Light Quarks on the Lattice

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2004 ~ present
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Introduction

Era of full QCD simulations

- 2+1 dynamical quarks ⇨ "true QCD"
- lighter quark mass ⇨ simulations at physical quark masses
- various formulations of quark actions
  - conventional (Wilson, Staggered, twisted-mass, ...)
  - "chiral" fermion (domain-wall, overlap, fixed-point, ...)
- good control over systematic errors
  - quark mass, volume, lattice spacing, ...
- more complicated observables

Several Reasons

- Faster computers (QCDOC, BlueGene/L, Clusters, ...)
- Improved Algorithms
  - Multiple-time scales for light quark (domain-decomposition, mass precondition, ...)
  - exact for odd flavors (Polynomial HMC, Rational HMC, ...)
- Theoretical developments
  - improvements (Wilson, twisted-mass, smearing, ...)
  - lattice chiral symmetry (domain-wall, overlap, fixed-point,...)
  - Lattice chiral perturbation theory (Staggered, Wilson, ...)

This Talk

Critical review of current full QCD simulations by several (but not all) groups including my owns.

Fermion actions

- Conventional action
  - Wilson-type fermion
  - twisted-mass fermion
  - Staggered fermion
- Action with improved chiral symmetry
  - Domain-wall fermion
  - Chirally improved fermion
- Action with “exact” chiral symmetry
  - Overlap fermion
  - Fixed point fermion

Staggered Fermions

Generalities

- Advantages
  - exact non-singlet U(1) chiral symmetry ⇨ smaller quark mass
  - single component/site ⇨ simulations are easier
  - $O(a^2)$ scaling violations
- Shortcomings
  - single field ⇨ 4 flavors ⇨ " taste " (irrelevant d.o.f)
    - N flavors $[\det D]^N \Rightarrow [\det D]^{N/4}$ "4-th root trick"
    - large taste breaking ⇨ improved actions with smearing

4-t root problem

$[\det D]^{1/4}$ may correspond to a non-local action

⇨ Violation of "locality",
a fundamental principle of (lattice) field theories

Current consensus:
Theory is non-local at finite lattice spacing.

Hope: non-localities vanish in the continuum limit.

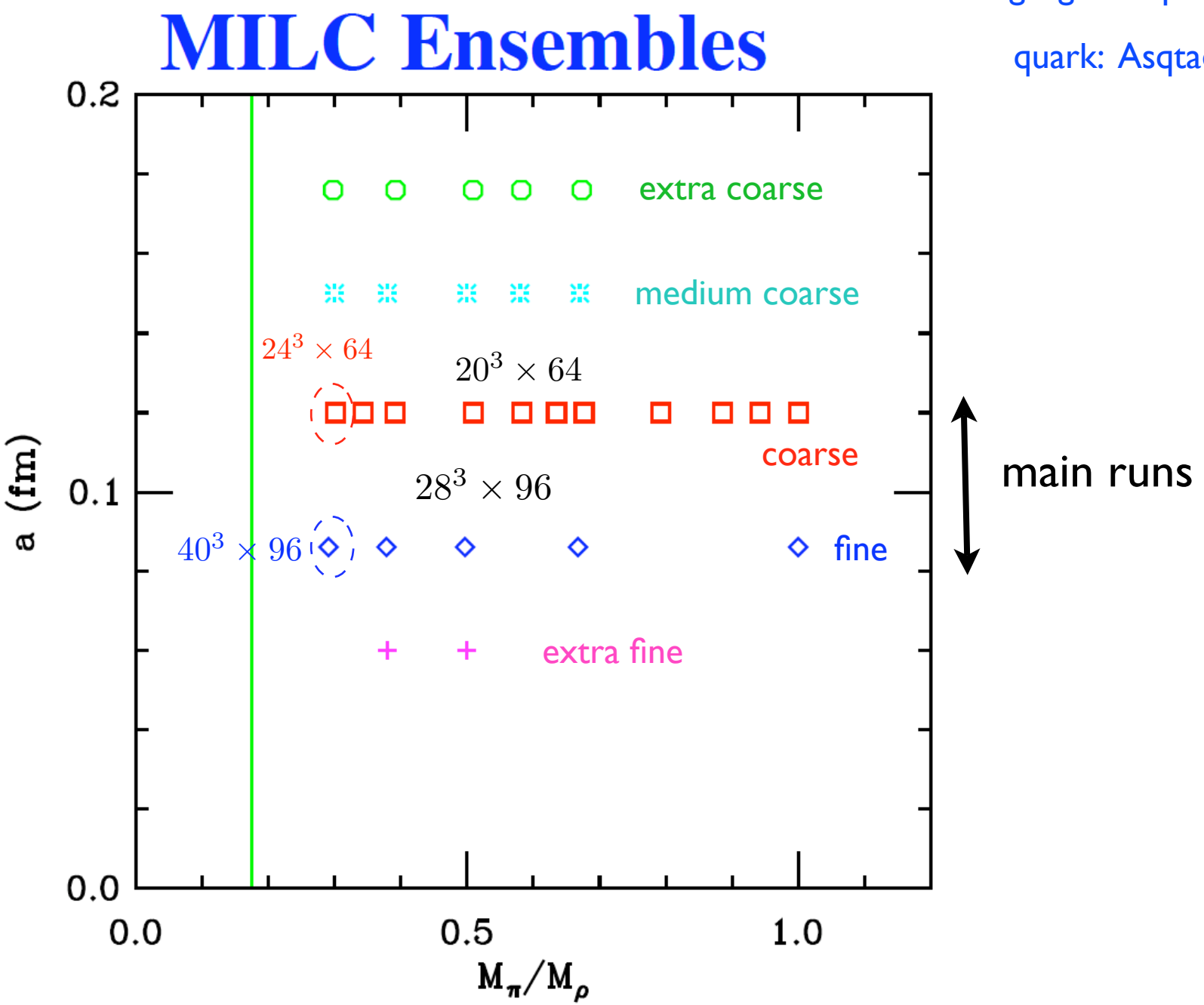
⇨ some class of non-local (lattice) theories belongs to the same universality class of (local) lattice QCD ?

Need further investigations.

