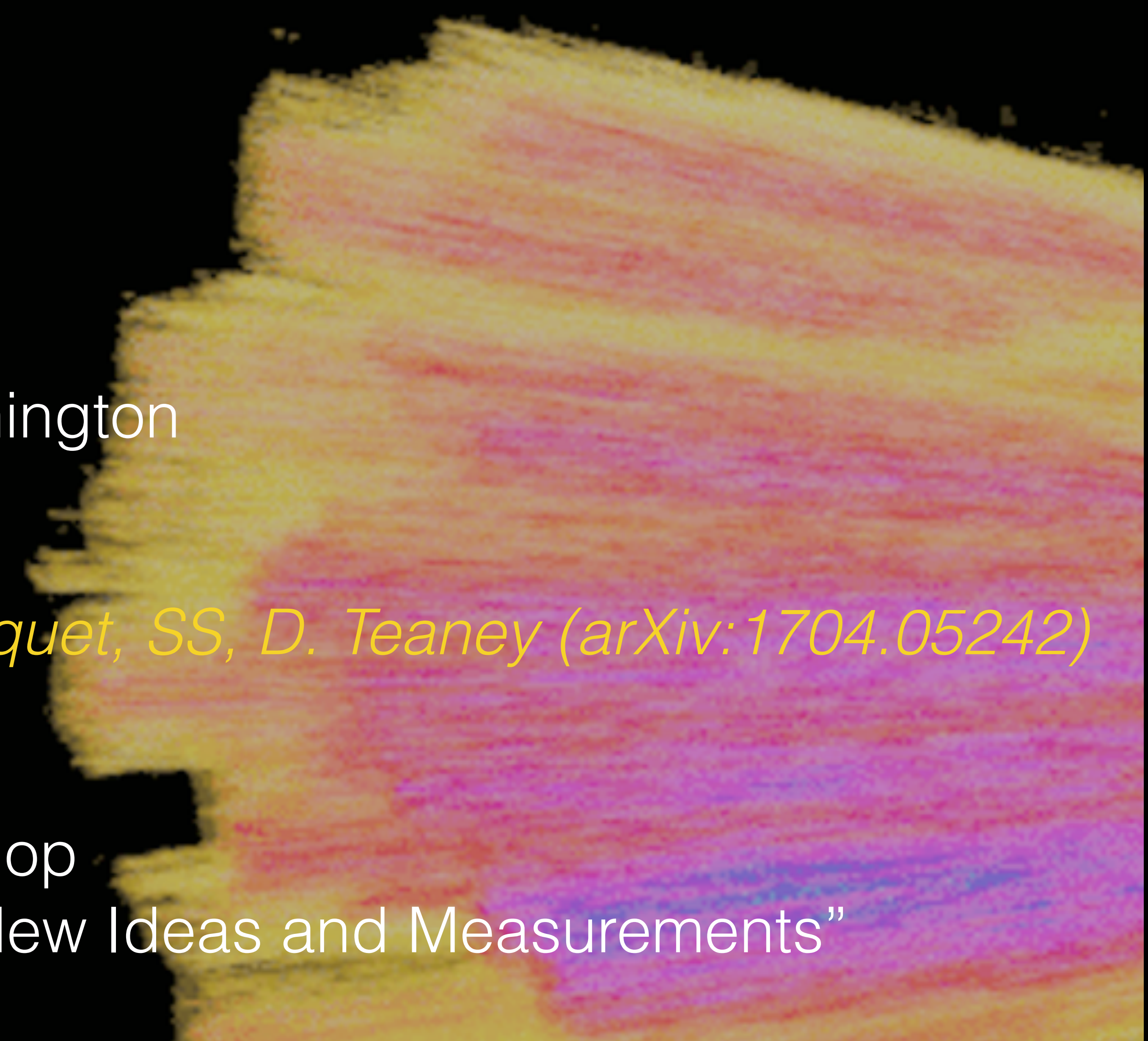
Event-by-event pre-equilibrium dynamics
— from gluon saturation towards the onset of hydrodynamics



Sören Schlichting | University of Washington

Based on
A. Kurkela, A. Mazeliauskas, J.-F. Paquet, SS, D. Teaney (arXiv:1704.05242)
(more in preparation)

RIKEN/BNL Research Center Workshop
"Saturation: Recent Developments, New Ideas and Measurements"
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UNIVERSITY *of* WASHINGTON

Outline

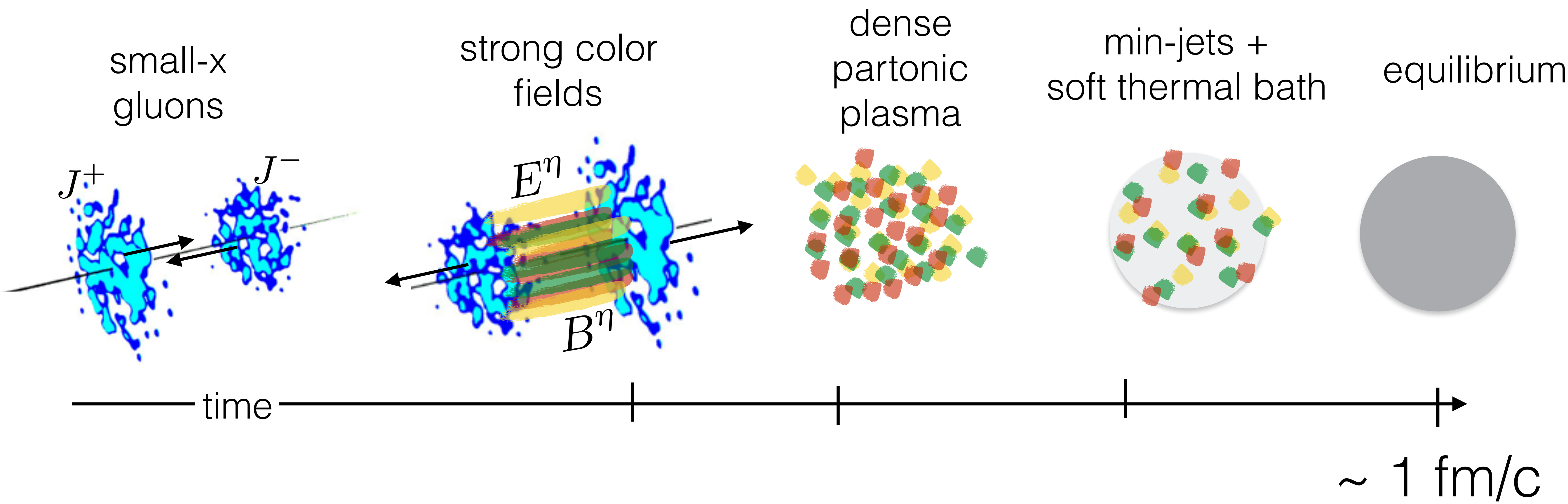
Microscopic early time dynamics & equilibration process

Macroscopic description of equilibration process

Event-by-event simulation of pre-equilibrium dynamics

Conclusions & Outlooks

Early time dynamics & equilibration process



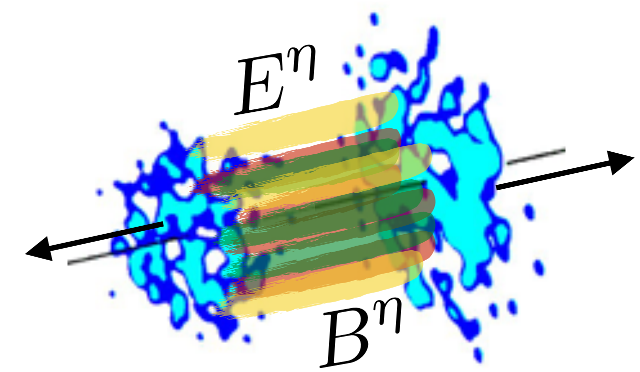
Starting from saturated nuclei before the collisions
sequence of processes that eventually leads to the formation
of an equilibrated QGP

Early time dynamics ($0<\tau<1/Q_s$)

Because of high phase-space density of gluons particle initial particle production and early time dynamics described in terms of classical field theory to leading order

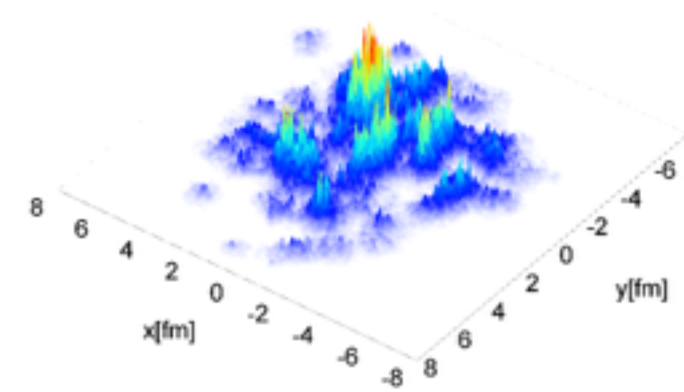
$$D_\mu F^{\mu\nu} = J^\nu$$

Strong boost invariant classical fields E^η, B^η created immediately after the collision



