# Higher-twist corrections to gluon TMD factorization 

I. Balitsky (in collaboration with A. Tarasov)

JLAB \& ODU

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## Particle production by gluon fusion in $p p$ scattering

Suppose we produce a scalar particle (e.g. Higgs) in a gluon-gluon fusion.

For simplicity, assume the vertex is local:

$$
\mathcal{L}_{\Phi}=g_{\Phi} \int d z \Phi(z) F^{2}(z), \quad F^{2} \equiv F_{\mu \nu}^{a} F_{a}^{\mu \nu}
$$



## Matrix element between hadron states $\Rightarrow \sum_{X}=1$

"Hadronic tensor"

$$
\begin{aligned}
W\left(p_{A}, p_{B}, q\right) & \stackrel{\text { def }}{=} \sum_{X} \int d^{4} x e^{-i q x}\left\langle p_{A}, p_{B}\right| F^{2}(x)|X\rangle\langle X| F^{2}(0)\left|p_{A}, p_{B}\right\rangle \\
& =\int d^{4} x e^{-i q x}\left\langle p_{A}, p_{B}\right| F^{2}(x) F^{2}(0)\left|p_{A}, p_{B}\right\rangle
\end{aligned}
$$

Double functional integral for $W$

$$
\begin{aligned}
& W\left(p_{A}, p_{B}, q\right)=\sum_{X} \int d^{4} x e^{-i q x}\left\langle p_{A}, p_{B}\right| F^{2}(x)|X\rangle\langle X| F^{2}(0)\left|p_{A}, p_{B}\right\rangle \\
& =\lim _{t_{i} \rightarrow-\infty}^{t_{f} \rightarrow \infty} \int d^{4} x e^{-i q x} \int^{\tilde{A}\left(t_{f}\right)=A\left(t_{f}\right)} D \tilde{A}_{\mu} D A_{\mu} \int^{\tilde{\psi}\left(t_{f}\right)=\psi\left(t_{f}\right)} D \tilde{\bar{\psi}} D \tilde{\psi} D \bar{\psi} D \psi \Psi_{p_{A}}^{*}\left(\overrightarrow{\tilde{A}}\left(t_{i}\right), \tilde{\psi}\left(t_{i}\right)\right) \\
& \times \Psi_{p_{B}}^{*}\left(\overrightarrow{\tilde{A}}\left(t_{i}\right), \tilde{\psi}\left(t_{i}\right)\right) e^{-i S_{\mathrm{CCD}}(\tilde{A}, \tilde{\psi})} e^{i S_{\mathrm{QCD}}(A, \psi)} \tilde{F}^{2}(x) F^{2}(y) \Psi_{p_{A}}\left(\vec{A}\left(t_{i}\right), \psi\left(t_{i}\right)\right) \Psi_{p_{B}}\left(\vec{A}\left(t_{i}\right), \psi\left(t_{i}\right)\right)
\end{aligned}
$$

"Left" $A, \psi$ fields correspond to the amplitude $\langle X| F^{2}(0)\left|p_{A}, p_{B}\right\rangle$, "right" fields $\tilde{A}, \tilde{\psi}$ correspond to amplitude $\left\langle p_{A}, p_{B}\right| F^{2}(x)|X\rangle$
The boundary conditions $\tilde{A}\left(t_{f}\right)=A\left(t_{f}\right)$ and $\tilde{\psi}\left(t_{f}\right)=\psi\left(t_{f}\right)$ reflect the sum over intermediate states $X$.

## Rapidity factorization for particle production

Sudakov variables:

$$
\begin{array}{ll}
p=\alpha p_{1}+\beta p_{2}+p_{\perp}, & p_{1} \simeq p_{A}, \quad p_{2} \simeq p_{B}, \quad p_{1}^{2}=p_{2}^{2}=0 \\
x_{*} \equiv p_{2} \cdot x=\sqrt{\frac{s}{2}} x^{+}, & x_{\bullet} \equiv p_{1} \cdot x=\sqrt{\frac{s}{2}} x^{-}
\end{array}
$$


"Projectile" fields: $|\beta|<b$
"Central" fields
"Target" fields: $|\alpha|<a$

We integrate over "central" fields in the background of projectile and target fields.

