Instabilities in non-abelian plasmas

Dietrich Bödeker [a] , Kari Rummukainen [b]

[a] Bielefeld U
[b] Oulu U & CERN

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Introduction

what we would like to understand

does an expanding system of quarks and gluons, created in a HIC

- thermalize?
- become isotropic?
- if so, on what time scale?

one should be able to answer this question theoretically in the limit of weak coupling

$$\alpha_{\rm s}(Q) \ll 1$$

Introduction

weak coupling $\rightarrow$ perturbation theory, Boltzmann equation

$gA_\mu \sim \partial_\mu \rightarrow$ naive perturbation theory breaks down due to large fields

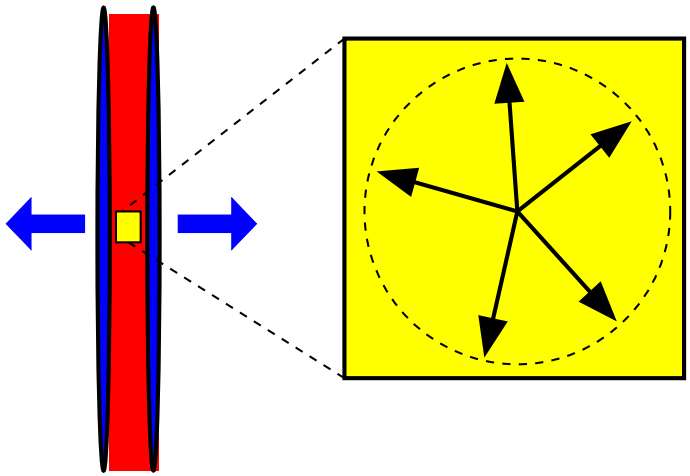
gluon saturation, classical field approximation

common lore:

- small x-saturation, CGC, gluons are produced with $p_\perp \sim Q \gg \Lambda_{\rm QCD}$
- right after collision when $t \sim Q^{-1}$: $f \sim g^{-2}$
- therafter: expansion $\rightarrow f < g^{-2}$

saturated gluon field may be crucial much longer!

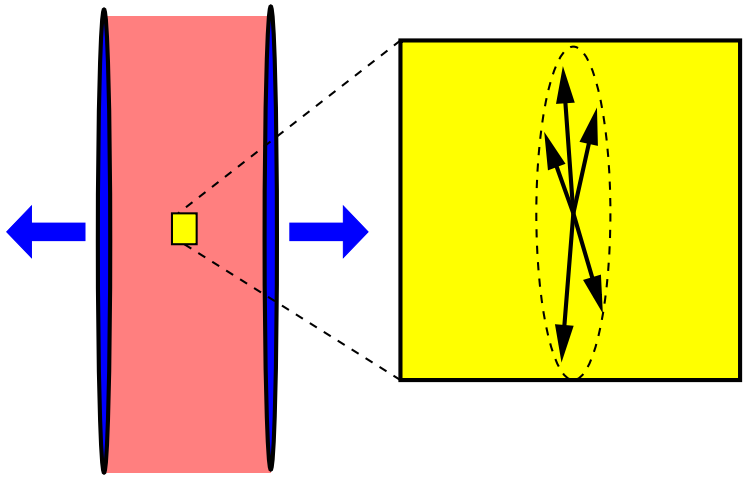
Anisotropic plasma



- $\tau \lesssim Q^{-1}$: production of "hard" gluons with

$$p \sim Q$$

(isotropic momentum distribution)

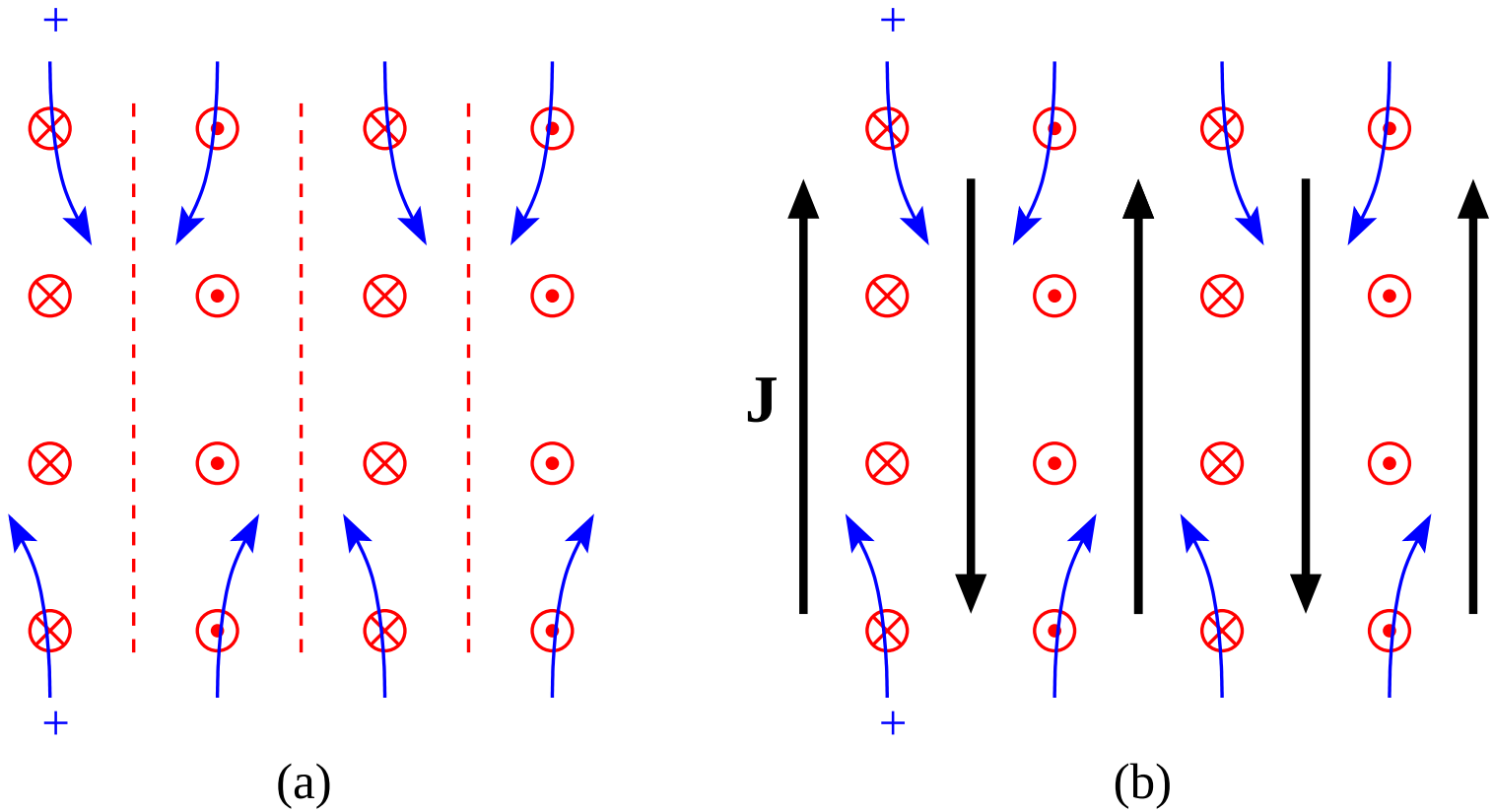


- expansion in z-direction $\longrightarrow$

$$p_z \ll p_\perp$$

Plasma instabilties

anisotropic plasmas are unstable [Weibel,...]



