RHIC Physics in the Context of the Standard Model: A Workshop and Symposium
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Structure Functions on the Lattice

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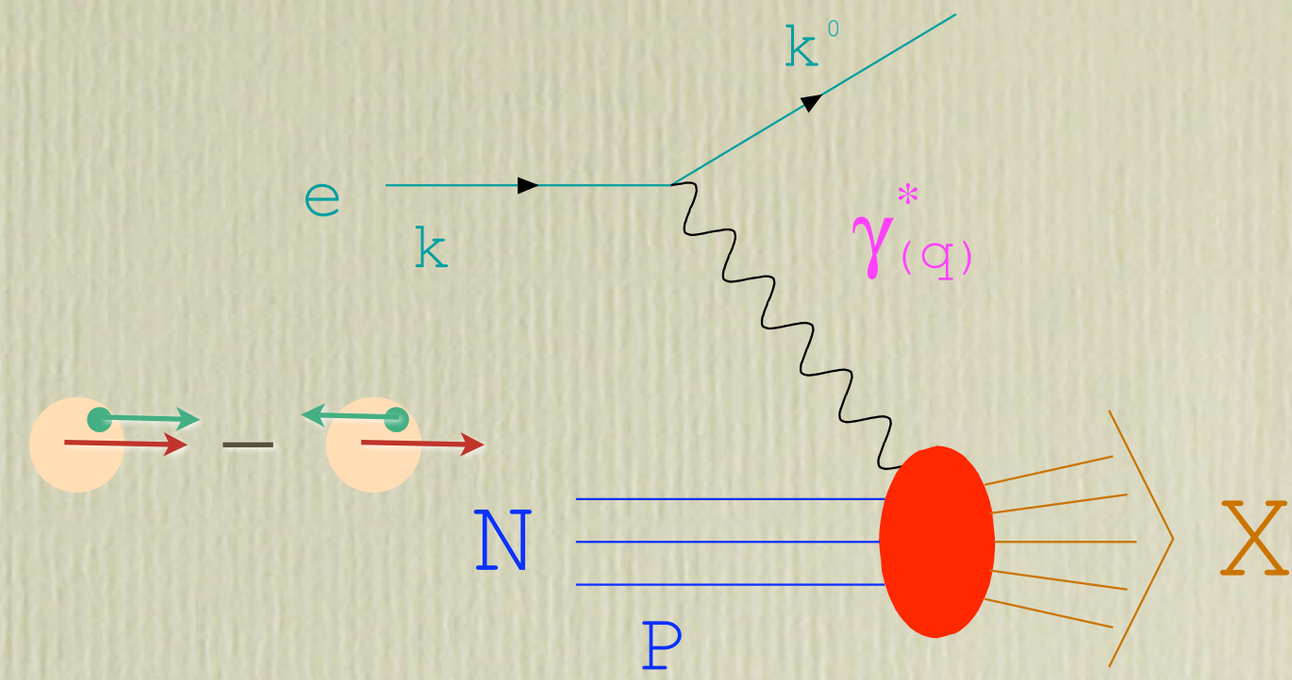
Moments of Structure Functions

$$2\int_0^1 dx x^{n-1} F_1(x,Q^2) \longrightarrow \langle x^n \rangle_q$$

$$2\int_0^1 dx x^{n} g_1(x,Q^2) \longrightarrow \langle x^n \rangle_{\Delta q}$$

$$2\int_0^1 dx x^{n} g_2(x,Q^2) \longrightarrow \langle x^n \rangle_{\Delta q}, d_n$$



- $\langle x^n \rangle_q$, $\langle x^n \rangle_{\Delta q}$,$d_n$ Nucleon matrix elements of local operators $\langle P,S|\mathcal{O}|P,S\rangle$

$$\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - trace\right]q$$

$$\mathcal{O}^{5q}_{\{\mu_1\mu_2\cdots\mu_n\}} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_5\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - trace\right]q$$

Transversity

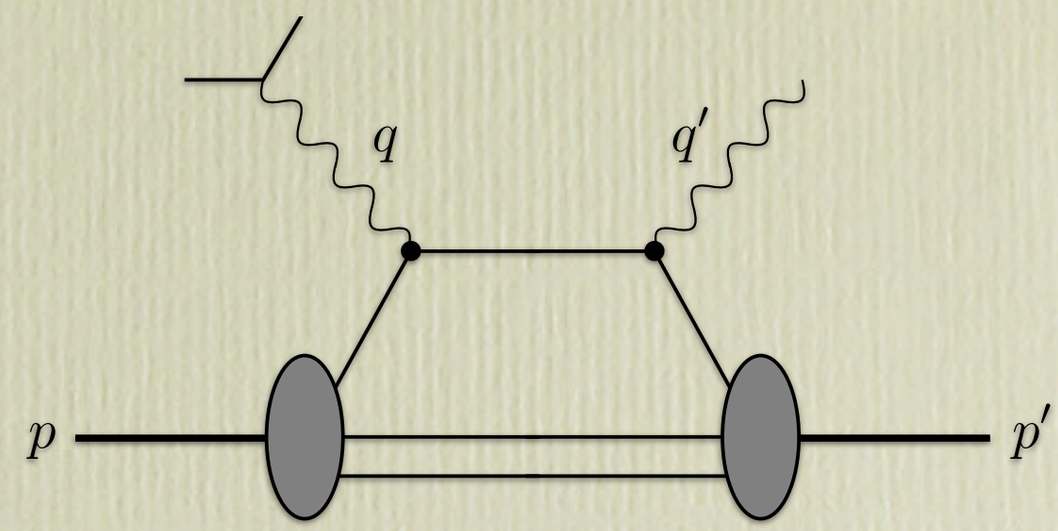
$$\int_0^1 dx x^n h_1(x, Q^2) \longrightarrow \langle x^n \rangle_{\delta q}$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \overline{q}[\left(\frac{i}{2}\right)^n \gamma_5 \sigma_{\rho\nu} \overleftrightarrow{D}_{\mu_1} \cdots \overleftrightarrow{D}_{\mu_n} - traces]q$$

- RHIC spin program

GPDs and Form Factors

Deeply virtual Compton scattering:



Euclidean Matrix elements:

$$\langle P'|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle$$
$$\sim \int dx\, x^{n-1}[H(x,\xi,t), E(x,\xi,t)]$$
$$\to A_{ni}(t), B_{ni}(t), C_n(t)$$

Lattice problems

- Broken Lorentz symmetry $\Longrightarrow$ higher moment operators mix with **lower** dimensional operators. Operators belonging in irreducible representations of $O(4)$ transform reducibly under the lattice Hyper-cubic group.

Continuum/Chiral extrapolation

- Continuum extrapolation ($a \to 0$).

  improved Wilson fermions: reduce or eliminate $\mathcal{O}(a)$ errors

  domain wall fermions have $\mathcal{O}(a^2)$

- Chiral extrapolation ($m_q \to 0$).

  Chiral perturbation theory

  Applicable at $a = 0$ for Wilson or KS fermions

  There exists modified χPT for $a \neq 0$

  Regular χPT at $a \neq 0$ for domain wall fermions

Lattice SF calculations

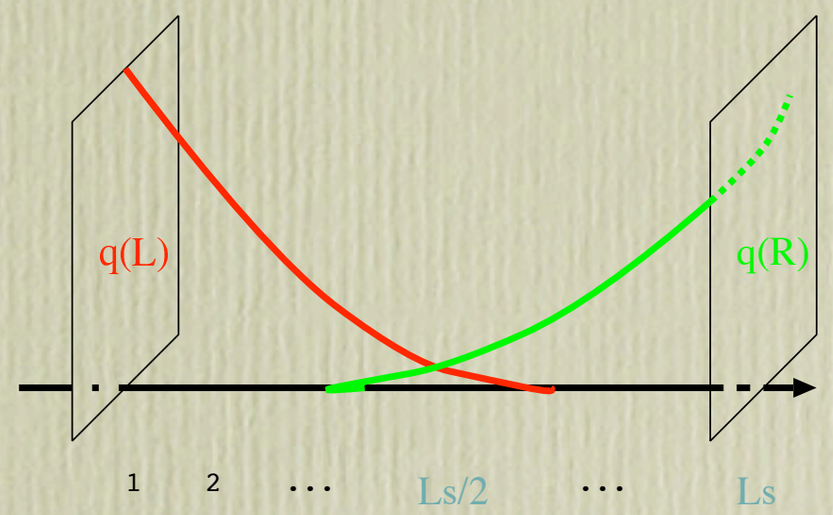
- QCDSF [Phys.Rev.D53 '96, Phys.Rev.D63 '01, hep-ph/0304249]
  - Structure Functions/Generalized PD and Form Factors
  - Wilson Fermions and Wilson/Clover
  - Quenched and Dynamical (with UKQCD)
  - Starting chiral fermions
- LHPC-SESAM [hep-lat/0201021, hep-lat/0312014]
  - Structure Functions/Generalized PD and Form Factors
  - Wilson Fermions
  - Quenched and Dynamical
  - Now doing DWF with dynamical Kogut-Susskind
- RBC [hep-lat/0209137, hep-lat/0309113]
  - Structure Functions and Form Factors
  - First to use Domain Wall Fermions
    - Reduced discretization errors
    - Can work close to the chiral limit
  - Quenched and Dynamical

Summary

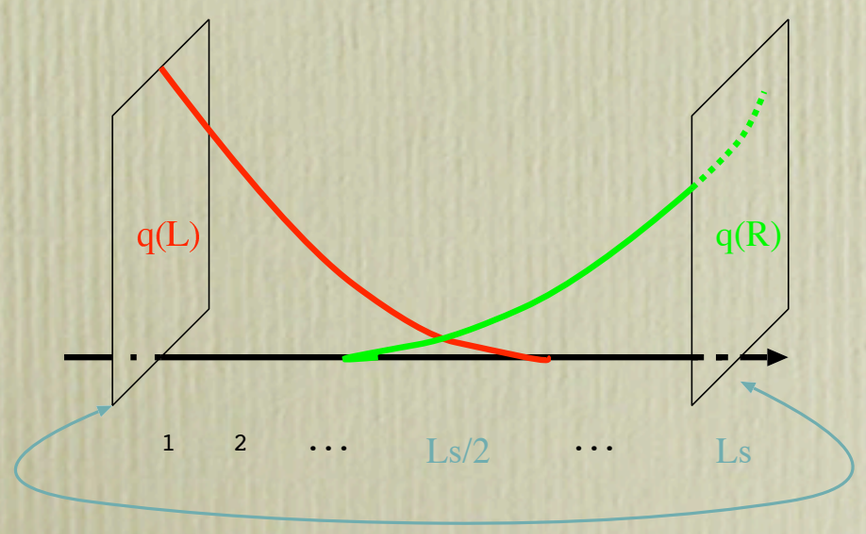
- Review work done with RBC
- Show some results from LHPC

Domain Wall Fermions for QCD

Formulate the 5D Wilson fermions with mass $M \neq 0$ in $s \epsilon [1, L_s]$



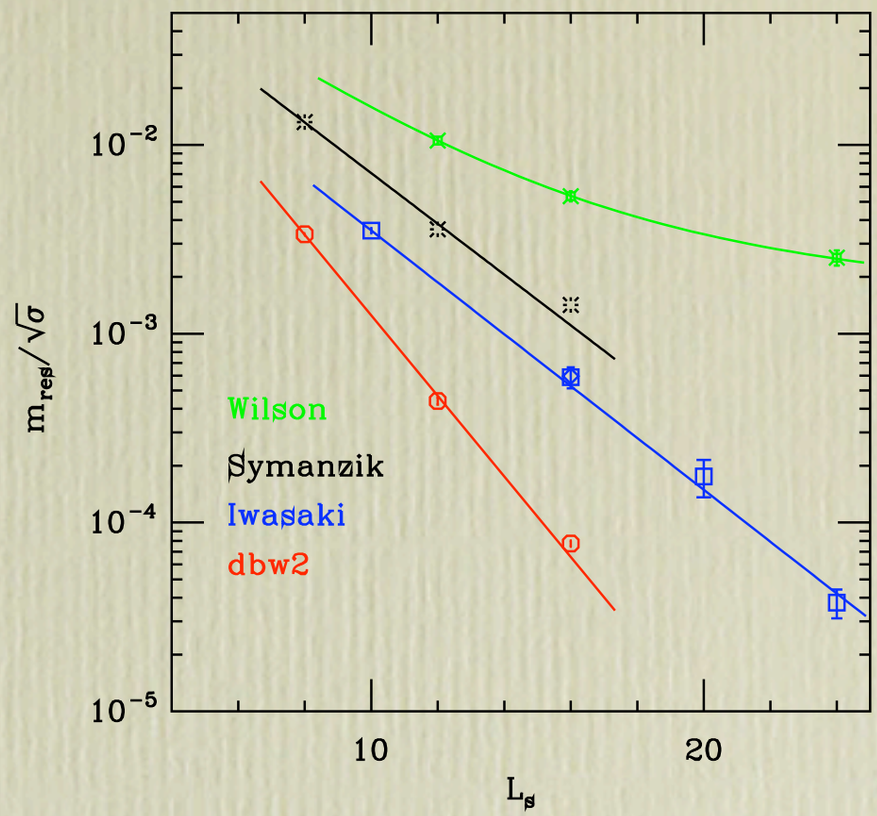
For $-2 < M < 0$, light chiral modes are bound on the walls.
Only one Dirac fermion without doublers remains.

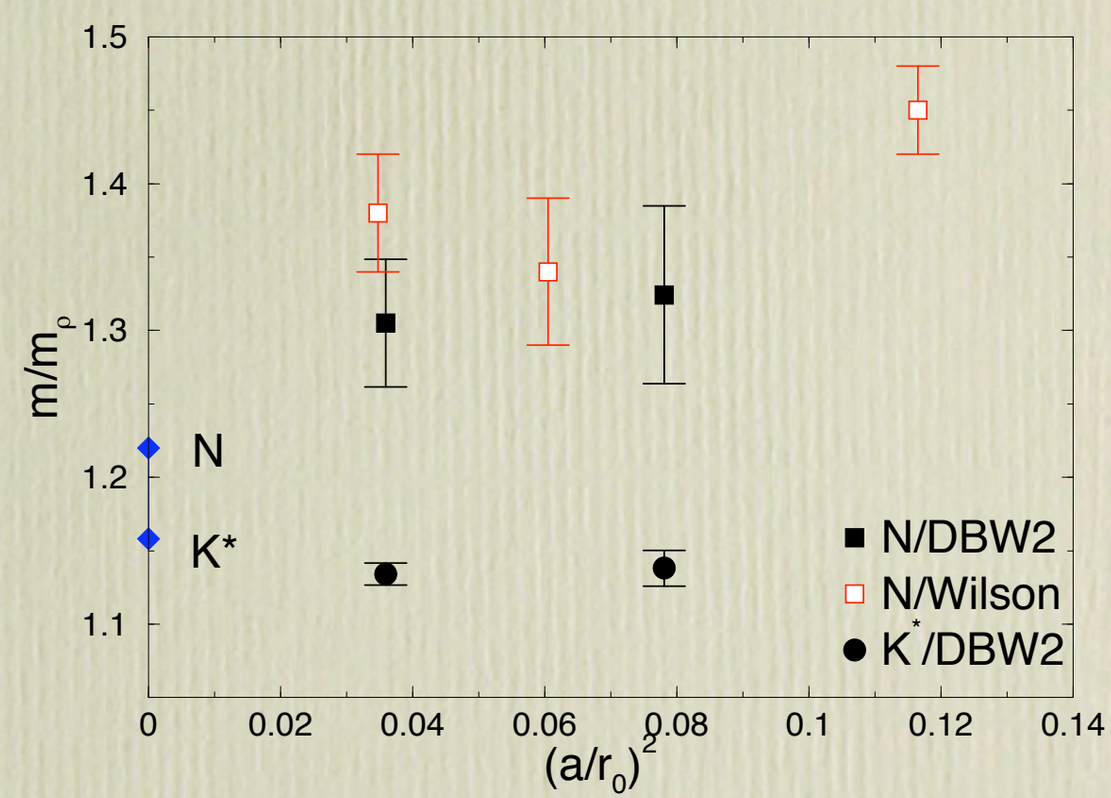


Fermion mass is introduced by explicitly coupling m_f of the walls. [Shamir,Furman & Shamir]