Simulations $N_f = 2 + 1$ MILC Collaboration

gauge: tadpole improved Symanzik

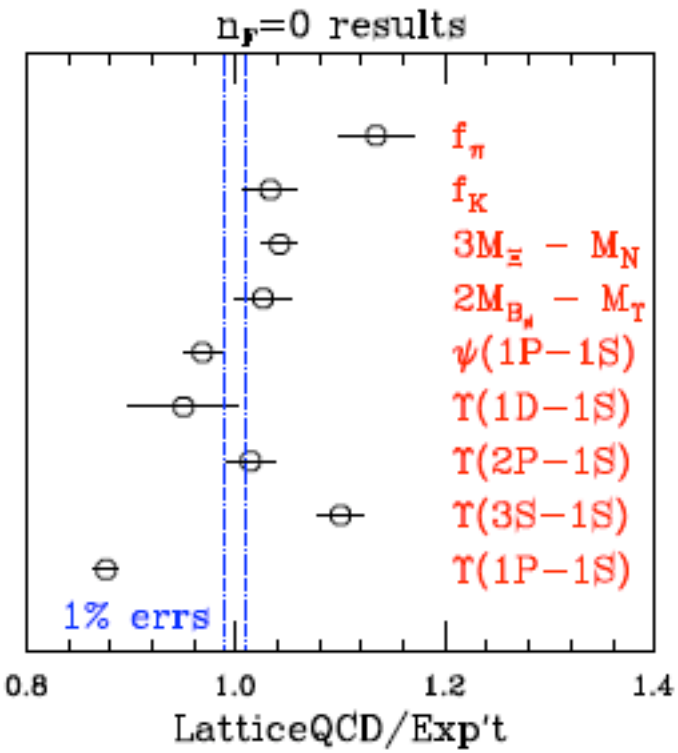
quark: Asqtad staggered



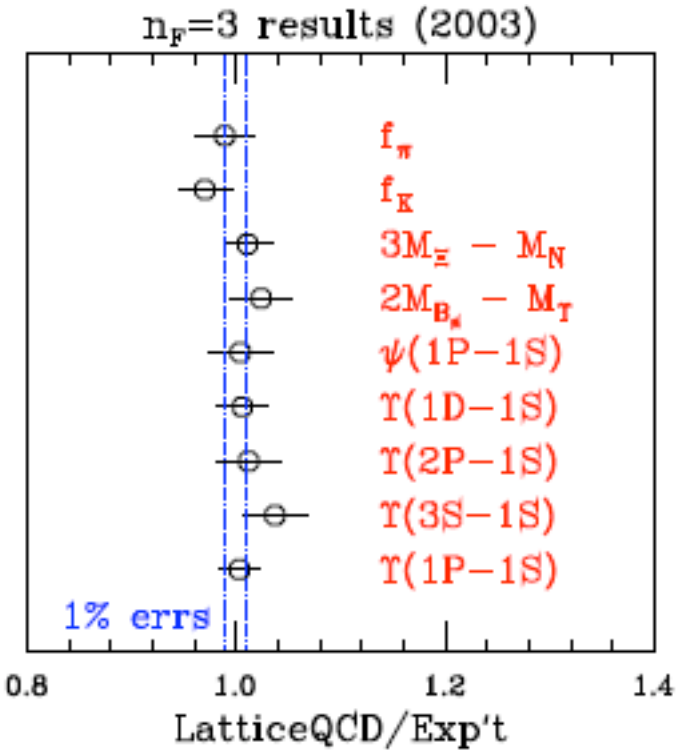
Results

Gottlieb, Lat2003

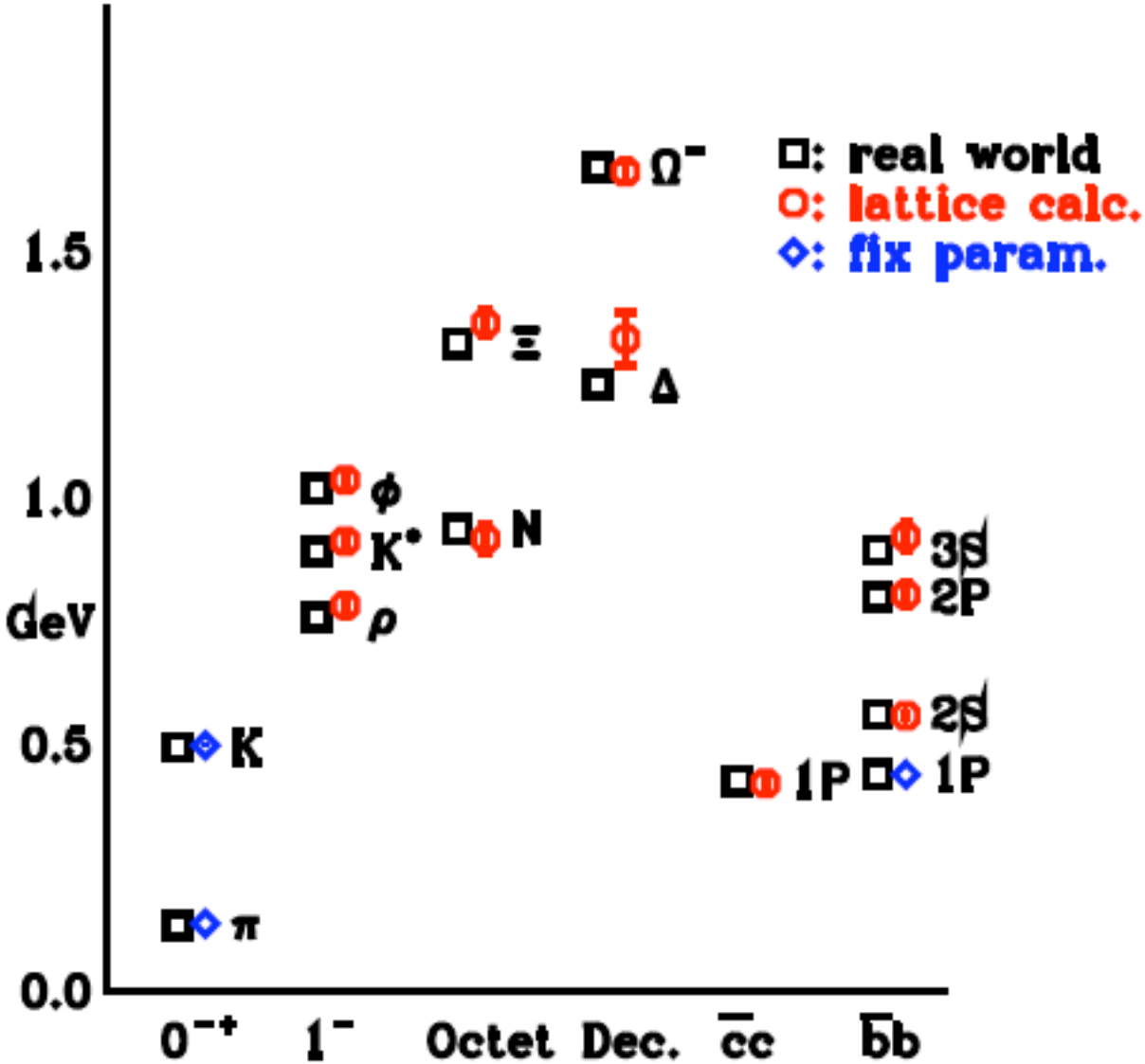
Quench



Full



Gottlieb, ILFTN2006



3-flavor lattice QCD reproduces Nature !?

Remarks

1. Full QCD simulation algorithm (R-algorithm) is "inexact".

check by exact algorithm (PHMC, RHMC) will be needed.

2. quark mass is much lighter than other simulations.

$$\begin{cases} a = 0.12\,\mathrm{fm} & m_\pi(\mathrm{NG})/m_\rho \simeq 0.3 \\ a = 0.09\,\mathrm{fm} & m_\pi(\mathrm{NG})/m_\rho \simeq 0.38 \end{cases}$$

$$m_\pi/m_\rho \simeq 0.18(\mathrm{exp.})$$

However other pions are heavier due to the taste symmetry violation.

$$\begin{cases} a = 0.12\,\mathrm{fm} & m_\pi(\text{taste singlet})/m_\rho \simeq 0.5 \\ a = 0.09\,\mathrm{fm} & m_\pi(\text{taste singlet})/m_\rho \simeq 0.47 \end{cases}$$

ChPT analysis to mixed action suggests valence quarks couple to "taste singlet" pion.

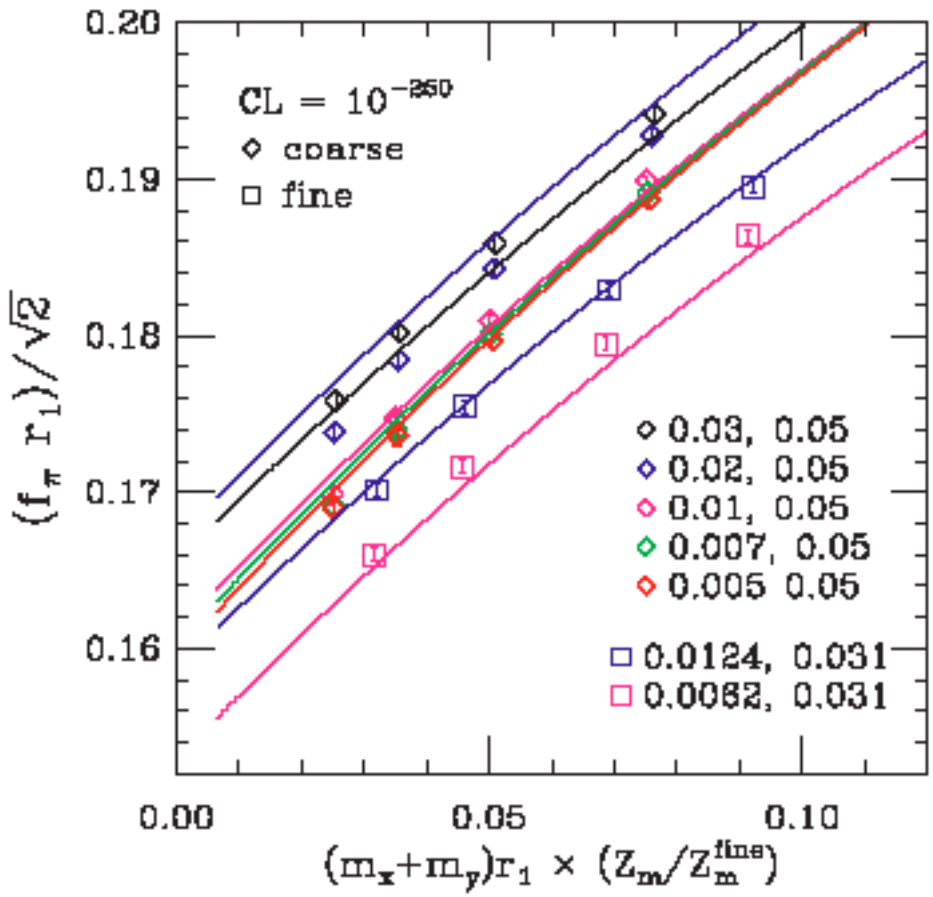
"effective" quark mass may not be so light.