exponential growth of small fluctuations of gauge fields/hard gluon color distribution

should play a role in HIC [Mrówczyński]

Saturation of plasma instabilties

QED: growth stops when hard particle momentum is changed by $O(1)$ within $t \sim k^{-1}$
$\rightarrow$ rapid isotropization

corresponds to gauge field amplitudes $gA \sim Q$

gluon self-interaction $\rightarrow$ QCD plasma instabilites could be very different from QED

growth *should* stop when EOM become non-linear, $gA \sim \partial \sim k \ll Q$

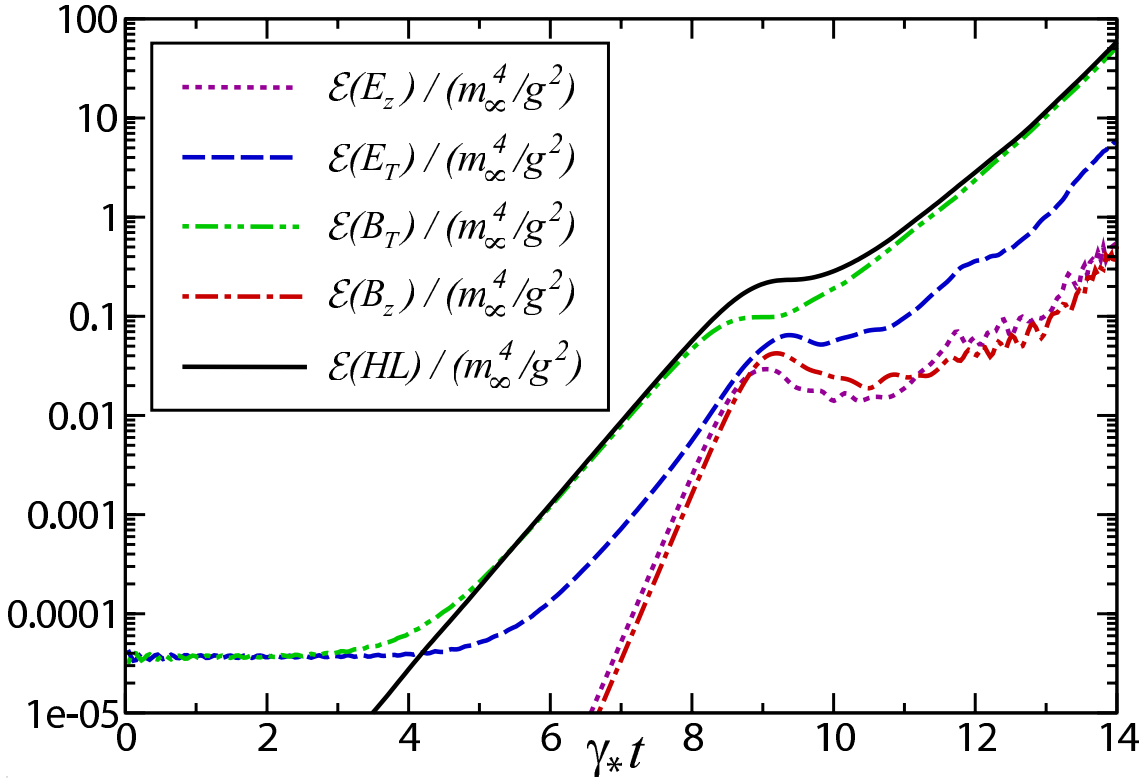
or maybe not? abelianization ? [Arnold,Lenaghan]

difficult non-linear problem!

Saturation of plasma instabilties

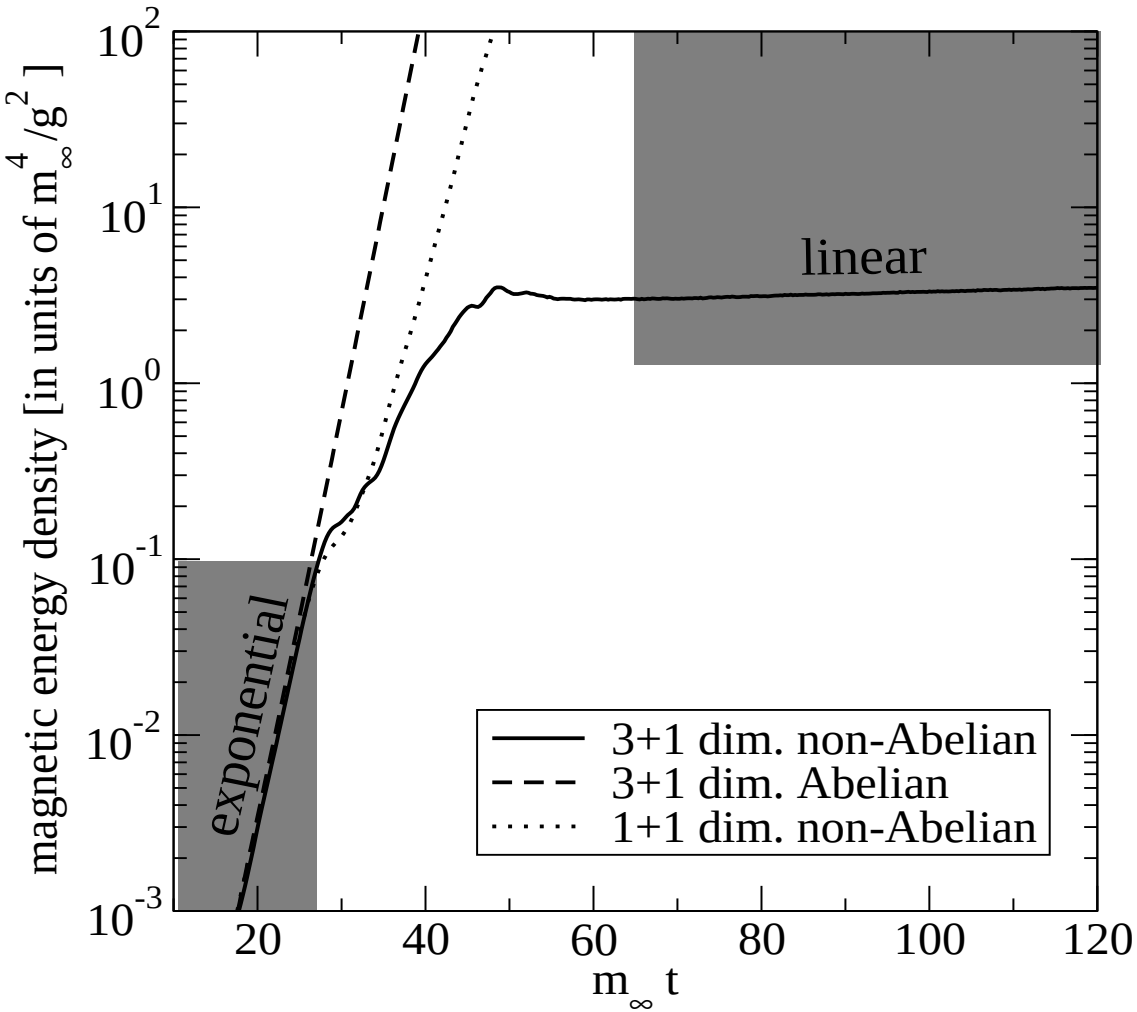
1+1 dimensional systems, $A_\mu(t,z)$, $\delta f(t,z,\mathbf{p})$

