



The use of Molecular Dynamics simulations for the interpretation of EXAFS spectra



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EXAFS Spectroscopy Laboratory





Centre of Excellence in Advanced Material Research and Technology







Outline

- Introduction
 - Motivation & problems
- MD-EXAFS approach
 - Concept State of the Art Basics of Molecular Dynamics (MD) EXAFS simulations Interpretation of results
- Examples of MD-EXAFS simulations ReO₃, ScF₃, BCC & FCC metals
- Conclusions





Introduction

- Characteristic times in X-ray absorption process vs thermal vibrations
- How local is X-ray absorption spectroscopy?
- EXAFS formalism beyond the first coordination shell: multiple-scattering effects
- Problems and possible solutions





Basics of X-ray absorption spectroscopy (XAS)







How local is X-ray absorption spectroscopy?





Contributions from outer shells are often available and contain useful structural information.

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EXAFS simulations beyond the first coordination shell

1. Multiple-scattering (MS) expansion (FEFF code):

$$\chi^{l}(k) = \sum_{n=2}^{\infty} \chi^{l}_{n}(k) \qquad \chi^{l}_{n}(k) = \sum_{i} A^{l}_{n}(k, R_{i}) \sin(2kR_{i} + \Phi^{l}_{n}(k, R_{i}) + 2\delta^{l}_{c}(k))$$

A.L. Ankudinov, B. Ravel, J.J. Rehr, S.D. Conradson, Phys. Rev. B 58 (1998) 7565-7576.

2. N-body expansion (GNXAS code):

$$\begin{split} \chi(k) &= \int 4\pi R^2 \rho_0 g_2(R) (\chi_2^{oio}(k) + \chi_4^{oioio}(k) + \ldots) dR \\ &+ \iiint 8\pi^2 R_1^2 R_2^2 \sin(\theta) \rho_0^2 g_3(R_1, R_2, \theta) \\ &\times (2\chi_3^{oijo}(k) + 2\chi_4^{oiojo}(k) + \chi_4^{oijio}(k) + \chi_4^{ojjio}(k) + \ldots) dR_1 dR_2 d\theta \\ &+ \iiint 8\pi^2 R_1^2 R_2^2 R_3^2 \sin(\theta) \rho_0^2 g_4(R_1, R_2, \theta, R_3, \Omega) \\ &\times (2\chi_4^{oijko}(k) + 2\chi_4^{oikjo}(k) + 2\chi_4^{ojjko}(k) + \ldots) dR_1 dR_2 d\theta dR_3 d\Omega \\ &+ \ldots \end{split}$$

EXAFS depends on atomic distribution functions!

A. Filipponi, A. Di Cicco, C. R. Natoli, Phys. Rev. B 52 (1995) 15122-15134.







Multiple-scattering terms and many body correlation functions



The rows group the different possible paths which give the same order contributions to the absorption coefficient, χ_2 , χ_3 or χ_4 . The columns group the paths corresponding to two, three, four body correlation functions g_i .

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EXAFS problems (1)

1) One needs to account for many-body correlations through the so-called multiple-scattering (MS) effects.







EXAFS problems (2)

2) A number of fitting parameters grows rapidly upon an increase of analyzed region size around the absorbing atom. This problem is especially relevant for <u>disordered</u> and <u>nanocrystalline</u> materials.



A. Kuzmin and J. Chaboy, IUCrJ 1 (2014) 571-589.





Possible solutions

The problem of disorder in EXAFS can be addressed using

- Molecular Dynamics (MD),
- Monte Carlo (MC),
- Reverse Monte Carlo (RMC)

simulations.