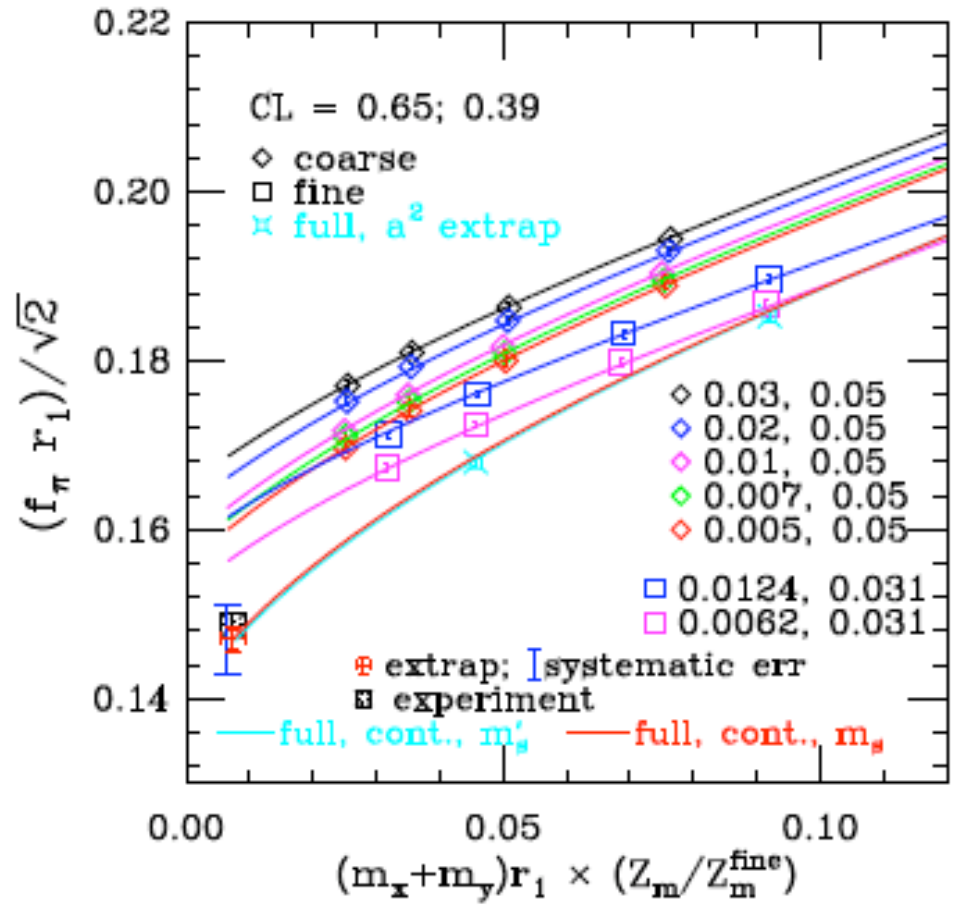
3. Analysis relies heavily on Staggered Chiral Perturbation theory(SChPT)

- to correct "unphysical(non-unitary)" contributions
- to correct "taste violations"
- chiral extrapolations
- continuum extrapolations
- Very complicated analysis
  - many parameters
  - many Partially Quenched data points
  - NLO(w log) + NNLO(w/o log)+NNNLO(w/o log)
  - it is hard to check by others

⇨ Bayesian fit

SChPT for f_π





Continuum ChPT fails.

SChPT works well. (488 dof)

4. Use of perturbative renormalization factors

1-loop 2-loop

$$m_s^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 76(0)(3)(7)(0)\ \mathrm{MeV},\ 87(0)(4)(4)(0)\ \mathrm{MeV}$$

estimate of perturbative errors

difference between 1-loop and 2-loop is larger than error estimate.

Non-perturbative calculations of renormalization factors will be needed.

Wilson-type fermions

Generalities

- Advantages
  - well-defined formulation (locality, no doubling,...)
  - straight-forward interpretation
  - reasonable computational cost
- Shortcoming
  - O(a) scaling violation
  - explicit violation of chiral symmetry
  - no protection against small eigenvalues
    large cost for small quark mass

⇨ O(a) improvement (Clover)

⇨ Improved Algorithm

Previous calculations

Many groups worked on $N_f = 2$ simulations

- Wilson
  - SESAM, APE, qq+q, ...
- Perturbative Clover
  - CP-PACS, ...
- Non-perturbative Clover
  - Alpha, UKQCD/QCDSF, JLQCD, ...

Except a few cases, quark masses are heavy: $m_\pi/m_\rho \geq 0.6$

Recent calculations

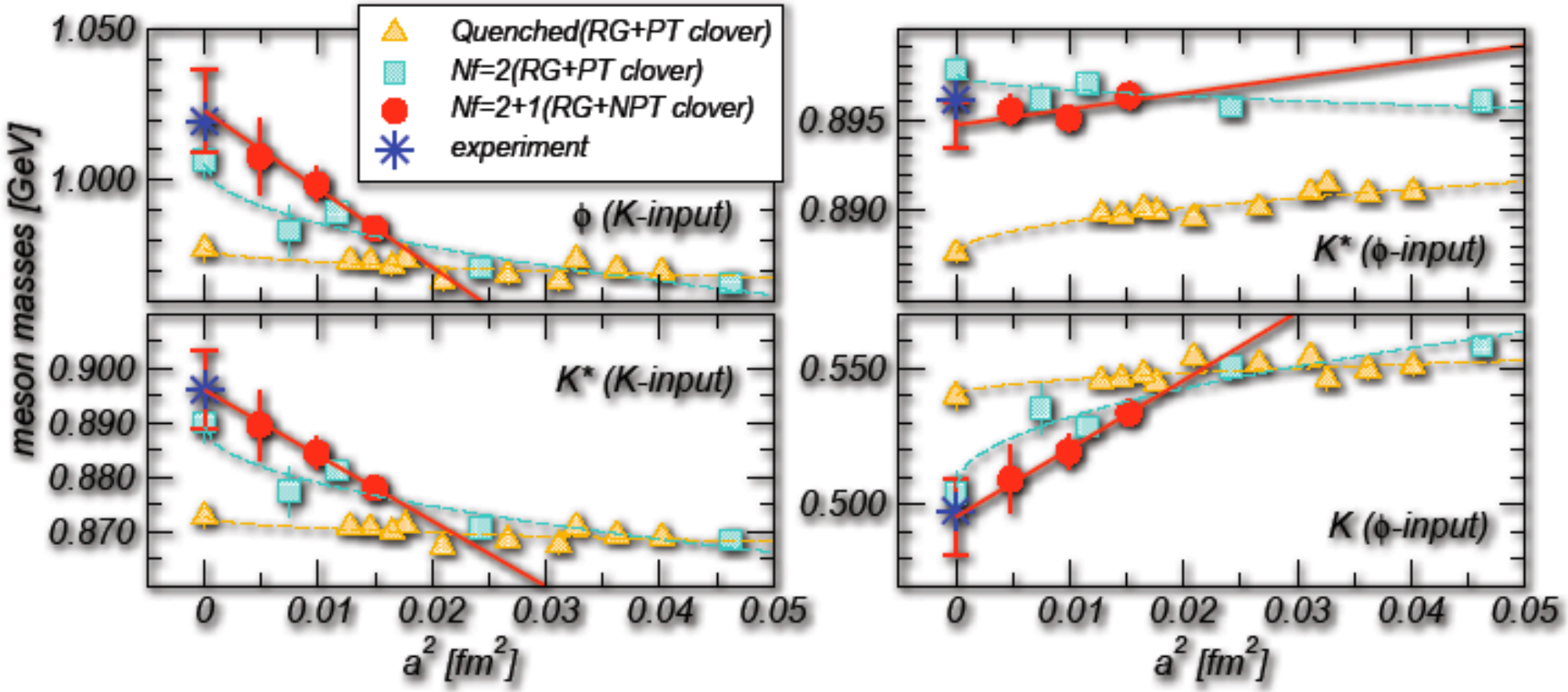
$N_f = 2 + 1$ CP-PACS/JLQCD Collaborations

- RG improved gauge/non-perturbative Clover fermion
- HMC + Polynomial HMC
- a=0.12/0.10/0.07 fm ⇨ continuum extrapolation
- L = 2 fm
- $m_\pi/m_\rho \simeq 0.60$–0.78 for light quarks
- perturbative renormalization/improvement for operators