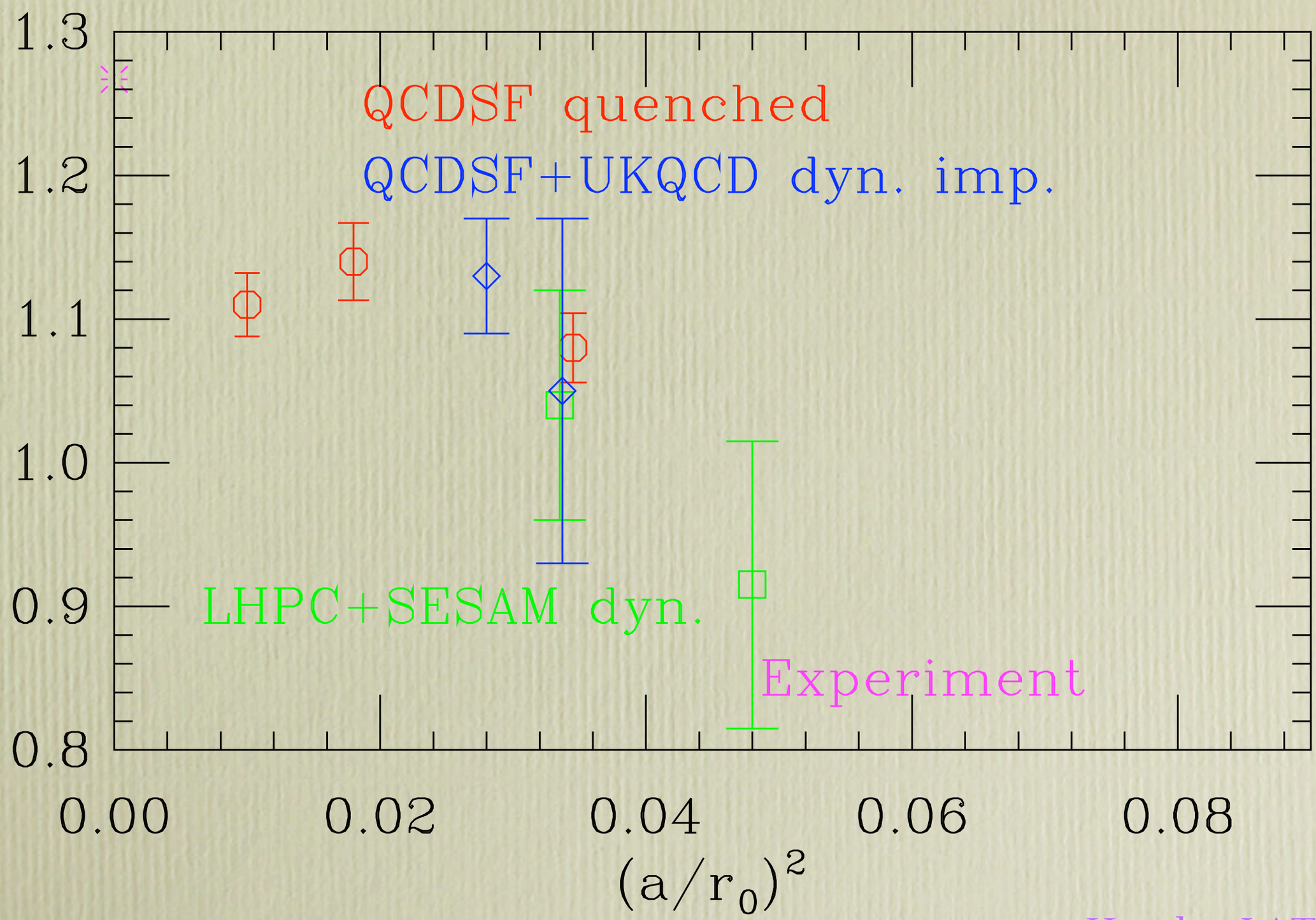
Why Domain Wall Fermions

- Excellent chiral properties
  at finite lattice spacing:
  - $L_s \rightarrow \infty$ Exact chiral symmetry
  - L_s finite: Exponentially small chiral symmetry breaking
  - Gauge action affects chiral symmetry
    [KO with RBC hep-lat/0211023]
  - Chiral extrapolations
- Simpler renormalization due to symmetry
- Can work close to the chiral limit
- Have $O(a^2)$ errors
- Excellent scaling properties



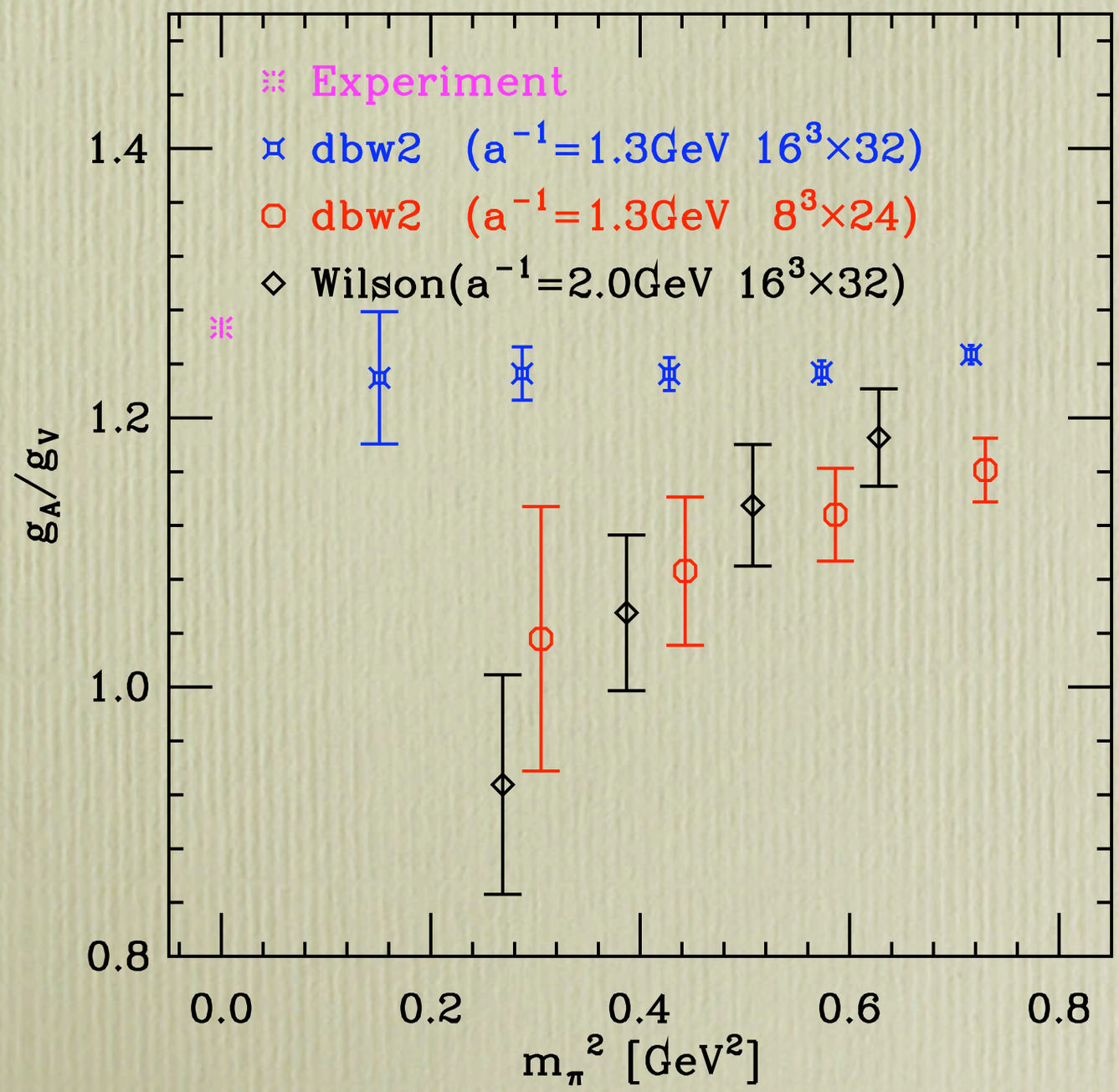


Axial Charge g_A



Horsley LAT02

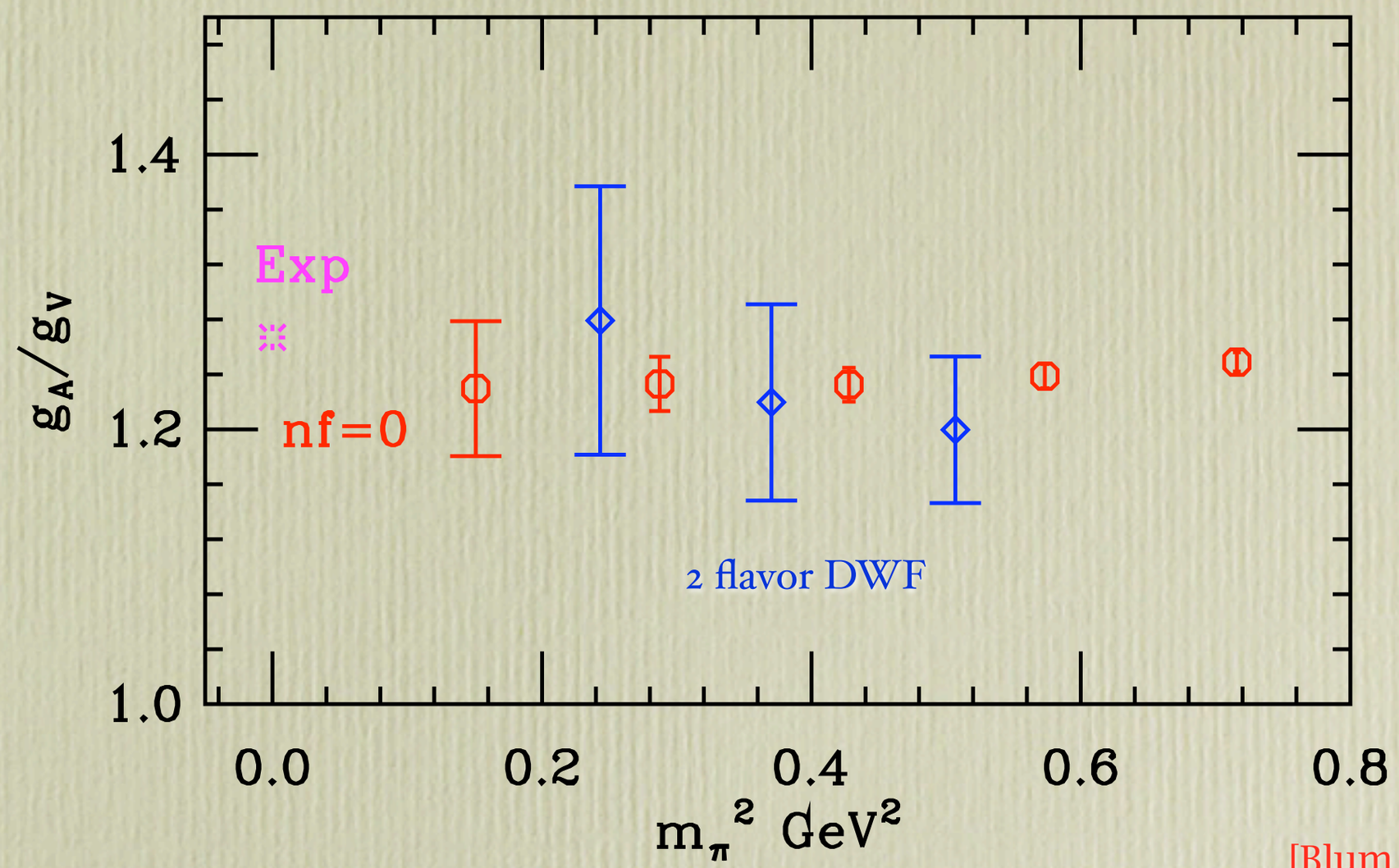
Finite Volume Effect



- Volumes: 1.1fm - 2.4fm

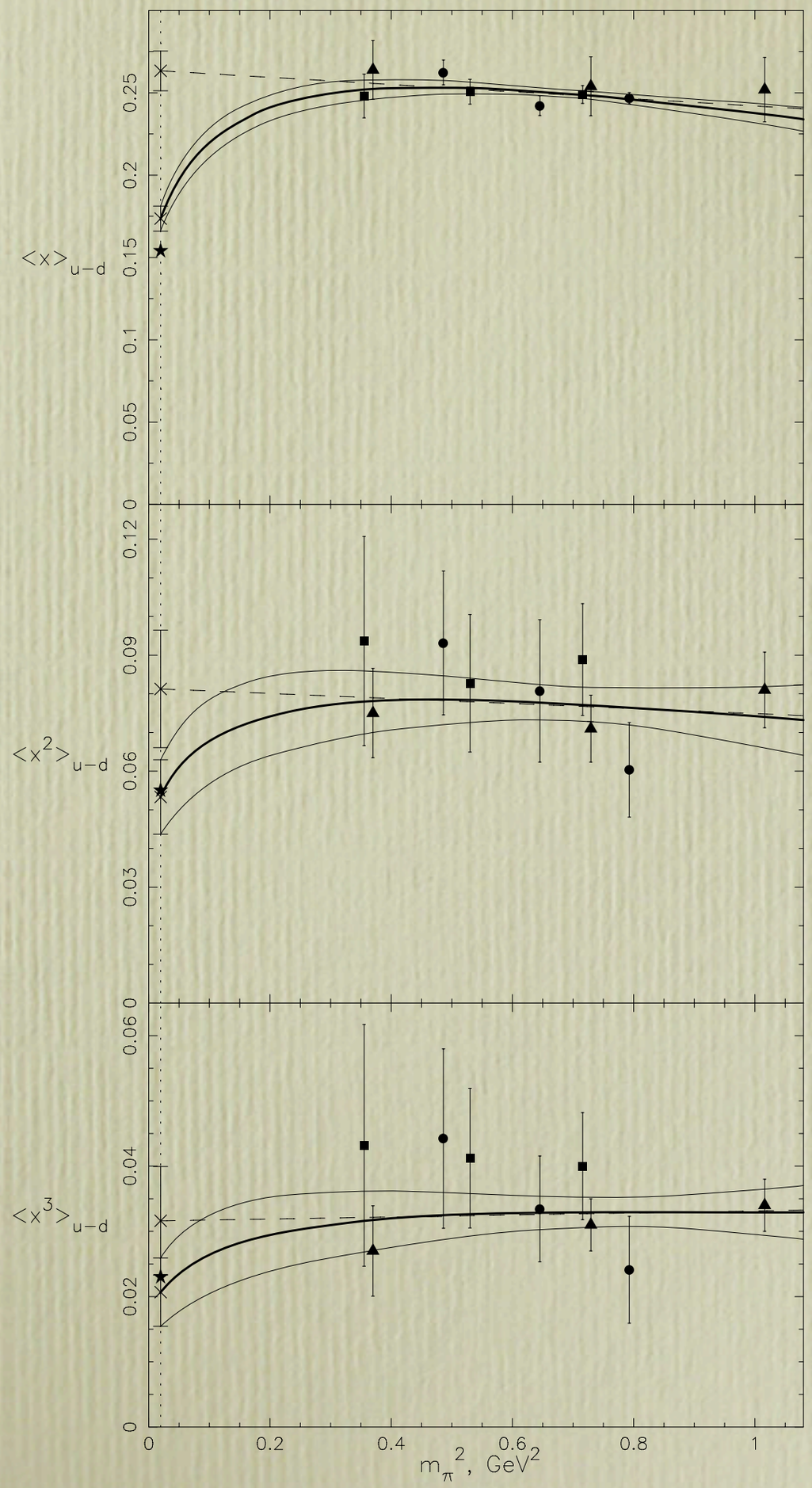
[Blum Ohta KO Sasaki '03]

Nucleon Axial Charge



[Blum KO Ohta Sasaki]

- Finite volume effect
- Experimental value: 1.2695(29)
- Quenched DWF: 1.212(24)$_{stat}$ (27)$_{norm}$ (linear fit)
- New dynamical result on the way (2 flavor and 2+1) [Blum, Lin, Sasaki, ...]