Configuration-averaged EXAFS

$$\chi_{\rm ca}^l(k) = \left\langle \chi^l(k) \right\rangle$$

Advantages of MD-EXAFS approach:

- ✓ has no structural fitting parameters
- ✓ allows interpretation of EXAFS spectra far beyond the first coordination shell
- ✓ allows a validation of the interatomic potential models





MD-EXAFS approach: Concept



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	VIVELEDAGA	Internet: http://www.dragon.lv/edaca
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face	X-ray absorption spectroscopy (X/ providing information on the local particular type. Today XAS is succ and disordered solids, liquids and temperature, pressure, etc. The siz of thermal and static disorder pres the excited photoelectron. Typical around the absorbing atom.	AS) at synchrotron radiation sources is a structural too atomic and electronic structure around an atom of a esfully applied to a study of crystalline, nancocrystalline gases in a wide range of external conditions defined by e of the region, probed by XAS, depends on the degree ent in a material and is limited by the mean-free path of y the information reach region extended up to 3-10 A
	An advantage of the XAS method is rise to multiple-scattering (MS) co Note that accurate account of both	s its sensitivity to many-atom distribution functions, giving ntributions, and to correlation effects in atom dynamics. effects is still challenging.
	The time-scale (about 10 ⁻¹⁵ ,10 ⁻¹⁶) the characteristic time (about 10 ⁻¹³) be considered as forcer at their process, and the total experiments the configurational average of all situation can be straightforwardly simulation with the extended X-ray the MD-EXAFS approach.	s) of the X-ray absorption process is much shorter than 10 ⁻¹⁴ s) of thermal vibrations. Therefore, the atoms may instantaneous positions during a single photoabsorption lay measured X-ray absorption spectrum corresponds to atomic positions over the time of the experiment. This y modelled combining the molecular dynamics (MD) absorption fine structure (EXAFS) calculations, known as
	Finally, the agreement between spectra can be used to validate the employed in the MD simulations.	the experimental and configuration-averaged EXAFS accuracy of the interatomic potential (force-field) models

EDACA code:

A. Kuzmin and R.A. Evarestov, J. Phys.: Condens. Matter 21 (2009) 055401.

A. Kuzmin and J. Chaboy, IUCrJ 1 (2014) 571-589.

A. Kuzmin, A. Anspoks, A. Kalinko, J. Timoshenko, Z. Phys. Chem. 230 (2016) 537-549.





First Molecular Dynamics Simulation

Phase Transition for a Hard Sphere System

B. J. ALDER AND T. E. WAINWRIGHT University of California Radiation Laboratory, Livermore, California (Received August 12 [1957))

J. Chem. Phys. 27, 1208 (1957); doi: <u>http://dx.doi.org/10.1063/1.1743957</u> J. Chem. Phys. 31, 459 (1959); doi: <u>http://dx.doi.org/10.1063/1.1730376</u>





Publications on Molecular Dynamics simulations





MD-EXAFS approach: State of the Art



Using MD results for EXAFS calculation





MD-EXAFS approach: History



[5] M. I. McCarthy, G. K. Schenter, M. R. Chacon-Taylor, J. J. Rehr, G. E. Brown, Jr., Phys. Rev. B 56 (1997) 9925-9936. Na⁺(H₂O)_n-MgO (100) interface





Basics of classical Molecular Dynamics (I)

Molecular dynamics (MD) simulation is a technique to simulate the motion of atoms under predefined conditions, such as **temperature (T)**, **pressure (P)**, etc.

The **NVT** ensemble is also called the *canonical* ensemble. In the **NVT** ensemble, the number (N) of particles (atoms) and the supercell size (V) are fixed, whereas the temperature (T) is controlled using a surrounding virtual heat bath (**thermostat**).

The **NPT** ensemble is also called *isothermal-isobaric* ensemble. In the **NPT** ensemble, the number (N) of particles (atoms) is fixed, whereas the temperature (T) and pressure (P) are controlled using a **thermostat** and a **barostat**.

• G. J. Martyna, D. J. Tobias, M. L. Klein, Constant pressure molecular dynamics algorithms, J. Chem. Phys. 101 (1994) 4177.

[•] J. C. Berendsen, J. P. M. Postma, W. F. van Gunsteren, A. DiNola, J. R. Haak, Molecular-Dynamics with Coupling to an External Bath, J. Chem. Phys. 81 (1982) 3684.

[•] G. J. Martyna, M. L. Klein, M. Tuckerman, Nosé-Hoover chains: The canonical ensemble via continuous dynamics, J. Chem. Phys. 97 (1992) 2635.