Results

Meson masses

Talk by Ishikawa

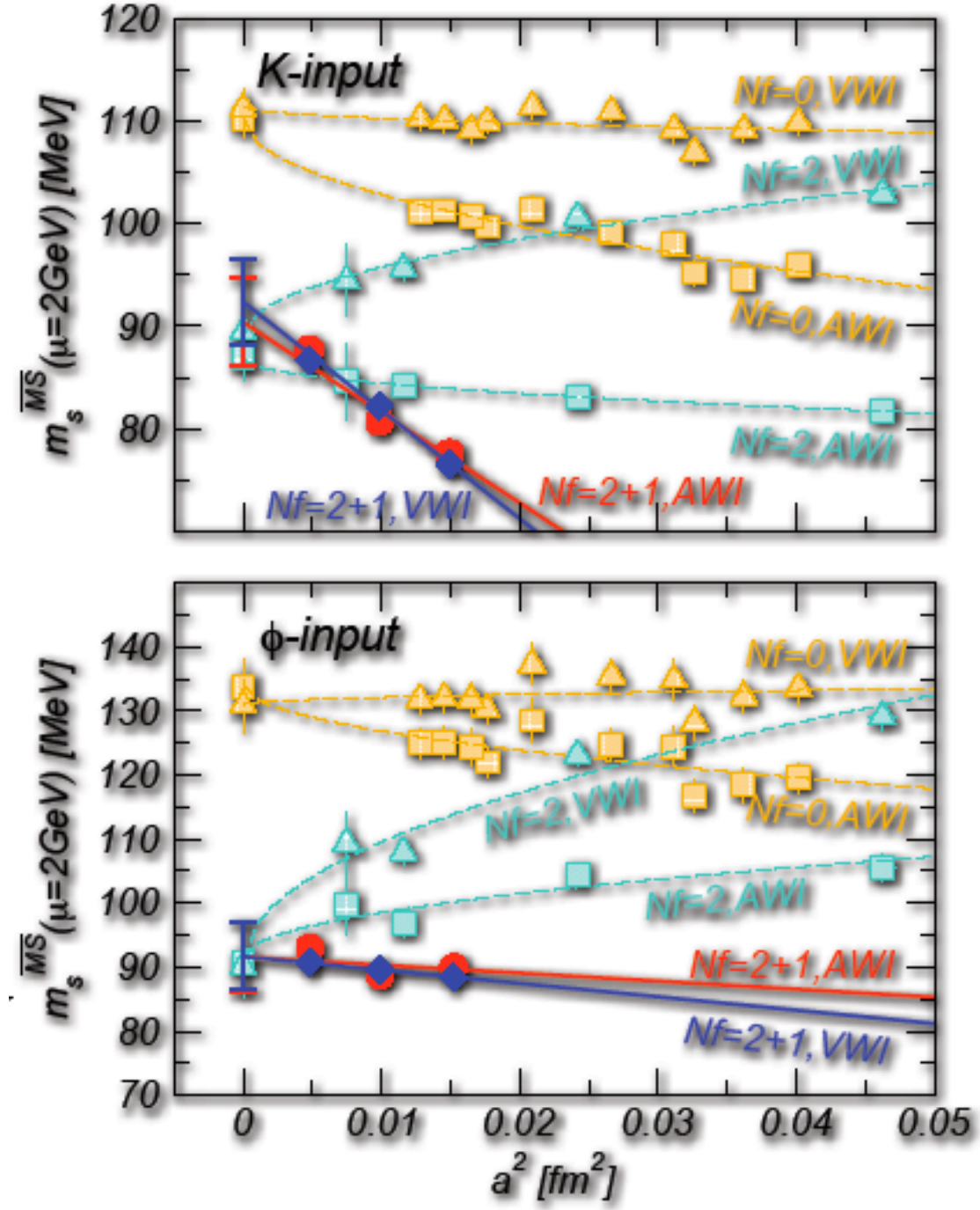


Input: $(m_\rho, m_\pi, m_K(m_\phi))$ for (a, m_l, m_s)

Meson masses agree with experimental values within large errors after continuum extrapolations.

strange quark mass

All definitions agree in the continuum limit.



(Nf=2, 0 : RG+clover(PT))

$$m_s^{\overline{\rm MS}}(2\,{\rm GeV}) = 90.3(3.7)\ {\rm MeV}$$

(AWI, combined K and ϕ inputs)

Staggered

$$m_s^{\overline{\rm MS}}(2\ {\rm GeV}) = 76(0)(3)(7)(0)\ {\rm MeV},\ 87(0)(4)(4)(0)\ {\rm MeV}$$

Remarks

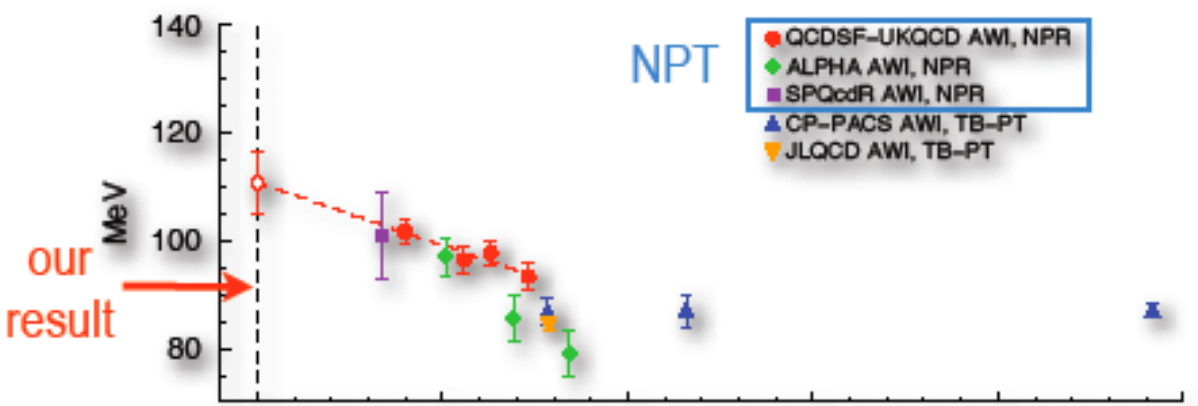
1. Light quark mass is heavy, "polynomial" chiral extrapolation

⇨ Simulation with lighter quark mass/use of (Wilson) ChPT will be needed for precision calculations to compare with staggered results.

⇨ New algorithm

2. Use of perturbative renormalization/improvement factors

$N_f = 2$

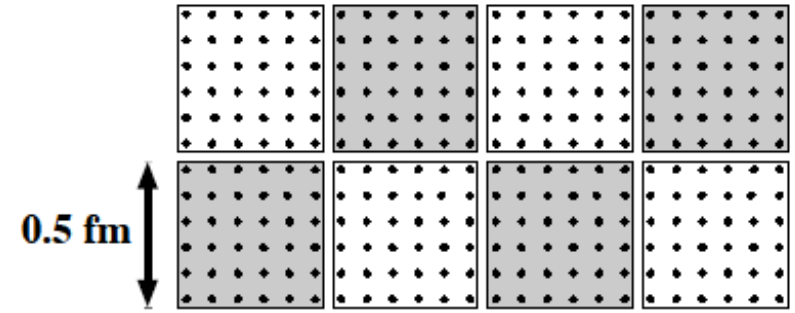


non-perturbative estimates will be needed.

hep-lat/0601004(QCDSF-UKQCD)

New Algorithm toward chiral limit

Domain-Decomposed HMC Luecsher



multi-scales + multi-time ⇨ lighter quark mass

Plaquette gauge/Wilson fermion

$N_f = 2$ Del Debbio,Giusti,Luescher,Petronzio,Tantalo

$a = 0.08$ fm, $La = 1.9$ fm: $m_\pi \simeq 294$ MeV

$a = 0.064$ fm, $La = 2.0$ fm: $m_\pi \simeq 450$ MeV

$N_f = 2 + 1$

Staggered

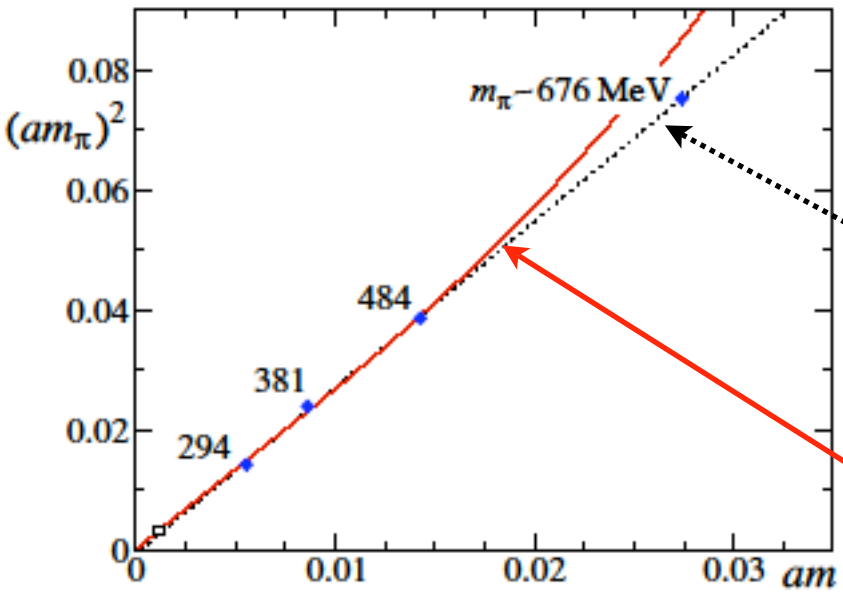
$a = 0.12$ fm, $m_\pi \simeq 253(\mathrm{NG}),420(\mathrm{singlest})$ MeV
$a = 0.09$ fm, $m_\pi \simeq 336(\mathrm{NG}),412(\mathrm{singlet})$ MeV

Comparable to or even lighter than staggered

This algorithm works well on finer and larger lattices !

