# Wavelet analysis of EXAFS spectra



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# **EXAFS data analysis**



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# Outline

- □ Fourier-transform of EXAFS data
- □Short-time Fourier transform
- □ Wavelet transform
- □ Wavelet transform of EXAFS data: examples

### **Fourier transform**

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PHYSICAL REVIEW LETTERS

1 November 1971

New Technique for Investigating Noncrystalline Structures: Fourier Analysis of the Extended X-Ray-Absorption Fine Structure\*

> Dale E. Sayerst and Edward A. Sternt‡ Department of Physics, University of Washington, Seattle, Washington 98105

> > and

Farrel W. Lytle Boeing Scientific Research Laboratories, Seattle, Washington 98124 (Received 16 July 1971)

$$\chi(k) = \sum_p \chi_p(k)$$

$$\chi_p(k) = S_0^2 \int_0^{+\infty} g_p(\mathbf{R}) \frac{f_p(k,R)}{kR^2} \sin\left(2kR + \phi_p(k)\right) dR$$



FIG. 1. Smoothed experimental EXAFS data for (a) crystalline and (b) amorphous Ge. Only the oscillatory part  $\chi$  of the absorption edge is shown.



FIG. 2. Fourier transformation of the data of Fig. 1.  $\varphi(r)$ , a radial structure function, compares amorphous and crystalline Ge. Numbers over the peaks indicate the measured distances in Å.

Taking into account sinusoidal nature of EXAFS signal, **Fourier transform** seems to be a natural way to get quick first impression on material structure.

$$FT_{\chi}(R) = (2\pi)^{-1/2} \int_{k_{\min}}^{k_{\max}} e^{2iRk} \chi(k) dk$$

# **EXAFS** equation



J. J. Rehr and R. C. Albers, Rev. Mod. Phys., 72 (2000).

2.4

### **Fourier transform: frequency**

$$\chi(k) = \sum_{p} S_{0}^{2} N_{p} \frac{f_{p}(k)}{kR_{p}^{2}} e^{-2k^{2}\sigma_{p}^{2}} \sin\left(2kR_{p} + \phi_{p}(k)\right) \rightarrow \sum_{p} \frac{S_{0}^{2} N_{p} \sin(2kR_{p})}{R_{p}^{2}}$$
$$FT_{\chi}(R) = (2\pi)^{-\frac{1}{2}} \int_{k_{min}}^{k_{max}} e^{2iRk} \chi(k) dk$$



### **Fourier transform: frequency**

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### **Fourier transform: window function**

$$\chi(k) = \sum_{p} S_{0}^{2} N_{p} \frac{f_{p}(k)}{kR_{p}^{2}} e^{-2k^{2}\sigma_{p}^{2}} \sin\left(2kR_{p} + \phi_{p}(k)\right) \rightarrow \sum_{p} \frac{S_{0}^{2} N_{p} \sin(2kR_{p})}{R_{p}^{2}}$$
$$FT_{\chi}(R) = (2\pi)^{-\frac{1}{2}} \int_{k_{min}}^{k_{max}} e^{2iRk} \chi(k) e^{\frac{-(k-k_{w})^{2}}{\sigma_{w}^{2}}} dk$$



### **Fourier transform: disorder factor**

$$\chi(k) = \sum_{p} S_{0}^{2} N_{p} \frac{f_{p}(k)}{kR_{p}^{2}} e^{-2k^{2}\sigma_{p}^{2}} \sin\left(2kR_{p} + \phi_{p}(k)\right) \rightarrow \sum_{p} \frac{S_{0}^{2} N_{p} \sin(2kR_{p}) e^{-2k^{2}\sigma_{p}^{2}}}{R_{p}^{2}}$$
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$$\chi(k) = \sum_{p} S_{0}^{2} N_{p} \frac{f_{p}(k)}{kR_{p}^{2}} e^{-2k^{2}\sigma_{p}^{2}} \sin\left(2kR_{p} + \phi_{p}(k)\right) \rightarrow \sum_{p} \frac{S_{0}^{2} N_{p} \sin\left(2kR_{p} + \phi_{p}(k)\right) e^{-2k^{2}\sigma_{p}^{2}}}{R_{p}^{2}}$$

Scattering **phase**  $\phi(k) = \phi_{scatterer}(k) + \phi_{central atom}(k)$ : different for different types of atoms and result in chemical sensitivity of EXAFS analysis Phase and amplitude depend on material and can be calculated, e.g., by FEFF

"Real Space Multiple Scattering Calculation of XANES," A.L. Ankudinov, B. Ravel, J.J. Rehr, and S.D. Conradson, Phys. Rev. B 58, 7565 (1998).