Basics of classical Molecular Dynamics (II)

During classical MD simulation the **Newton's equations** of motion are numerically integrated for a set of atoms starting from a given **initial configuration**. This is performed via numerical integration by discretizing time into small intervals called the **time step** (Δ **t**). The time step must be smaller than the fastest vibrational frequency in the system, typically in the order of 1 femtosecond ($\approx 10^{-15}$ s).

The interactions between the atoms, i.e. the interatomic forces, can be calculated based on **empirical potentials** or **first-principles theory** (*ab initio*).

Machine Learning (ML) potentials become popular nowadays.

These interatomic forces determine the acceleration of the atoms, allowing the positions and velocities propagate towards the next time step. Repeating this procedure many times yields **a series of snapshots**, describing the trajectory of the system in phase space, which can be analyzed to extract the desired properties, including EXAFS/XANES.







Temperature in the classical MD simulation

 $\rightarrow 0$ K, $v_i \rightarrow 0$

$$\langle E_k \rangle = \left(\frac{1}{2} \sum_{i=1}^N (m_i \vec{v}_i)^2 \right) = \frac{3}{2} N k_B T \qquad T$$

١



T

The Debye temperature (θ_D) and characteristic temperature (T_0) of zero-point vibration. The unit for temperature is K.

Element	$ heta_D$	T_0
Be	1440	481.06
Al	428	142.98
Ti	420	140.31
Ta	240	80.18
Pb	105	35.08

Y. Yang and Y. Kawazoe, EPL 98 (2012) 66007

N.B. The magnitude of zero-point vibrations at T_0 equals that of the excited vibrations.

Measurements are often performed using cryostats!





Quantum effects

 $\Lambda_{\rm th} = \frac{h}{\sqrt{2\pi m k_{\rm p} T}}$

Quantum effects are expected to become significant when the thermal de Broglie wavelength

is much larger than the interatomic distance *d*.

Typical interatomic distance in solids is about $d \approx 1-3$ Å.

```
For example, at T = 300 K:

\Lambda_{th} = 1 \text{ Å} for a H atom (m<sub>H</sub> = 1 amu)

\Lambda_{th} = 0.24 \text{ Å} for a O atom (m<sub>O</sub> = 16 amu)

\Lambda_{th} = 0.12 \text{ Å} for a Zn atom (m<sub>Zn</sub> = 65 amu)

\Lambda_{th} = 0.09 \text{ Å} for a Sn atom (m<sub>Sn</sub> = 119 amu)

\Lambda_{th} = 0.06 \text{ Å} for a U atom (m<sub>H</sub> = 238 amu)
```



All atoms, except for the lightest ones such as H, He, can be considered as "point" particles at **sufficiently** high temperature (or $d >> \Lambda_{th}$), and classical mechanics can be used to describe their motion.





Interatomic potentials

Empirical potentials

- Depend on a set of parameters which must be determined in advance by fitting to some physical properties (structure, phonons, elastic constants, etc).
- Must be validated using experiment or more accurate simulation.
- MD simulations are fast and can be done using desktop computer or a server.

Machine Learning potentials

- Must be trained using ab initio (DFT) simulations.
- Must be validated using experiment.
- MD simulations are slower than with empirical potentials but much faster than doing ab initio simulations. Server or cluster are required.

Ab initio (DFT) potentials

- Calculated from first principles.
- Must be validated using experiment.
- MD simulations are slow. Supercomputer is required.





Interatomic potentials or force-fields

as a consequence of the Born-Oppenheimer separation of electrons and nuclei motion



Two-body

External field

Three-body



https://openkim.org/

When more accurate description is needed, electronic behavior can be obtained from first-principles calculations using ab initio Molecular Dynamics (AIMD).





Molecular Dynamics Programs

https://en.wikipedia.org/wiki/Comparison_of_software_for_molecular_mechanics_modeling

LAMMPS	- a Large-scale Atomic/Molecular Massively Parallel Simulator. https://www.lammps.org/
GULP	- the General Utility Lattice Program. https://gulp.curtin.edu.au/gulp/
DL_POLY	- a general purpose serial and parallel molecular dynamics simulation package. https://www.scd.stfc.ac.uk/Pages/DL_POLY.aspx
СР2К	 a program to perform atomistic and molecular simulations of solid state, liquid, molecular, and biological systems. (also ab initio) https://www.cp2k.org/

Important: MD trajectory must be saved as *.XYZ file with a specific structure.