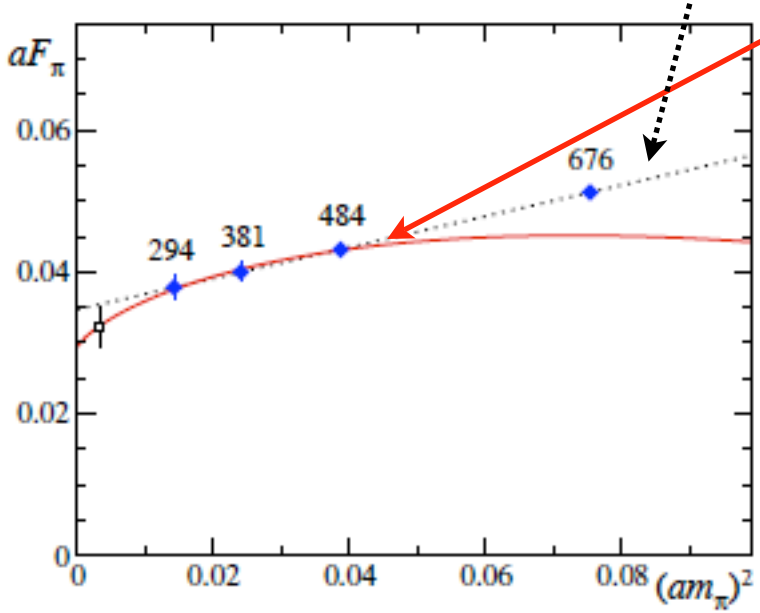
$(m_\pi a)^2$ vs. $m_q a$

a =0.08 fm



Both show linear quark mass dependence.

$f_\pi a$ vs. $(m_\pi a)^2$



Also consistent with ChPT if heaviest point is omitted.

Need more studies for definite conclusion.

perturbative renormalization

Further remarks on Wilson-type with DDHMC

- autocorrelations
- operator mixing problems B_K

  ⇨ Mixed actions ? (overlap valence ?)
- non-perturbative renormalizations/improvements are not so easy

New project using DDHMC

PACS-CS collaboration

New cluster at University of Tsukuba
14 TFlops peak

Target $N_f = 2 + 1$

- RG action/non-perturbative Clover (same as before)
- DDHMC + (UV-filtered) PHMC
- a=0.1, 0.07 fm
- L= 2.4 fm or 3.2fm
- much lighter quark masses $m_\pi/m_\rho = 0.4,\ 0.3,\ 0.2\ (?)$
- 10000 trajs. for 100 independent configurations
- non-perturbative renormalizations/improvements if possibles
- B_K ?

Test runs

$a = 0.1\text{fm},\ La = 1.6\ \text{fm}$

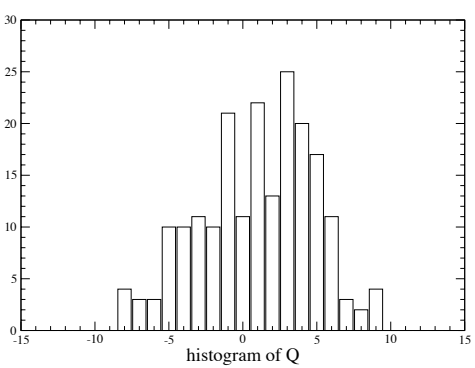
β	κ_{ud}	κ_s	$m_{PS}/m_V(LL)$	$\tau_{\rm int}[P]$	#mult/traj	$\tau_{\rm int}[P]\cdot$#mult/10^3
1.83	0.13655	0.1371	0.7773(14)	56(46)	10040(44)	562(462)
1.90	0.13580	0.1364	0.7628(51)	7.2(1.5)	60288(126)	434(90)
1.90	0.13610	0.1364	0.7392(37)	4.7(0.7)	65997(99)	310(46)
1.90	0.13640	0.1364	0.7172(40)	5.7(1.1)	92239(168)	526(101)
1.90	0.13660	0.1364	0.6908(59)	4.6(0.8)	101641(205)	468(81)
1.90	0.13680	0.1364	0.6694(87)	5.4(1.4)	114052(346)	616(160)
1.90	0.13700	0.1364	0.6397(72)	6.6(1.8)	148625(626)	981(268)
1.90	0.13700	0.1364	0.624(7)	10.6(2.7)	20284(61)	215(55)
1.90	0.13741	0.1364	0.512(11)	16.0(6.6)	31776(171)	508(210)
1.90	0.13759	0.1364	0.458(18)	34.9(23.2)	52686(465)	1839(1222)
1.90	0.13770	0.1364	0.413(22)	36.8(23.6)	69342(990)	2552(1636)

Test runs

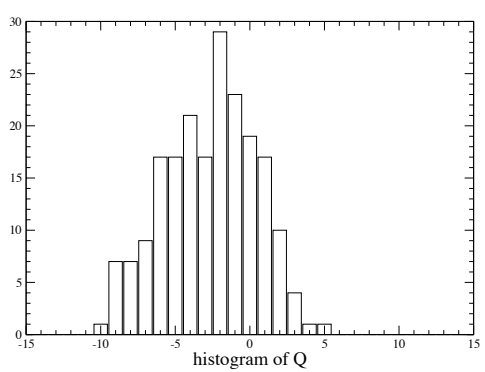
CP-PACS/JLQCD

Test runs

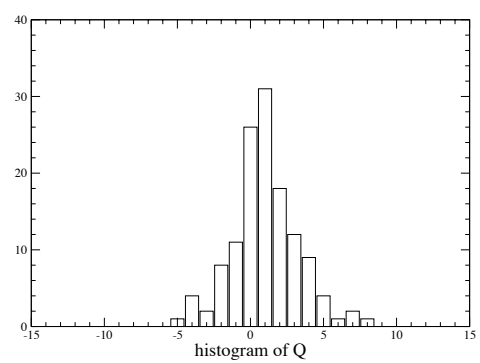
Topological charge distribution



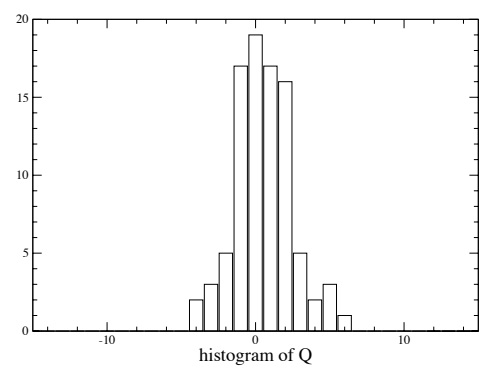
$m_\pi/m_\rho \simeq 0.62$



$m_\pi/m_\rho \simeq 0.51$



$m_\pi/m_\rho \simeq 0.46$



$m_\pi/m_\rho \simeq 0.41$

Cost estimates based on test runs

$a = 0.1\text{fm},\ La = 3.2\ \text{fm}$

	m_π/m_ρ	m_π (MeV)	SR1100	PACS-CS	SAP+GCR
volume			$16^3 \times 32$	$32^3 \times 64$	
sustained speed			13.8 Gflops	4 Tflops	
days for 10000 trajs.	0.62	660	96	11	8
	0.5	480	148	16	11
	0.4	350	243	27	17
	0.3	250	333	37	20
total days			820	91	56

- L^5 dependence assumed
- SAP+GCR to BiCGStab acceleration factor is 3

Encouraging !

Twisted-mass Fermion

Generalities

Variant of Wilson fermion

- Advantages
  - protection against small eigenvalues ⇨ easier at lighter quark
  - automatic (off-shell) O(a) improvement
  - simple operator mixing for B_K
- Shortcomings
  - explicit flavor symmetry breaking
  - difficult for strange quark

European Twisted Mass Collaboration

$N_f = 2$ DBW2 gauge action, $a \simeq 0.17$, 0.13 fm

Symanzik gauge action, $a \simeq 0.09$ fm

N_f=2+1+1 $(u, d) + s + c$

Symanzik gauge action, $a \simeq 0.19$, 014 fm

No physical results yet.