## Rapidity factorization for particle production

After integration over $C$ fields

$$
\begin{aligned}
& W\left(p_{A}, p_{B}, q\right) \\
& =\int d^{4} x e^{-i q x} \int^{\tilde{A}\left(t_{f}\right)=A\left(t_{f}\right)} D \tilde{A}_{\mu} D A_{\mu} \int^{\tilde{\psi}_{a}\left(t_{f}\right)=\psi_{a}\left(t_{f}\right)} D \bar{\psi}_{a} D \psi_{a} D \tilde{\bar{\psi}}_{a} D \tilde{\psi}_{a} \\
& \quad \times e^{-i S_{\mathrm{CCD}}\left(\tilde{A}, \tilde{\psi}_{a}\right)} e^{i S_{\mathrm{QCD}}\left(A, \psi_{a}\right)} \Psi_{p_{A}}^{*}\left(\overrightarrow{\tilde{A}}\left(t_{i}\right), \tilde{\psi}_{a}\left(t_{i}\right)\right) \Psi_{p_{A}}\left(\vec{A}\left(t_{i}\right), \psi\left(t_{i}\right)\right) \\
& \quad \times \int^{\tilde{B}\left(t_{f}\right)=B\left(t_{f}\right)} D \tilde{B}_{\mu} D B_{\mu} \int^{\tilde{\psi}_{b}\left(t_{f}\right)=\psi_{b}\left(t_{f}\right)} D \bar{\psi}_{b} D \psi_{b} D \tilde{\bar{\psi}}_{b} D \tilde{\psi}_{b} \\
& \quad \times e^{-i S_{\mathrm{CCD}}\left(\tilde{B}, \tilde{\psi}_{b}\right)} e^{i S_{\mathrm{QCD}}\left(B, \psi_{b}\right)} \Psi_{p_{B}}^{*}\left(\overrightarrow{\tilde{B}}\left(t_{i}\right), \tilde{\psi}_{b}\left(t_{i}\right)\right) \Psi_{p_{B}}\left(\vec{B}\left(t_{i}\right), \psi_{b}\left(t_{i}\right)\right) \\
& \quad \times e^{S_{\mathrm{eff}(U, V, \tilde{U}, \tilde{V})}^{\mathcal{O}}\left(q, x, y ; A, \psi_{a}, \tilde{A}, \tilde{\psi}_{a} ; B, \psi_{b}, \tilde{B}, \tilde{\psi}_{b}\right)}
\end{aligned}
$$

$\mathcal{O}$ - sum of the connected diagrams for $F^{2}(x) F^{2}(0)$ in the background fields
$S_{\text {eff }}$ - effective action (sum of disconnected diagrams $=e^{S_{\text {eff }}}$.

## Approximations for projectile and target fields

At the tree level $\beta=0$ for $A, \tilde{A}$ fields and $\alpha=0$ for $B, \tilde{B}$ fields $\Leftrightarrow$ $A=A\left(x_{\bullet}, x_{\perp}\right), \tilde{A}=\tilde{A}\left(x_{\bullet}, x_{\perp}\right)$ and $B=B\left(x_{*}, x_{\perp}\right), \tilde{B}=\tilde{B}\left(x_{\bullet}, x_{\perp}\right)$.