Decoherence of classical fields occurs on a time scale $\tau \sim 1/Q_s$ where quasi-particle description starts to become applicable

-> Basis for microscopic initial state calculations (IP-Glasma)



IP-Glasma

Challenge to understand subsequent equilibration process

- need quantum corrections (beyond NLO)
- effects of plasma instabilities

Equilibration process at weak coupling

Equilibration proceeds as three step process described by "bottom-up" scenario

Baier, Mueller, Schiff, Son PLB502 (2001) 51-58

Phase I: Evolution towards classical-attractor

quasi-particle description becomes applicable

Phase II: Mini-jets undergo a radiative break-up cascade

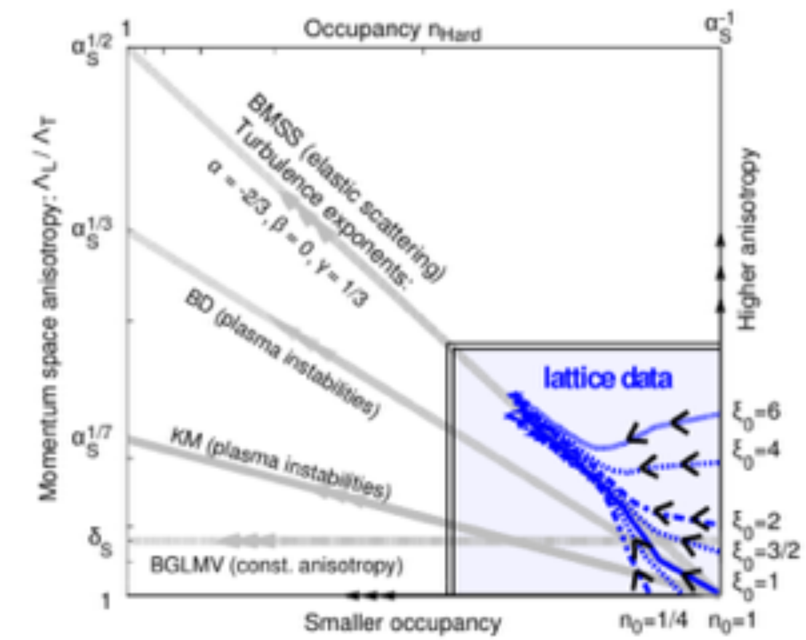
formation of soft thermal bath

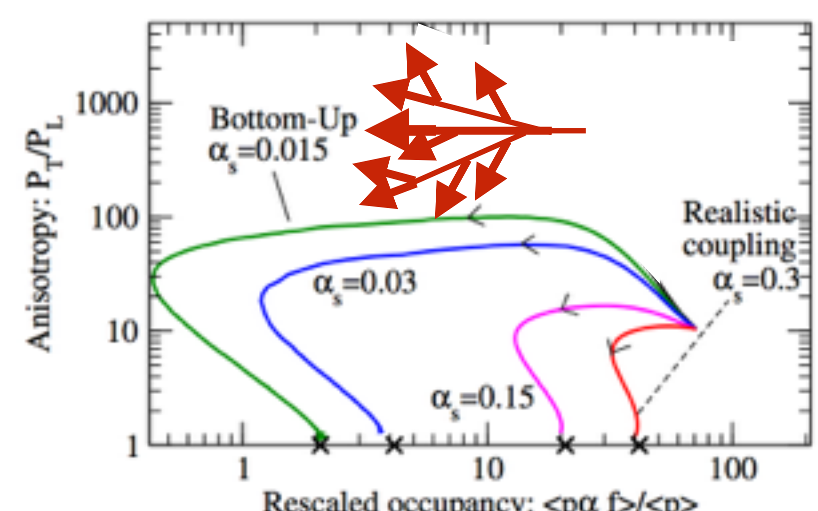
Phase III: Quenching of mini-jets in soft thermal bath

isotropization of plasma

Equilibration time determined by the time-scale for a mini-jet Q_s to loose all its energy

Berges,Boguslavski,SS, Venugopalan, PRD 89 (2014) no.7, 074011





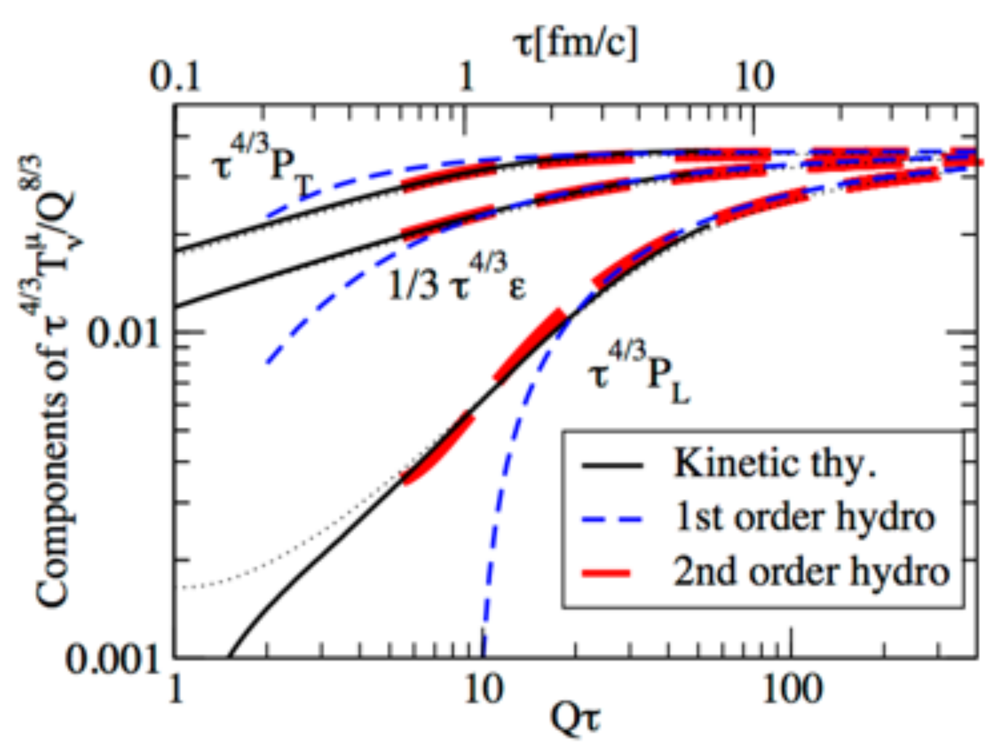
Kurkela, Zhu PRL 115 (2015) 182301

Hydrodynamic behavior

Extrapolations from weak-coupling limit to realistic values of α_s (~0.3) at RHIC & LHC energies yield results consistent with phenomenological estimates

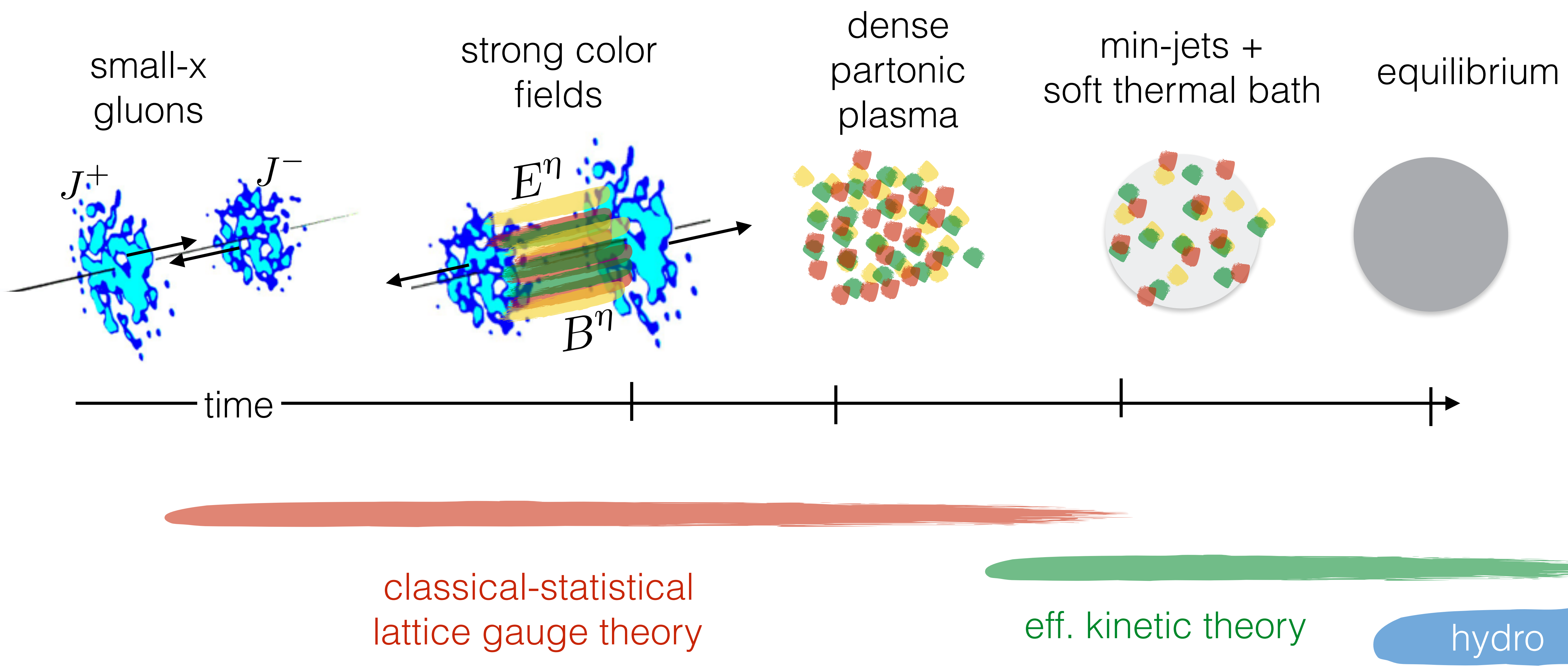
Viscous hydrodynamics applicable on time scales ~1 fm/c

