Fluctuations of the gluon distribution at small x: correlation of multiplicity and transverse momentum fluctuations

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Saturation: Recent Developments,
New Ideas and Measurements
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based on A.D. & V. Skokov,
arXiv:1704.05917

Observables at small x

average over stochastic small-x color fields:

$$\langle O[A^+]\rangle = \frac{1}{Z}\int \mathcal{D}A^+\, W[A^+]\, O[A^+]$$

$$W[A^+]:\ \text{JIMWLK weight functional}$$

In the MV model:

$$\begin{aligned} W[A^+] &= e^{-S[A^+]}\,, \\ S[A^+] &= \int dx^- d^2x_\perp \frac{\mathrm{tr}\,\nabla_\perp^2 A^+(x^-,x_\perp)\nabla_\perp^2 A^+(x^-,x_\perp)}{2\mu^2(x^-)} \end{aligned}$$

Examples:

- $O[A^+] = g^2\,\mathrm{tr}\,A^+(x_\perp)A^+(y_\perp)$ (cov. gauge) gluon distribution
- $O[A^+] = g^2\,\mathrm{tr}\,A^i(x_\perp)A^i(y_\perp)$ (L.C. gauge) WW gluon distribution
- $O[A^+] = \dfrac{1}{N_c}\,\mathrm{tr}\,V(x_\perp)V^\dagger(y_\perp)$ dipole forw. scatt. amplitude

$$V(x_\perp) = \mathcal{P}e^{-ig\int dx^- A^+(x^-,x_\perp)}$$

Idea: integrate out fluctuations which do not affect observable O[A^+]

→ obtain effective action / potential for that observable

(we work directly with MV / JIMWLK action w/o introducing “dipole picture”)

$$e^{-V_{\text{eff}}[X(q)]} = \int \mathcal{D}A^+(q)\, W[A^+(q)]\, \delta(X(q) - O[A^+(q)])$$

$$\frac{\delta V_{\text{eff}}[X(k)]}{\delta X(q)} = 0 \to X_s(q) \equiv \langle X(q) \rangle$$

(at large Nc; or else do proper Legendre transform)

for $O[A^+] = g^2\,\mathrm{tr}\,|A^+(q)|^2$ in MV model :

$$V_{\text{eff}}[X(q)] = \int \frac{d^2q}{(2\pi)^2} \left[\frac{q^4}{g^4\mu^2} X(q) - \frac{1}{2} A_\perp N_c^2 \log X(q) \right]$$

$$X_s(q) = \frac{1}{2} N_c^2 A_\perp \frac{g^4\mu^2}{q^4} \qquad ✓$$

(cov. gauge gluon distribution at q>Qs)

same result for non-local Gaussian model by Iancu, Itakura & McLerran (approx. to JIMWLK), provided

at q > Q_s : $\mu^2 \to \mu_0^2 \left(\frac{q^2}{Q_s^2} \right)^{1-\gamma}$

γ = 0.63, BFKL anom. dim. in presence of absorptive (saturation) boundary (Mueller & Triantafyllopoulos)

Aside: constrained effective potential introduced originally to compute potential for Polyakov loop in YM at T>0:

$$\exp(-V\mathcal{V}(\ell)) = \int \mathcal{D}A_\mu \, \delta(\ell - \frac{1}{N}\mathrm{tr}\,\overline{\mathbf{L}}) \, \exp(-\frac{1}{g^2}S(A))$$

for Higgs:
L. O'Raifeartaigh, A. Wipf and H. Yoneyama: Nucl. Phys. B 271 (1986) 653

for Polyakov loop:
C. Korthals Altes: Nucl. Phys. B 420 (1994) 637

to two loops and general eigenvalues:
A. Dumitru, Y. Guo and C. P. Korthals Altes: Phys. Rev. D 89, 016009 (2014)

Fluctuations:

instead of $X(q) = X_s(q) + \delta X(q)$,

$$\begin{aligned}\Delta V_{\text{eff}}[\delta X(q)] &= \frac{1}{2}\int \frac{d^2 l}{(2\pi)^2}\frac{d^2 k}{(2\pi)^2}\,\delta X(l)\left\{\frac{\delta}{\delta X(l)}\frac{\delta}{\delta X(k)}V_{\text{eff}}[\delta X(q)]\right\}\delta X(k)\\ &= \frac{1}{2}\int \frac{d^2 q}{(2\pi)^2}\frac{\delta X(q)^2}{X_s(q)^2}\frac{1}{2}N_c^2 A_\perp\end{aligned}$$

$$Z_{1\ \text{loop}} = \int \mathcal{D}\,\delta X(q)\ e^{-V_{\text{eff}}[\delta X(q)]} = e^{-\frac{1}{2}\,\text{tr}\,\log\left(\frac{1}{2}\frac{N_c^2 A_\perp}{X_s(q)^2}\right)}$$

let's go with $X(q) = X_s(q)\,\eta(q)$ (canonical dim. of η is zero)

$$\begin{aligned}\Delta V_{\text{eff}}[\eta(q)] &\equiv V_{\text{eff}}[\eta(q)] - V_{\text{eff}}[\eta(q)=1]\\ &= \frac{1}{2}N_c^2 A_\perp \int \frac{d^2 q}{(2\pi)^2}\left[\eta(q) - 1 - \log\eta(q)\right]\end{aligned}$$

- field redefinition $\varphi = \log\eta$ yields Liouville action/potential (in q space!) with $R<0$

