step size

T=0: no $m_{ud}^{sim} = m_{ud}^{phys}$ data.

$$\left.\frac{p}{T^4}\right|_{(\beta, m_{ud}^{phys})} = \left.\frac{p}{T^4}\right|_{(\beta, m_{ud}^{sim})} - N_t^4 \int_{m_{ud}^{sim}}^{m_{ud}^{phys}} dm_{ud} \langle \bar\psi\psi_{ud} \rangle.$$

Extrapolation only needed for $\langle \bar\psi\psi_{ud} \rangle (m_{ud}^{phys}) \leftarrow m_{ud}^{sim} = \{3, 5, 7, 9\} \times m_{ud}^{phys}$.

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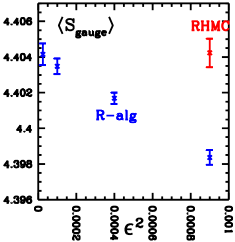

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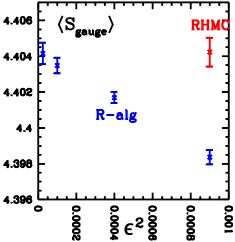

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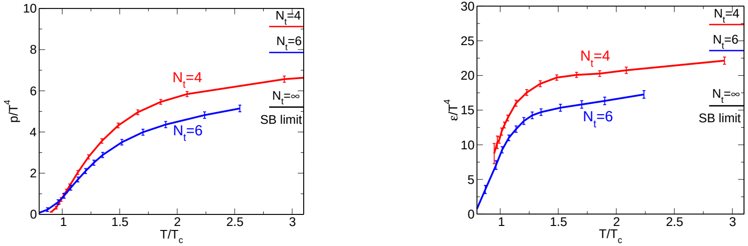
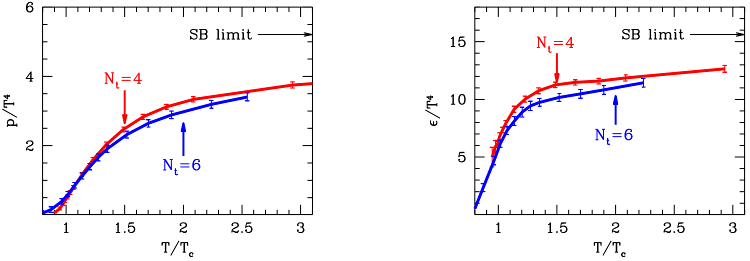
Simulation Procedure

- many β's (16 pts for $N_t = 4$, 14 pts for $N_t = 6$)
- given β, $m_s^{sim} = m_s^{phys}(\beta)$ fixed. $\rightarrow m_{ud}^{phys} = m_s^{phys}/25$.
- $T \neq 0$:
  - $m_{ud}^{sim} = m_{ud}^{phys}$.
  - $N_s = 3N_t$.
- $T = 0$:
  - $m_{ud}^{sim} = \{3, 5, 7, 9\} \times m_{ud}^{phys}$
  - keeping $L_s m_\pi > 3$
- finite size effect?
  - less than stat. error for several β's checked

EoS



Multiplied with c_{SB}/c_{N_t} ($c_{SB} = \lim_{N_t \to \infty} c_{N_t}$), $N_t = 4$(red), 6(blue).



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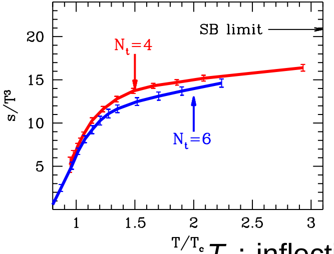
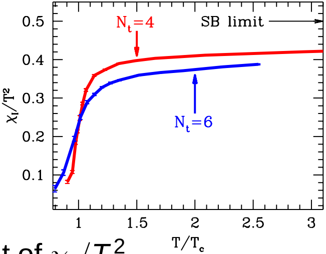
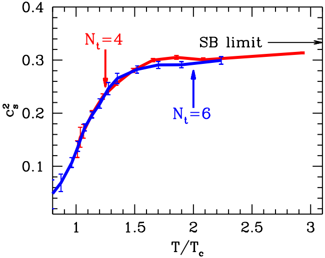
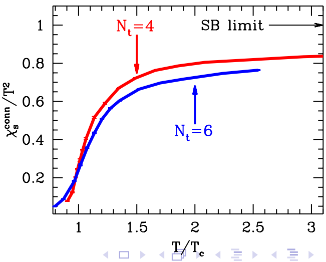