Similar to strong coupling picture viscous hydrodynamics becomes applicable when pressure anisotropies are still O(1)



Kurkela, Zhu PRL 115 (2015) 182301

Early time dynamics & equilibration process



By combination of weak-coupling methods a complete description of early-time dynamics can be achieved

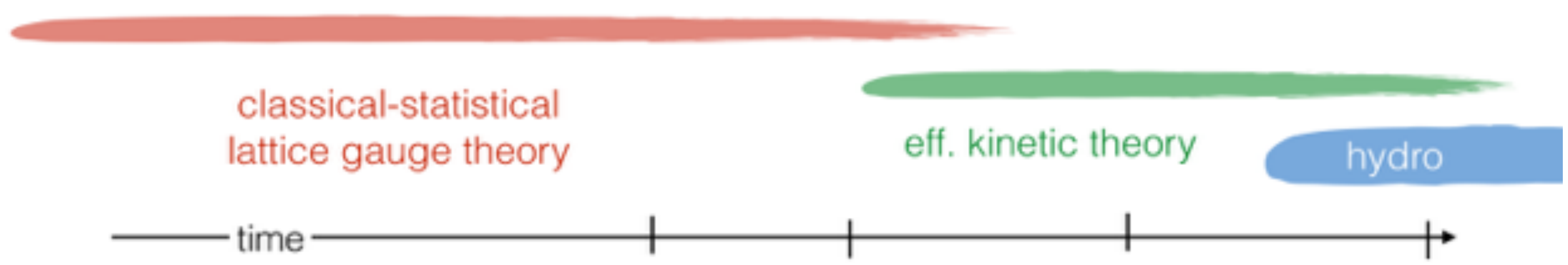
Event-by-event pre-equilibrium dynamics

Goal: Obtain event-by-event initial conditions including weakly coupled pre-equilibrium evolution

-> Eliminate uncertainties in extraction of QGP transport properties due to artificial time scale τ_{Hydro} when hydro simulation starts

Challenge: Different degrees of freedom relevant at different times

classical fields, quasi-particles, energy-momentum tensor



Brute force calculation extremely challenging (CYM f(x,p), 3+2+1D EKT)

Ultimately we are only interested in calculation of energy-momentum tensor

Exploit memory loss to use macroscopic degrees of freedom for description of pre-equilibrium dynamics

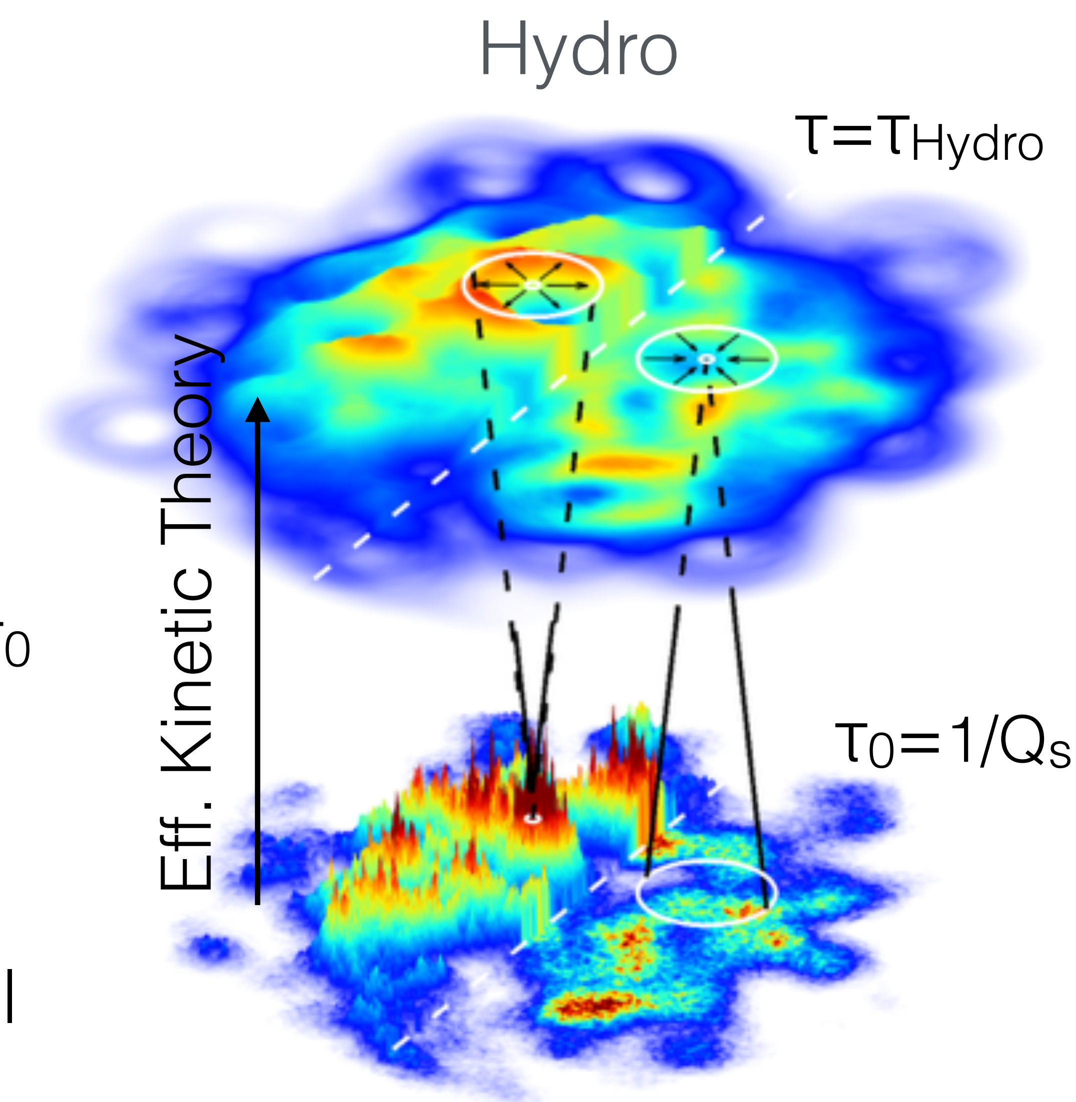
Macroscopic pre-equilibrium evolution

Extract energy-momentum tensor $T^{\mu\nu}(x)$ from classical statistical lattice simulation

Evolve $T^{\mu\nu}$ from initial time $\tau_0 \sim 1/Q_s$ to hydro initialization time τ_{Hydro} using eff. kinetic theory description

Causality restricts contributions to $T^{\mu\nu}(x)$ to be localized from causal disc $|x-x_0| < \tau_{Hydro}-\tau_0$ useful to decompose into a local average $T^{\mu\nu}_{BG}(x)$ and fluctuations $\delta T^{\mu\nu}(x)$

Since in practice size of causal disc is small $\tau_{Hydro}-\tau_0 << R_A$ fluctuations $\delta T^{\mu\nu}(x)$ around local average $T^{\mu\nu}_{BG}(x)$ are small and can be treated in a linearized fashion