NB: because of boundary conditions $\tilde{A}\left(t_{f}\right)=A\left(t_{f}\right)$ and $\tilde{\psi}\left(t_{f}\right)=\psi\left(t_{f}\right)$ for the purpose of calculating the integral over central fields one can set

$$
\begin{aligned}
& A\left(x_{\bullet}, x_{\perp}\right)=\tilde{A}\left(x_{\bullet}, x_{\perp}\right), \\
& \text { and } \\
& B\left(\psi_{*}\left(x_{\bullet}, x_{\perp}\right)=\tilde{\psi}_{\perp}\left(x_{\bullet}, x_{\perp}\right)\right. \\
& \tilde{B}\left(x_{*}, x_{\perp}\right), \\
& \psi_{b}\left(x_{*}, x_{\perp}\right)=\tilde{\psi}_{b}\left(x_{*}, x_{\perp}\right)
\end{aligned}
$$

The fields $A, \psi$ and $\tilde{A}, \tilde{\psi}$ do not depend on $x_{*} \Rightarrow$ if they coincide at $x_{*}=\infty \Rightarrow$ they coincide everywhere.
Similarly,
$B, \psi_{b}$ and $\tilde{B}, \tilde{\psi}_{b}$ do not depend on $x_{\bullet} \Rightarrow$
if they coincide at $x_{\bullet}=\infty$ they should be equal.

## $F_{\mu \nu}^{2}(C)$ in the tree approximation


$F_{\mu \nu}(C)=$ sum of tree diagrams in external $A$ and $B$ fields

## $F_{\mu \nu}(C)$ in the tree approximation


$F_{\mu \nu}(C)=$ sum of tree diagrams in external $\tilde{A}, A$ and $\tilde{B}, B$ fields with sources $\tilde{J}_{\mu}=D^{\mu} F_{\mu \nu}(\tilde{A}+\tilde{B})$ and $J_{\mu}=D^{\mu} F_{\mu \nu}(A+B)$

Since $\tilde{A}=A$ and $\tilde{B}=B$ the sources and background fields are the same to the left and to the right of the cut
$\Rightarrow$
$F_{\mu \nu}(C)$ is a sum of diagrams with retarded Green functions
(F. Gelis, R. Venugopalan)

## $F_{\mu \nu}(C)$ in the tree approximation

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## Classical solution

The sum of diagrams with retarded Green functions $\Leftrightarrow$ solution of classical YM equations

$$
D^{\nu} F_{\mu \nu}^{a}=\sum_{f} g \bar{\psi}^{f} t^{a} \gamma_{\mu} \psi^{f}, \quad\left(\not P+m_{f}\right) \psi^{f}=0
$$

with boundary conditions

$$
\begin{array}{ll}
A_{\mu}(x) \stackrel{x_{*} \rightarrow-\infty}{=} \bar{A}_{\mu}\left(x_{\bullet}, x_{\perp}\right), & \psi(x) \stackrel{x_{*} \rightarrow-\infty}{=} \psi_{a}\left(x_{\bullet}, x_{\perp}\right) \\
A_{\mu}(x) \stackrel{x_{\bullet} \rightarrow-\infty}{=} \bar{B}_{\mu}\left(x_{*}, x_{\perp}\right), & \psi(x) \stackrel{x_{\bullet} \rightarrow-\infty}{=} \psi_{b}\left(x_{*}, x_{\perp}\right)
\end{array}
$$

following from $C_{\mu}, \psi_{c} \xrightarrow{t \rightarrow-\infty} 0$.
The projectile and target fields satisfy YM equations

$$
\begin{aligned}
& D^{\nu} F_{\mu \nu}^{a}=\sum_{f} g \bar{\psi}_{a}^{f} t^{a} \gamma_{\mu} \psi_{a}^{f}, \quad\left(\not P+m_{f}\right) \psi_{a}^{f}=0 \\
& D^{\nu} F_{\mu \nu}^{a}=\sum_{f} g \bar{\psi}_{b}^{f} t^{a} \gamma_{\mu} \psi_{b}^{f}, \quad\left(\not P+m_{f}\right) \psi_{b}^{f}=0
\end{aligned}
$$

Method of solution: start with $\bar{A}_{\mu}+\bar{B}_{\mu}$ and correct by computing Feynman diagrams (with retarded propagators) with a source $J_{\nu}=D^{\mu} F^{\mu \nu}(U+V)$