- simpler problem
- physically motivated



1+1 dimensional simulations show continued growth [Romatschke,Rebhan,Strickland]

The fate of instabilities in 3+1 dimensions



gaussian random initial conditions

growth saturates at $k \sim gA$

[Arnold, Moore, Yaffe]

not understood:

- why are 1+1 and 3+1 dimensions so different?
- how much do the initial conditions matter?

Are plasma instabilities relevant?

Bottom up thermalization [Baier, Mueller, Son, Schiff]

assumption: boost invariant occupation number $f = f(\tau, \eta - y, \mathbf{p}_\perp)$

p_z is broadened by elastic scattering $\rightarrow$ (in the early stage)

$$p_z \sim Q(Q\tau)^{-1/3}$$

unstable modes at saturation: field strength $\sim k^2/g$

random walk through many domains of size k^{-1} $\rightarrow$

$$p_z \sim Q(Q\tau)^{-1/4}$$

more isotropic than in the early stage of bottom up [DB], only lower limit on p_z

Are plasma instabilities relevant?

exponential growth of fluctuations only when

$$\text{growth rate} \qquad \gamma \gtrsim \frac{1}{\tau} \qquad (= \text{ expansion rate })$$

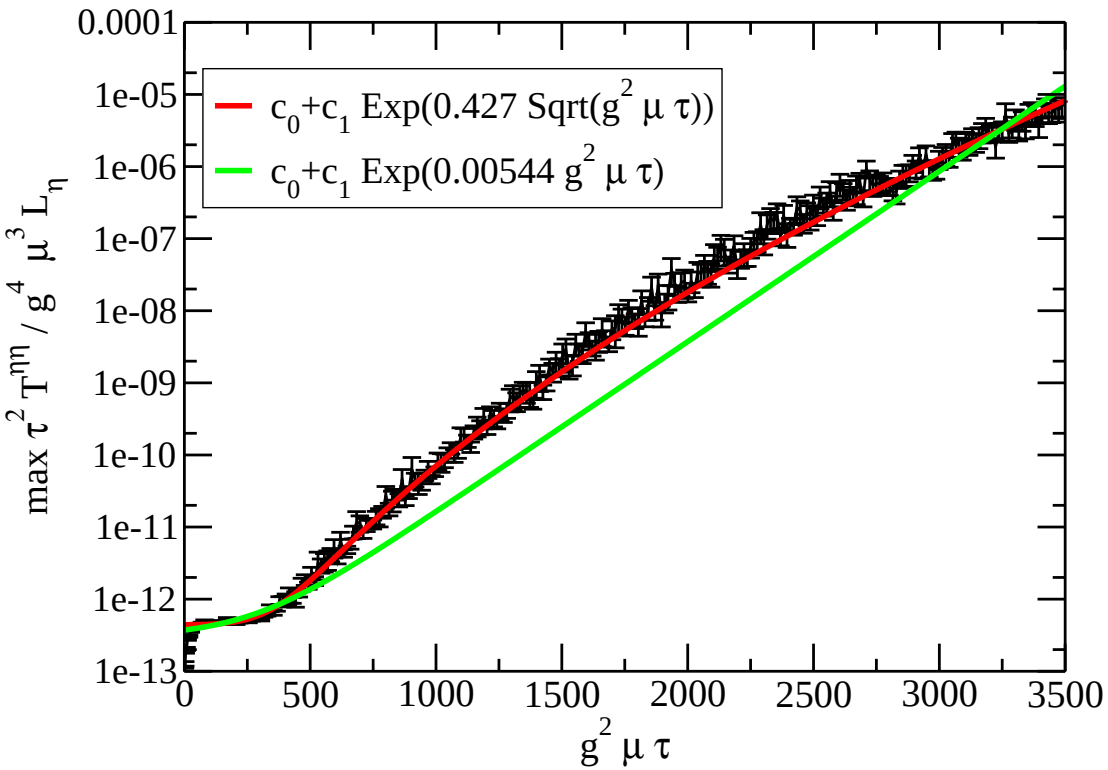
parametrically in Bottom Up

$$\gamma \sim \frac{Q}{\sqrt{Q\tau}}$$

OK for $Q\tau \gtrsim 1$

how big is big?

Are plasma instabilities relevant?



expanding system of SU(2) gauge fields on the lattice [Romatschke, Venugopalan]

analytic estimates: unstable modes grow when $Q\tau \sim 20$ [Rebhan,Romatschke]

Simulation of strongly anisotropic plasmas

system with "hard" classical particles + soft classical fields

neglect backreaction on hard particle momentum distribution $\bar{f}(\mathbf{p})$ "hard loop effective theory"

$$(D_\mu F^{\mu\nu})^a = g \int \frac{d^3p}{(2\pi)^3} v^\nu f^a,$$

$$(v \cdot Df)^a + g v^\mu F^a_{\mu i} \frac{\partial \bar{f}}{\partial p^i} = 0$$

we use the W field method, SU(2) gauge group

$$W^a(x, \mathbf{v}) \equiv 4\pi g \int_0^\infty \frac{dp p^2}{(2\pi)^3} f^a(x, \mathbf{v}p)$$

- weak sphaleron rate in thermal equilibrium (isotropic $\bar{f}$) [Bödeker,Moore,Rummukainen]
- plasma instabilities in SU(2) [Arnold,Moore,Yaffe]