Keegan,Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171
Kurkela, Mazeliauskas, Paquet, SS, Teaney (in preparation)

Macroscopic pre-equilibrium evolution

Effective kinetic description needs phase-space distribution $f(\tau, p, x)$

Memory loss: Details of initial phase-space distribution become irrelevant as system approaches local equilibrium

Can describe evolution of $T^{\mu\nu}$ in kinetic theory in terms of a representative phase-space distribution

$$f(\tau, p, x) = f_{BG}(Q_s(x)\tau, p/Q_s(x)) + \delta f(\tau, p, x)$$

where f_{BG} characterizes typical momentum space distribution, and δf can be chosen to represent local fluctuations of initial energy momentum tensor, e.g. energy density $\delta T^{\tau\tau}$ and momentum flow $\delta T^{\tau i}$

Energy perturbations:

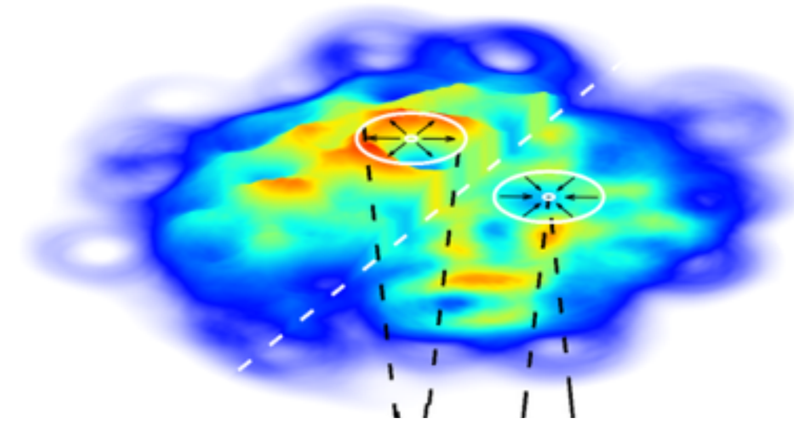
$$\delta f_s(\tau_0, p, x) \propto \frac{\delta T^{\tau\tau}(x)}{T^{\tau\tau}_{BG}(x)} \times \frac{\partial}{\partial Q_s(x)} f_{BG}\Big(\tau_0, p/Q_s(x)\Big)$$

local amplitude

representative form of phase-space distribution

Macroscopic pre-equilibrium evolution

Energy-momentum tensor on the hydro surface can be reconstructed directly from initial conditions according to

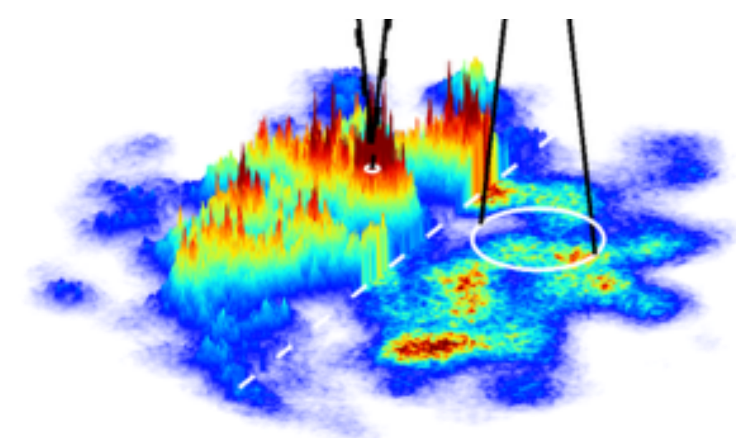


$$T^{\mu\nu}(\tau,x) = T^{\mu\nu}_{BG}\Big(Q_s(x)\tau\Big) + \int_{Disc} G^{\mu\nu}_{\alpha\beta}\Big(\tau,\tau_0,x,x_0,Q_s(x)\Big)\ \delta T^{\alpha\beta}(\tau_0,x_0)$$

non-equilibrium evolution of (local) average background

non-equilibrium Greens function of energy-momentum tensor

Effective kinetic theory simulations only need to be performed once to compute background evolution and Greens functions



Scaling variables

Background evolution and Greens functions still depend on variety of variables, $Q_s(x)$, α_s, …

-> Identify appropriate scaling variables to reduce complexity

Since ultimately evolution will match onto visc. hydrodynamics, check wether hydrodynamics admits scaling solution

1st order hydro: $$T^{\tau\tau}(\tau) = T^{\tau\tau}_{Ideal}(\tau)\Big(1 - \frac{8}{3}\frac{\eta/s}{T_{eff}\tau} + ...\Big)$$

where $T^{\tau\tau}_{Ideal}(\tau)$ is the Bjorken energy density and $T_{eff} = \tau^{-1/3} \lim_{\tau\to\infty} T(\tau)\tau^{1/3}$

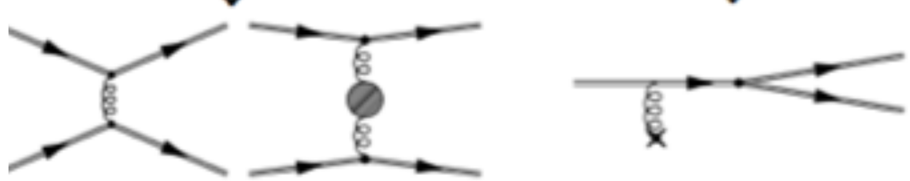
Natural candidate for scaling variable is $x_s = T_{eff}\tau/(\eta/s)$

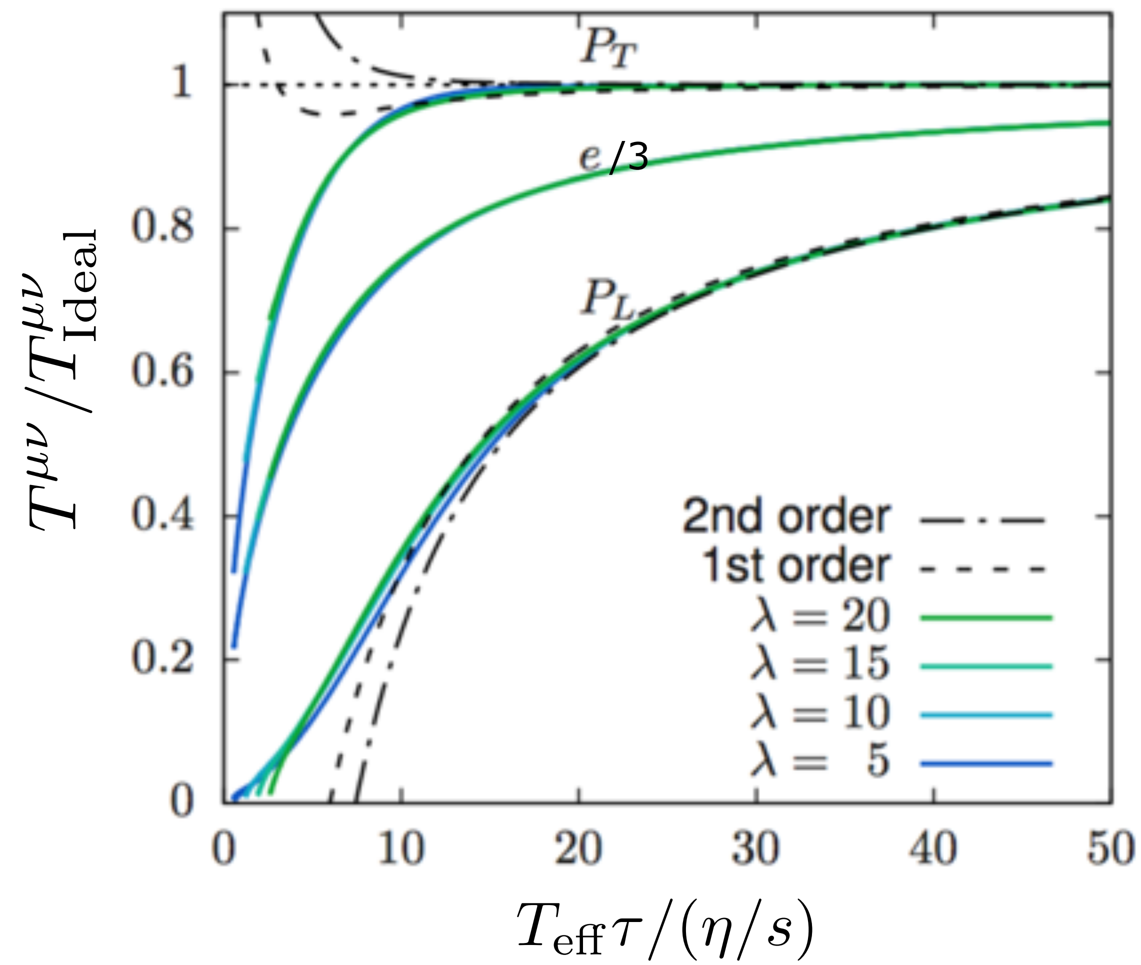
Background — Scaling variables

Numerical simulation of background evolution in effective kinetic theory

$$\left(\partial_\tau - \frac{p_z}{\tau}\right) f(\tau, |\mathbf{p}_\perp|, p_z) = \mathcal{C}[f]$$

$$\mathcal{C}[f] = \underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]} + \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]}$$