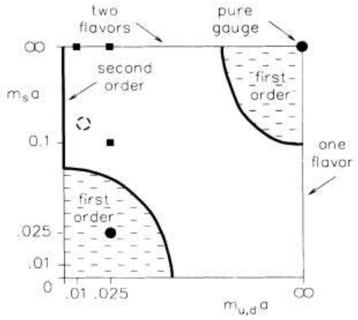


T_c: inflection point of χ_I/T^2.



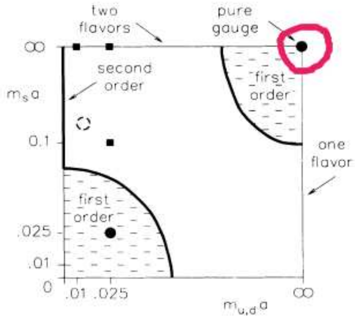


$m_s - m_{ud}$ phase diagram



Columbia 1990.

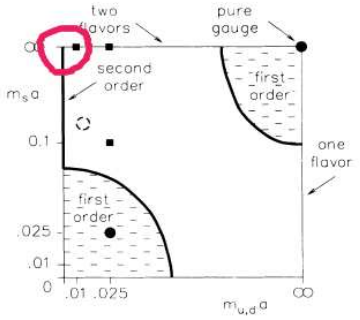
$m_s - m_{ud}$ phase diagram



Non-lattice approach

- $m_s = m_{ud} \to \infty$: global Z_3 symmetry
  - 3d, Z_3 symmetric spin model:
  - low T: symmetric phase: confined
  - high T: broken phase: deconfined
  - 1st order transition
    (Yaffe & Svetitsky)
- $1/m = 0+$
  - small perturbation by external field
    $h \propto e^{-\beta m}$ (Banks & Ukawa)
  - 1st order transition

$m_s - m_{ud}$ phase diagram



Non-lattice approach

- chiral symmetry: (Pisarski & Wilczek)
  - $m_s \to \infty$, $m_{ud} = 0$: $N_f = 2$: 2nd order with $O(4)$
  - $m_s = m_{ud} \to 0$: $N_f = 3$: 1st order
  - $m = 0+$: 1st order

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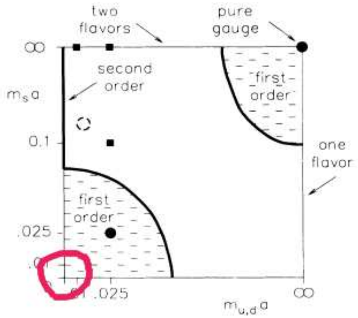

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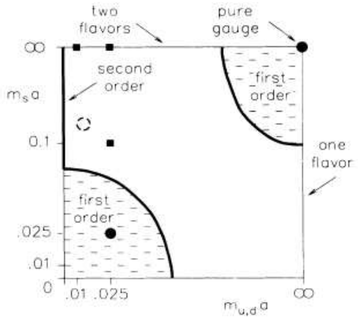
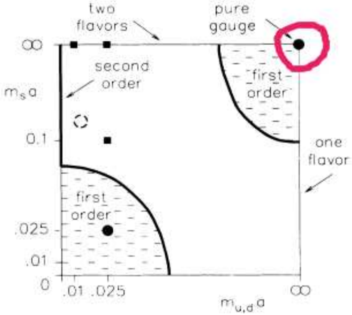