## Power counting for background fields

To get the relative strength of Lorentz components of background $A\left(x_{\bullet}, x_{\perp}\right)$ and $B\left(x_{*}, x_{\perp}\right)$ fields, we compare the typical term in the leading contribution to possible higher-twist corrections Leading-twist operator

$$
\begin{aligned}
& \left\langle p_{A}\right| U_{*}^{m i}\left(x_{\bullet}, x_{\perp}\right) U_{*}^{m j}(0)\left|p_{A}\right\rangle \sim s^{2}\left(g_{i j}^{\perp}+\frac{x_{i}^{\perp} x_{j}^{\perp}}{x_{\perp}^{2}}\right) \\
& U_{* i}^{a}\left(z_{\bullet}, z_{\perp}\right) \equiv\left(\left[-\infty \bullet, z_{\bullet}\right]_{z_{*}}^{\bar{A}_{*}}\right)^{a b} \bar{A}_{* i}^{b}\left(z_{\bullet}, z_{\perp}\right)
\end{aligned}
$$

Higher-twist operator

$$
\begin{aligned}
& d^{m n l}\left\langle p_{A}\right| U_{*}^{m i}\left(x_{\bullet}, x_{\perp}\right) U_{*}^{n k}\left(x_{\bullet}^{\prime}, x_{\perp}\right) U_{*}^{l j}(0)\left|p_{A}\right\rangle \\
& \sim s^{3}\left(g_{i j}^{\perp} \frac{x^{k}}{x_{\perp}^{2}}+g_{i k}^{\perp} \frac{x^{j}}{x_{\perp}^{2}}+g_{j k}^{\perp} \frac{x^{i}}{x_{\perp}^{2}}+\frac{x_{i}^{\perp} x_{j}^{\perp} x^{k}}{x_{\perp}^{2}}\right)
\end{aligned}
$$

$\Rightarrow U_{* k} \sim s \frac{x_{k}}{x_{\perp}^{2}} \sim s m_{\perp}$. Similarly, $V_{\bullet k} \sim s \frac{x_{k}}{x_{\perp}^{2}} \sim s m_{\perp}$
(For the purpose of power counting $m_{\perp} \sim q_{\perp}, m_{N}$ )

In this way we get the relative strength of the background gluon fields in projectile and target:

$$
\begin{aligned}
& A_{*}\left(x_{\bullet}, x_{\perp}\right) \sim s, \quad A_{\bullet}\left(x_{\bullet}, x_{\perp}\right) \sim m_{\perp}^{2}, \quad A_{i}\left(x_{\bullet}, x_{\perp}\right) \sim m_{\perp}, \\
& B_{*}\left(x_{*}, x_{\perp}\right) \sim m_{\perp}^{2}, \quad B_{\bullet}\left(x_{*}, x_{\perp}\right) \sim s, \quad B_{i}\left(x_{*}, x_{\perp}\right) \sim m_{\perp}
\end{aligned}
$$

for the gluon fields and

$$
\begin{aligned}
& \hat{p}_{1} \psi_{a}\left(x_{\bullet}, x_{\perp}\right) \sim m_{\perp}^{5 / 2}, \quad \gamma_{i} \psi_{a}\left(x_{\bullet}, x_{\perp}\right) \sim m_{\perp}^{3 / 2}, \quad \hat{p}_{2} \psi_{a}\left(x_{\bullet}, x_{\perp}\right) \sim s \sqrt{m_{\perp}}, \\
& \hat{p}_{1} \psi_{b}\left(x_{\bullet}, x_{\perp}\right) \sim s \sqrt{m_{\perp}}, \quad \gamma_{i} \psi_{b}\left(x_{\bullet}, x_{\perp}\right) \sim m_{\perp}^{3 / 2}, \quad \hat{p}_{2} \psi_{b}\left(x_{\bullet}, x_{\perp}\right) \sim m_{\perp}^{5 / 2}
\end{aligned}
$$

for quark fields.
We see that there large background fields $A_{*}, B_{\bullet} \sim s$ so the linear term $J_{\nu} \sim D^{\mu} F_{\mu \nu}(A+B) \sim s^{2} \Rightarrow$ the expansion in powers of $J_{\nu}$ is bad.