Non-equilibrium evolution of background $T^{\mu\nu}$ is a unique function of x_s

Scaling property extends beyond hydrodynamic regime in the relevant range of (large) couplings

Greens functions

Greens functions describe evolution of energy/momentum perturbations on top of a (locally) homogenous boost-invariant background

-> Description of perturbations in Fourier space

Decomposition in a complete basis of tensors leaves a total of 10 independent functions, e.g. for energy perturbations

energy response

$$\tilde{G}^{\tau\tau}_{\tau\tau}(\tau,\tau_0,\mathbf{k}) = \tilde{G}^{s}_{s}(\tau,\tau_0,|\mathbf{k}|)\ ,$$

momentum response

$$\tilde{G}^{\tau i}_{\tau\tau}(\tau,\tau_0,\mathbf{k}) = \frac{\mathbf{k}^i}{|\mathbf{k}|}\tilde{G}^{v}_{s}(\tau,\tau_0,|\mathbf{k}|)\ ,$$

shear stress response

$$\tilde{G}^{ij}_{\tau\tau}(\tau,\tau_0,\mathbf{k}) = \tilde{G}^{t,\delta}_{s}(\tau,\tau_0,|\mathbf{k}|)\ \delta^{ij} + \tilde{G}^{t,k}_{s}(\tau,\tau_0,|\mathbf{k}|)\ \frac{\mathbf{k}^i\mathbf{k}^j}{|\mathbf{k}|^2}\ ;$$

Numerically computed in eff. kinetic theory by solving linearized Boltzmann equation on top of non-equilibrium background

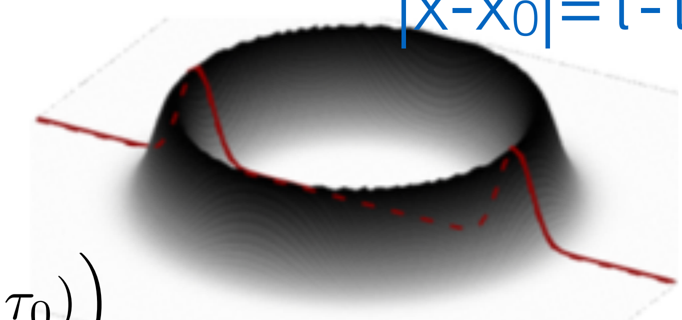
$$\left(\partial_\tau + \frac{i\mathbf{p}_\perp\mathbf{k}_\perp}{p} - \frac{p_z}{\tau}\right)\ \widetilde{\delta f}(\tau,|\mathbf{p}_\perp|,p_z;\ \mathbf{k}_\perp) = \delta\mathcal{C}[f,\widetilde{\delta f}]$$

and computing appropriate moments of δf

Greens functions

Free-streaming:

coordinate space



Energy-momentum perturbations propagate as a concentric wave traveling at the speed of light

energy/momentum response:

$$G_s^{s/v}(\tau,\tau_0,\mathbf{x}-\mathbf{x}_0) = \frac{1}{2\pi(\tau-\tau_0)}\delta\Big(|\mathbf{x}-\mathbf{x}_0| - (\tau-\tau_0)\Big)$$

Hydrodynamic response functions in the limit of small wave-number k $(\tau-\tau_0) << 1$ and large times $x_s >> 1$:

(c.f. Vredevoogd,Pratt PRC79 (2009) 044915, Keegan,Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171)

energy response: $$\tilde{G}_s^s(\tau,\tau_0,k) = \tilde{G}_s^s(\tau,\tau_0,k=0)\left(1-\frac{1}{2}k^2(\tau-\tau_0)^2\,\tilde{s}_s^{(2)}+...\right),$$

momentum response: $$\tilde{G}_s^v(\tau,\tau_0,k) = \tilde{G}_s^s(\tau,\tau_0,k=0)\left(-ik(\tau-\tau_0)\,\tilde{s}_v^{(1)}+...\right),$$

shear response: determined by hydrodynamic constitutive relations

$$\tilde{G}_s^s(\tau,\tau_0,k=0) = \left(\frac{T^{\tau\tau}(\tau_0)}{T^{\tau\tau}(\tau)}\right)\left(\frac{3T^{\tau\tau}(\tau)-T^\eta_\eta(\tau)}{3T^{\tau\tau}(\tau_0)-T^\eta_\eta(\tau_0)}\right)$$

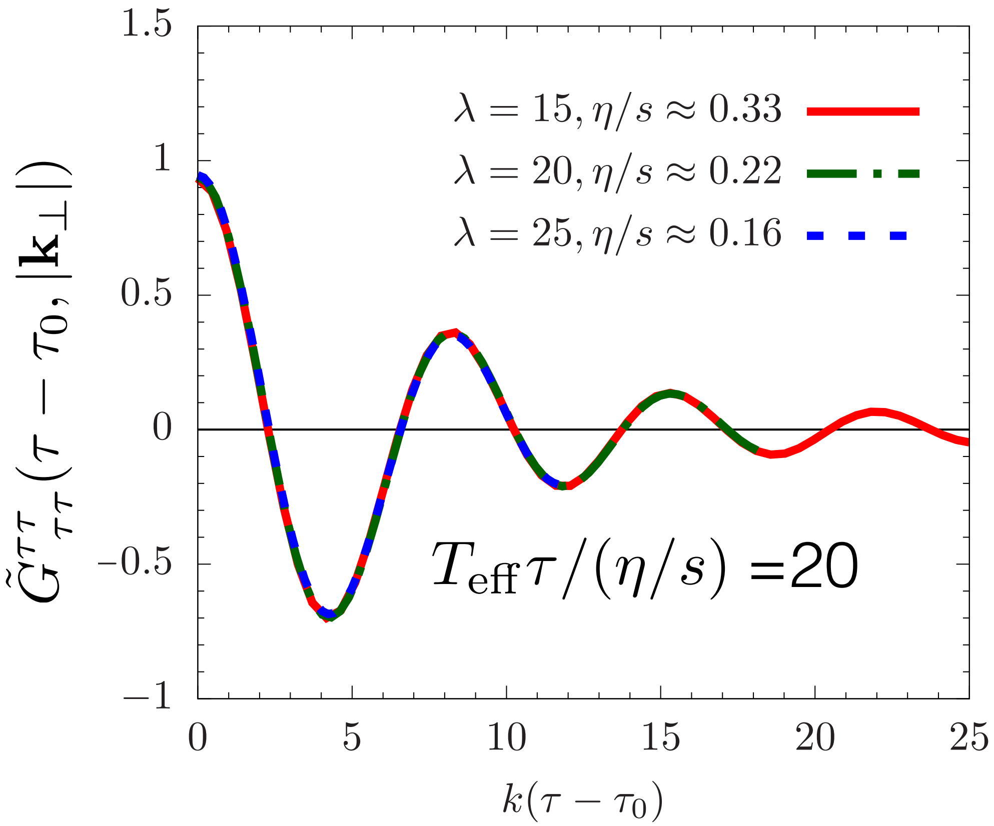
background evolution

$$\tilde{s}_v^{(1)} = \frac{1}{2} \qquad \tilde{s}_s^{(2)} = \frac{1}{2}\left(1+\frac{2}{3x_s}\right)$$