Simulation of strongly anisotropic plasmas

we expand W, $\bar{f}$ in spherical harmonics

$$\bar{f}(p\mathbf{v}) = \sum_{l}^{L_{\rm asym}} \bar{f}_l(p) Y_{l0}(\mathbf{v})$$

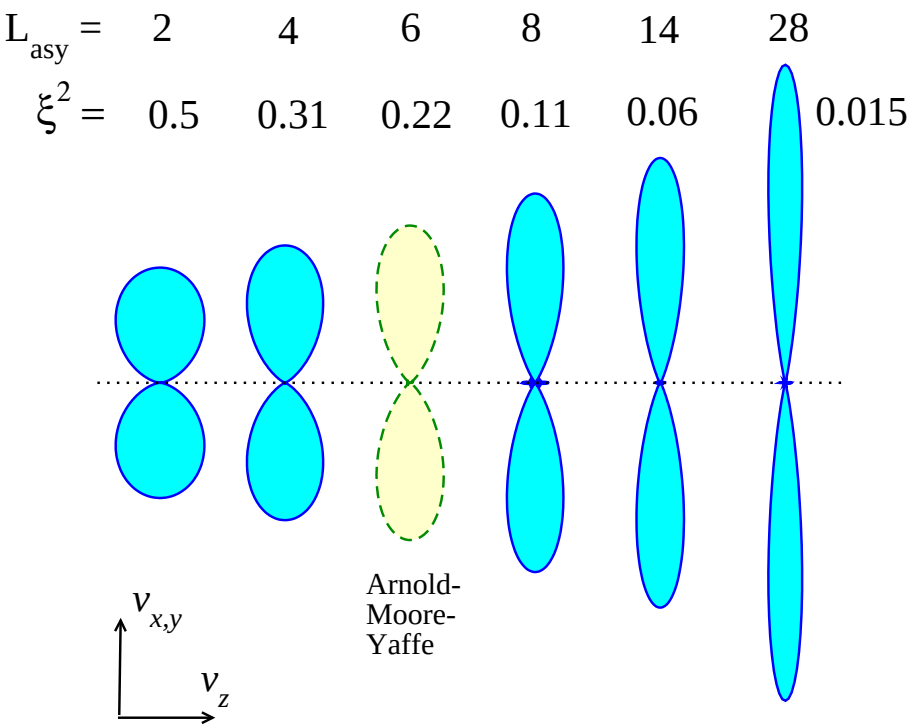
$$W(x,\mathbf{v}) = \sum_{l,m}^{L_{\rm max}} W_{lm}(x) Y_{lm}(\mathbf{v})$$

we use similar techniques as Arnold, Moore, Yaffe but

- 5 different values for the anisotropy, both weaker and much stronger
- large lattices (up to 240^3) with a large number of W-fields (up to $L_{\rm max} = 240$, i.e., 14250 fields in addition to A_i^a)

Simulation of strongly anisotropic plasmas

for each L_{asym} we try to maximally localize the distribution along the xy-plane



