QCD phase diagram

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RBRC 1999-2004
QCD – asymptotically free field theory

At short distances – can see fundamental constituents.

QCD is a quantum field theory:

\[ S = \int d^4x \left[ \sum_{f=1}^{N_f} \bar{q}_f \left( i \partial_\mu + g A_\mu - m_f \right) q_f - \frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} \right]; \]

Predictive power: i.e., for every physical quantity it gives a recipe to calculate it.

... Whether we are ready to do the calculation is a different matter.

Strongly interacting theory (confinement) — hadrons. QCD generates the mass as we know it.
QCD thermodynamics

Applications:
- Neutron stars (large density, low $T$)
- Heavy-ion collisions (large $T$, large density)

QCD allows first principle calculations

Questions: phases, phase diagram, as function of $T, \mu_B, \ldots$
QCD thermodynamics

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QCD allows first principle calculations

Questions: phases, phase diagram, as function of $T$, $\mu_B$, ...

Early expectations  \[ \Rightarrow \]

Natural scale:

\[ kT \sim \frac{\hbar c}{1\text{fm}} = 0.2 \text{ GeV}. \]

\[ (T \sim 10^{12} \text{K}) \]

or

\[ \rho_B \sim 1\text{fm}^{-3}. \]

\[ U \sim \alpha_s/r \]

\[ U/K \sim \alpha_s \ll 1 \]

Asymptotic freedom

QGP  \[ r \sim 1/T \rightarrow 0 \]

\[ K \sim T \]

\[ \rho_B \sim 1\text{fm}^{-3}. \]

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Asymptotic freedom

QCD phase diagram – p. 3/12
Contemporary view

“Minimal” phase diagram

$T$, GeV

$\mu_B$, GeV

QGP

critical point

vacuum

nuclear matter

color s.c. quark matter

CFL quark matter

E
Contemporary view

“Minimal” phase diagram

T, GeV

$\alpha_s \ll 1$

$\mu_B$, GeV

QGP

vacuum

Lattice simulations

E

critical point

Only models

Empirical nuclear physics

nuclear matter

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“Minimal” phase diagram
Contemporary view

“Minimal” phase diagram
Role of chiral symmetry and $m_q \neq 0$

Ferromagnet

$H = 0$

$T_c$

$T$

$M$

$T$, GeV

0.1

0

1

$\mu_B$, GeV

hot QGP

2nd order (lattice MC)

3cr. point

vacuum

nuclear matter

cold quark matter

1st order (models: MIT bag, NJL, RM, ...)

$m_q = 0$

QCD phase diagram – p. 5/12
Role of chiral symmetry and $m_q \neq 0$

Ferromagnet at $H \neq 0$:

no phase transition, but crossover

Why all phases (can be) connected? Because $\not\exists$ order parameters:
no chiral symmetry ($m_q \neq 0$);
\[ \ldots \]
Role of chiral symmetry and \( m_q \neq 0 \)

Ferromagnet at \( H \neq 0 \):

\[ \langle M \rangle \]

no phase transition, but crossover

Why all phases (can be) connected? Because \( \nexists \) order parameters:

- no chiral symmetry \( (m_q \neq 0) \);
- ...

No phase boundary does not mean same physics:

- (compare water-vapor, gas-plasma)
Critical point in known liquids

Critical point ∃ in many liquids (critical opalescence).

Water:
Locating the QCD critical point

![QCD phase diagram](image)
Locating the QCD critical point

QCD phase diagram – p. 7/12
Experiments can scan the phase diagram by changing $\sqrt{s}$: RHIC.

Signatures: event-by-event fluctuations.

Susceptibilities diverge $\Rightarrow$ fluctuations grow towards the critical point.
Critical point on the lattice

Several approaches:

- **Reweighting:** Fodor-Katz
  - 2001: $\mu_B \sim 725$ MeV
  - 2004: $\mu_B \sim 360$ MeV (smaller $m_q$ and $V$)

- **Taylor expansion:** Bielefeld-Swansea (to $\mu^6$)
  - 2003: $\mu_B \sim 420$ MeV
  - 2005: $300$ MeV $\lesssim \mu_B \lesssim 500$ MeV

- **Taylor expansion:** Gavai-Gupta (to $\mu^8$)
  - $\mu_B \sim 180$ MeV (more precisely $> 180$ MeV)

- **Imaginary $\mu$:** Philipsen-deForcrand, Lombardo, *et al*
  - Sensitive to $m_s$, perhaps $\mu_B \gg 300$ MeV

- **Fixed density:** deForcrand, Kratochvila
  - $?(N_f = 4, \text{ small volumes})$
Shape and size of singular region

Important for experimental scan:

We only need to hit the ‘petal’. Maximum $\xi$ is anyway limited by finite $\tau$ to $\xi \sim 2-3$ fm, hence $\chi/\chi_0 \sim 4-9$.

(from Hatta, Ikeda)

- Wide in $\mu_B$, slim in $T$. (Note scale: $\mu_B : T \sim 15 : 1$.)
- $\chi/\chi_0 > 2.5$ in the interval $\Delta \mu_B \sim 150$ MeV;
- $\chi/\chi_0 > 4$ --- $\Delta \mu_B =$?
- Lattice?
Magnitude of the effect

Scaling and universality of critical phenomena: \( \chi \sim \xi^{\text{power}} \) (power \( \approx 2 \)).

How big can \( \xi \) grow?

Limited by:
- Finite size of the system \( \xi < 6 \) fm.
- Finite time: \( \tau \sim 10 \) fm.

\[ \xi \sim \tau^{1/z} \]

\( z > 1 \) – dynamical critical exponent.

Dynamic universality class of liquid-gas phase transition, i.e., \( z \approx 3 \):

- Critical mode – diffusive: \( \omega \sim iDq^2 \),
- \( D = \frac{\lambda_B}{\chi_B} \sim \xi^{-1} \rightarrow 0 \quad 2 + 1 = 3 \).

(Son, MS, PRD70:056001,2004)
Near the critical point (for CERES acceptance) one expects:

\[
\sim 2\% \times \left( \frac{G}{300 \text{ MeV}} \right)^2 \left( \frac{\xi_\sigma}{3 \text{ fm}} \right)^2
\]

\((\xi_\sigma = 1/m_\sigma)\)

Signal one is looking for: *non-monotonic* dependence on \(\sqrt{s}\).

Also: low transverse momenta (thermal, soft)), \(p_T < 3T \sim 500 \text{ MeV} – \text{where critical point correlations are (cf. CERES data – } p_T < 1.5 \text{ GeV).} \)
Current status and summary notes

Phase diagram of QCD at nonzero baryon density is under active theoretical investigation: much progress in lattice calculations.

Still a lot to be done to narrow down the prediction for the critical point.

Heavy ion collision experiments can discover the critical point by observing non-monotonous signatures.

Needed:
- Accelerator with variable $\sqrt{s}$ to scan phase diagram
- Detector with sufficient acceptance and p.id. at $p_T \lesssim 500$ MeV to measure fluctuations (of mean $p_T$, ratios, etc.);
- measure $\mu_B$, $T$ of freezeout.

RHIC is the ideal machine to do this