## Way out: gauge transformation

Gauge transformation with a matrix $\Omega(x)$ such that

$$
\Omega\left(x_{*}, x_{\bullet}, x_{\perp}\right) \xrightarrow{x_{*} \rightarrow-\infty}\left[x_{\bullet},-\infty_{\bullet}\right]_{x}^{\bar{A}_{*}}, \quad \Omega\left(x_{*}, x_{\bullet}, x_{\perp}\right) \xrightarrow{x_{\bullet} \rightarrow-\infty}\left[x_{*},-\infty_{*}\right]_{x}^{\bar{B}_{\bullet}}
$$

The matrix $\Omega$ is obtained in a following way:

$$
\mathcal{A}_{\mu}\left(x_{*}, x_{\bullet}\right)=\Omega^{\dagger}\left(x_{*}, x_{\mathbf{\bullet}}\right) \partial_{\mu} \Omega\left(x_{*}, x_{\mathbf{\bullet}}\right)
$$

is a solution of our YM equation in 2 longitudinal dimensions.


## Way out: gauge transformation

Gauge transformation with a matrix $\Omega(x)$ such that

$$
\Omega\left(x_{*}, x_{\bullet}, x_{\perp}\right) \xrightarrow{x_{*} \rightarrow-\infty}\left[x_{\bullet},-\infty \infty_{\bullet}\right]_{x}^{\bar{A}_{*}}, \quad \Omega\left(x_{*}, x_{\bullet}, x_{\perp}\right) \xrightarrow{x_{\bullet} \rightarrow-\infty}\left[x_{*},-\infty_{*}\right]_{x}^{\bar{B}_{\bullet}}
$$

The initial conditions turn to

$$
\begin{aligned}
& A_{\mu}(x) \stackrel{x_{*} \rightarrow-\infty}{=} U_{\mu}\left(x_{\bullet}, x_{\perp}\right), \quad \psi(x) \stackrel{x_{*} \rightarrow-\infty}{=} \Sigma_{a}\left(x_{\bullet}, x_{\perp}\right) \\
& A_{\mu}(x) \stackrel{x_{\bullet} \rightarrow-\infty}{=} V_{\mu}\left(x_{*}, x_{\perp}\right), \quad \psi(x) \stackrel{x_{\bullet} \rightarrow-\infty}{=} \Sigma_{b}\left(x_{*}, x_{\perp}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
U_{\mu}\left(x_{\bullet}, x_{\perp}\right) & \equiv \frac{2}{s} p_{2 \mu} U_{\bullet}\left(x_{\bullet}, x_{\perp}\right)+U_{\mu_{\perp}}\left(x_{\bullet}, x_{\perp}\right) \\
V_{\mu}\left(x_{*}, x_{\perp}\right) & \equiv \frac{2}{s} p_{1 \mu} V_{*}\left(x_{*}, x_{\perp}\right)+V_{\mu_{\perp}}\left(x_{*}, x_{\perp}\right), \\
U_{i}\left(x_{\bullet}, x_{\perp}\right) & \equiv \frac{2}{s} \int_{-\infty}^{x_{\bullet}} d x_{\bullet}^{\prime} U_{* i}\left(x_{\bullet}^{\prime}, x_{\perp}\right), \quad V_{i}\left(x_{*}, x_{\perp}\right) \equiv \frac{2}{s} \int_{-\infty}^{x_{*}} d x_{*}^{\prime} V_{\bullet i}\left(x_{*}^{\prime}, x_{\perp}\right) \\
\Sigma_{a}\left(z_{\bullet}, z_{\perp}\right) & \equiv\left[-\infty_{\bullet}, z_{\bullet}\right]_{z} \psi\left(z_{\bullet}, z_{\perp}\right), \quad \Sigma_{b}\left(z_{*}, z_{\perp}\right) \equiv\left[-\infty_{*}, z_{*}\right]_{z} \psi\left(z_{*}, z_{\perp}\right)
\end{aligned}
$$