$$\chi(k) = \sum_{p} S_0^2 N_p \frac{f_p(k)}{kR_p^2} e^{-2k^2 \sigma_p^2} \sin\left(2kR_p + \phi_p(k)\right) \to \sum_{p} \frac{S_0^2 N_p \sin\left(2kR_p + \phi_p(k)\right) e^{-2k^2 \sigma_p^2}}{R_p^2}$$

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Kuzmin et al, Cent. Eur. J. Phys. • 9(2) • 2011 • 502-509

$$\chi(k) = \sum_{p} S_0^2 N_p \frac{f_p(k)}{kR_p^2} e^{-2k^2 \sigma_p^2} \sin\left(2kR_p + \phi_p(k)\right) \to \sum_{p} \frac{S_0^2 N_p \sin\left(2kR_p + \phi_p(k)\right) e^{-2k^2 \sigma_p^2}}{R_p^2}$$



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![](_page_21_Figure_2.jpeg)

$$\chi(k) = \sum_{p} S_0^2 N_p \frac{f_p(k)}{kR_p^2} e^{-2k^2 \sigma_p^2} \sin\left(2kR_p + \phi_p(k)\right) \to \sum_{p} \frac{S_0^2 N_p \sin\left(2kR_p + \phi_p(k)\right) e^{-2k^2 \sigma_p^2}}{R_p^2}$$

![](_page_22_Figure_2.jpeg)

$$\chi(k) = \sum_{p} S_0^2 N_p \frac{f_p(k)}{kR_p^2} e^{-2k^2 \sigma_p^2} \sin\left(2kR_p + \phi_p(k)\right) \to \sum_{p} \frac{S_0^2 N_p \sin\left(2kR_p + \phi_p(k)\right) e^{-2k^2 \sigma_p^2}}{R_p^2}$$

![](_page_23_Figure_2.jpeg)

![](_page_24_Figure_1.jpeg)

![](_page_24_Figure_2.jpeg)

P. A. Lee, P. H. Citrin, P. Eisenberger, B. M. Kincaid, Rev. Mod. phys. 53 (1981) 769-806.

Kuzmin et al, Cent. Eur. J. Phys. • 9(2) • 2011 • 502-509

$$\chi(k) = \sum_{p} S_{0}^{2} N_{p} \frac{f_{p}(k)}{kR_{p}^{2}} e^{-2k^{2}\sigma_{p}^{2}} \sin\left(2kR_{p} + \phi_{p}(k)\right) \rightarrow \sum_{p} \frac{S_{0}^{2} N_{p} f_{p}(k) \sin\left(2kR_{p} + \phi_{p}(k)\right) e^{-2k^{2}\sigma_{p}^{2}}}{kR_{p}^{2}}$$
$$FT_{\chi}(R) = (2\pi)^{-\frac{1}{2}} \int_{k_{min}}^{k_{max}} e^{2iRk} \chi(k) k^{2} e^{\frac{-(k-k_{w})^{2}}{\sigma_{w}^{2}}} dk$$

![](_page_25_Figure_2.jpeg)

$$\chi(k) = \sum_{p} S_{0}^{2} N_{p} \frac{f_{p}(k)}{kR_{p}^{2}} e^{-2k^{2}\sigma_{p}^{2}} \sin\left(2kR_{p} + \phi_{p}(k)\right) \rightarrow \sum_{p} \frac{S_{0}^{2} N_{p} f_{p}(k) \sin\left(2kR_{p} + \phi_{p}(k)\right) e^{-2k^{2}\sigma_{p}^{2}}}{kR_{p}^{2}}$$
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![](_page_26_Figure_2.jpeg)

$$\chi(k) = \sum_{p} S_{0}^{2} N_{p} \frac{f_{p}(k)}{kR_{p}^{2}} e^{-2k^{2}\sigma_{p}^{2}} \sin\left(2kR_{p} + \phi_{p}(k)\right) \rightarrow \sum_{p} \frac{S_{0}^{2} N_{p} f_{p}(k) \sin\left(2kR_{p} + \phi_{p}(k)\right) e^{-2k^{2}\sigma_{p}^{2}}}{kR_{p}^{2}}$$
$$FT_{\chi}(R) = (2\pi)^{-\frac{1}{2}} \int_{k_{min}}^{k_{max}} e^{2iRk} \chi(k) k^{2} e^{\frac{-(k-k_{w})^{2}}{\sigma_{w}^{2}}} dk$$