LHPC-SESAM:

diamonds - quenched,

squares - dynamical

QCDSF:

quenched - triangles

[hep-lat/0201021]

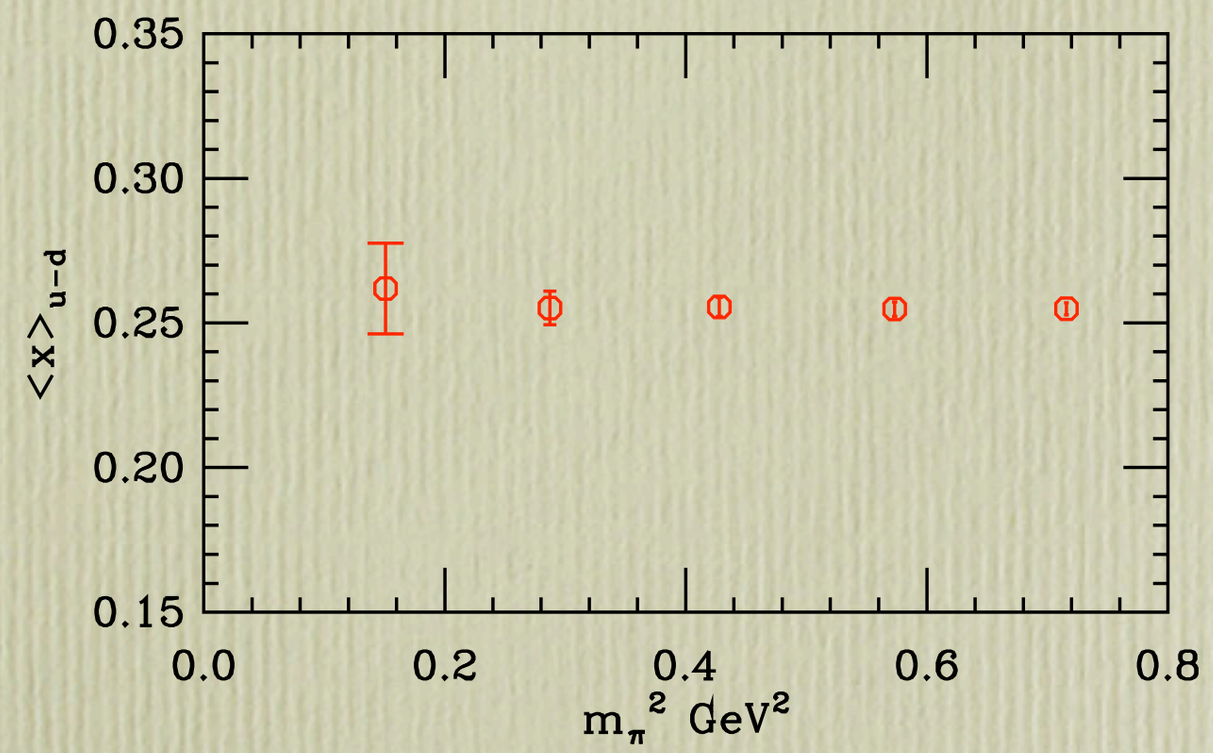
$$\langle x \rangle_{u-d} \sim a_1 \Big[1 - \frac{(3g_A^2+1)m_\pi^2}{(4\pi f_\pi)^2} \ln\Big(\frac{m_\pi^2}{m_\pi^2+\mu^2}\Big)\Big] + b_1 m_\pi^2.$$

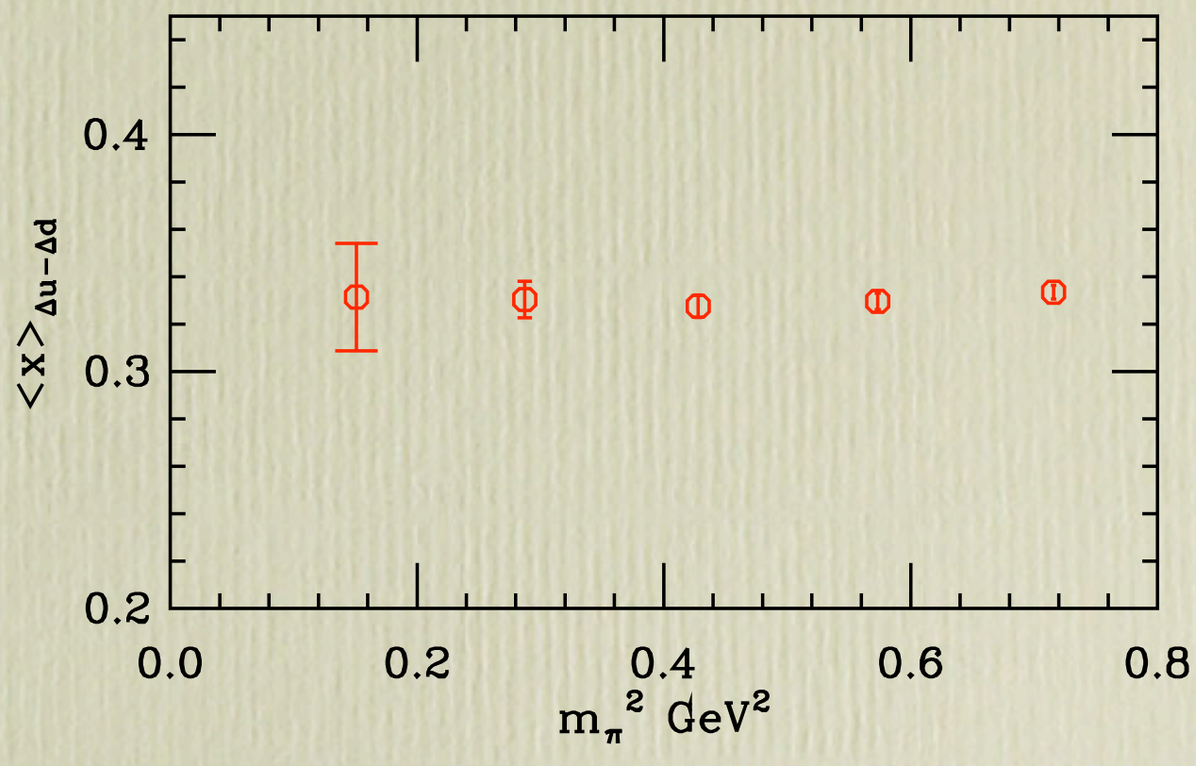
Where $\mu = 550 MeV$

The log coefficient is valid for full QCD

[Detmold et.al. Phys.Rev.D87 2001]

RBC calculation





- Unrenormalized
- No curvature
- Use NPR for renormalization

Renormalization

$$\langle N|\mathcal{O}|N\rangle_{ren}(\mu) = Z_{\mathcal{O}}(a\mu)\langle N|\mathcal{O}|N\rangle_{lat}(a)$$

$Z_{\mathcal{O}}(a\mu)$ can be computed:

- Perturbatively in $\overline{\text{MS}}$
- Non-perturbatively (ex. RI/MOM), perturbative matching to $\overline{\text{MS}}$

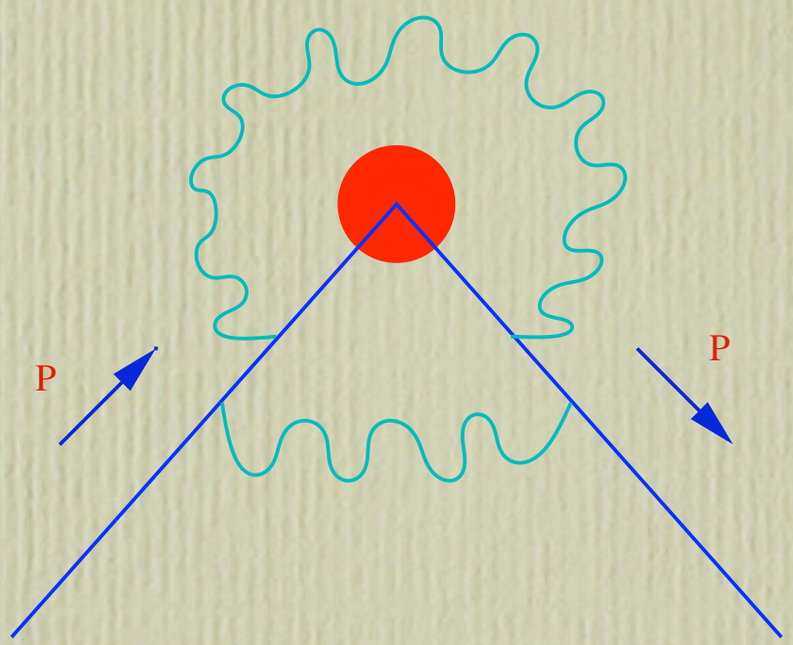
Lattice complications:

Broken symmetries (ex. Lorentz, chiral symmetry) $\Longrightarrow$ operator mixing

NPR needed when mixing with lower dimensional operators occurs.

... domain wall fermions

In Landau gauge we compute off-shell matrix elements of the $\mathcal{O}$.



$$Tr V_{\mathcal{O}}(p^2)\Gamma\Big|_{p^2=\mu^2}\frac{Z_{\mathcal{O}}}{Z_q}=1$$

- $V_{\mathcal{O}}(p^2)$ the amputated vertex
- Γ a projector

This defines the MOM scheme. Extrapolate to $m_f \to 0$ and we have the RI scheme.

$$Z_{rgi}=\frac{Z(\mu^2)}{C(\mu^2)}$$

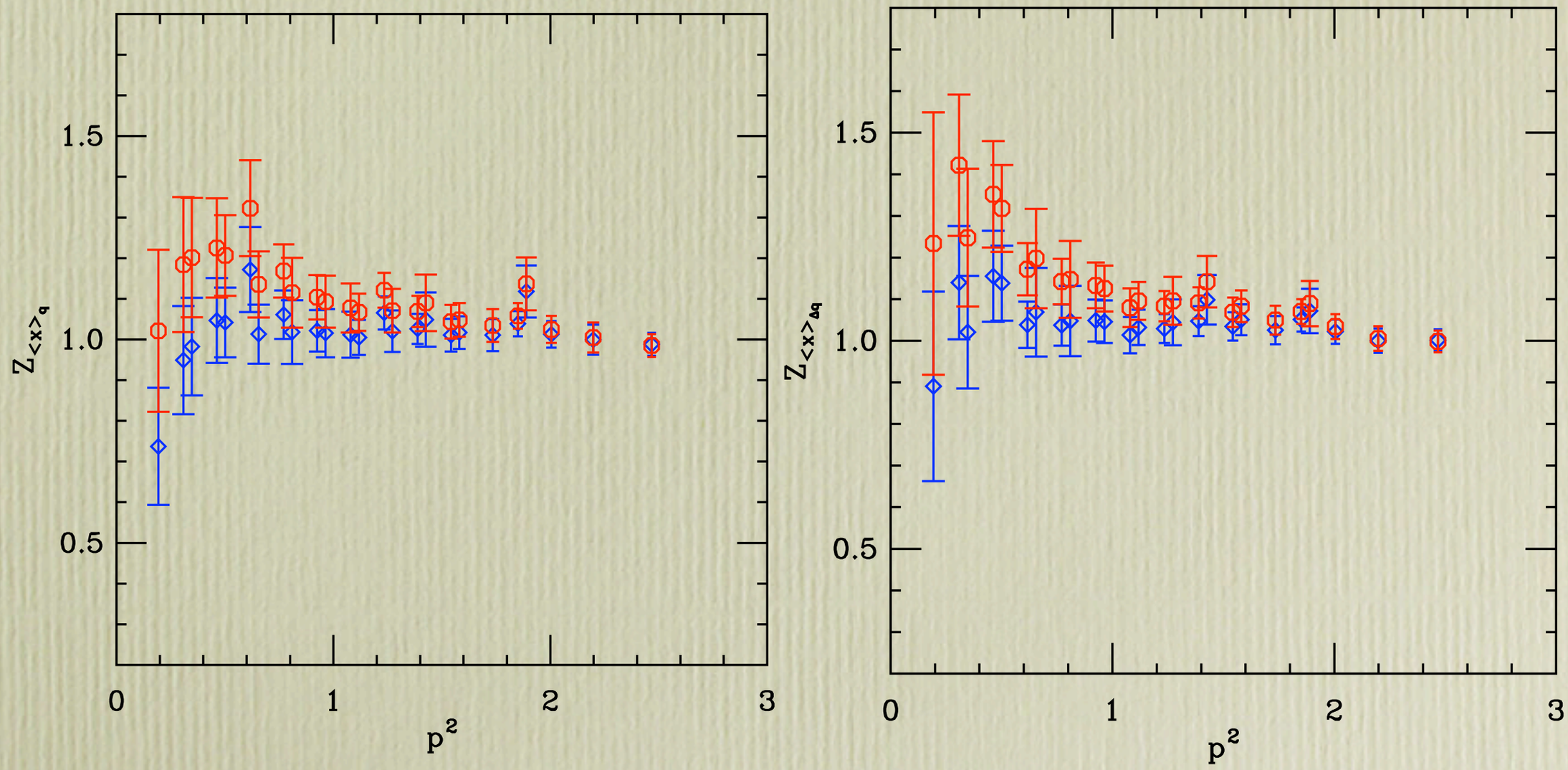
- $C(\mu^2)$ the operator running (in continuum PT)
- Z_{rgi} scale invariant if

$$\Lambda_{QCD} \ll \mu^2 \ll a^{-1}$$

- Requires perturbative matching to $\overline{\text{MS}}$

Introduced by Martinelli et.al. Nuc.Phys.B445 81 (1995)