New initial conditions look like the projectile fields in the light-like gauge $p_{2}^{\mu} A_{\mu}=0$ and target fields in the light-like gauge $p_{1}^{\mu} A_{\mu}=0$

## Classical solution with new background fields

Now all background fields are $\sim m_{\perp}$

$$
\begin{aligned}
& U_{i}\left(x_{\bullet}, x_{\perp}\right) \sim m_{\perp}, \quad U_{\bullet}\left(x_{\bullet}, x_{\perp}\right) \sim m_{\perp}^{2}, \quad U_{*}=0 \\
& V_{i}\left(x_{*}, x_{\perp}\right) \sim m_{\perp}^{2}, \quad V_{*}\left(x_{*}, x_{\perp}\right) \sim m_{\perp}^{2}, \quad V_{\bullet}=0
\end{aligned}
$$

and we have to solve

$$
D^{\nu} F_{\mu \nu}^{a}=\sum_{f} g \bar{\psi}^{f} t^{a} \gamma_{\mu} \psi^{f}, \quad\left(\not P+m_{f}\right) \psi^{f}=0
$$

with boundary conditions

$$
\begin{array}{ll}
A_{\mu}(x) \stackrel{x_{*} \rightarrow-\infty}{=} U_{\mu}\left(x_{\bullet}, x_{\perp}\right), & \psi(x) \stackrel{x_{*} \rightarrow-\infty}{=} \Sigma_{a}\left(x_{\bullet}, x_{\perp}\right) \\
A_{\mu}(x)^{x} \stackrel{\rightarrow-\infty}{=} V_{\mu}\left(x_{*}, x_{\perp}\right), & \psi(x) \stackrel{x_{\bullet} \rightarrow-\infty}{=} \Sigma_{b}\left(x_{*}, x_{\perp}\right)
\end{array}
$$

Now we start with $U_{\mu}+V_{\mu}$ and computie Feynman diagrams (with retarded propagators) with a source $J_{\nu}=D^{\mu} F^{\mu \nu}(A+B) \sim m_{\perp}^{3}$

## Expansion at small momentum transfer

The solution of YM equations in general case (scattering of two "color glass condensates") is yet unsolved problem.
Fortunately, for our case of particle production with $\frac{q_{\perp}}{Q} \ll 1$ we can use this small parameter and construct the approximate solution as a series in $\frac{q_{\perp}}{Q}$.

Example:
$A_{\bullet}=U \bullet+\int d z\left(x\left|\frac{1}{p^{2}+i \epsilon p_{0}} p_{\bullet}\right| z\right)\left[U_{j}, V^{j}\right](z)=U \bullet+\frac{1}{2} \int d z\left(x\left|\frac{1}{\alpha-\frac{p_{1}^{2}}{\beta s}+i \epsilon}\right| z\right)\left[U_{j}, V^{j}\right](z)$
The characteristic $\alpha \geq \alpha_{q}$ and $\beta \geq \beta_{q}$ so $\alpha \gg \frac{p_{1}^{2}}{\beta s}$

$$
\Rightarrow\left(x\left|\frac{1}{\alpha-\frac{p_{\perp}^{2}}{\beta s}+i \epsilon}\right| z\right)=\left(x\left|\frac{1}{\alpha+i \epsilon}\right| z\right)+\left(x\left|\frac{1}{\alpha+i \epsilon} \frac{p_{\perp}^{2}}{\beta s} \frac{1}{\alpha+i \epsilon}\right| z\right)+\ldots
$$