Domain-wall Fermion

Generalities

size of 5th dimension

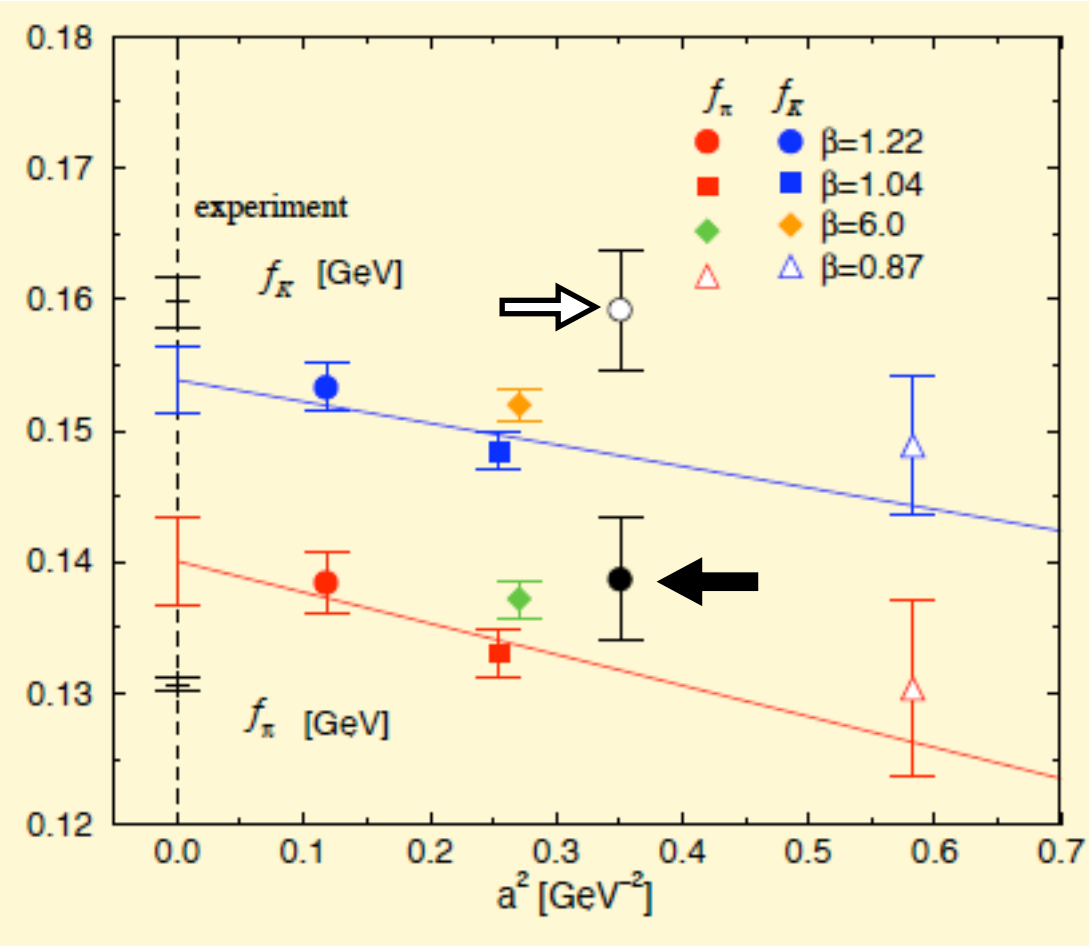
- Advantages
  - exact chiral symmetry at $N_s \to \infty$
    - no O(a) scaling violation
    - renormalization is simple, no extra operator mixings
    - small quark mass
- Shortcomings
  - large computational cost $\simeq N_s \times$ ordinary fermions
  - at $N_s \neq \infty$, small chiral symmetry violation $\sim e^{-cN_s}$
    - small O(a) scaling violation
    - possibilities of small operator mixings

$N_f = 2$

RBC Collaboration

QCDSP

DBW2 gauge action, $a = 0.12$ fm, $La = 2$ fm, $N_s = 12$



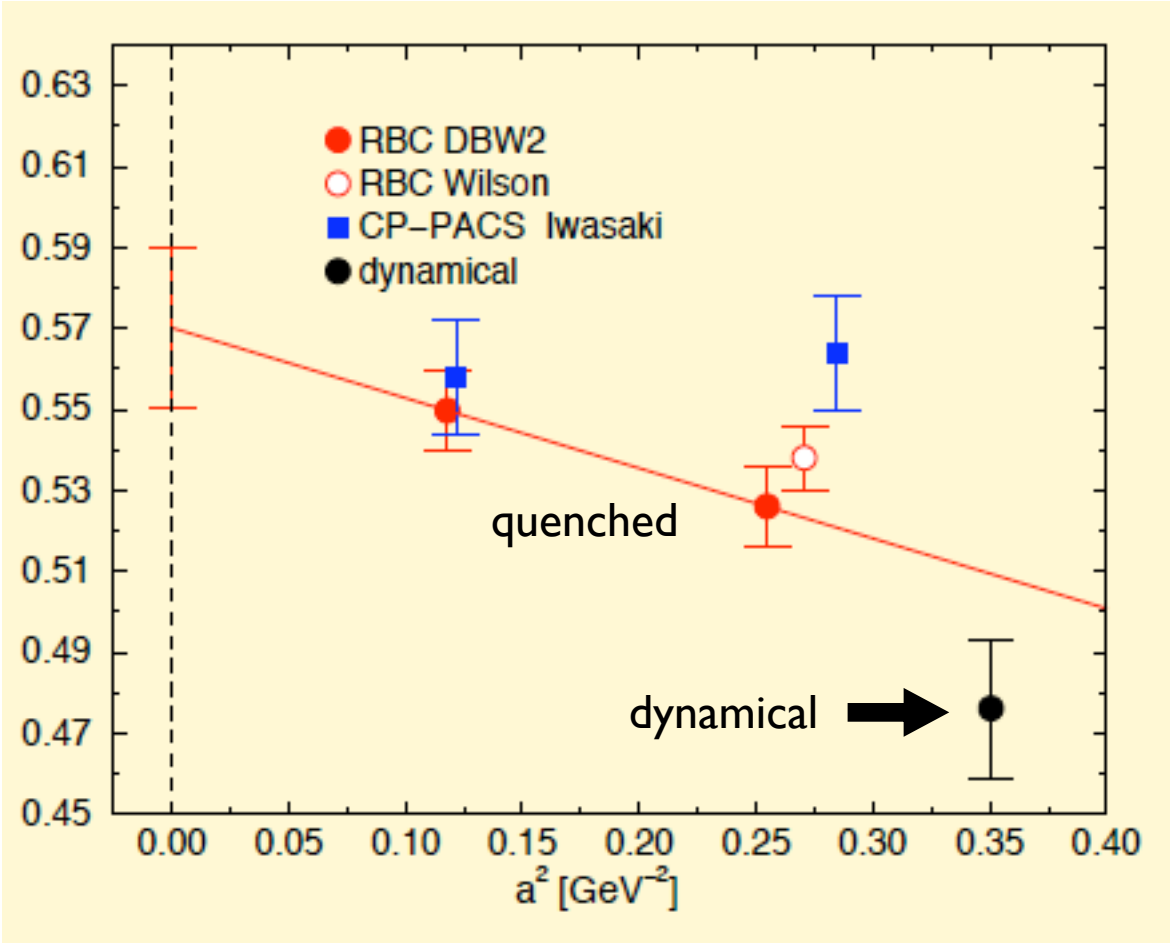
f_π, f_K

Closer to experimental values than quenched results ?
Smaller scaling violation ?

need continuum extrapolations to confirm these

	$N_F = 2$	experiment	$N_F = 0$
f_π	134(4)	130.7	129.0(50)
f_K	157(4)	160	149.7(36)
f_K / f_π	1.18(1)	1.224	1.118(25)

B_K



smaller than quenched results ?

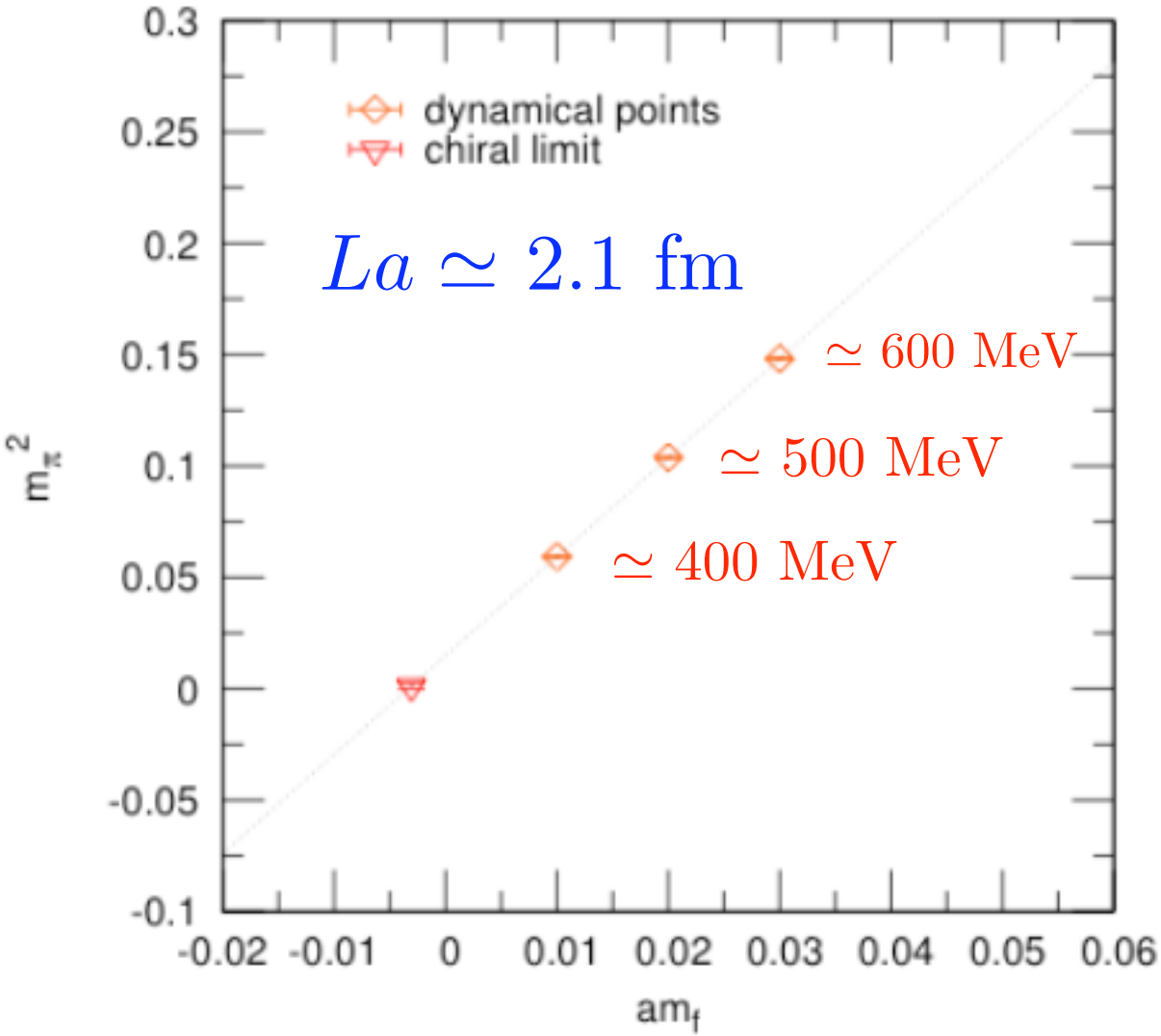
⇩

need a continuum extrapolation to confirm that this is really a dynamical quark effect

non-perturbative renormalization (RI-MOM)