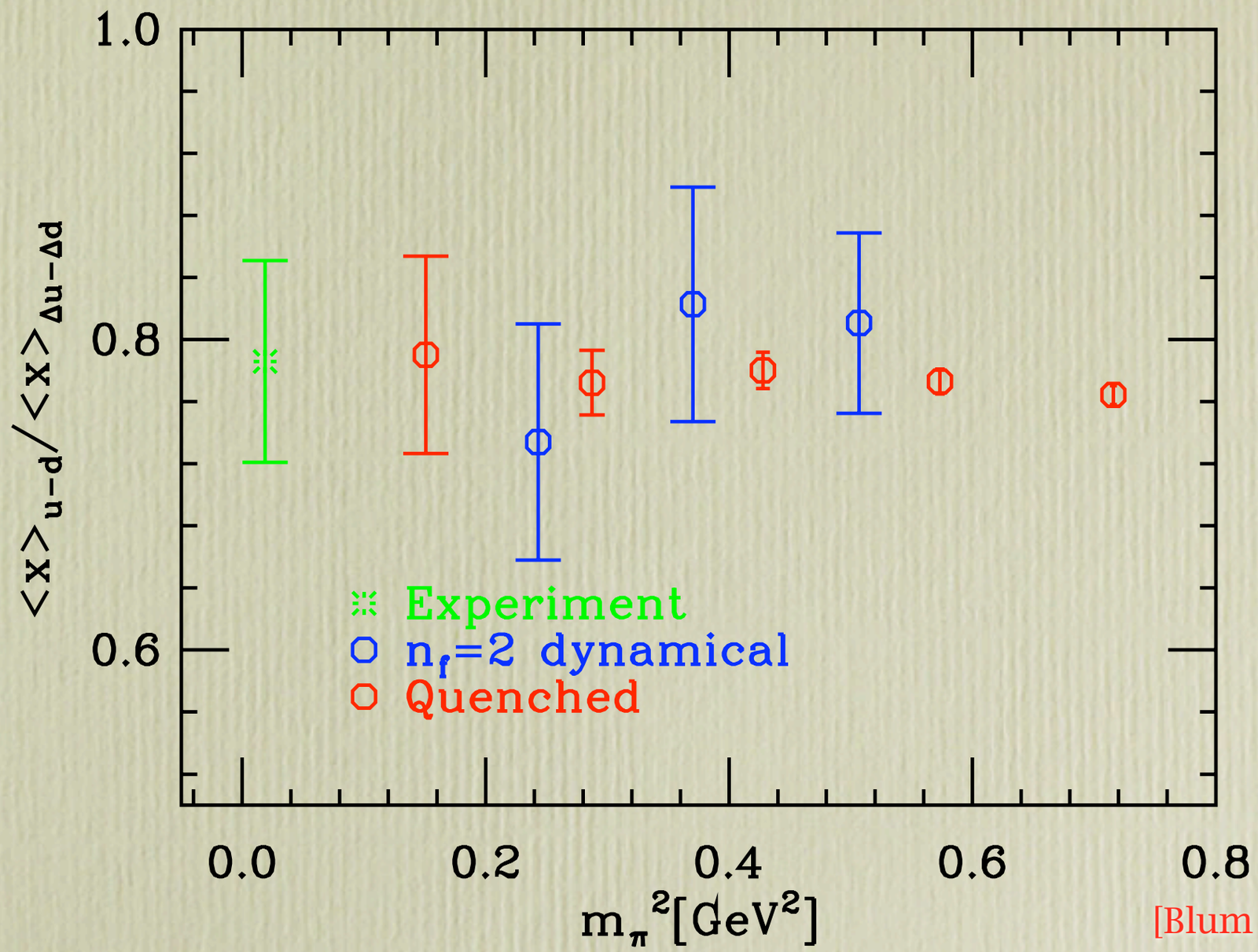
Renormalization factors



- Chiral Symmetry implies equality

Ratio of first moments

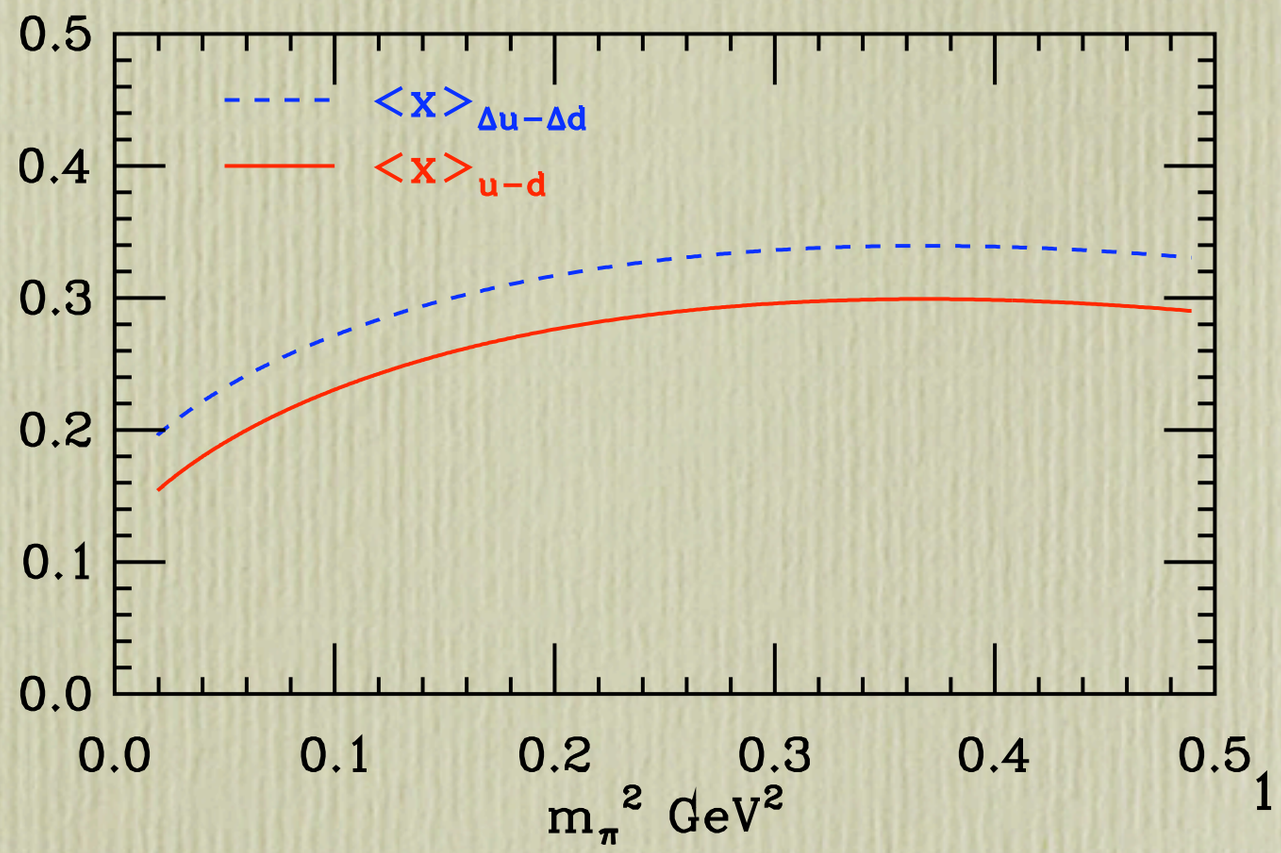
(polarized and unpolarized)



[Blum KO Ohta '05]

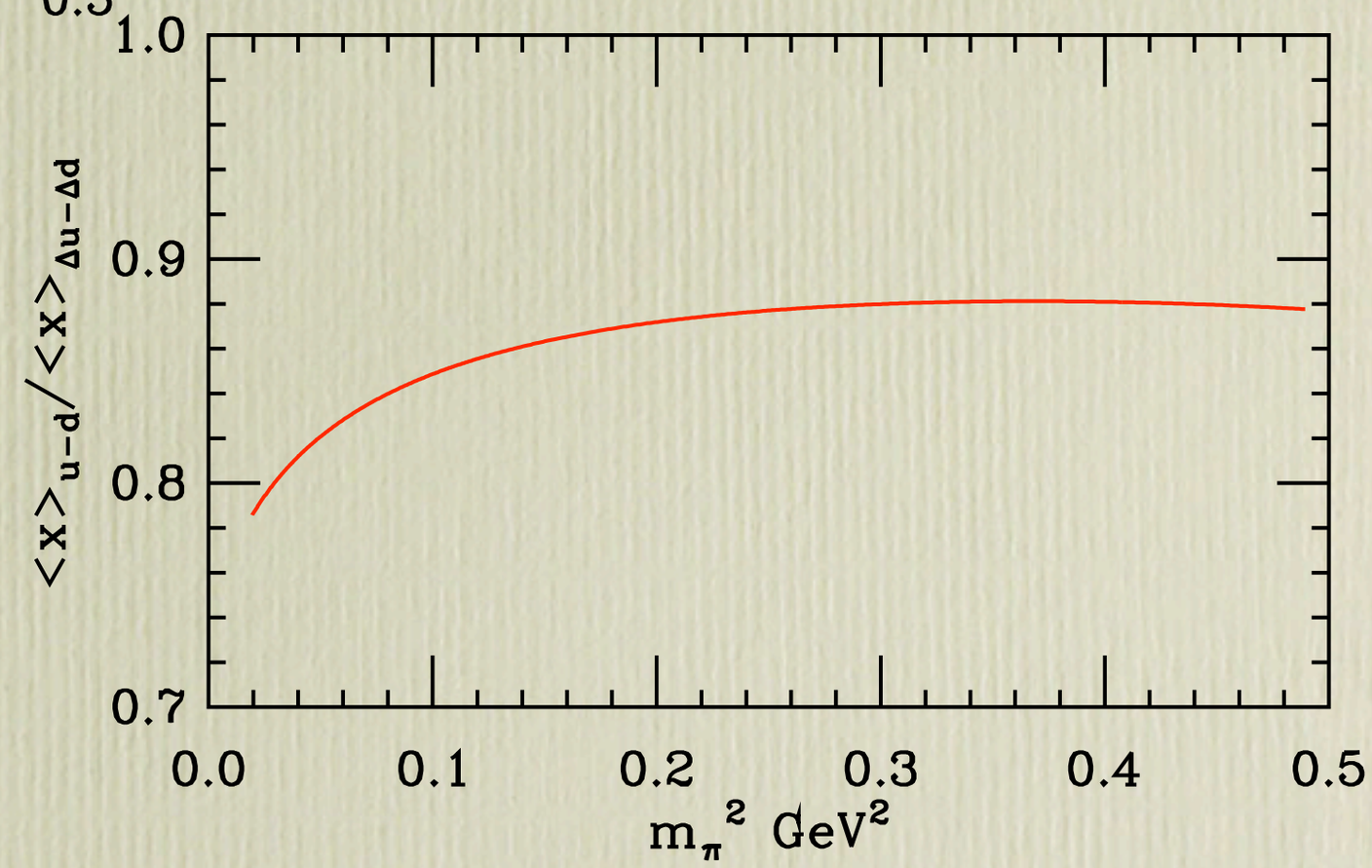
- No curvature observed down to **400MeV** pions (Quenched)
- Renormalization constant cancels in the ratio for DWF
- Ratio agrees with experimental expectations

Chiral Log



Individual matrix element: large variation

Ratio: small variation



The g_2 structure function

$$2\int_0^1 dx x^n g_2(x,Q^2) = \frac{1}{2}\frac{n}{n+1}\sum_{q=u,d}[e^q_{2,n}(\mu^2/Q^2, g(\mu))d^q_n(\mu) - 2e^q_{1,n}(\mu^2/Q^2, g(\mu))\langle x^n\rangle_{\Delta q}(\mu)],$$

$\langle x^n\rangle_{\Delta q}(\mu) \rightarrow$ Twist 2

$d^q_n(\mu) \rightarrow$ Twist 3

$d^q_n(\mu)$ estimations:

- Negligible $\Longrightarrow$ Wandzura - Wilczek relation of g_1 and g_2
- Need not be small in a confining theory [Jaffe and Ji Phys.Rev.D43,91].

Twist Three

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1}-S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n}+\cdots-traces]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^n \gamma_5\gamma_{[\sigma}\overleftrightarrow{D}_{\mu_1]}\cdots\overleftrightarrow{D}_{\mu_n} - traces\right]q$$

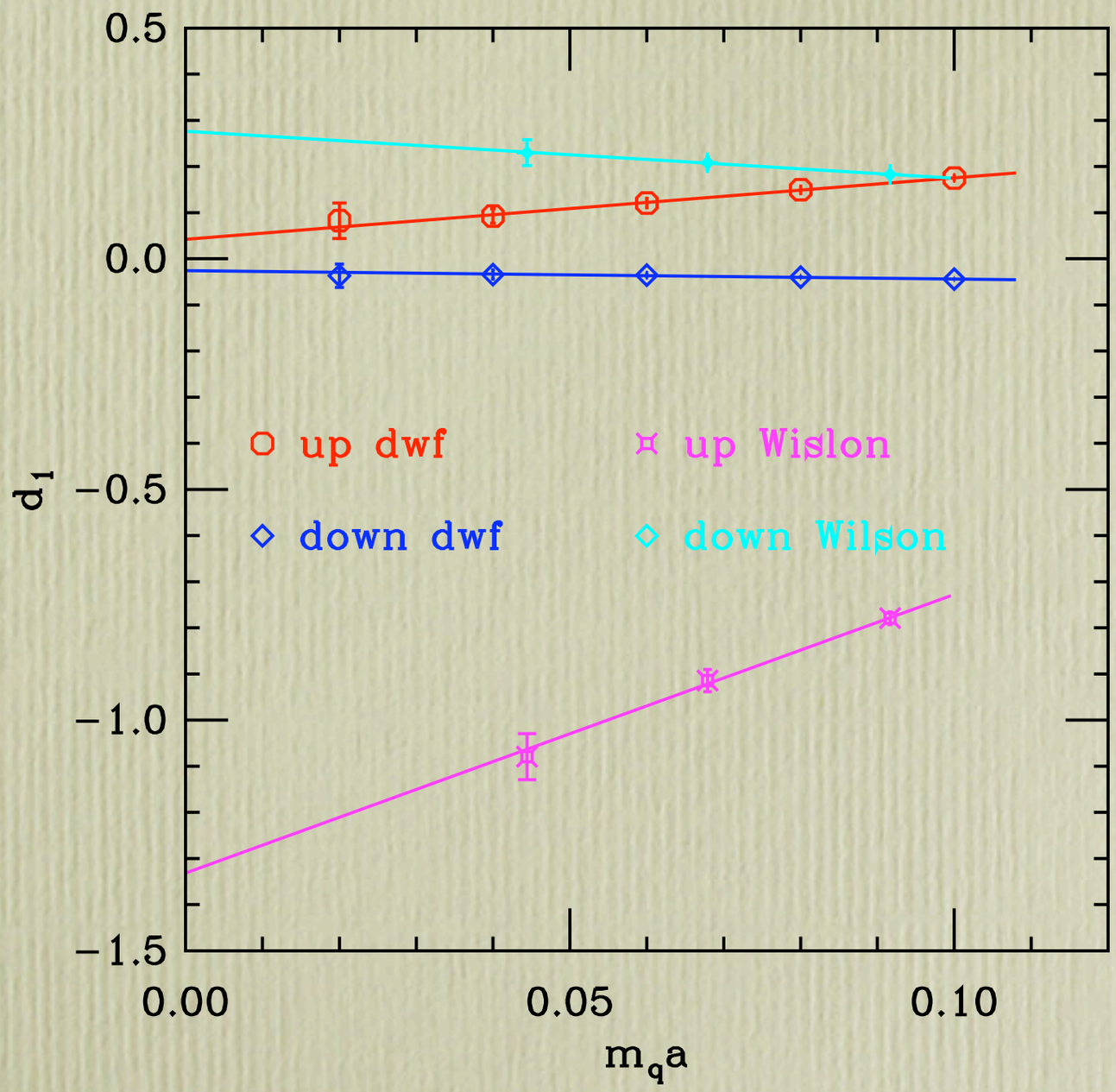
Measure:

$$\mathcal{O}^{[5]q}_{34} = \frac{1}{4}\bar{q}\gamma_5\left[\gamma_3\overleftrightarrow{D}_4 - \gamma_4\overleftrightarrow{D}_3\right]q \quad \rightarrow \quad d^q_1$$

- Hyper-cubic group representation: $\mathbf{6}^+_1$
- Momentum: $\vec{P}=0$
- Renormalization: Multiplicative (DWF Chiral symmetry)

chiral symmetry breaking causes mixing with

$$\mathcal{O}^{\sigma q}_{34} = \bar{q}\gamma_5\sigma_{34}q$$



Note:

- Unrenormalized
- Disagreement with the Wilson results

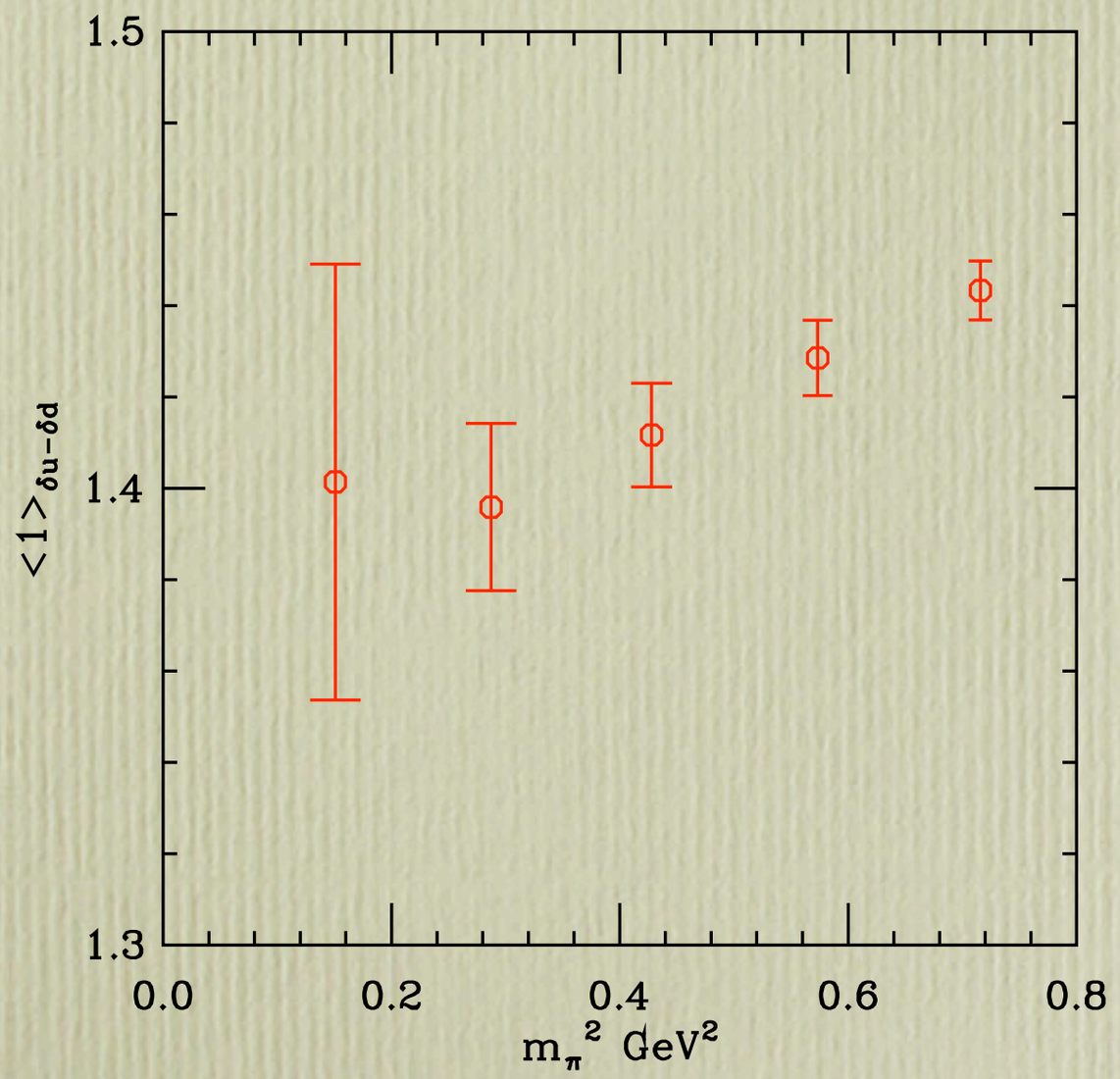
  Power divergent mixing

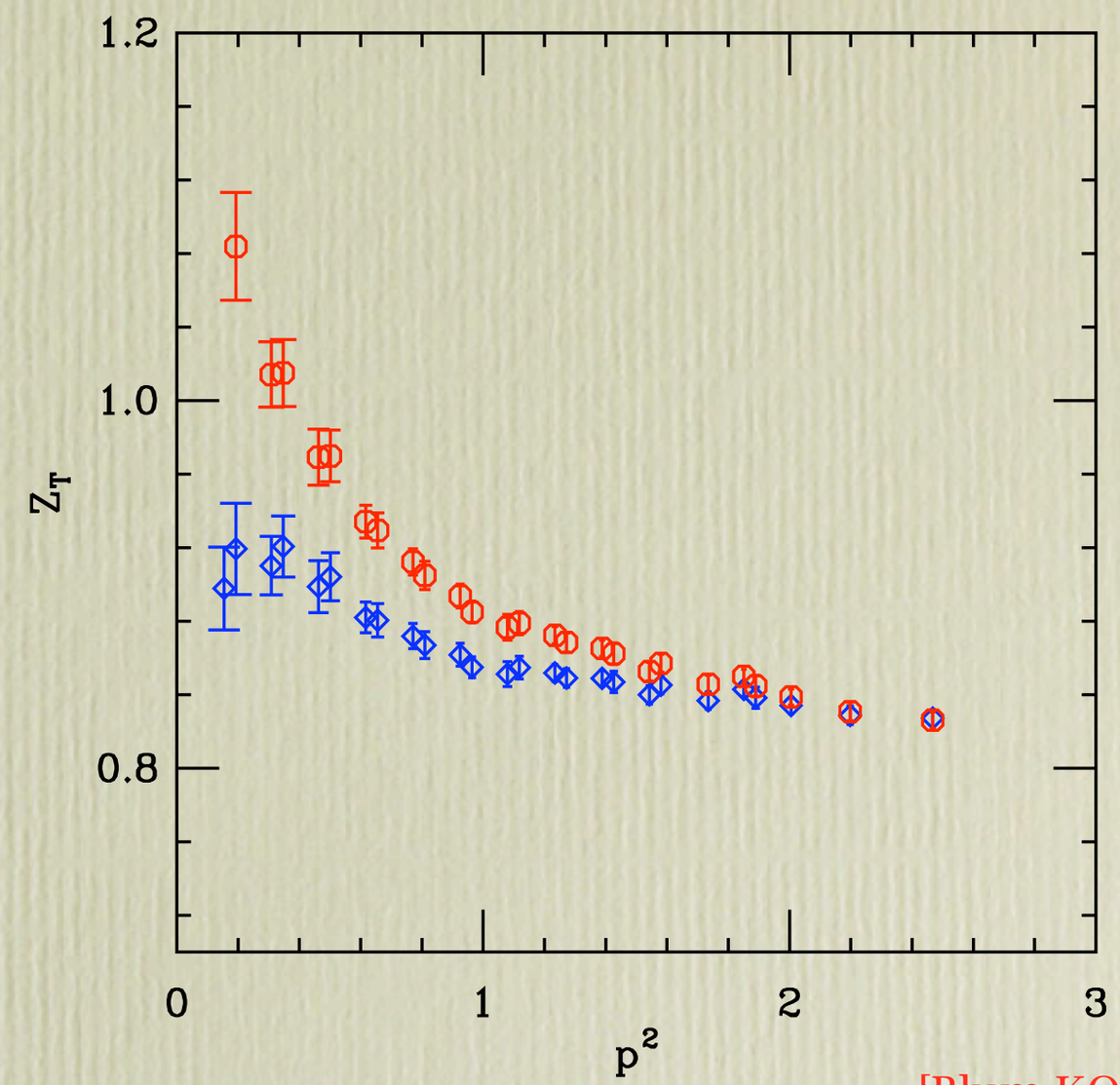
  [LHPC-SESAM: hep-lat/0201021]
- Small at chiral limit

[Blum KO Ohta '05]

plateaus

Transversity





[Blum KO Ohta '05]

- Linear extrapolation: 1.193(30) at 2GeV $\overline{\text{MS}}$

LHPC collaborators (SF project)

- R. Edwards (Jlab)
- G. Fleming (Yale)
- P. Hagler (Vrije Universiteit)
- J. Negele (MIT)
- A. Pochinsky (MIT)
- D. Renner (UofA)
- D. Richards (Jlab)
- W. Schroers (DESY)

THE HYBRID ACTION PROGRAM

- Domain wall fermions for valence (with hyp smeared links)
  - Chiral symmetry ($O(a^2)$ errors better scaling)
  - Ward Identities (renormalization, power divergent mixing)
- Kogut-Susskind 2+1 Dynamical flavors
  - Improved KS action (Asqtad: $O(a^4, g^2 a^2)$) [KO, Sugar, Toussaint '99]
  - MILC has generated lattices: Ready to milk the MILC
- Light quark masses: Lightest pion $m_\pi \sim 250$MeV
- Volumes: 2.6 to 3.2 fm
- Future: Continuum extrapolation
  - MILC lattice spacings: a=0.125fm, 0.09fm
  - a=0.06fm in 1 - 2 years

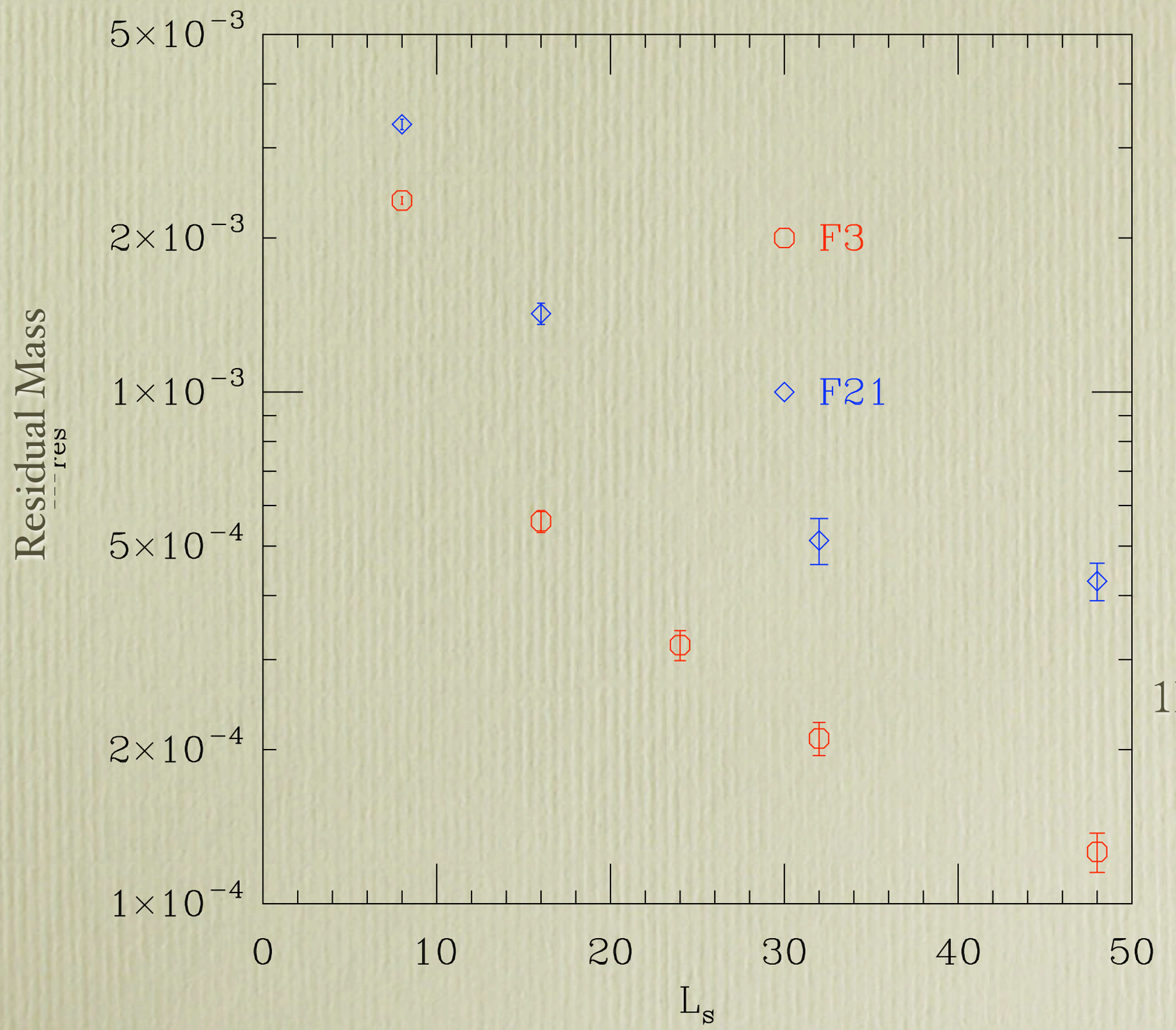
Chiral symmetry breaking

$$\Delta_\mu \langle \mathcal{A}^a_\mu(x)\mathcal{O}\rangle = 2\, m_f\, \langle J^a_5(x)\mathcal{O}\rangle + 2\langle J^a_{5q}(x)\mathcal{O}\rangle + i\langle \delta^a_x \mathcal{O}\rangle$$

- The size of $\langle J^a_{5q}(x)\mathcal{O}\rangle$ measures chiral symmetry breaking
- Let's use for the operator $\mathcal{O} = J^a_5(0)$
- Assume at long distances $J^a_{5q} \sim J^a_5$
- The proportionality constant is the residual mass

$$M_{\text{res}} = \left. \frac{\sum_{x,y}\langle J^a_{5q}(y,t)\, J^a_5(x,0)\rangle}{\sum_{x,y}\langle J^a_5(y,t)\, J^a_5(x,0)\rangle} \right|_{t\geq t_{min}}$$

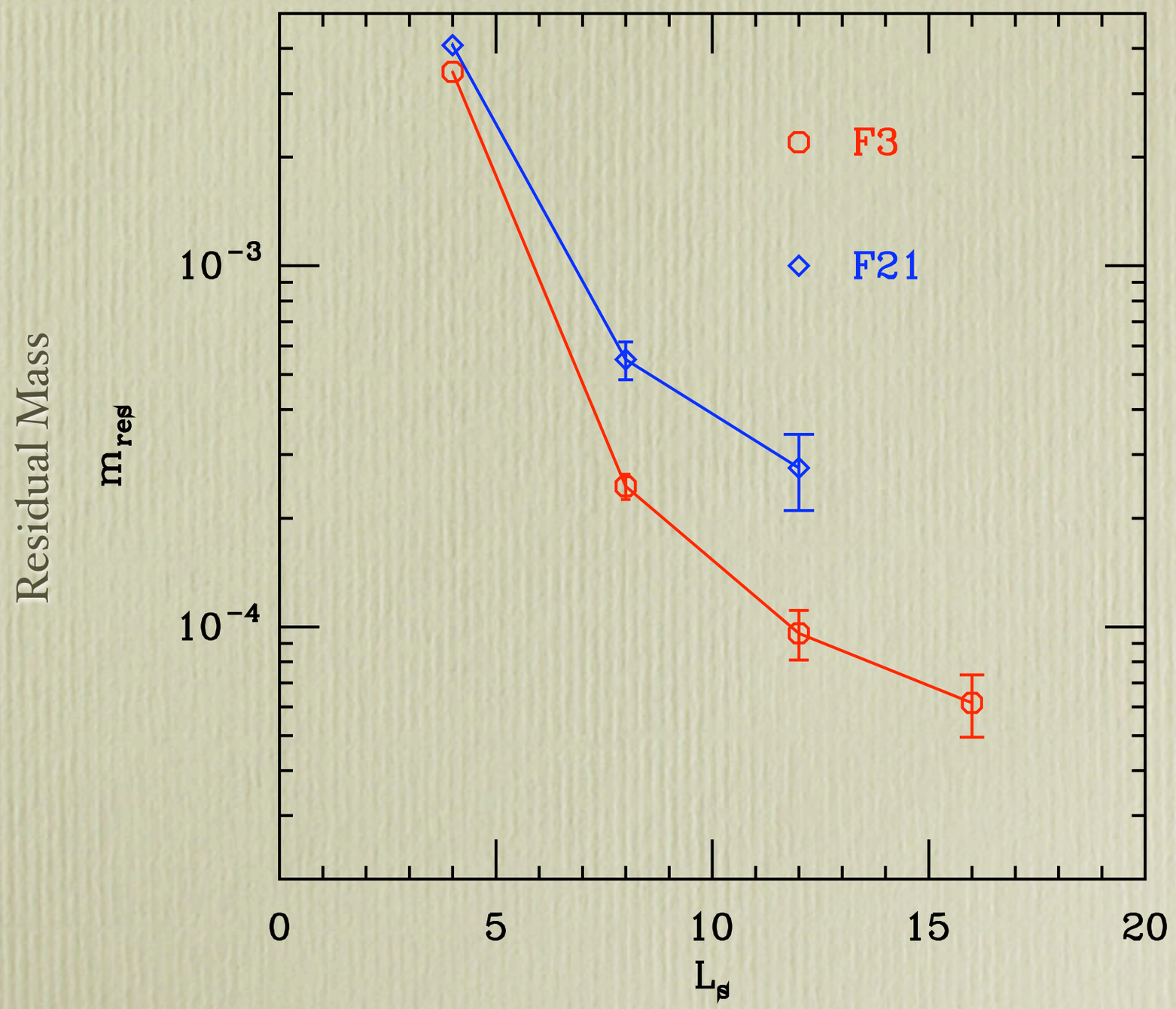
Residual Mass vs Ls



a=0.125fm

At Ls = 16:
1MeV < m_{res} <2.5MeV

Residual Mass vs Ls



a=0.09fm

At Ls = 12:
0.2MeV<m_{res} <0.7MeV

The 4D effective operator

With a little algebra we get

$$\mathcal{P}^{-1}\frac{1}{D_{dwf}(1)}D_{dwf}(m)\mathcal{P} = \begin{bmatrix} D_{ov}(m) & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ -(1-m)T^{-L_s/2+1}\frac{1}{T^{-L_s/2}+T^{L_s/2}} & 1 & 0 & 0 & \cdots & \cdots & 0 \\ -(1-m)T^{-L_s/2+2}\frac{1}{T^{-L_s/2}+T^{L_s/2}} & 0 & 1 & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \cdots & \vdots \\ -(1-m)\frac{1}{T^{-L_s/2}+T^{L_s/2}} & 0 & \cdots & \cdots & 1 & 0 & \cdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ -(1-m)T^{L_s/2-1}\frac{1}{T^{-L_s/2}+T^{L_s/2}} & 0 & \cdots & \cdots & \cdots & 0 & 1 \end{bmatrix}$$