![](_page_27_Figure_2.jpeg)

$$\chi(k) = \sum_{p} S_{0}^{2} N_{p} \frac{f_{p}(k)}{kR_{p}^{2}} e^{-2k^{2}\sigma_{p}^{2}} \sin\left(2kR_{p} + \phi_{p}(k)\right) \rightarrow \sum_{p} \frac{S_{0}^{2} N_{p} f_{p}(k) \sin\left(2kR_{p} + \phi_{p}(k)\right) e^{-2k^{2}\sigma_{p}^{2}}}{kR_{p}^{2}}$$
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![](_page_28_Figure_2.jpeg)

$$\chi(k) = \sum_{p} S_{0}^{2} N_{p} \frac{f_{p}(k)}{kR_{p}^{2}} e^{-2k^{2}\sigma_{p}^{2}} \sin\left(2kR_{p} + \phi_{p}(k)\right) \rightarrow \sum_{p} \frac{S_{0}^{2} N_{p} f_{p}(k) \sin\left(2kR_{p} + \phi_{p}(k)\right) e^{-2k^{2}\sigma_{p}^{2}}}{kR_{p}^{2}}$$
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![](_page_29_Figure_2.jpeg)

# FT-EXAFS: summary

- There are many structure-unrelated factors that can affect number, position and shape of FT peaks
- Chemical sensitivity of FT-EXAFS is very limited: e.g., Ni-O and Ni-Ni peaks in FT-EXAFS look very similar, since information about location of spectral components in k-space is lost

![](_page_31_Figure_0.jpeg)

![](_page_31_Figure_1.jpeg)

No information in FT-EXAFS about location of spectral components in k-space

$$\chi(k) = \sum_{p} N_{p} e^{-2(k-k_{0})^{2}\sigma_{p}^{2}} \sin(2kR_{p})$$
$$FT_{\chi}(R) = (2\pi)^{-\frac{1}{2}} \int_{k_{min}}^{k_{max}} e^{2iRk} \chi(k) k^{2} e^{\frac{-(k-k_{w})^{2}}{\sigma_{w}^{2}}} dk$$

![](_page_32_Figure_2.jpeg)

$$\chi(k) = \sum_{p} N_{p} e^{-2(k-k_{0})^{2}\sigma_{p}^{2}} \sin(2kR_{p})$$
$$FT_{\chi}(R) = (2\pi)^{-\frac{1}{2}} \int_{k_{min}}^{k_{max}} e^{2iRk} \chi(k) k^{2} e^{\frac{-(k-k_{w})^{2}}{\sigma_{w}^{2}}} dk$$

![](_page_33_Figure_2.jpeg)

$$\chi(k) = \sum_{p} N_{p} e^{-2(k-k_{0})^{2}\sigma_{p}^{2}} \sin(2kR_{p})$$
$$FT_{\chi}(R) = (2\pi)^{-\frac{1}{2}} \int_{k_{min}}^{k_{max}} e^{2iRk} \chi(k)k^{2} e^{\frac{-(k-k_{w})^{2}}{\sigma_{w}^{2}}} dk$$

![](_page_34_Figure_2.jpeg)

12/7/2023

$$\chi(k) = \sum_{p} N_{p} e^{-2(k-k_{0})^{2}\sigma_{p}^{2}} \sin(2kR_{p})$$
$$FT_{\chi}(R) = (2\pi)^{-\frac{1}{2}} \int_{k_{min}}^{k_{max}} e^{2iRk} \chi(k) k^{2} e^{\frac{-(k-k_{w})^{2}}{\sigma_{w}^{2}}} dk$$

![](_page_35_Figure_2.jpeg)

![](_page_35_Figure_3.jpeg)

$$\chi(k) = \sum_{p} N_{p} e^{-2(k-k_{0})^{2}\sigma_{p}^{2}} \sin(2kR_{p})$$
$$FT_{\chi}(R) = (2\pi)^{-\frac{1}{2}} \int_{k_{min}}^{k_{max}} e^{2iRk} \chi(k) k^{2} e^{\frac{-(k-k_{w})^{2}}{\sigma_{w}^{2}}} dk$$

![](_page_36_Figure_2.jpeg)

$$\chi(k) = \sum_{p} N_{p} e^{-2(k-k_{0})^{2}\sigma_{p}^{2}} \sin(2kR_{p})$$
$$FT_{\chi}(R) = (2\pi)^{-\frac{1}{2}} \int_{k_{min}}^{k_{max}} e^{2iRk} \chi(k) k^{2} e^{\frac{-(k-k_{w})^{2}}{\sigma_{w}^{2}}} dk$$

![](_page_37_Figure_2.jpeg)

Result depends on the width of window function. What is the optimal width? - it depends on frequency

### **Morlet wavelet transform**

![](_page_38_Figure_1.jpeg)

![](_page_38_Figure_2.jpeg)

Window function width  $\sim \sigma_0^2 R_0/R$  - different for different frequencies!