Interpretation of results

VESTA a 3D visualization program for structural models, volumetric data such as electron/nuclear densities, and crystal morphologies.

http://jp-minerals.org/vesta/en/



 OVITO
 a scientific visualization and data analysis solution for atomistic and other particle-based models.

 https://www.ovito.org/
 Image: Covito of the sector o





MD-EXAFS simulation related parameters

- Equilibration time
- Proper averaging (number of configurations & time step)
- The simulation box size must be large enough to avoid boundary condition artifacts such as, for example, artificial correlations (simulation box size > 2R_{max})
- Multiple-scattering series truncation problem in FEFF8.5L:
 NLEG = 8 default 8 order * (bond length 2-3 Å) / 2 = 8-12 Å
 - CRITERIA4.02.5* defaultcritcw=4.0%critpw=2.5%CRITERIA0* use all paths, if possible (cw and pw criteria turned off)
- A "configuration" average over the spectra of multiple absorbing atoms (for nanoparticles)
 CFAVERAGE iphabs nabs rclabs

iphabs potential index for the type of absorbing atoms over which to make the configuration average **nabs** the configuration average is made over the first nabs absorbers in the `feff.inp' file of type iphabs. **rclabs** radius to make a small atom list from a bigger one allowed in `feff.inp'.





Non-structural parameters in MD-EXAFS simulation

The configuration-averaged EXAFS spectrum $\chi_{ca}(k)$ is calculated **uniquely** from the results of the MD simulation.

 E_0 and S_0^2

Comparison with the experiment requires two more parameters:







Examples of MD-EXAFS simulations







Cubic perovskite-type ReO₃



A. Kalinko, R.A. Evarestov, A. Kuzmin, J. Purans, J. Phys.: Conf. Ser. 190 (2009) 012080.





Force-field model for ReO₃ used in the MD simulations

	Exper.*	Calc.	
Lattice constant (Å)	3.75	3.75	
Bulk Modulus (GPa)	211	198	
Elastic constants (Gpa)			
C ₁₁	572	503	
C ₁₂	7	46]	for central
C ₄₄	68	46 ∫	pairwise forces



$$U_{ij} = -De^{\frac{-n(r_{ij}-r_{ij})}{2r_{ij}}}$$

 Atom
 Z, e

 Re
 3.36

 O
 -1.12

 Buckingham potential, cutoff 20 Å

The parameters of the force-field model for cubic ReO₃

Pair of atoms	A, eV	ρ , Å	C , $eVÅ^2$
Re–O	1194.24	0.3426	98.10
0–0	3224.54	0.2652	47.40

Covalent	exponential	potential,	cutoff 2 Å
Pair of atoms	D	n	r_0 , Å
Re–O	4.8283	6.2107	1.875

 * T. Chatterji, P. F. Henry, R. Mittal, S. L. Chaplot, Phys. Rev. B 78 (2008) 134105;
 N. Tsuda, Y. Sumino, I. Ohno, T. Akahane, J. Phys. Soc. Jpn. 41 (1976) 1153–1158.





1200

Force-field parameters were optimized to reproduce:

- structural parameters,
- elastic constants,
- ab initio phonon frequencies.

Phonon dispersion curves calculated using frozen phonon (LCAO, CRYSTAL06) method.

Phonon dispersion curves calculated by the force-field (GULP 3.1) method.

R. A. Evarestov, A. Kalinko, A. Kuzmin, M. Losev, J. Purans, Integr. Ferroelectrics 108 (2009) 1-10.





Molecular Dynamics simulations of ReO₃

using the rigid-ion model as implemented in the GULP3.1 code*



Cubic perovskite ReO_3 Space group No. 221, *Pm*-3*m* Lattice parameter $a_0 = 3.7477$ Å

* J.D. Gale and A.L. Rohl, Mol. Simul. 9 (2003) 291.