propellor shaped distribution

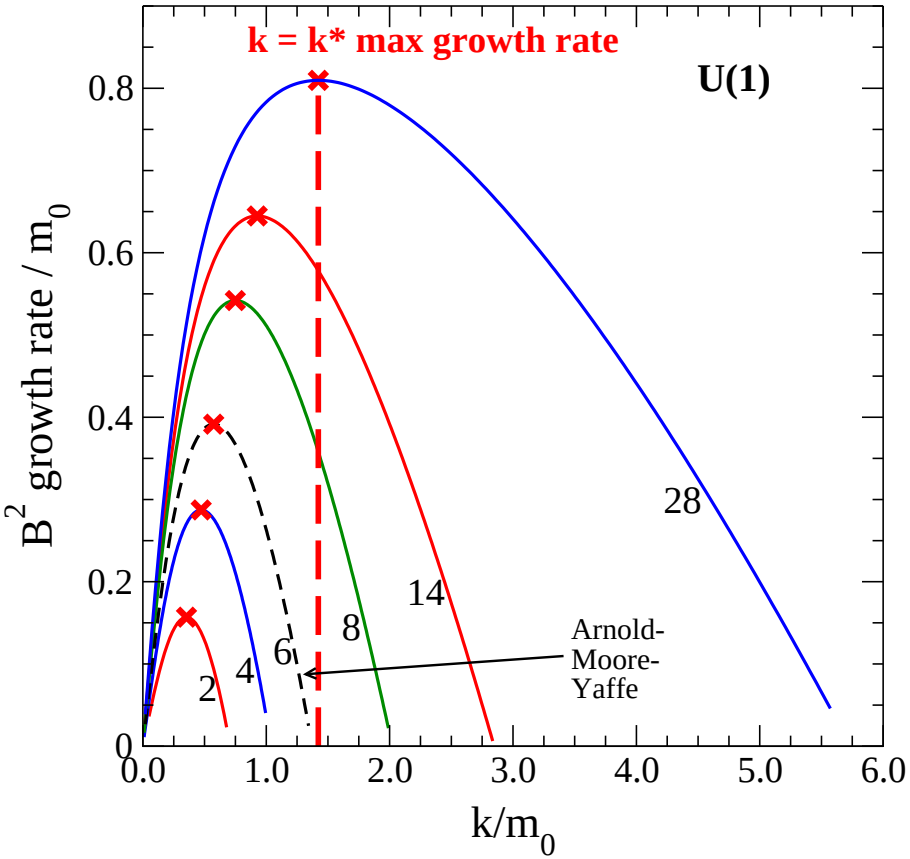
asymmetry parameter

$\xi^2 \equiv \frac{v_z^2}{\mathbf{v}^2}$

$\xi = 0.5, \ldots, 0.015$

one needs $L_{\mathrm{max}} > L_{\mathrm{asym}}$

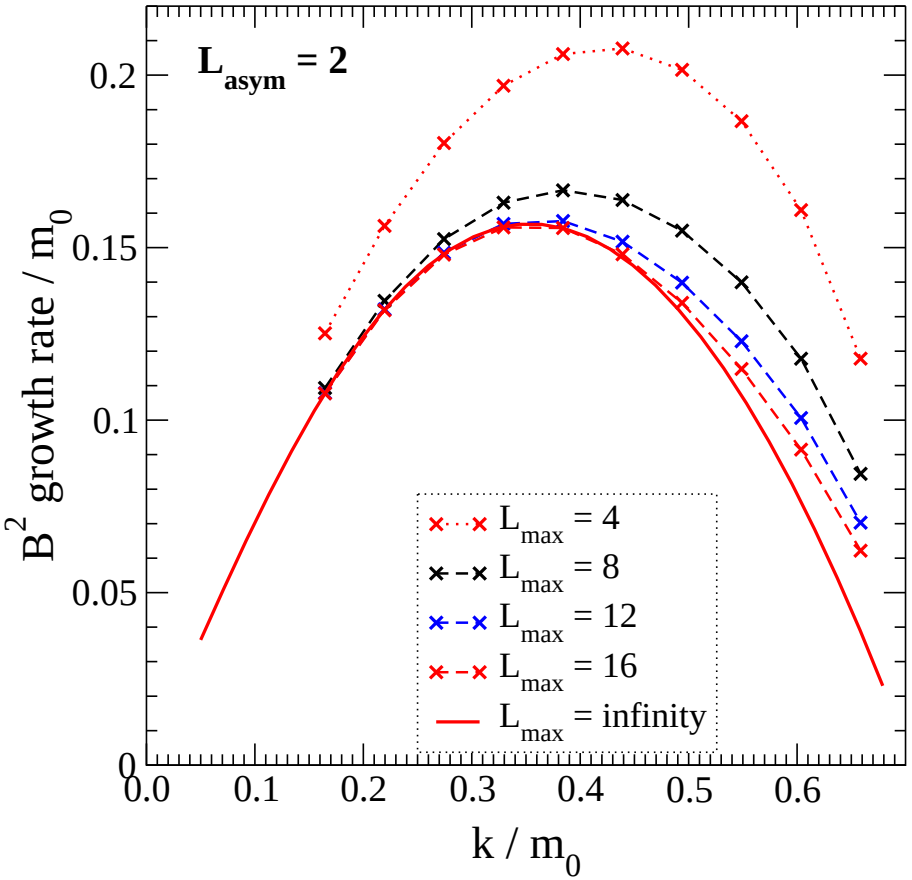
Growth rate in weak field regime

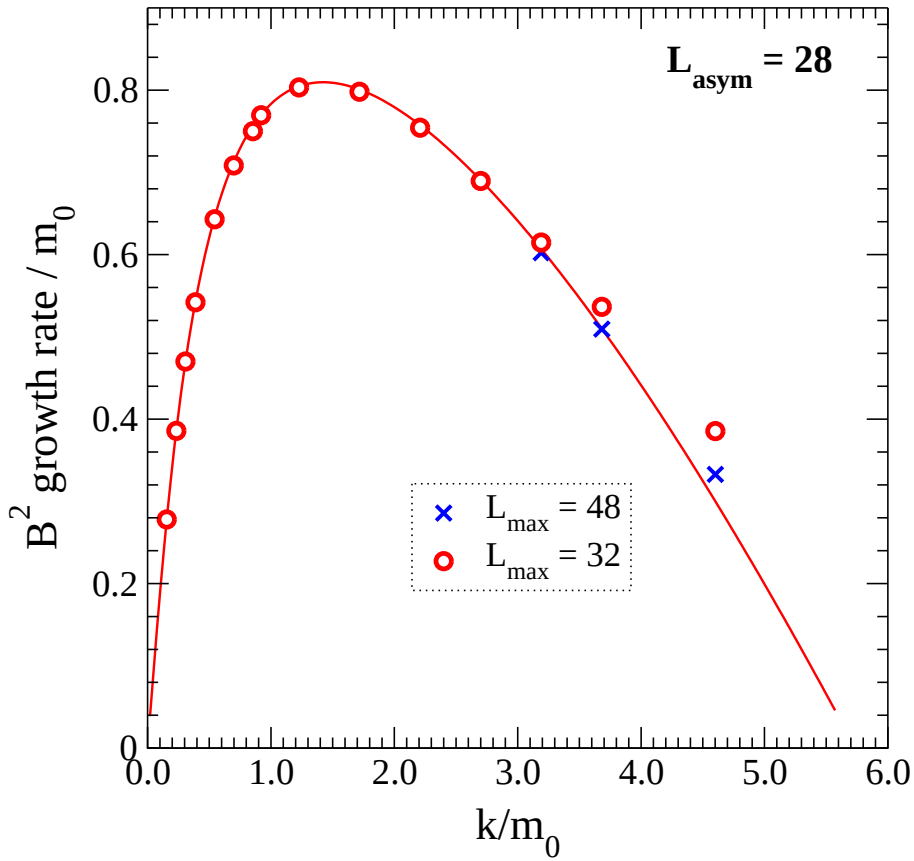


growth rate as a function of k

much larger range of unstable modes for large asymmetry

Growth rate in weak field regime





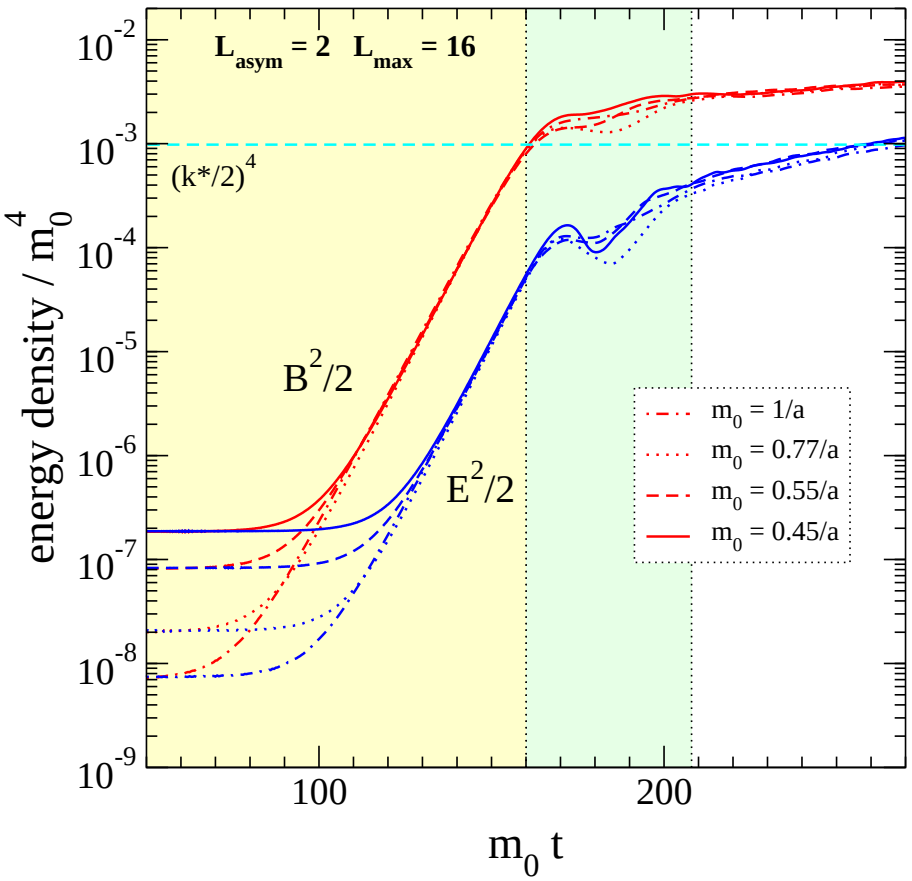
$L_{\max}$ cutoff effects appear to be small