### **Morlet wavelet transform**

![](_page_39_Figure_1.jpeg)

![](_page_39_Figure_2.jpeg)

Timoshenko et al, Z. Phys. Chem. 2016; 230(4): 551–568

#### Resolution different for different frequencies!

### More about wavelet transform

$$WT_{\chi}(k,R) = (R/R_0)^{\frac{1}{2}} \int_{k_{min}}^{k_{max}} \phi\left(\frac{k'-k}{R_0/R}\right) \chi(k') k'^2 dk'$$

• Some other popular choices for mother wavelet function:

![](_page_40_Figure_3.jpeg)

https://georgemdallas.wordpress.com/2014/05/14/wavelets-4-dummies-signal-processing-fourier-transforms-and-heisenberg/

• It is invertible:

$$\chi(k) = \frac{1}{C} \int_{k_{min}}^{k_{max}} \int_{R_{min}}^{R_{max}} WT(k', R) \left( \frac{R}{R_0} \right)^{\frac{1}{2}} \phi^* \left( \frac{k' - k}{R_0 / R} \right) \frac{dR}{R_0} dk'$$
$$C = \int_0^\infty \frac{\left| \hat{\phi}(R) \right|^2}{R} dR$$

Single-scattering and multiple-scattering effects in Re L<sub>3</sub>-edge EXAFS spectra for ReO<sub>3</sub>

![](_page_41_Figure_2.jpeg)

### Discrimination of metallic and non-metallic species (hcp-Ni vs. Ni<sub>3</sub>C)

Ni clusters in piezoresistive Ni:a-C:H thin films may assume fcc-Ni, hcp-Ni or Ni<sub>3</sub>C, dependently on film deposition conditions. hcp-Ni and Ni<sub>3</sub>C is very hard to distinguish by XRD, and also Ni-Ni EXAFS is very similar.

![](_page_42_Figure_3.jpeg)

![](_page_42_Figure_4.jpeg)

![](_page_42_Figure_5.jpeg)

Fig. 4. Backscattering amplitude functions (BAF) of C and Ni with Ni<sub>3</sub>C and that of Ni with hcp-Ni. Each BAF was calculated with FEFF6 by assuming single coordination shells with Ni and C, respectively. CN - coordination number.

Uhlig et al, Diamond & Related Materials 34 (2013)

Fig. 3. Theoretical  $\chi(k)$  and  $\chi(R)$  spectra of hcp-Ni (JCPDS #00-045-1027) and Ni<sub>3</sub>C (JCPDS #01-072-1467).

![](_page_42_Picture_9.jpeg)

Discrimination of metallic and non-metallic species (hcp-Ni vs. Ni<sub>3</sub>C)

![](_page_43_Figure_2.jpeg)

Struis et al, J. Phys. Chem. C 2009, 113

### Analysis of multi-element compounds (Pt on Mg(In)(Al)O<sub>x</sub> support)

1. Pt precursor (acetylacetonate) was impregnated on Mg(In)(Al)O<sub>x</sub> mixed oxide. Analysis of first coordination shell is straightforward. But what are the distant peaks in FT-EXAFS?

![](_page_44_Figure_3.jpeg)

![](_page_44_Figure_4.jpeg)

Filez et al, Analytical chemistry 87 (2015)

### Analysis of multi-element compounds (Pt on Mg(In)(Al)O<sub>x</sub> support)

![](_page_45_Figure_2.jpeg)

Pt-O feature has disappeared after reduction. But XANES does not look like that in metallic Pt...

WT-EXAFS has Pt-Pt feature, but is different from that in metallic Pt: indicates alloying with In from the support.

![](_page_45_Figure_5.jpeg)

Filez et al, Analytical chemistry 87 (2015)

### Analysis of bimetallic systems; Ramsauer-Townsend effect

![](_page_46_Figure_2.jpeg)

Timoshenko, Keller, Frenkel, J.Chem. Phys. 146 (2017)

Analysis of bimetallic systems; chemical sensitivity and size sensitivity

![](_page_47_Figure_2.jpeg)

![](_page_47_Figure_3.jpeg)

Pd K-edge, cuboctahedral particle

Timoshenko, Keller, Frenkel, J.Chem. Phys. 146 (2017)

### Analysis of bimetallic systems; sensitivity to structural motifs

![](_page_48_Figure_2.jpeg)

Timoshenko, Keller, Frenkel, J.Chem. Phys. 146 (2017)

# **Summary and conclusions**

- Wavelet transform is an integral transformation, similar to Fourier transform, but provides information not only about signal frequencies, but also about the location of different spectral components in the original space.
- Wavelet transform provides a more informative way for visualization of EXAFS spectra: allows one to distinguish signal from artefacts, to discriminate between contributions of different species.
- Fitting of EXAFS data in wavelet space allows one to exploit more efficiently the information, encoded in EXAFS features, and may stabilize the fit.

# Thank you for your attention!