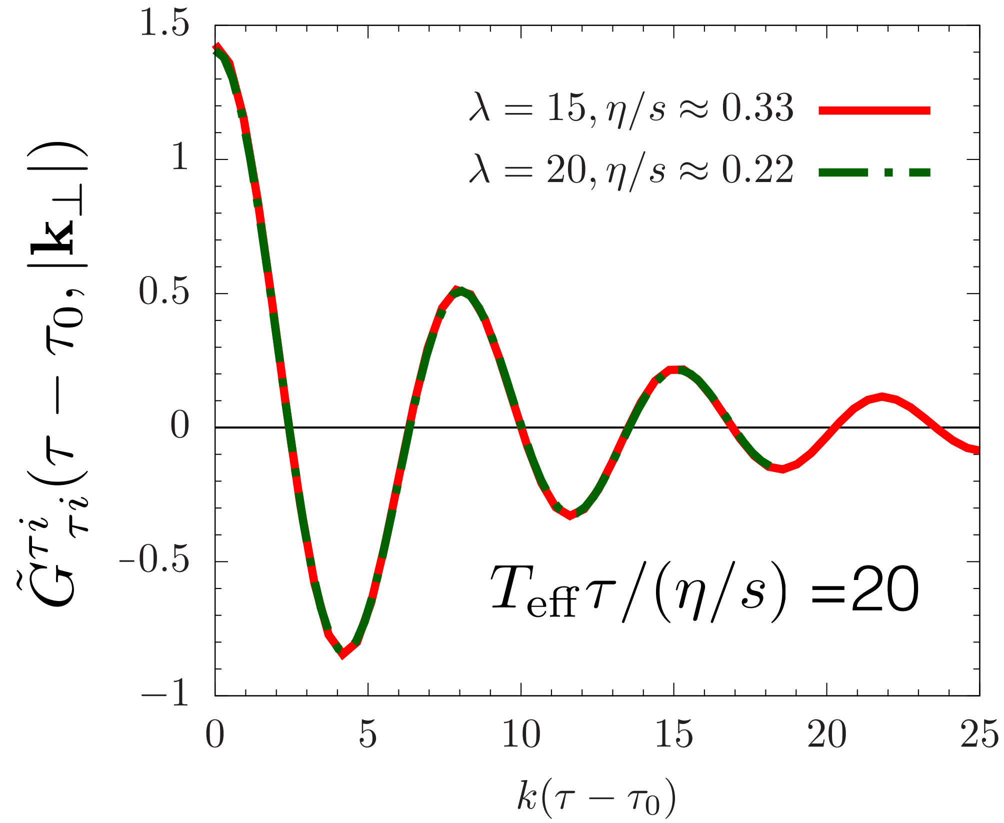
"long wave-length constants"

Greens functions — Scaling variables

Energy response
to energy perturbation



Momentum response
to momentum perturbations



Non-equilibrium Greens functions show universal scaling
in $x_s = T_{eff}\tau/(\eta/s)$ and $k(\tau - \tau_0)$ beyond hydro limit

Satisfy hydrodynamic constitutive relations for sufficiently large
times $x_s >> 1$ and long wave-length $k\,(\tau-\tau_0) << 1$

Scaling variables

Scaling properties ensure that pre-equilibrium evolution of energy momentum tensor can be expressed in terms of

Background: $T^{\mu\nu}_{BG}(x_s)$ Greens-functions: $G^{\mu\nu}_{\alpha\beta}\left(x_s, \frac{x-x_0}{\tau-\tau_0}\right)$

which are computed once and for all in numerical kinetic theory simulation

Since dependence of coupling constant α_s has been re-expressed in terms of physical parameter η/s, can now perform event-by-event simulations for variety of physical parameters

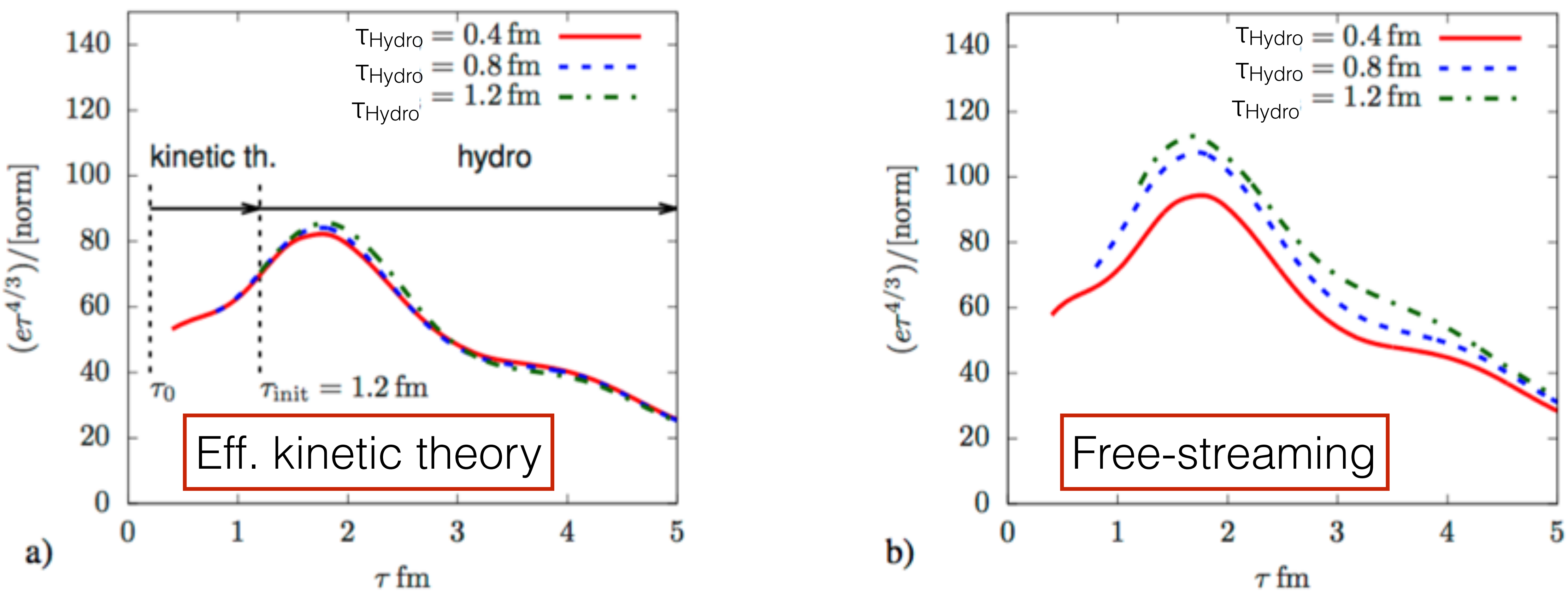
Event-by-event pre-equilibrium evolution

1) Evolve IP-Glasma initial conditions to early time τ_0 = 0.2 fm/c

2) Macroscopic pre-equilibrium evolution to hydro initialization time τ_{Hydro}

3) Hydrodynamic evolution from τ_{Hydro} ($\eta/s = 2/(4\pi)$ | conformal EoS)

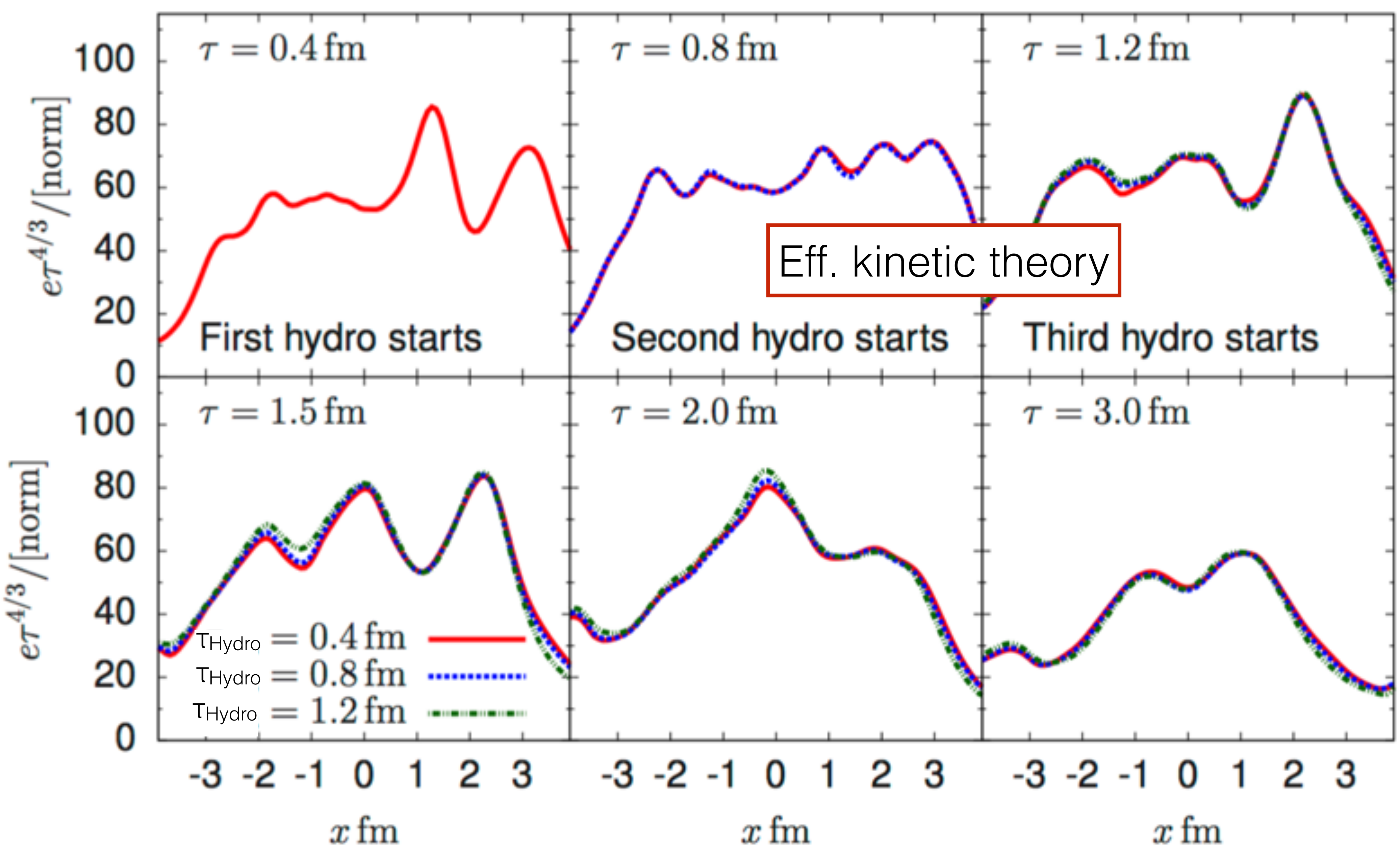
Energy density in the central region of Pb+Pb collision