NVT/NPT ensembles Leap-frog Verlet integration Supercell **5x5x5** (500 atoms) T = 300 K, 423 K, 573 K Simulation time: 20 ps equilibration + 20 ps production Time step: 0.5 fs 4000 atomic configurations





Thermal disorder from MD simulations in ReO₃



A. Kalinko, R.A. Evarestov, A. Kuzmin, J. Purans, J. Phys.: Conf. Ser. 190 (2009) 012080.





Comparison between experimental and configuration-averaged Re L_3 -edge EXAFS signals and their Fourier transforms in ReO₃



Index	Path	Path	Path	Half-path
		type	degener.	length $(Å)$
1	$Re_0-O_1-Re_0$	SS	6	1.875
2	$\operatorname{Re}_0-\operatorname{O}_1-\operatorname{O}_1-\operatorname{Re}_0$	DS	24	3.201
3	$\mathrm{Re}_0\text{-}\mathrm{Re}_2\text{-}\mathrm{Re}_0$	SS	6	3.750
4	$\operatorname{Re}_0-\operatorname{O}_1-\operatorname{O}_1''-\operatorname{Re}_0$	DS	6	3.750
5	$Re_0-O_1-Re_2-Re_0$	DS	12	3.750
6	$\mathrm{Re}_0\text{-}\mathrm{O}_1\text{-}\mathrm{Re}_0\text{-}\mathrm{O}_1''\text{-}\mathrm{Re}_0$	TS	6	3.750
7	$\operatorname{Re}_0-\operatorname{O}_1-\operatorname{Re}_0-\operatorname{O}_1-\operatorname{Re}_0$	TS	6	3.750
8	$\operatorname{Re}_0-\operatorname{O}_1-\operatorname{Re}_2-\operatorname{O}_1-\operatorname{Re}_0$	TS	6	3.750
9	$\operatorname{Re}_0-\operatorname{O}_1-\operatorname{Re}_0-\operatorname{O}_1'-\operatorname{Re}_0$	TS	24	3.750
10	$Re_0-O_3-Re_0$	SS	24	4.193
11	$Re_0-O_1-O_3-Re_0$	DS	48	4.360
12	$\operatorname{Re}_0-\operatorname{O}_1-\operatorname{O}_1-\operatorname{Re}_0$	TS	24	4.527
13	$\operatorname{Re}_0-\operatorname{O}_1-\operatorname{O}_3-\operatorname{O}_1-\operatorname{Re}_0$	TS	24	4.527
14	$\mathrm{Re}_0\text{-}\mathrm{Re}_4\text{-}\mathrm{Re}_0$	SS	12	5.303
15	$\mathrm{Re}_0-\mathrm{O}_5-\mathrm{Re}_0$	SS	30	5.625



A. Kalinko, R.A. Evarestov, A. Kuzmin, J. Purans, J. Phys.: Conf. Ser. 190 (2009) 012080.





Validation of interatomic potentials for bcc tungsten using the W L₃-edge EXAFS

$$\sigma_{\mathbf{th}}^{\mathbf{2}} = \langle u_{\mathrm{A}}^{2} \rangle + \langle u_{\mathrm{B}}^{2} \rangle - 2 \langle u_{\mathrm{A}} u_{\mathrm{B}} \rangle$$



I. Jonane, A. Anspoks, A. Kuzmin, Modelling Simul. Mater. Sci. Eng. 26 (2018) 025004.





Validation of Molecular Dynamics simulations: How accurate are force-fields? EAM vs 2NN-MEAM





Supercell: 7*a* × 7*a* × 7*a* 686 atoms

MD GULP code NVT T=300 K ∆t=0.5 fs 20 ps + 20 ps

EDACA + FEFF8.50L





Embedded Atom Model (EAM): M.W. Finnis and J.E. Sinclair, Phil. Mag. A 50 (1984) 45-55. 2nd Nearest-Neighbor Modified EAM (2NN-MEAM): B.J. Lee, M. Baskes, H. Kim, Y. Koo Cho, Phys. Rev. B 64 (2001) 184102.