$$\mathcal{P} = \begin{bmatrix} P_- & P_+ & \cdots & 0 \\ 0 & P_- & P_+ \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & P_+ \\ P_+ & 0 & \cdots & P_- \end{bmatrix} \qquad L = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ -T^{-L_s+1}M_+ & 1 & 0 & 0 & \cdots \\ -T^{-L_s+2}M_+ & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -T^{-1}M_+ & 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$M_- = P_- - m\,P_+ \qquad M_+ = P_+ - m\,P_-$$

$$T^{-1} = \frac{1+H_T}{1-H_T} \qquad H_T = \gamma_5 D$$

$$\boxed{D_{ov}(m) = \frac{1+m}{2} + \frac{1-m}{2}\gamma_5\mathcal{E}_{L_s}[\gamma_5 D(M_5)]}$$

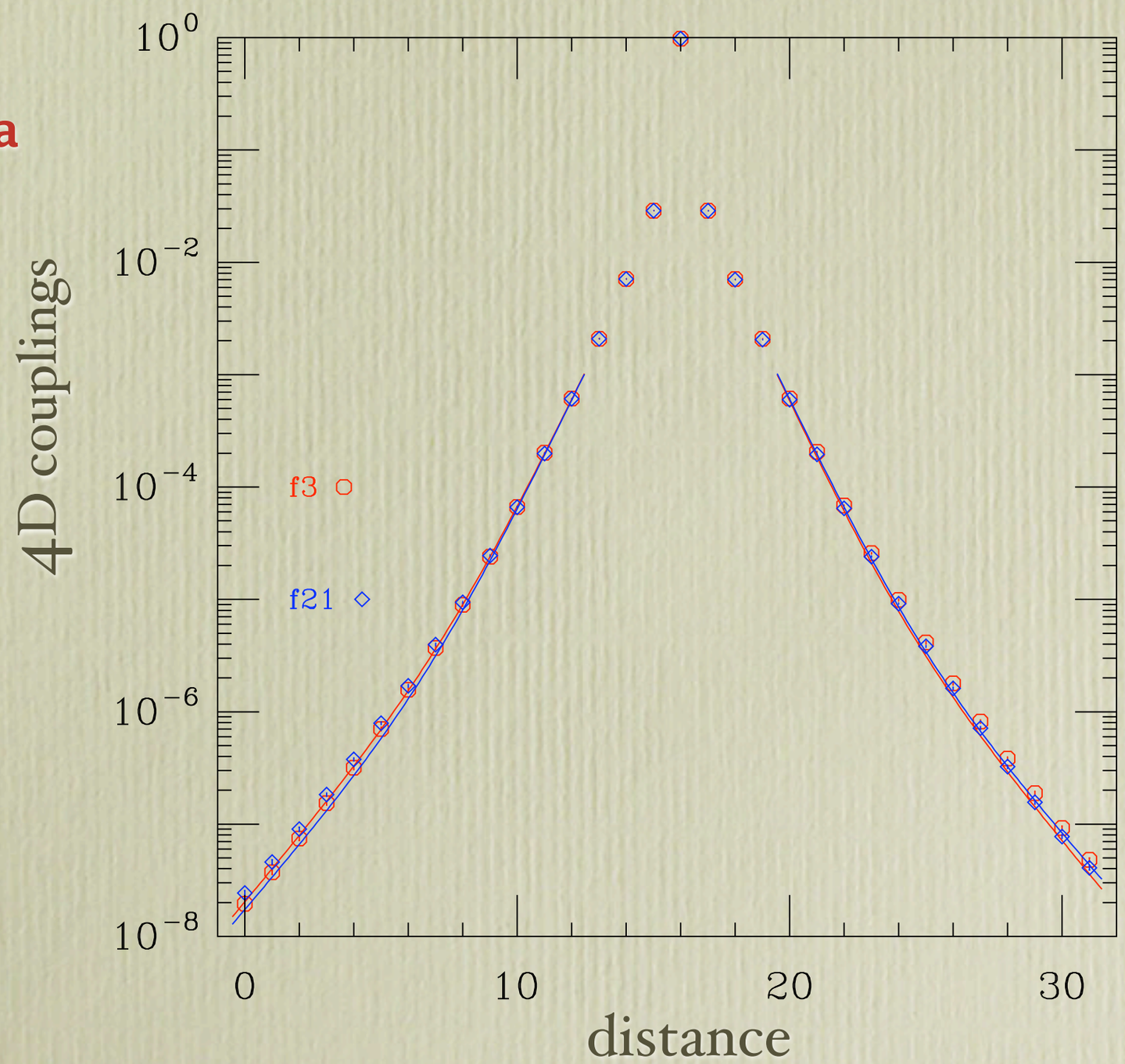
$$\varepsilon_{L_s} = \frac{T^{-L_s}-1}{T^{-L_s}+1} = \frac{(1+H_T)^{L_s}-(1-H_T)^{L_s}}{(1+H_T)^{L_s}+(1-H_T)^{L_s}}$$

$$D = (b_5+c_5)\frac{D_w}{2+(b_5-c_5)D_w} = \alpha\frac{D_w}{2+a_5 D_w}$$

- Overlap: $\alpha=2$, $a_5=0$ (Borici)
- DWF: $\alpha=1$, $a_5=1$ (Shamir)