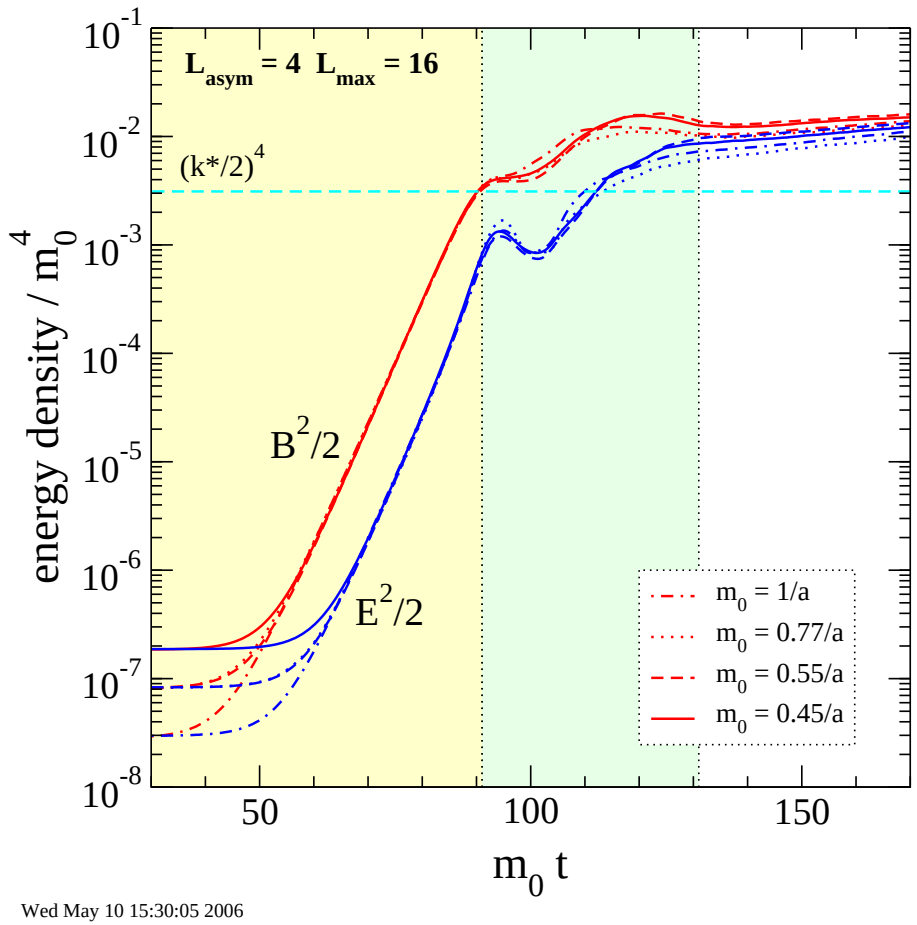
Results: growth of energy for weak anisotropy

little growth beyond weak field regime

lattice UV modes far from saturated

consistend with Arnold, Moore, Yaffe ($L_{\rm asym} = 6$)