Consistent description of pre-equilibrium dynamics ensures smooth transition to hydrodynamics at times $\tau > \tau_{Hydro}$

Event-by-event pre-equilibrium evolution

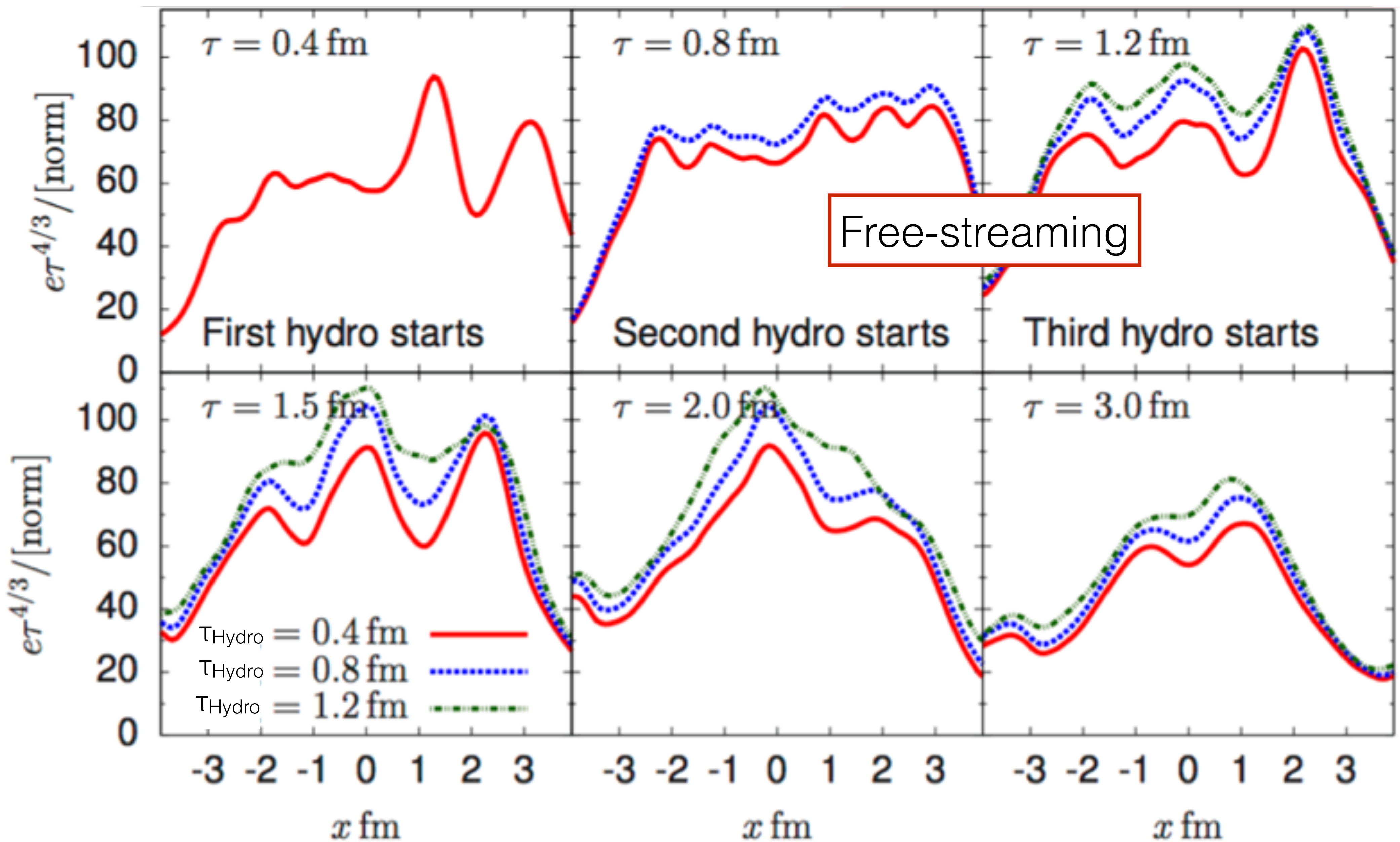
Energy density profile in Pb+Pb collision



No sensitivity to switching time τ_{Hydro} from pre-equilibrium to hydro

Event-by-event pre-equilibrium evolution

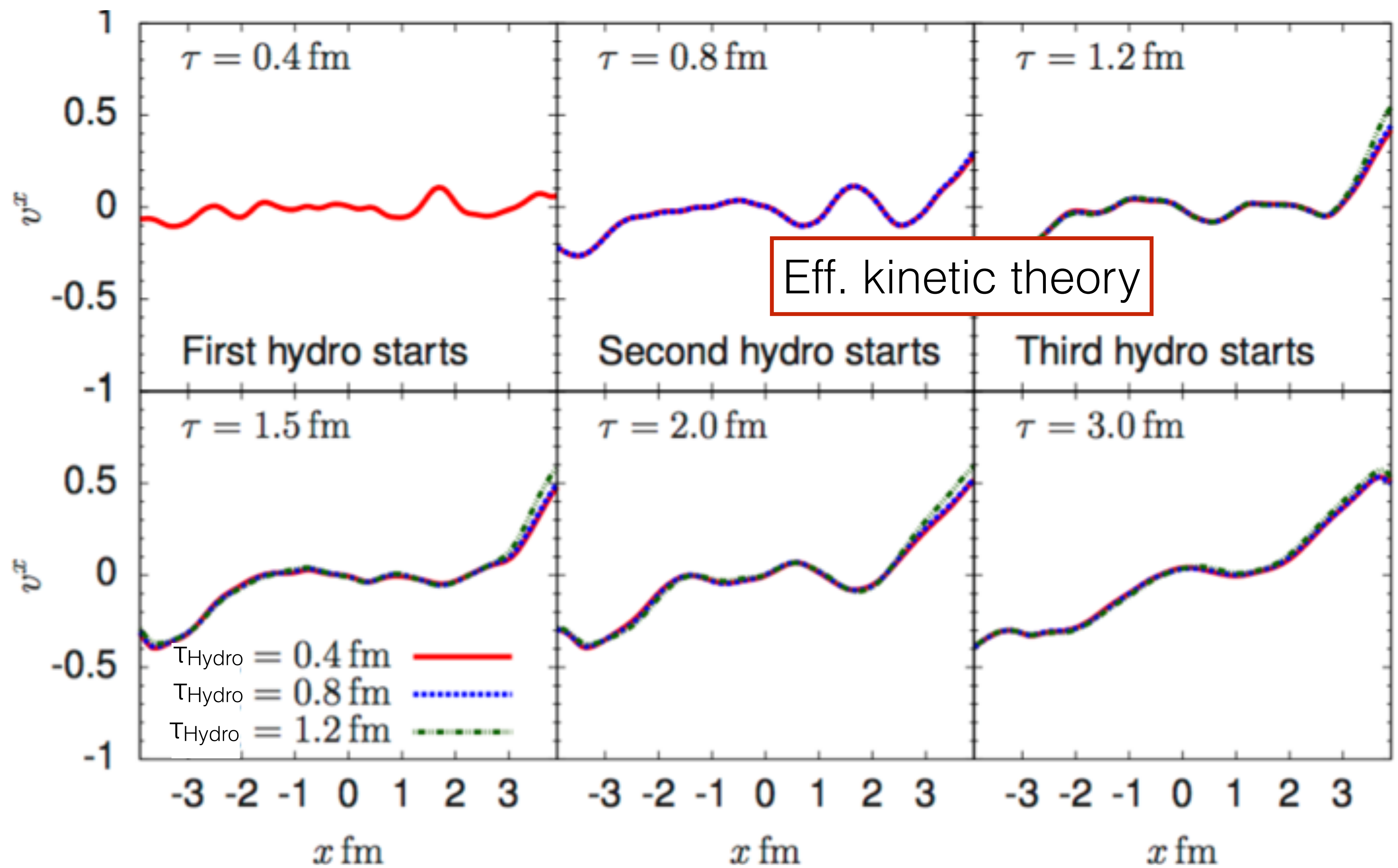
Energy density profile in Pb+Pb collision