Locality of the 4D action

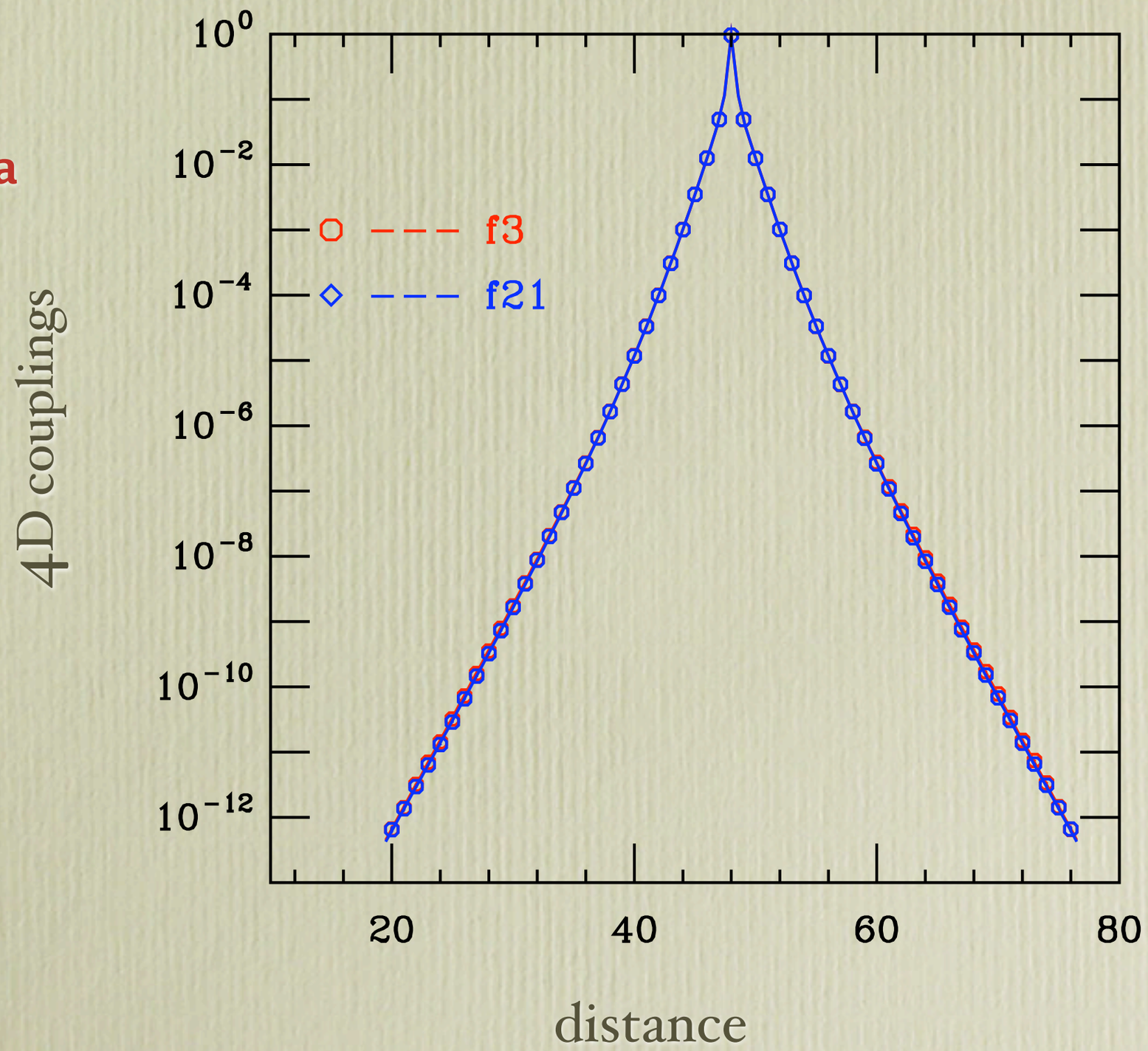
Localization: ~ **1.5a**



a=0.125fm

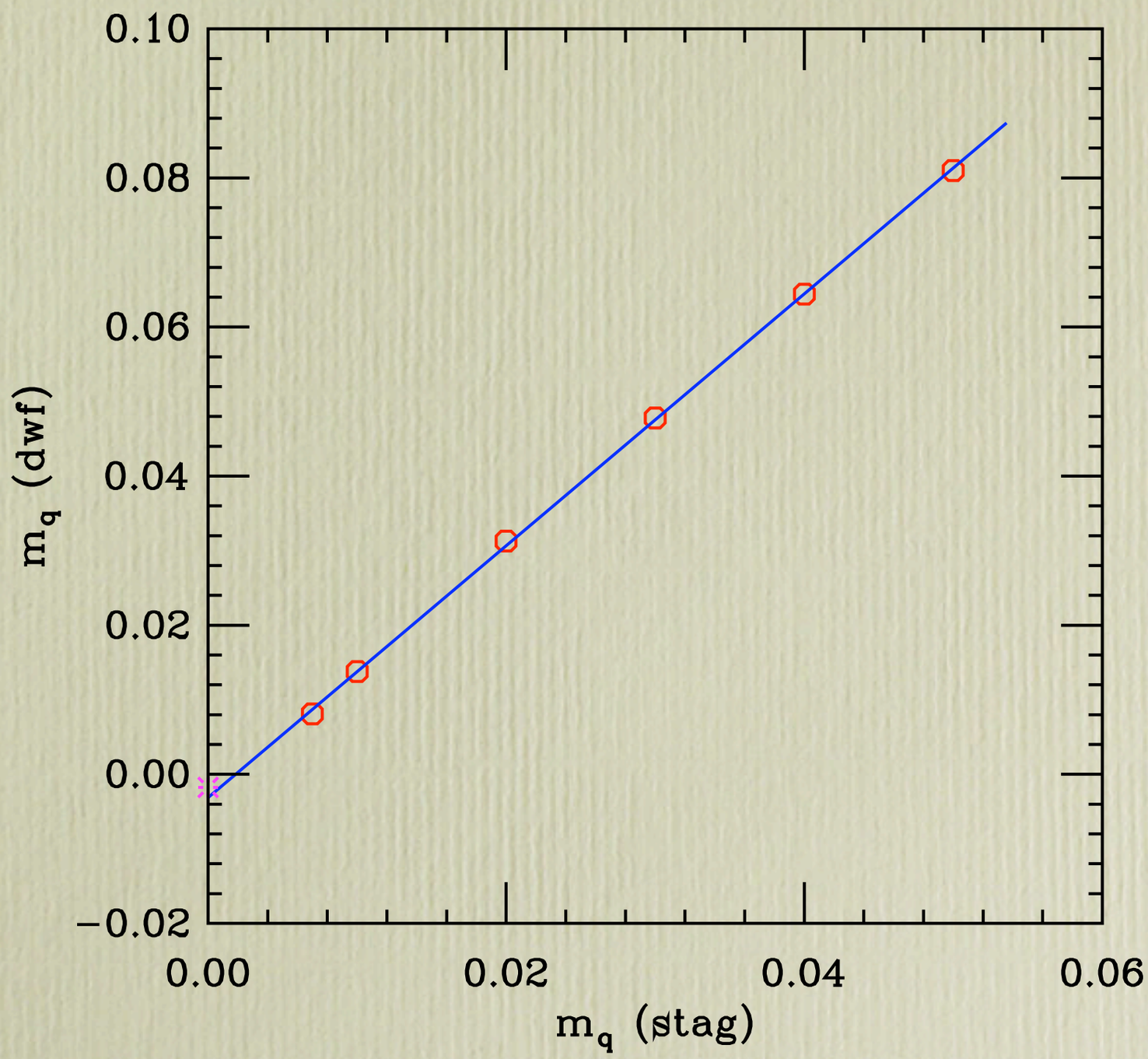
Locality of the 4D action

Localization: ~ **1.3a**



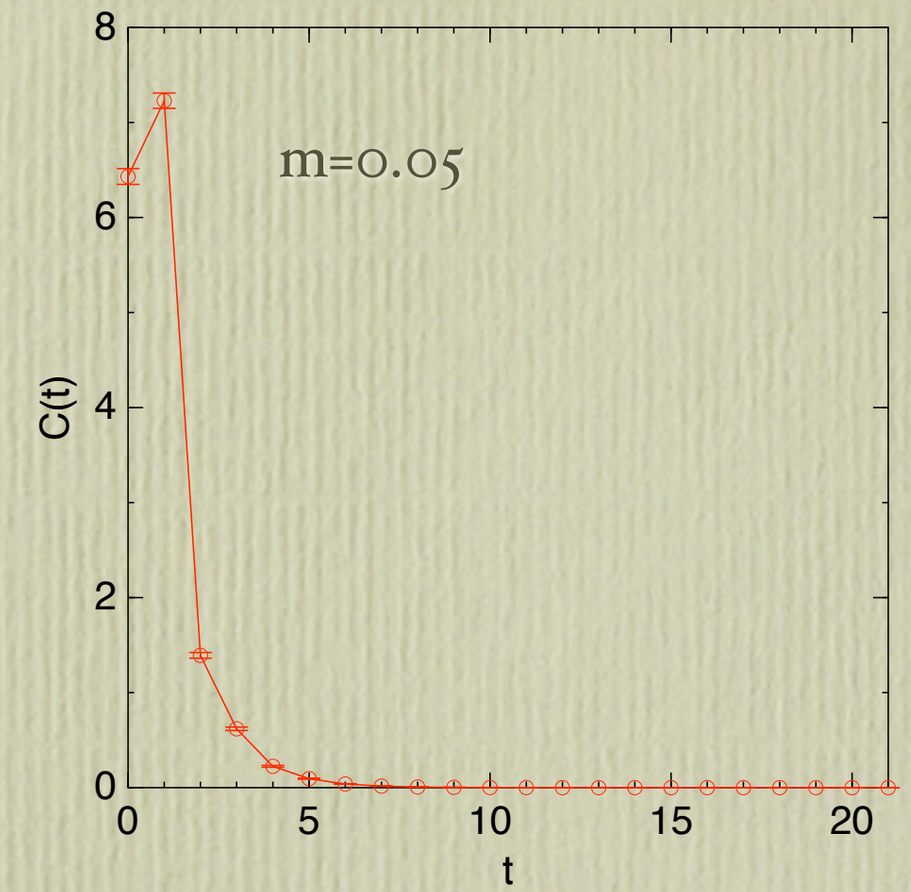
a=0.09fm

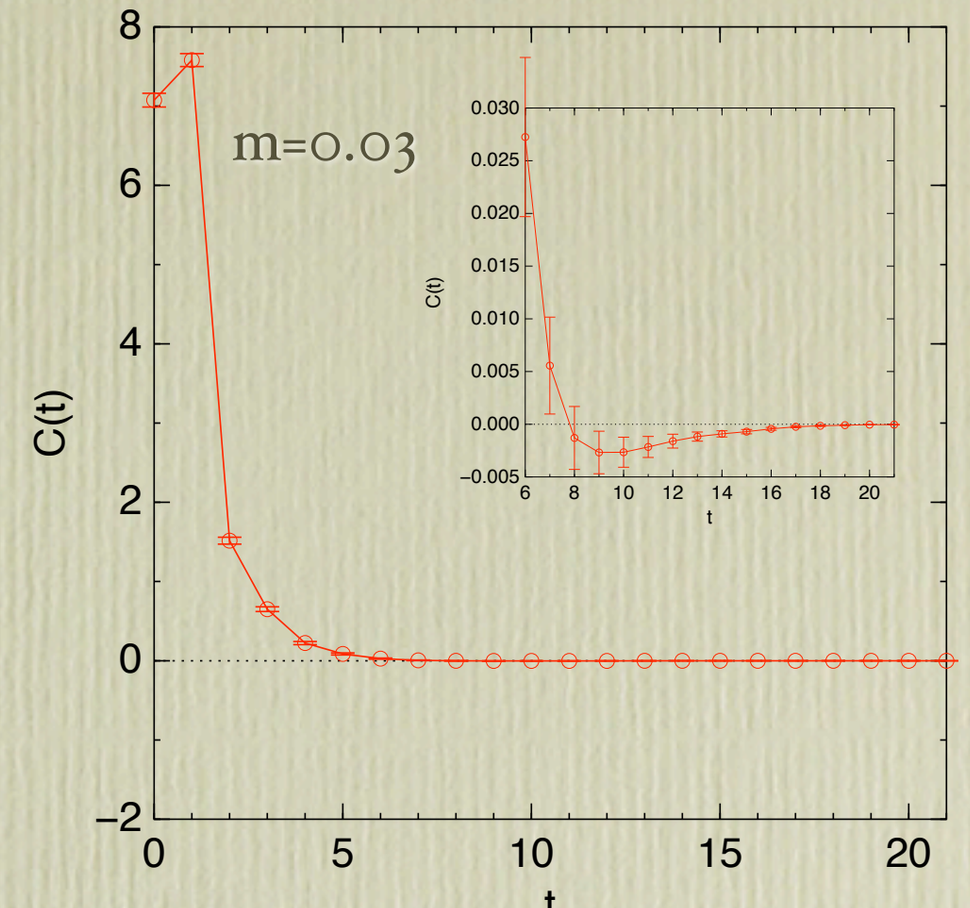
The DWF quark masses

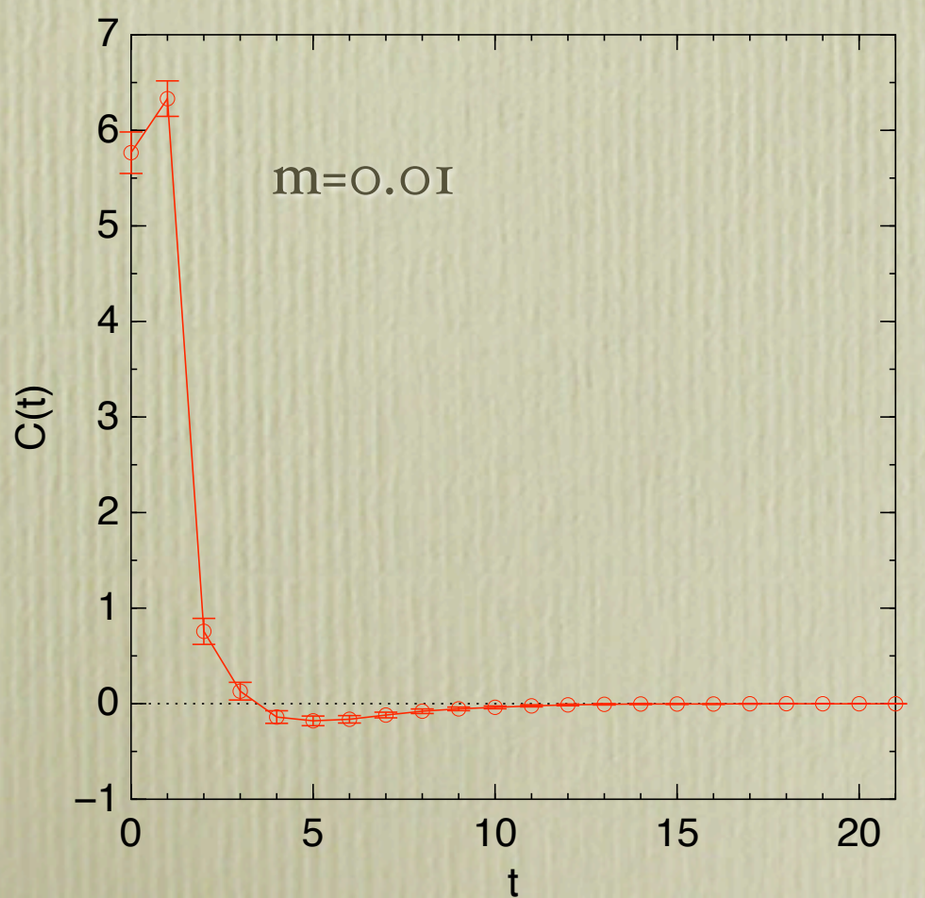


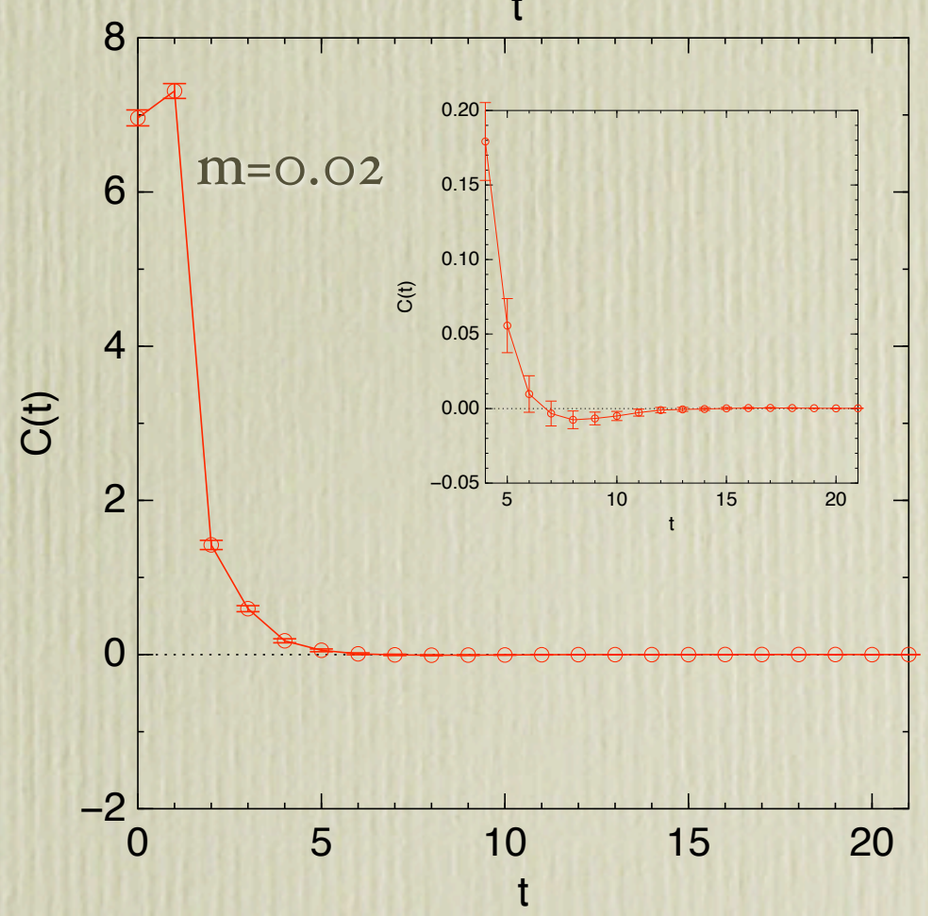
a=0.125fm

IsoVector scalar correlator









χPT calculation: Prelovsek LAT 05

F_K/F_π

Beane, Bedaque, KO, Savage in preparation

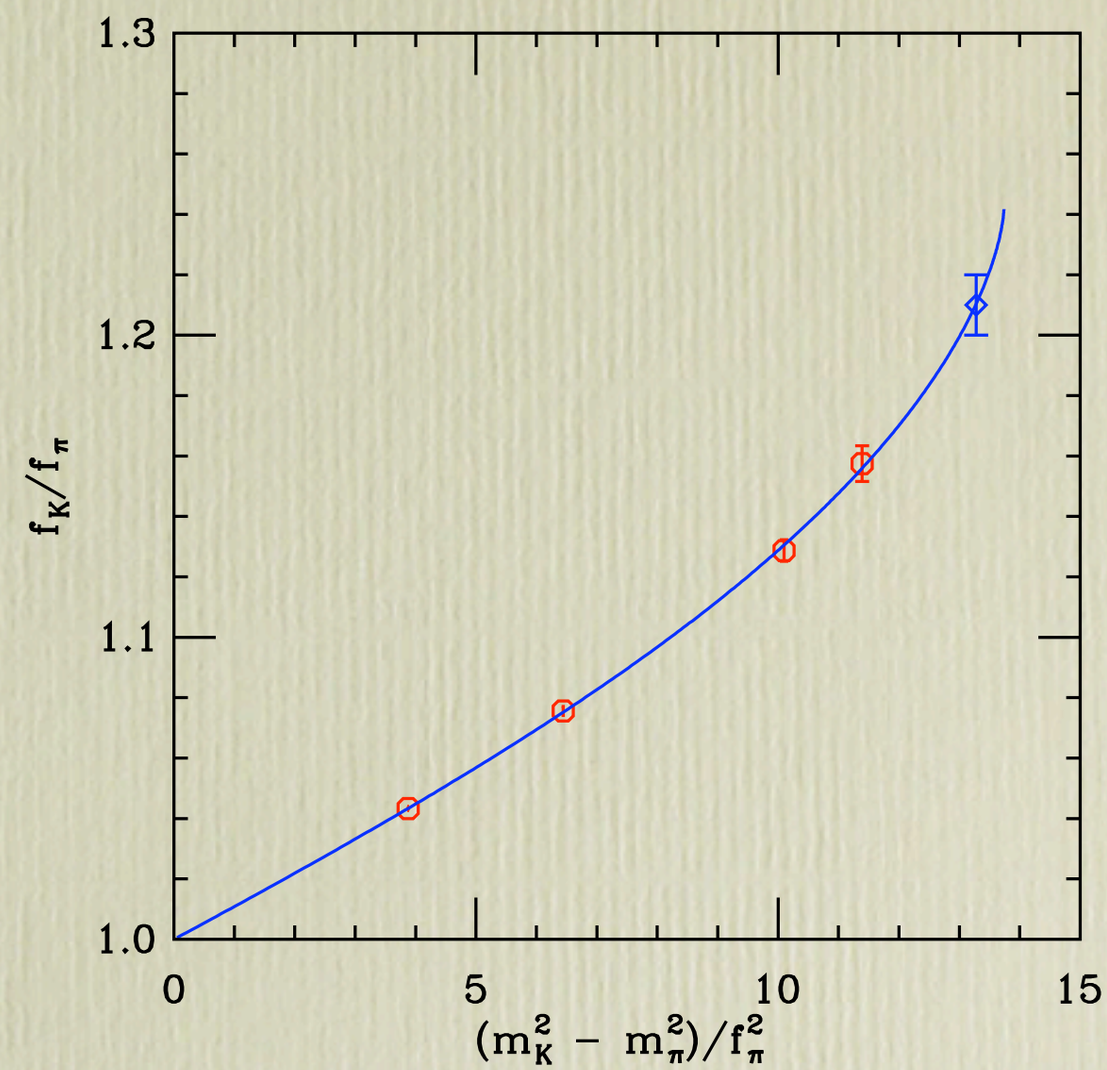
Gasser-Leutwyler:

$$\frac{f_K}{f_\pi} = 1 + \frac{5}{4} l_\pi(\mu) - \frac{1}{2} l_K(\mu) - \frac{3}{4} l_\eta(\mu) + \frac{8}{f^2}\left(m_K^2 - m_\pi^2\right) L_5(\mu)$$

$$l_i(\mu) = \frac{1}{16\pi^2}\frac{m_i^2}{f^2}\log\left(\frac{m_i^2}{\mu^2}\right)$$

$$\frac{f_K}{f_\pi} = 1.210(10)$$

$$\left.\frac{f_K}{f_\pi}\right|_{\text{exp.}} = 1.223(12)$$



Result comparable with MILC $\left.\frac{f_K}{f_\pi}\right|_{\text{MILC}} = 1.210(4)(13)$