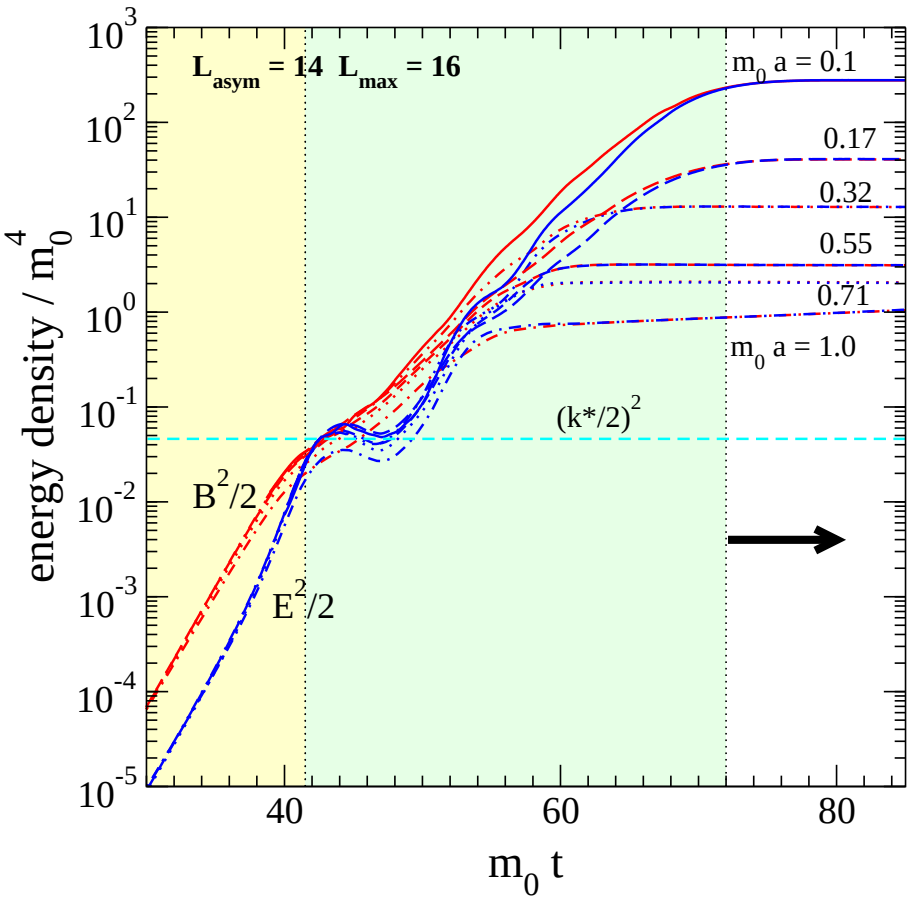


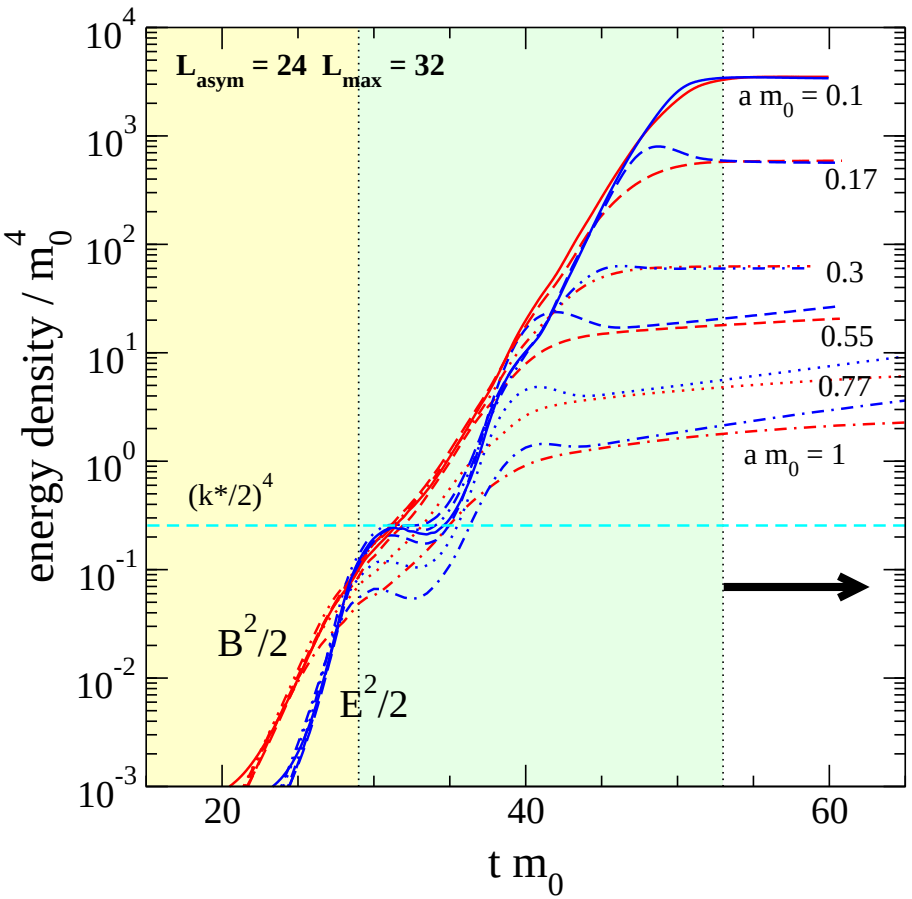
Results: growth of energy for strong anisotropy

quasi-exponential growth beyond weak field regime

stops when lattice UV modes saturate, $aA \sim 1$

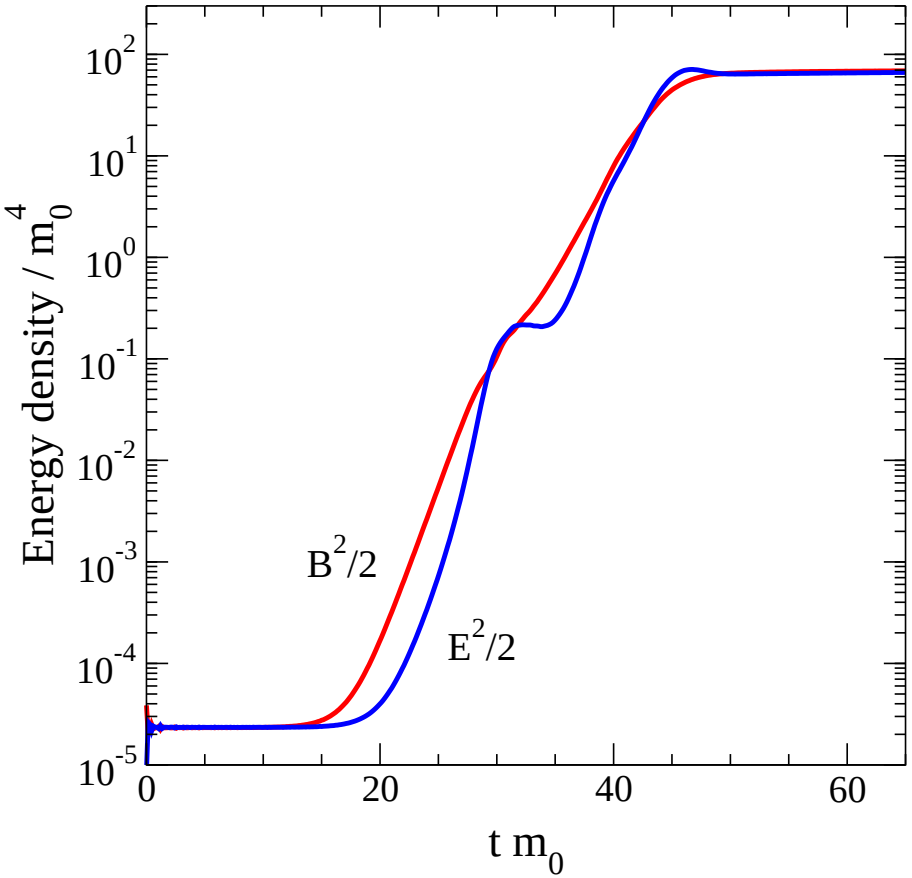
how far does it continue when $a \to 0$?