$N_t = 4$
Columbia 1990

Lattice approach

$m_s - m_{ud}$ phase diagram



Lattice approach

- $m_s = m_{ud} \to \infty$:
  - 1st order confirmed by lattice
    - 1st order (Columbia)
    - 2nd order (Ape)
    - 1st order by Finite Size Scaling (Fukugita et al)

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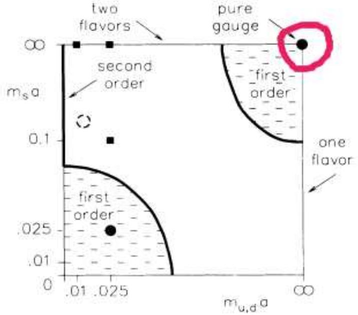

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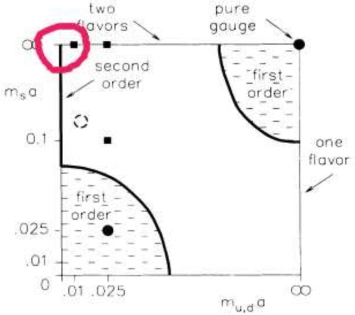
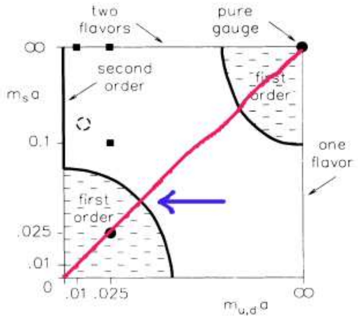


Lattice approach

- $m_s \to \infty$, $m_{ud} = 0$: $N_f = 2$:
  - Wilson fermion confirmed $O(4)$ scaling
  - Staggered: no $O(4)$, $O(2)$ scaling observed.
    1st order ? (Pisa)
  - possible $\Delta\tau$ artifact (Philipsen Lattice2005)

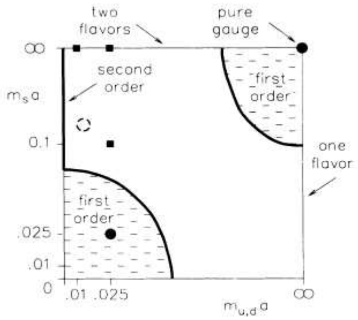
$m_s - m_{ud}$ phase diagram



Lattice approach

- $m_s = m_{ud}$: $N_f = 3$: end point (Bielefeld-Swansea)
  - $m_{\pi,c} = 290$ MeV [standard, $N_t = 4$]
  - $m_{\pi,c} = 67$ MeV [p4, $N_t = 4$]
  - Large discretization error!
    $\rightarrow$ needs investigation on a dependence

$m_s - m_{ud}$ phase diagram



- Very important to determine the order at the physical point: all $\mu \neq 0$ physics depend on this.
- Position of the physical point: open question in the continuum limit.
- Very demanding to explore all the mass range.
- Perhaps doable to determine the order at the physical point.

Strategy

- Exact algorithm (RHMC)
- Finite size scaling
- Correct quark masses
  - tune m_s and m_u so that ratios of m_π, m_K, f_K take physical values.
  - LCP$_2$: $m_u = m_s/27.3$, $m_s \rightarrow$
- Continuum limit