$N_f = 2+1$ on-going project on QCDOC RBC/UKQCD Collaborations

Iwasaki gauge action $N_s = 16$, $a \simeq 0.13$ fm, $La \simeq 2.1$ fm (finsih), 3.1 fm (running)



a little heavier than others

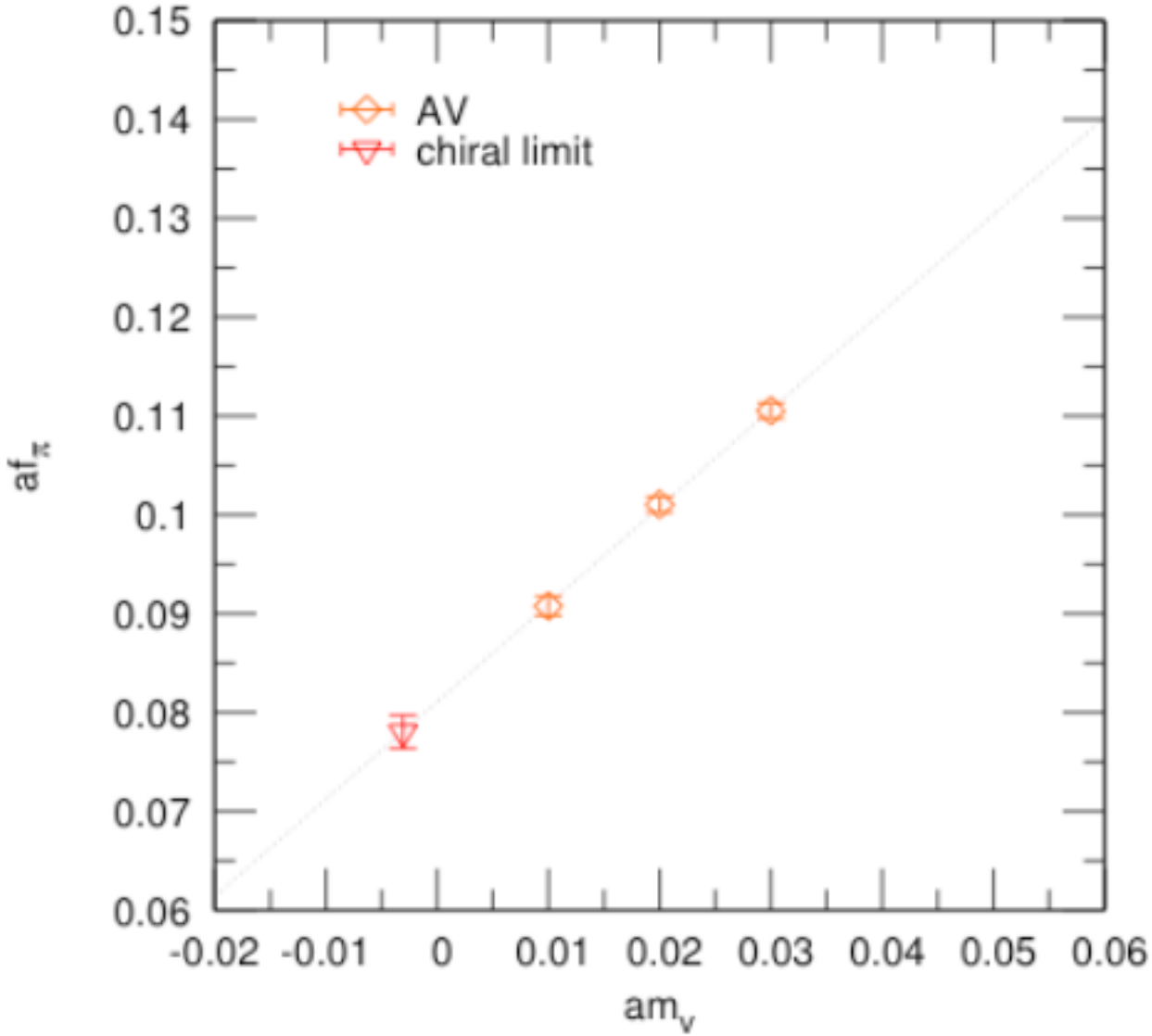
$m_\pi \simeq 290$ MeV (Wilson)

$m_\pi \simeq 250(\mathrm{NG})/420(\mathrm{S})$ MeV (staggered)

⇩

future: smaller quark mass ?

f_π



$f_\pi = 0.781(17) \simeq 125 \text{ MeV}$ (linear chiral extrapolation)

Comments

- DWF at $N_s \neq \infty$ is not exact chiral, but a highly impoved fermion.
  - it is important to check how small effects are
  - operator mixings
- continuum extrapolations are necessary for precision calculations.
  - confirm expected $O(a^2)$ scaling behaviour
  - small O(a) effect ?

Overlap Fermion

$$D_{ov} = 1 + \gamma_5 \mathrm{sign}(H_W) = 1 + \gamma_5 \frac{H_W}{\sqrt{H_W^2}}$$

H_W: negative mass Wilson op.

- satisfies GW relation
  - equivalent to domain-wall fermion at $N_s \to \infty$
  - exact "lattice" chiral symmetry
- low eiganmodes of H_W (main problem)
  - non-locality for $H_W \simeq 0$ (theoretical)
  - computational cost of sign function (technical)
  - reflection/relraction, level-crossing of low modes (technical)

⇨ huge computational cost !

Overlap project (JLQCD2 Collaboration)

Blue Gene/L at KEK (10 racks, 57.3 TFlops)

Strategy

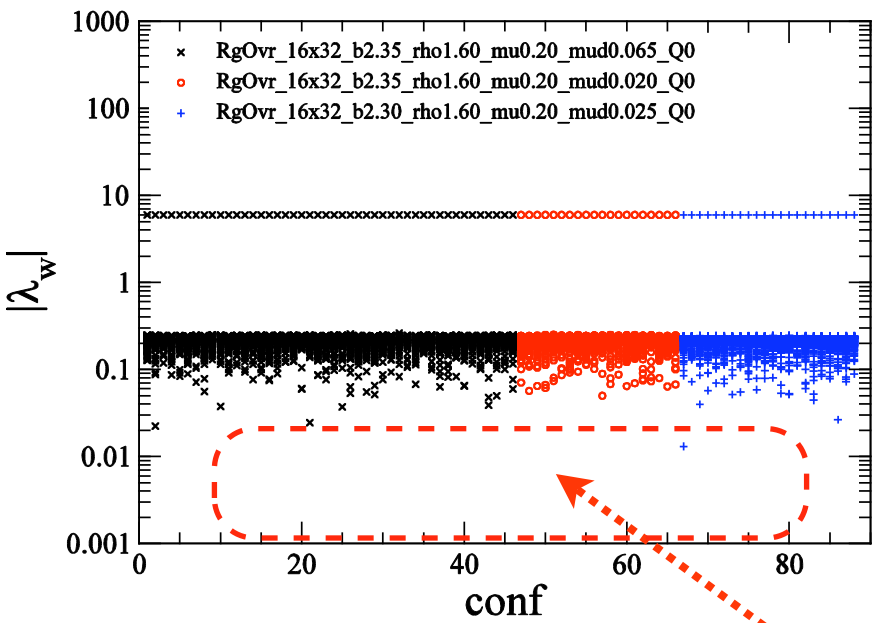
- Fix topology by topology conserving gauge action (Iwasaki + $\det H_W$)
  - $\Rightarrow$ suppress low eigen-modes
    - Locality is OK
    - No reflection/refraction
    - low cost of sign function
- exact GW relation (up to 10^{-8})