Non-trivial result as free-streaming evolution yields similar profiles but incorrect normalization

Event-by-event pre-equilibrium evolution

Pre-equilibrium flow profile in Pb+Pb collision:



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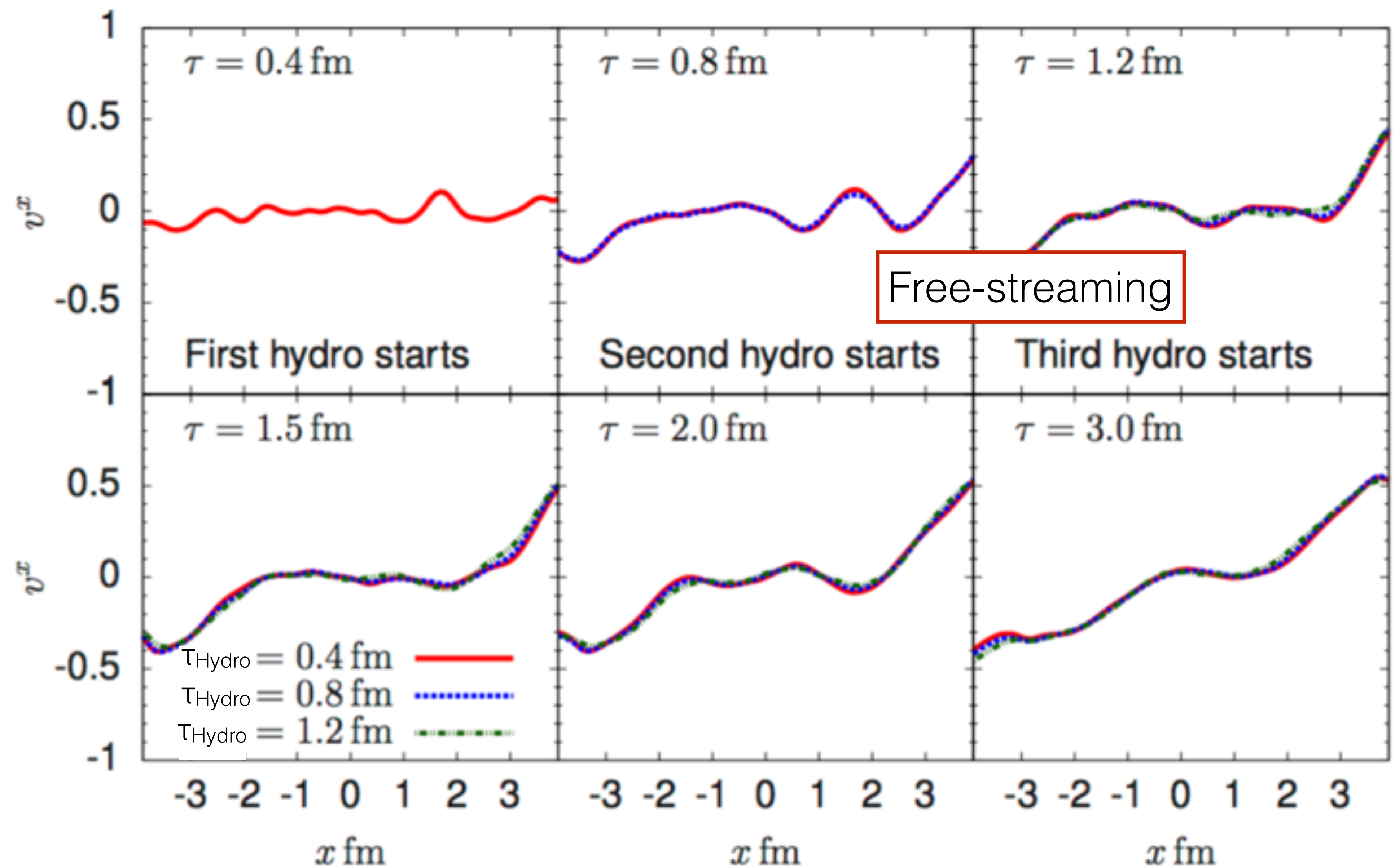
Event-by-event pre-equilibrium evolution

Pre-equilibrium flow profile in Pb+Pb collision:

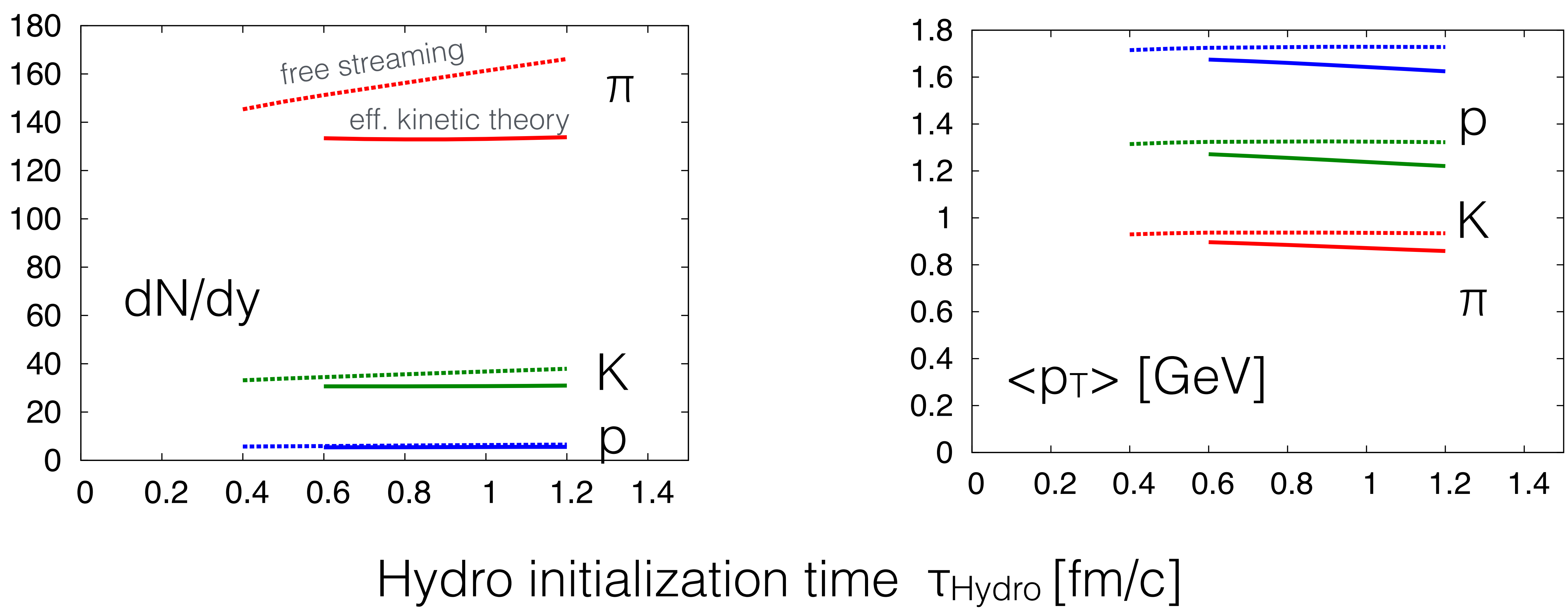


Universality of long wave-length response leads to very similar results for free-streaming and effective kinetic theory

(c.f. Vredevoogd,Pratt PRC79 (2009) 044915, Keegan,Kurkela, Mazeliauskas, Teaney JHEP 1608 (2016) 171)

Event-by-event pre-equilibrium evolution

Hadronic observables in single Pb+Pb event:



Very little to no sensitivity to switching time τ_{Hydro} from pre-equilibrium to hydro for dN/dy, $\langle p_T \rangle$, $\langle v_2 \rangle$, …

Conclusions & Outlook

Significant progress in understanding early time dynamics of heavy-ion collisions from weak-coupling perspective

Development of macroscopic description which enables event-by-event description and can be used in phenomenological modeling of heavy-ion collisions

Description in macroscopic framework is completely general and can be used beyond weak coupling limit

-> Direct comparisons with strong coupling limit should be possible

Several interesting directions beyond bulk phenomenology are starting to be explored

topological transitions, quark production & anomalous transport, photon production