and in the leading order in $p_{\perp} / p_{\|}$we get

$$
\begin{aligned}
& A_{\bullet}(x)=U_{\bullet}\left(x_{\bullet}, x_{\perp}\right)+\frac{1}{2} \int d z\left(x\left|\frac{1}{\alpha+i \epsilon}\right| z\right)\left[U_{j}, V^{j}\right](z) \\
& =U_{\bullet}\left(x_{\bullet}, x_{\perp}\right)-\frac{i}{2} \int_{-\infty}^{x_{\bullet}} d x_{\bullet}^{\prime}\left[U_{j}\left(x_{\bullet}^{\prime}, x_{\perp}\right), V^{j}\left(x_{*}, x_{\perp}\right)\right]
\end{aligned}
$$

## Gluon fields in the leading order in $p_{\perp}^{2} / p_{\|}^{2} \sim q_{\perp}^{2} / Q^{2}$

With the expansion

$$
\frac{1}{p^{2}+i \epsilon p_{0}}=\frac{1}{p_{\|}^{2}-p_{\perp}^{2}+i \epsilon p_{0}}=\frac{1}{p_{\|}^{2}}-\frac{1}{p_{\|}^{2}+i \epsilon p_{0}} p_{\perp}^{2} \frac{1}{p_{\|}^{2}+i \epsilon p_{0}}+\ldots
$$

the dynamics in transverse space is trivial.
Gluon fields : $\quad F_{\bullet i}^{(-1)}=V_{\bullet i}, \quad F_{* i}^{(-1)}=U_{* i}$,

$$
\begin{aligned}
& F_{* \bullet}^{(-1)}=U_{* \bullet}+V_{* \bullet}-\frac{i s}{2} U_{j}^{a b} V^{b j} \\
& F_{\bullet i}^{(0) a}=U_{\bullet i}^{a}-i U_{\bullet}^{a b} V_{i}^{b}-\frac{i}{2(\alpha+i \epsilon)} \tilde{L}_{i}^{(0)}-\mathcal{D}_{i}^{a b} V_{j}^{b c} \frac{1}{2(\alpha+i \epsilon)} U^{c j}, \\
& F_{* i}^{(0) a}=V_{* i}^{a}-i V_{*}^{a b} U_{i}^{b}-\frac{i}{2(\beta+i \epsilon)} \tilde{L}_{i}^{(0)}-\mathcal{D}_{i}^{a b} U_{j}^{b c} \frac{1}{2(\beta+i \epsilon)} V^{c j}, \\
& F_{i k}^{(0)}=U_{i k}+V_{i k}-i\left[U_{i}, V_{k}\right]-i\left[V_{i}, U_{k}\right]
\end{aligned}
$$

where

$$
\begin{aligned}
L_{i}^{(0) a} & =-i U^{j a b} V_{j i}-i V^{j a b} U_{j i}-i \mathcal{D}_{j}^{a b}\left(U^{j b c} V_{i}^{c}+V^{j b c} U_{i}^{c}\right) \\
& -\frac{2 i}{s}\left(U_{* \bullet}^{a b} V_{i}^{b}-V_{* \bullet}^{a b} U_{i}\right)+\bar{\Sigma}_{a} t^{a} \gamma_{i} \Sigma_{b}+\bar{\Sigma}_{b} t^{a} \gamma_{i} \Sigma_{a}
\end{aligned}
$$