I. Jonane, A. Anspoks, A. Kuzmin, Modelling Simul. Mater. Sci. Eng. 26 (2018) 025004.

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Simultaneous MSRD and MSD determination from EXAFS



MD: MSD(EAM)=0.0023 Å² MSD(MEAM)=0.0029 Å²

RMC: MSD(RMC)=0.0039 Å²

X-ray diffraction: MSD=0.0061 Å² [1] MSD=0.0022 Å² [2]

Lattice dynamics calculations: MSD=0.0023 Å² [3] MSD=0.0018 Å² [4]

C.R. Houska, J. Phys. Chem. Solids 25 (1964) 359.
 T. Paakkari, Acta Crystallogr. A 30 (1974) 83.
 P.C. Fine, Phys. Rev. 56 (1939) 355.
 L. Dobrzynski, P. Masri, J. Phys. Chem. Solids 33 (1972) 1603.

 $MSRD_{AB} = MSD_A + MSD_B - 2\varphi\sqrt{MSD_A}\sqrt{MSD_B}$

 $\mathbf{R} \rightarrow \infty, \boldsymbol{\varphi} \rightarrow \mathbf{0} \& \mathbf{MSRD}_{AB} \rightarrow \mathbf{MSD}_A + \mathbf{MSD}_B$



I. Jonane, A. Anspoks, A. Kuzmin, Modelling Simul. Mater. Sci. Eng. 26 (2018) 025004.





Cubic perovskite ScF₃ - the effect of the supercell size



D. Bocharov, M. Krack, Yu. Rafalskij, A. Kuzmin, J. Purans, Comp. Mater. Sci. 171 (2020) 109198.





6

Ab initio molecular dynamics simulations of negative thermal expansion in ScF_3 : the effect of the supercell size





NpT AIMD simulations were performed using CP2K code.

code. https://www.cp2k.org/

D. Bocharov, M. Krack, Yu. Rafalskij, A. Kuzmin, J. Purans, Comp. Mater. Sci. 171 (2020) 109198.





Ab initio molecular dynamics simulations of negative thermal expansion in ScF_3 : the effect of the supercell size



Small supercell size introduces artefacts into RDFs and results in inaccurate RDFs and BADFs.

D. Bocharov, M. Krack, Yu. Rafalskij, A. Kuzmin, J. Purans, Comp. Mater. Sci. 171 (2020) 109198.

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Validation of Machine Learning potentials



A. V. Shapeev, D. Bocharov, A. Kuzmin, Comput. Mater. Sci. 210 (2022) 111028.





Validation of moment tensor potentials for fcc and bcc metals using EXAFS spectra





NVT MD simulations were performed by LAMMPS using Moment Tensor Potentials (MTPs) [1,2] as implemented in the MLIP-2 software package [3].

MTP potentials were trained on-the-fly using the results of AIMD by VASP.

A.V. Shapeev, Multisc. Model. Simul. 14 (2016) 1153–1173.
 K. Gubaev, E.V. Podryabinkin, G.L. Hart, A.V. Shapeev, Comput. Mater. Sci. 156 (2019) 148–156.

[3] I.S. Novikov, K. Gubaev, E.V. Podryabinkin, A.V. Shapeev, Mach. Learn.: Sci. Technol. 2 (2020) 025002.

https://github.com/materialsvirtuallab/mlearn







Conclusions

- Molecular Dynamics is a natural way to include disorder (static and dynamic) into EXAFS simulations taking into account multiple-scattering effects.
- The MD-EXAFS approach allows one to interpret EXAFS spectra based on the multiple-scattering formalism far beyond the first coordination shell.
 Good for EXAFS!
- No structural fitting parameters is used in the MD-EXAFS approach. The structural model is uniquely defined from the results of the MD simulation. But your EXAFS results will be as good as the MD ones.
- EXAFS spectra can be used as an additional physical property for interatomic potential/theory validation. Good for Molecular Dynamics!
- Care should be taken regarding the temperature. At low temperature, special treatment is required, e.g. based on path-integral methodology*.
 - * D. Marx, M. Parrinello, J. Chem. Phys. 104 (1996) 4077-4082





Thank you for your attention!