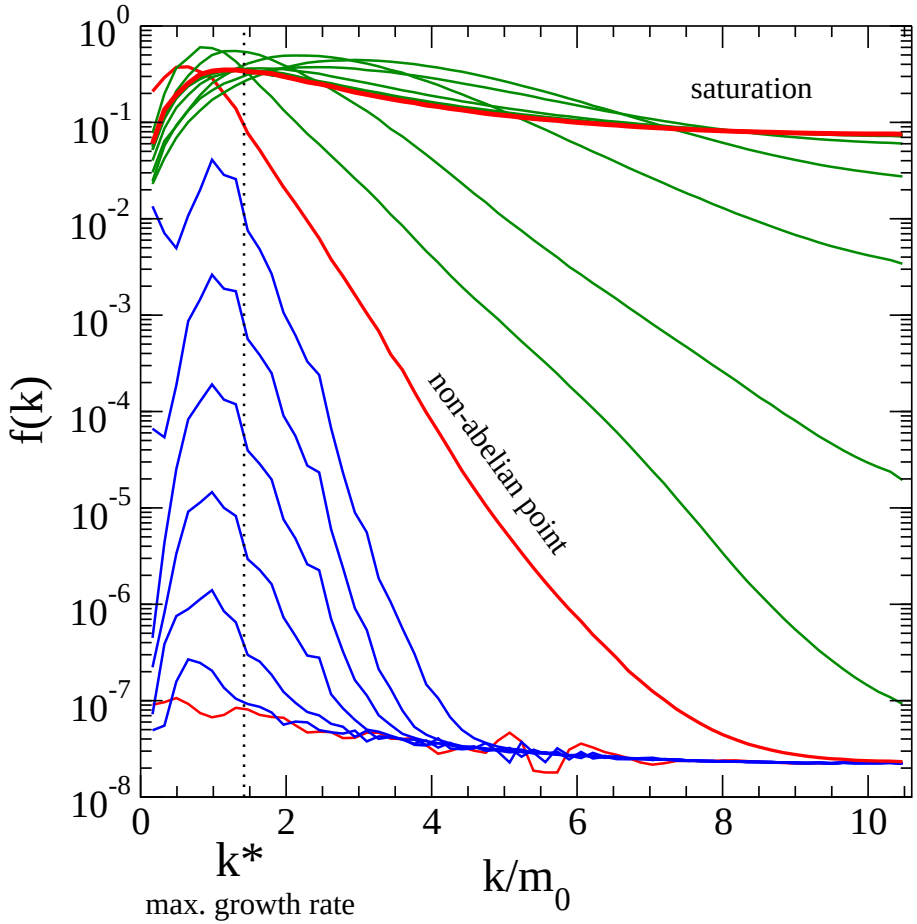




Coulomb gauge spectrum

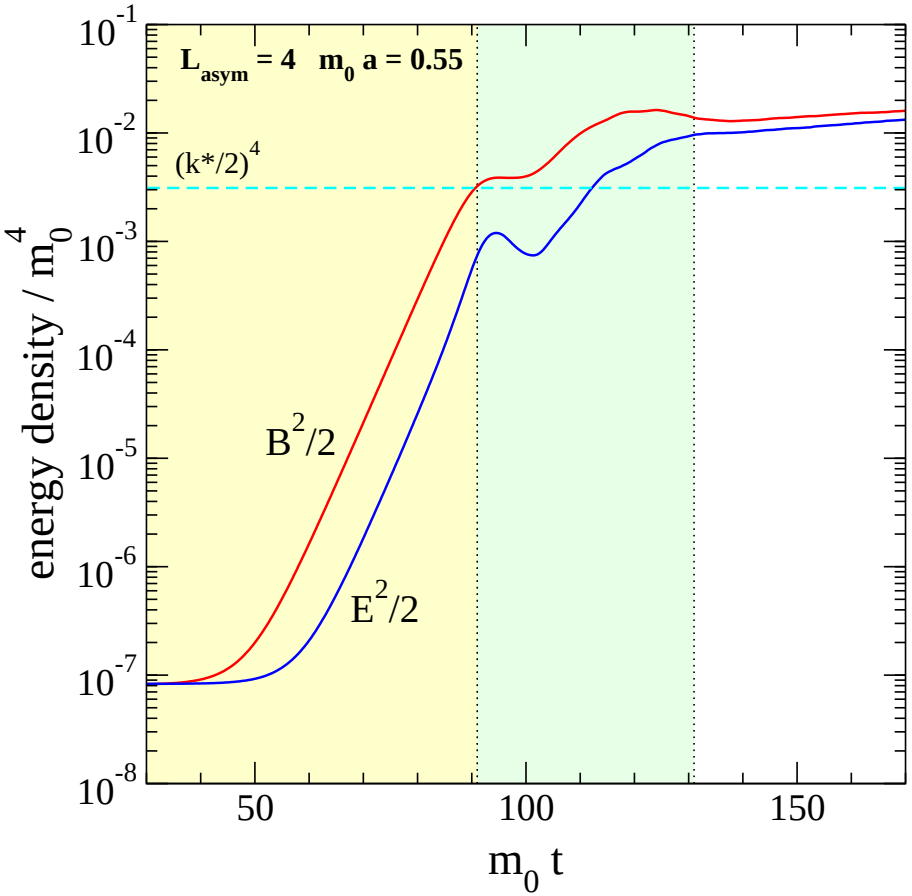
gauge fixed to Coulomb gauge, $f_{\rm soft}(\mathbf{k}) \propto kA(\mathbf{k})$

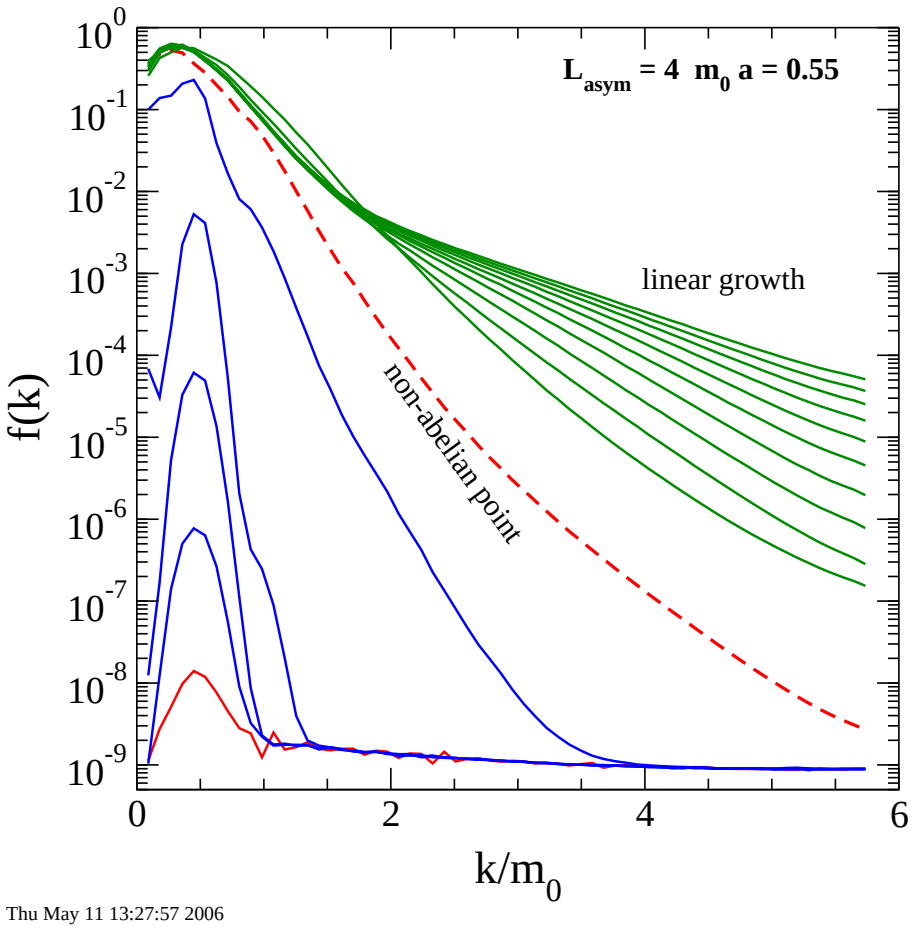




Coulomb gauge spectrum for weak anisotropy

weak anisotropy remains IR dominated





UV avalanche?

Rapid growth of UV modes first observed by Dumitru, Nara, Strickland

Romatschke, Venugopalan: UV modes become unstable when backreaction sets in?

we find UV avalanche without backreaction

dependence on initial conditions?

dependence on volume?

more work is necessary!