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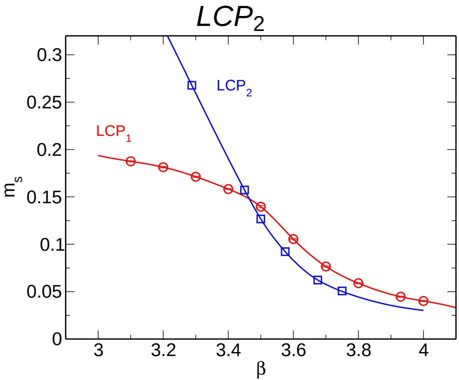
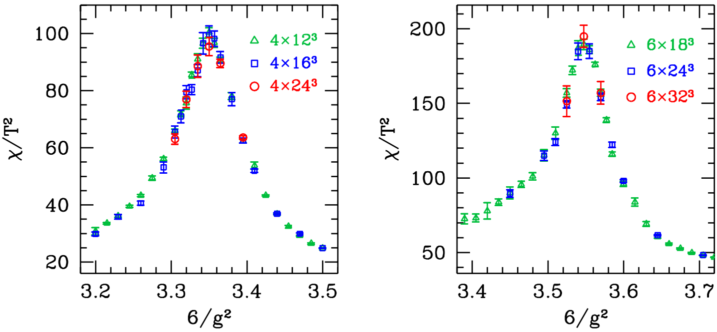


Finite Size Scaling of chiral susceptibility χ_{ud}

- $N_s/N_t = 3 - 5(6)$.
- No volume dependence found.
- How about in the continuum limit ?



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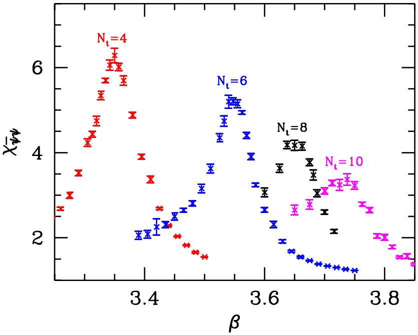


Continuum limit of the peak hight

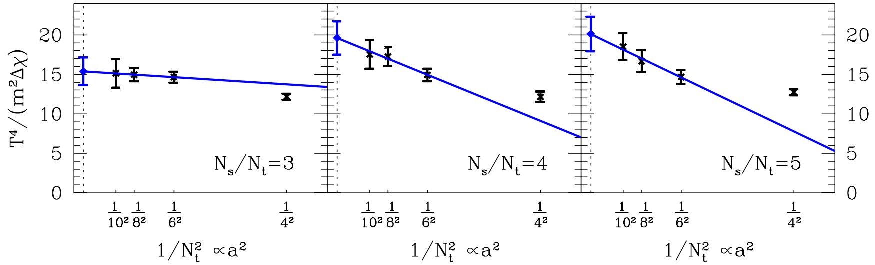
χ has power divergence, which should be subtracted.

$$\Delta\chi = \chi(T \neq 0) - \chi(T = 0).$$

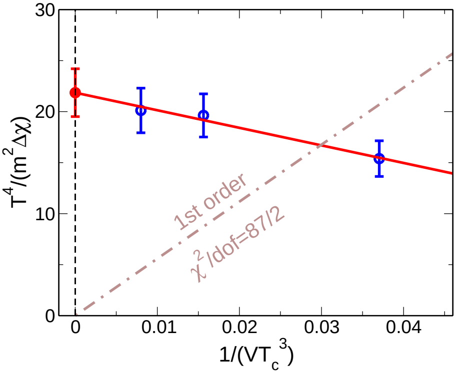
To make RG invariant quantity, mass squared is multiplied

$$m_{ud}^2 \Delta\chi$$

$$T_c^4/(m_{ud}^2 \Delta\chi).$$



Finite Size Scaling after continuum extrapolation



- Susceptibility does not diverge as $V \to \infty$
- $\to$ Crossover

Summary

- Stout-link smearing improvement was used to reduce the taste violation. Works quite well.

EoS

- The equation of state was calculated with $N_t = 4$ and 6 lattices, using LCP_1 (approximation).
- For the reliable continuum extrapolation, $N_t = 8$ simulation is needed.

Order of the transition

- Fine tuned LCP_2 was used.
- Continuum limit of chiral susceptibility was obtained using $N_t = 4, 6, 8, 10$.
- Finite Size Scaling was applied.
- Cross-over was found for the physical point.

T_c ?