## Gluon fields in the leading order in $p_{\perp}^{2} / p_{\|}^{2} \sim q_{\perp}^{2} / Q^{2}$

With the expansion

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\frac{1}{p^{2}+i \epsilon p_{0}}=\frac{1}{p_{\|}^{2}-p_{\perp}^{2}+i \epsilon p_{0}}=\frac{1}{p_{\|}^{2}}-\frac{1}{p_{\|}^{2}+i \epsilon p_{0}} p_{\perp}^{2} \frac{1}{p_{\|}^{2}+i \epsilon p_{0}}+\ldots
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& F_{* i}^{(0) a}=V_{* i}^{a}-i V_{*}^{a b} U_{i}^{b}-\frac{i}{2(\beta+i \epsilon)} \tilde{L}_{i}^{(0)}-\mathcal{D}_{i}^{a b} U_{j}^{b c} \frac{1}{2(\beta+i \epsilon)} V^{c j}, \\
& F_{i k}^{(0)}=U_{i k}+V_{i k}-i\left[U_{i}, V_{k}\right]-i\left[V_{i}, U_{k}\right],
\end{aligned}
$$

We integrate over $\alpha$ without cutoff $\alpha>\sigma$ since the contour over $\alpha$ can be removed from the pole to the region of large $\alpha$ (if there is no pinch). Similarly, we integrate over all $\beta$ 's.
(Different from SCET where they keep the cutoffs $\alpha>\sigma_{b}$ and $\beta>\sigma_{a}$ ).

## TMD factorization with power corrections

## At the tree level

$$
\begin{aligned}
& F^{2}(x)=\frac{8}{s} U_{*}^{a i}(x) V_{\bullet i}^{a}(x)+2 f^{m a c} f^{m b d} \Delta^{i j, k l} U_{i}^{a}(x) U_{j}^{b}(x) V_{k}^{c}(x) V_{l}^{d}(x)+\ldots \\
& \Delta^{i j, k l} \equiv g^{i j} g^{k l}-g^{i k} g^{j l}-g^{i l} g^{j k}
\end{aligned}
$$

$\Rightarrow$ in the region $s \gg Q^{2} \gg Q_{\perp}^{2}$

$$
W\left(p_{A}, p_{B}, q\right)=\frac{64 / s^{2}}{N_{c}^{2}-1} \int d^{2} x_{\perp} e^{i(q, x) \perp} \frac{2}{s} \int d x_{\bullet} d x_{*} e^{-i \alpha_{q} x_{\bullet}-i \beta_{q} x_{*}}
$$

$$
\times\left\{\left\langle p_{A}\right| U_{*}^{m i}\left(x_{\bullet}, x_{\perp}\right) U_{*}^{m j}(0)\left|p_{A}\right\rangle\left\langle p_{B}\right| \mathcal{F}_{\bullet i}^{n}\left(x_{*}, x_{\perp}\right) \mathcal{F}_{\bullet j}^{n}(0)\left|p_{B}\right\rangle\right.
$$

$$
-\frac{N_{c}^{2}}{N_{c}^{2}-4} \frac{\Delta^{i j, k l}}{Q^{2}} \int_{-\infty}^{x_{\bullet}} d \underset{S}{2} x_{\bullet}^{\prime} d^{a b c}\left\langle p_{A}\right| U_{* i}^{a}\left(x_{\bullet}, x_{\perp}\right) U_{* j}^{b}\left(x_{\bullet}^{\prime}, x_{\perp}\right) U_{* r}^{c}(0)\left|p_{A}\right\rangle
$$

$$
\left.\times \int_{-\infty}^{x_{\bullet}} d \frac{2}{s} x_{*}^{\prime} d^{m n l}\left\langle p_{B}\right| V_{\bullet k}^{m}\left(x_{*}, x_{\perp}\right) V_{\bullet l}^{n}\left(x_{*}^{\prime}, x_{\perp}\right) V_{\bullet r}^{n}(0)\left|p_{B}\right\rangle+x \leftrightarrow 0\right\}
$$

The correction is $\sim \frac{Q_{\perp}^{2}}{Q^{2}}$ (easy to see e.g. on a quark target).

1 Conclusions

- Higher-twist power correction to particle production by gluon fusion at $s \gg q^{2} \gg q_{\perp}^{2}$ is calculated
2 Outlook
- Power corrections to Drell-Yan and SIDIS
- Factorization at the one-loop level (and match to evolution equations for gluon TMDs).

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## Thank you for attention!