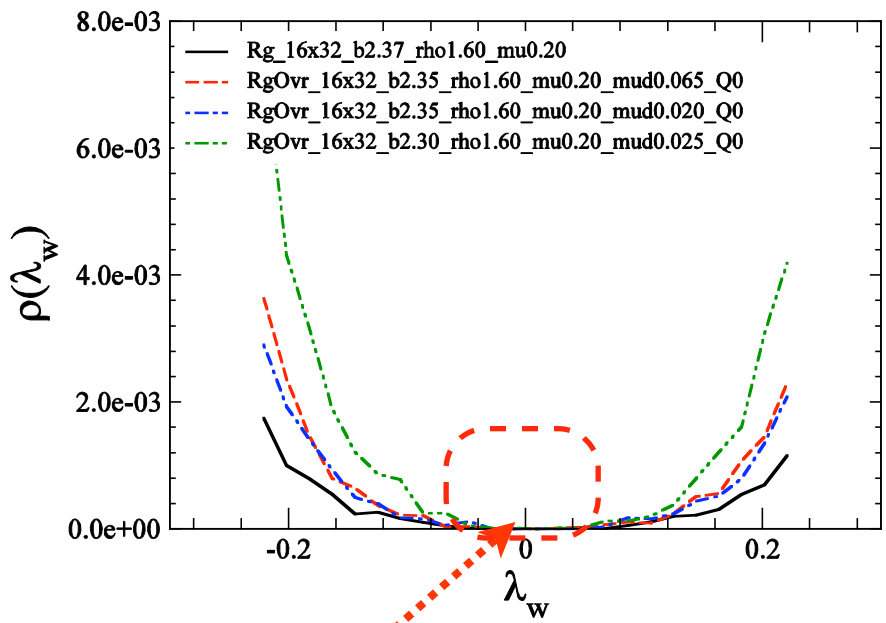
Test runs on a half rack(512 nodes)

$16^3 \times 32,\ a = 0.11\ \mathrm{fm},\ m_q = m_s/4 \sim m_s$

- No near zero-modes of H_W
- 23~47 min/trajectory on a half rack
  - 50 trajectories/day on one rack ⟶ 1500 trajectories/month
    - a few thousand traj. by summer (lattice 06)

eigenvalue distributions with dynamical overlap





No small eigen-modes

Plans

- $N_f = 2,\ a = 0.125\ \mathrm{fm},\ La = 2\ \mathrm{fm},\ m_q = m_s/5 \sim m_s$
  - check of ChPT, spectrum, decay constant
  - weak matrix elemnts: $B_K,\ K \to \pi l \nu,\ \ldots$
  - algorithmic improvement: ex. 5-dim solver $\longrightarrow 3\text{–}5\ \mathrm{speed\ up}$
- larger volume or finer lattice spacing $24^3 \times 48$
- epsilon regime
- $N_f = 2 + 1\ (\mathrm{HMC}\ !)$

Comments

Fixed topology is OK ? $\Longrightarrow$ Large volume may help ?

Fixed point fermion

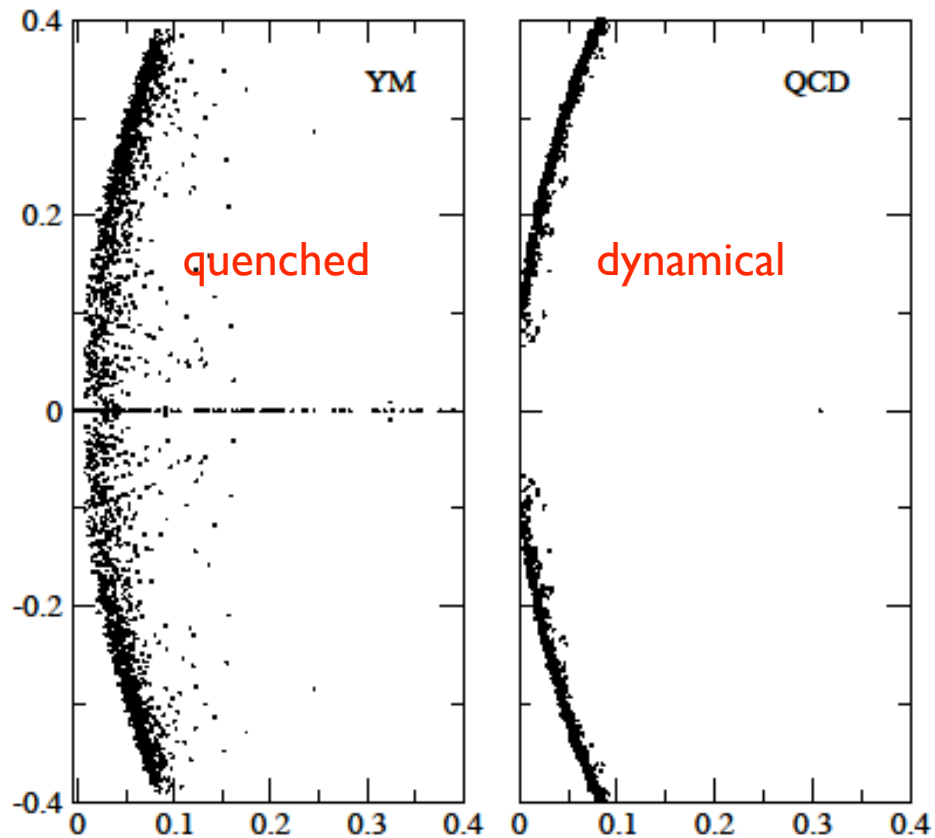
- Fixed point of RG transformation in QCD
  - GW relation, local
  - no cut-off effect in the classical limit
  - small cut-off effect expected at quantum level
- complicated structure of FP action
  - parametrization/truncation ⇨ parametrized FP fermion
  - simpler approximation ⇨ chirally improved fedrmion

Parametrized FP action

Hasenfratz, Hasenfratz, Niedermayer

- FP gauge + FP fermion
  - tiny violation of GW relation
- Global update algorithm $\text{cost} \propto (\text{volume})^2$
- test $N_f = 2+1,\ a = 0.15\ \text{fm},\ La = 1.2\ \text{fm},\ m_\pi \simeq 300\ \text{MeV}(?)$

complex eigenvalues spectrum

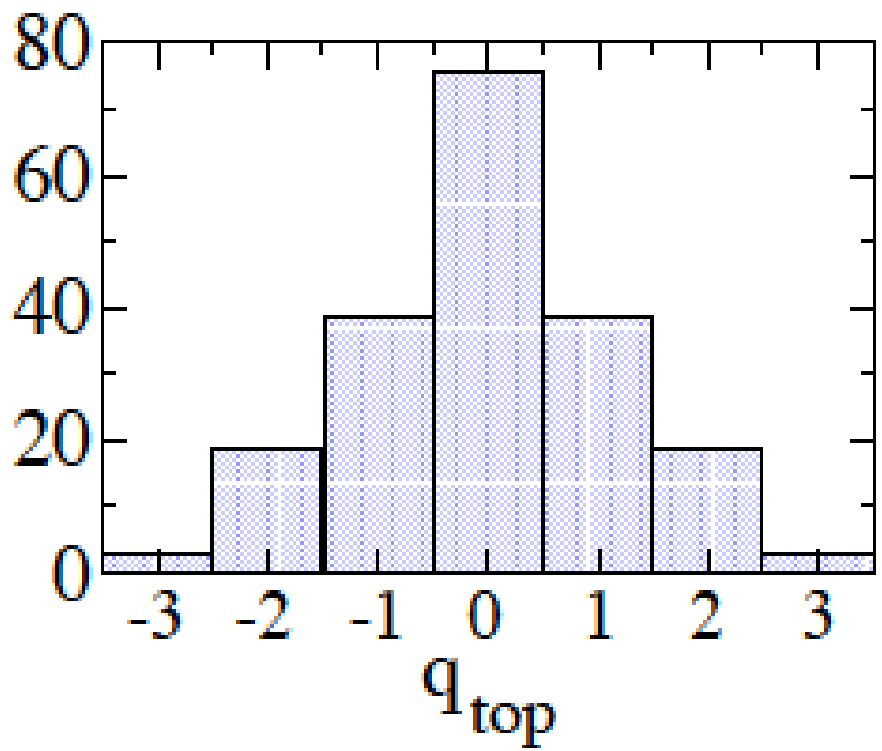


Chirally improved fermion

Bern-Graz-Regensburg (BGR) collaboration

- Tadpole improved Symanzik gauge + chirally improved fermion
  - small violation of GW relation
  - HMC update is possible
- **test** $N_f = 2,\ a = 0.11 \sim 0.14\ \mathrm{fm},\ La = 0.9 \sim 1.6\ \mathrm{fm}$
  - $m_\pi \simeq 600 \sim 1000\ \mathrm{MeV}\ (m_\pi/m_\rho \simeq 0.64 \sim 0.72)$

symmetric distribution of topological charge



Summary

- Era of full QCD: many new results are expected
  - refinement of staggered fermion results MILC
  - much lighter quark mass of Wilson/clover fermions PACS-CS, Europe
  - $N_f = 2 + 1$ dynamical domain-wall fermion RBC/UKQCD
  - dynamical overlap $(N_f = 2 \rightarrow N_f = 2 + 1)$ JLQCD2
  - other fermions (twisted-mass, FP, chirally-improved) ETMC, HHN, BGR
- a year or two from now, we can obtain more precise results on basic quantities, by comparing results from different fermions.
- more complicated quantities in full QCD (predictions)