Need much higher precision to see effects of Mixed χPT Baer et.al.'05

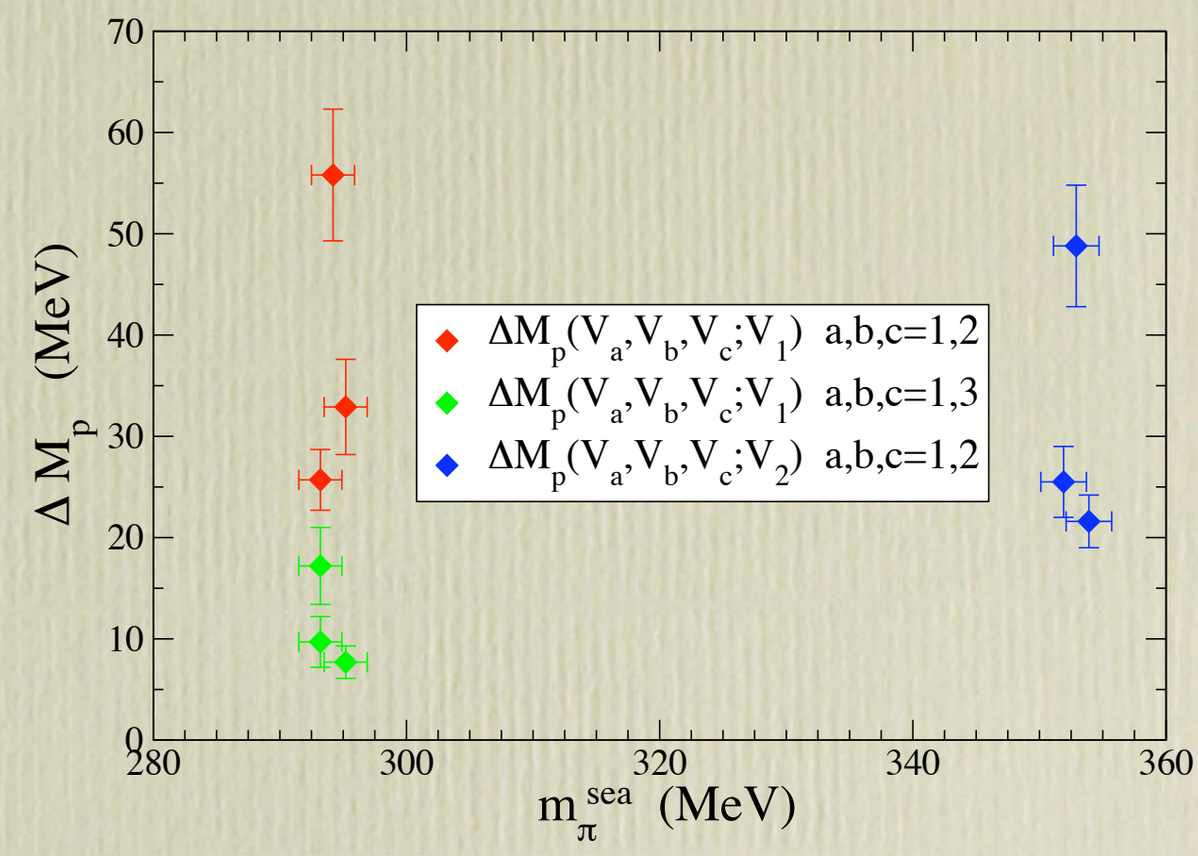
Strong Isospin Breaking

Beane, KO, Savage hep-lat/0605015

$$M_n - M_p|^{d-u} = \frac{2}{3}\left(2\overline{\alpha} - \overline{\beta}\right)\left(\frac{1-\eta}{1+\eta}\right) m_\pi^2$$

MILC: $\eta = m_u/m_d = 0.43(1)(8)$

Extraction	$M_n - M_p\|^{d-u}$ (MeV) at $m_\pi^{\text{phys.}}$
LO $\mathcal{O}(m_q)$	$1.96 \pm 0.92 \pm 0.37$
NLO $\mathcal{O}(m_q^{3/2})$	$2.26 \pm 0.57 \pm 0.42$

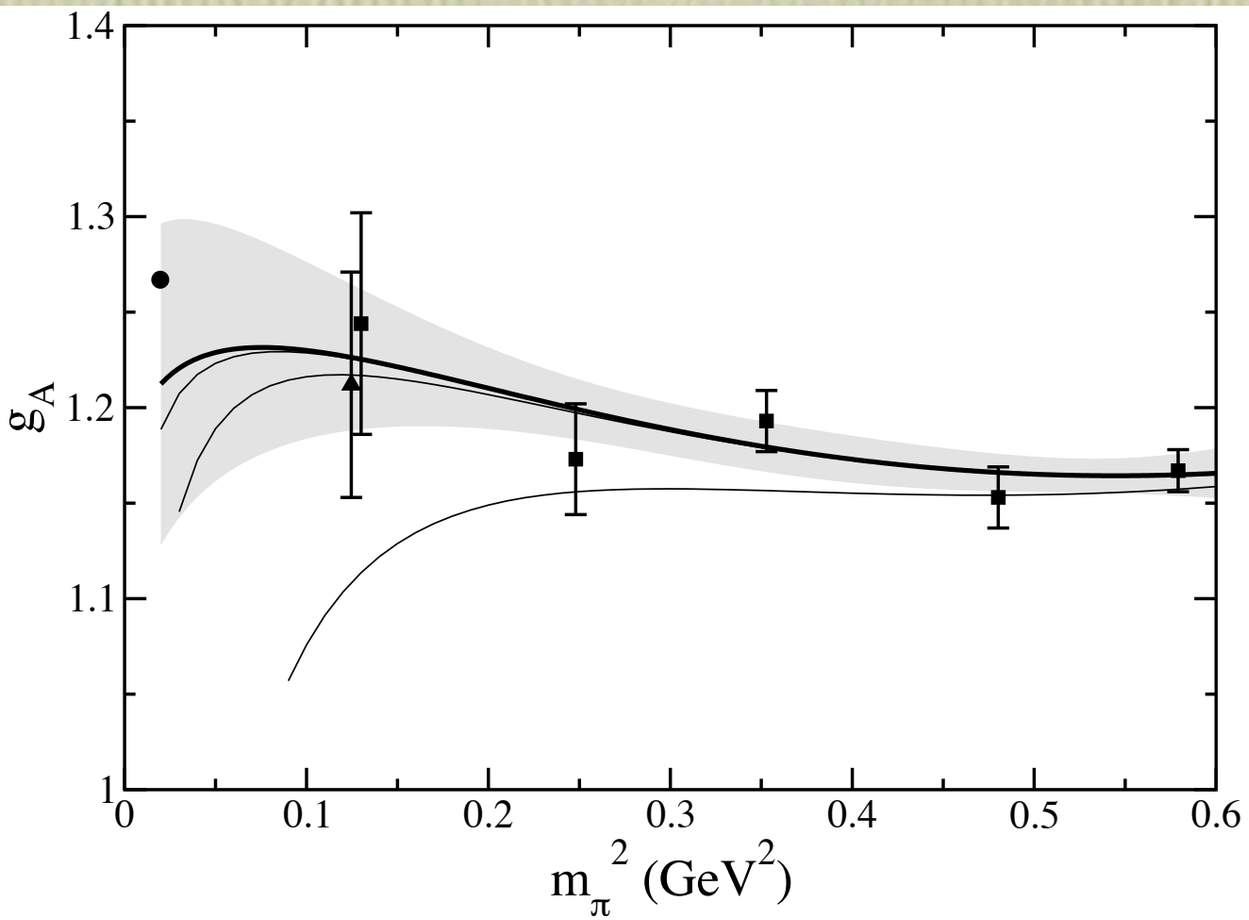


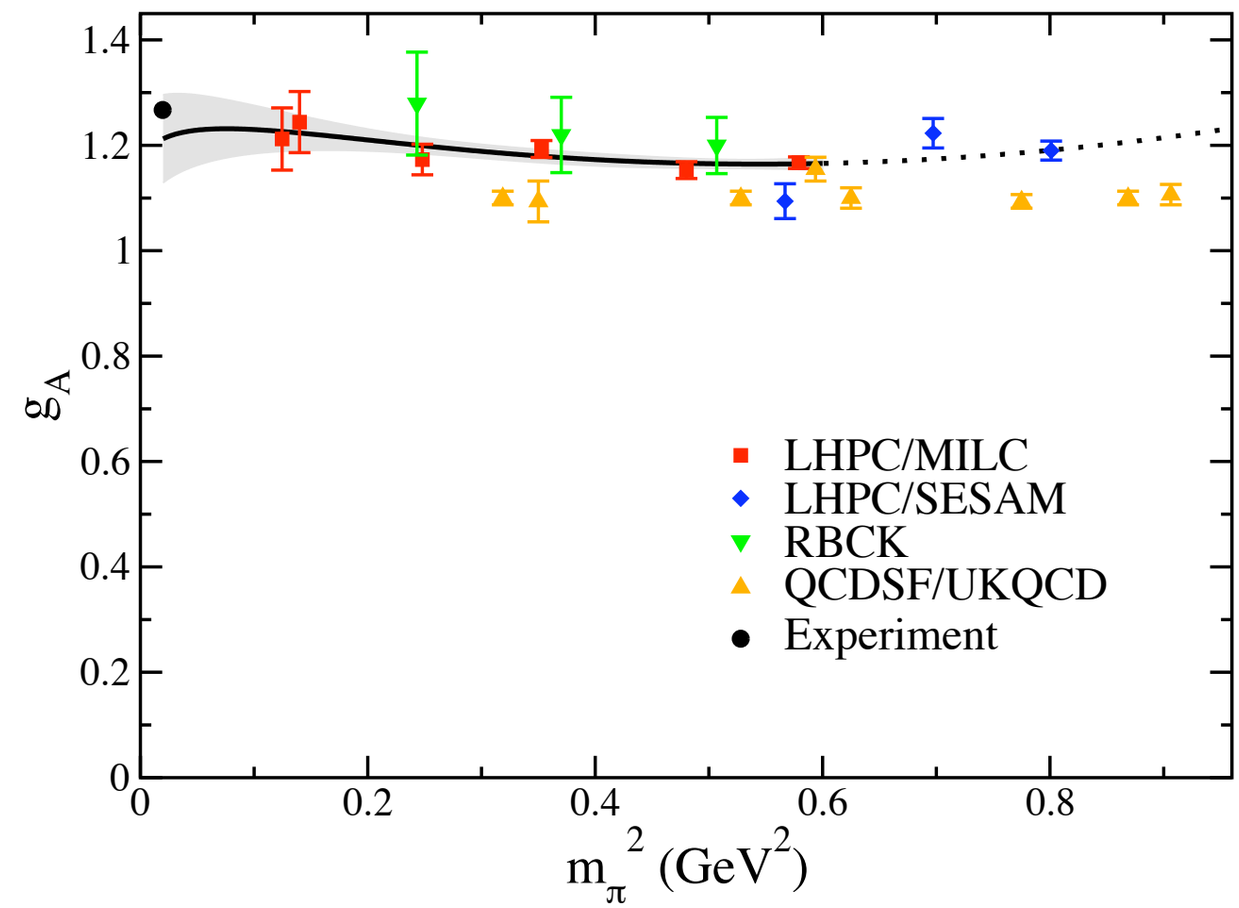
Extraction	$\frac{1}{3}\left(2\overline{\alpha} - \overline{\beta}\right)$ (l.u.)	$\overline{\alpha} + \overline{\beta}$ (l.u.)	g_1	$\|g_{\Delta N}\|$	χ^2/dof
LO $\mathcal{O}(m_q)$	0.198 ± 0.093	2.07 ± 0.08	$--$	$--$	0.56
NLO $\mathcal{O}(m_q^{3/2})$	0.229 ± 0.058	3.4 ± 1.1	-0.10 ± 0.35	0.60 ± 0.66	0.21

Exp. value: $M_n - M_p = 1.2933317(5)$ MeV $\xrightarrow[\text{Gasser Leutwyler '82}]{\text{minus EM part}}$ $M_n - M_p = 2.05(30)$ MeV

Nucleon Axial charge

LHPC

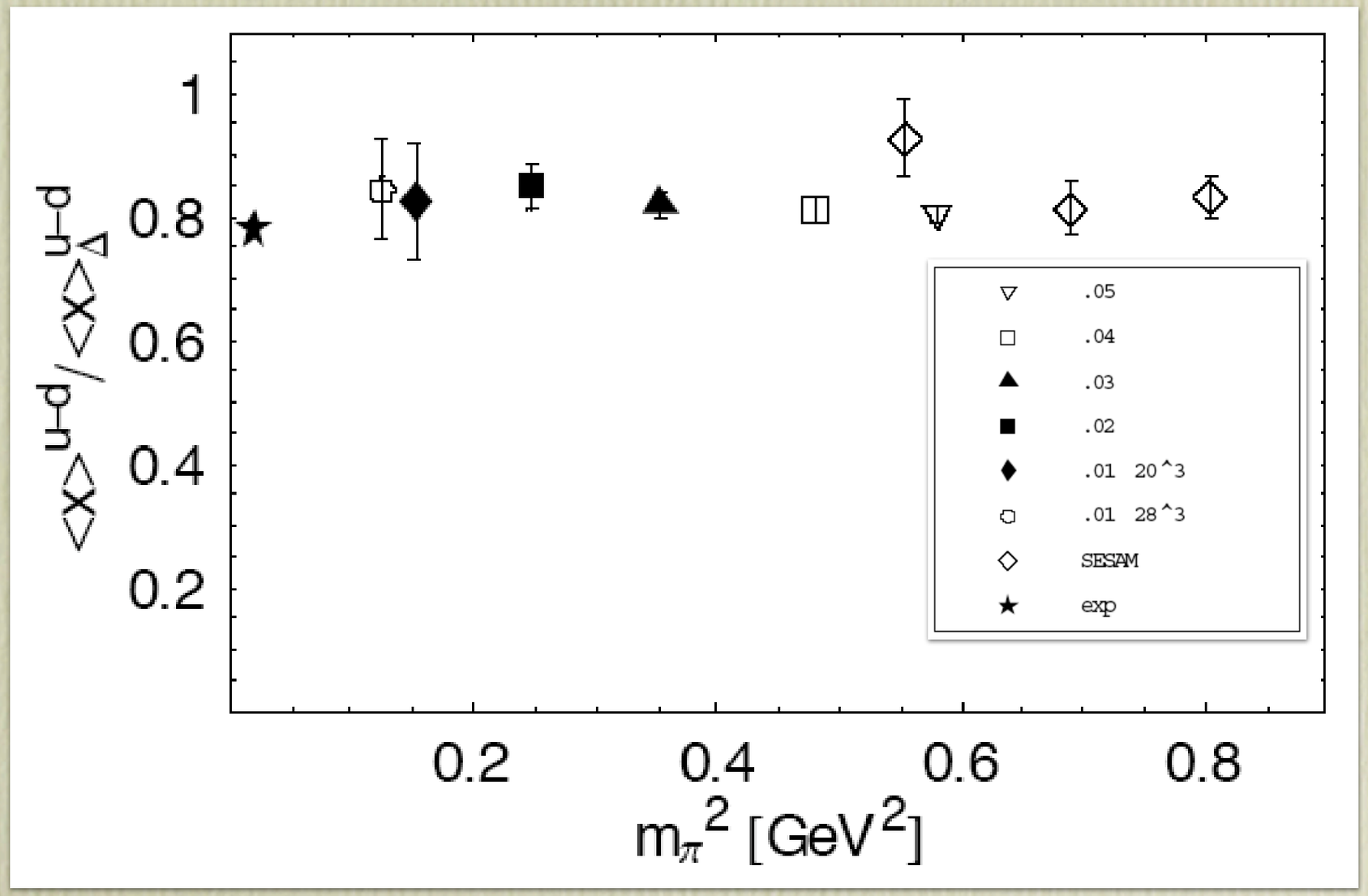




- Non-perturbatively renormalized
- $g_A(m_\pi=140\text{MeV}) = 1.22(8)$

Quark momentum fraction ratio $\langle x \rangle_q^{u-d} / \langle x \rangle_{\Delta q}^{u-d}$

LHPC/MILC LHPC/SESAM



Future

- Approach the chiral limit: Resolve the puzzle!
- Few moments of GPDs computed with dynamical fermions

  LHPC in preparation / QCDSF clover/Wilson
- Precision calculation of axial and tensor couplings
- Wilson fermions? (improved dynamical algorithms)