Fluctuations: correlation of gluon number and transv. momentum

$$\Delta N_g[\eta(q)] = \int \frac{d^2q}{(2\pi)^2} q^2 X_s(q)\ [\eta(q) - 1]$$

$$\Delta \overline{q^2}[\eta(q)] = \frac{\Delta N_g[\eta(q)]}{\int \frac{d^2q}{(2\pi)^2} X_s(q)\eta(q)}$$

$$\Delta N_g \simeq N_c A_\perp \Delta \overline{q^2}$$

for small amplitude fluctuations

ΔN_g increases with $\Delta \overline{q^2}$

Ansatz: $$\eta(q > Q_s) = 1 + \eta_0 \left(\frac{Q_s^2}{q^2}\right)^a \Theta\left(Q^2 - q^2\right)$$

- a > 0: fluctuation "know" about scale Q_s
- a → 0: scale invariant fluctuation

maximize ΔN_g (and $\Delta \overline{q^2}$), minimize "penalty" $\Delta S[\eta(q)] \equiv \Delta V_{\rm eff}[\eta(q)]$

→ MV model: 0<a<1

(because for *a=0*: $\Delta N_g \sim \log Q^2$, $\Delta S \sim Q^2$)

IIM model (anom. dim.) a ≈ 0

→ Mueller picture where Qs corresponds to an absorptive boundary

$$\eta_0 \sim \Delta S \frac{\left(Q^2/Q_s^2\right)^a}{N_c^2 A_\perp Q^2}$$

(ΔS ~ log p^{-1} where p is probability of fluctuation)

MC simulation:

- MV: generate random configurations of $A^+(x^-,x_\perp)$ according to $S_{MV}[A^+]$
- JIMWLK: stochastic evolution (at fixed α_s) of associated Wilson line to $\alpha_s Y=1$

$$X(q) = \int d^2b \int d^2r\, e^{-iqr}\, \mathrm{tr}\left[V_Y\left(b-\frac{r}{2}\right)V_Y^\dagger\left(b+\frac{r}{2}\right)-1\right]$$

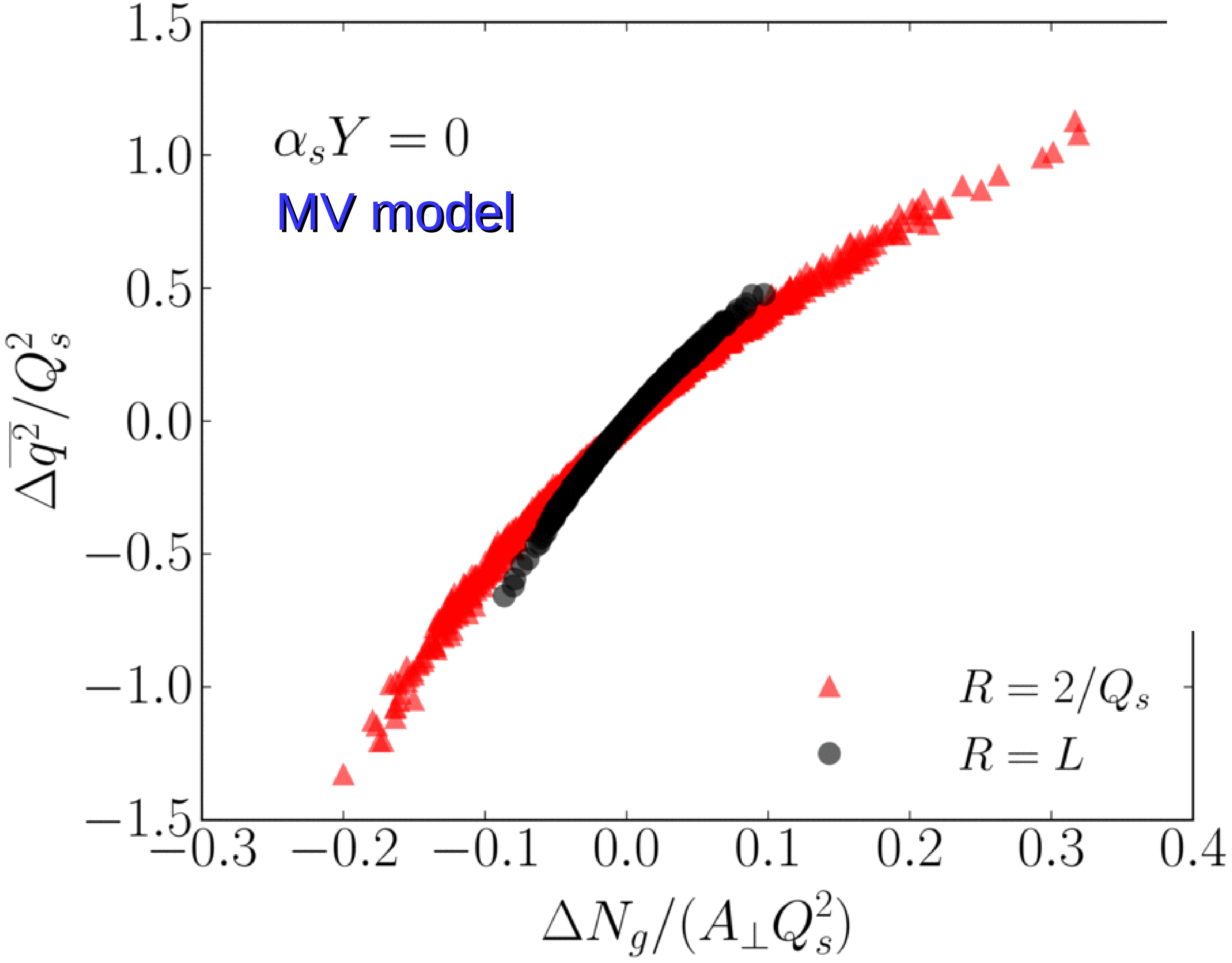
(integrated over entire 2d lattice)

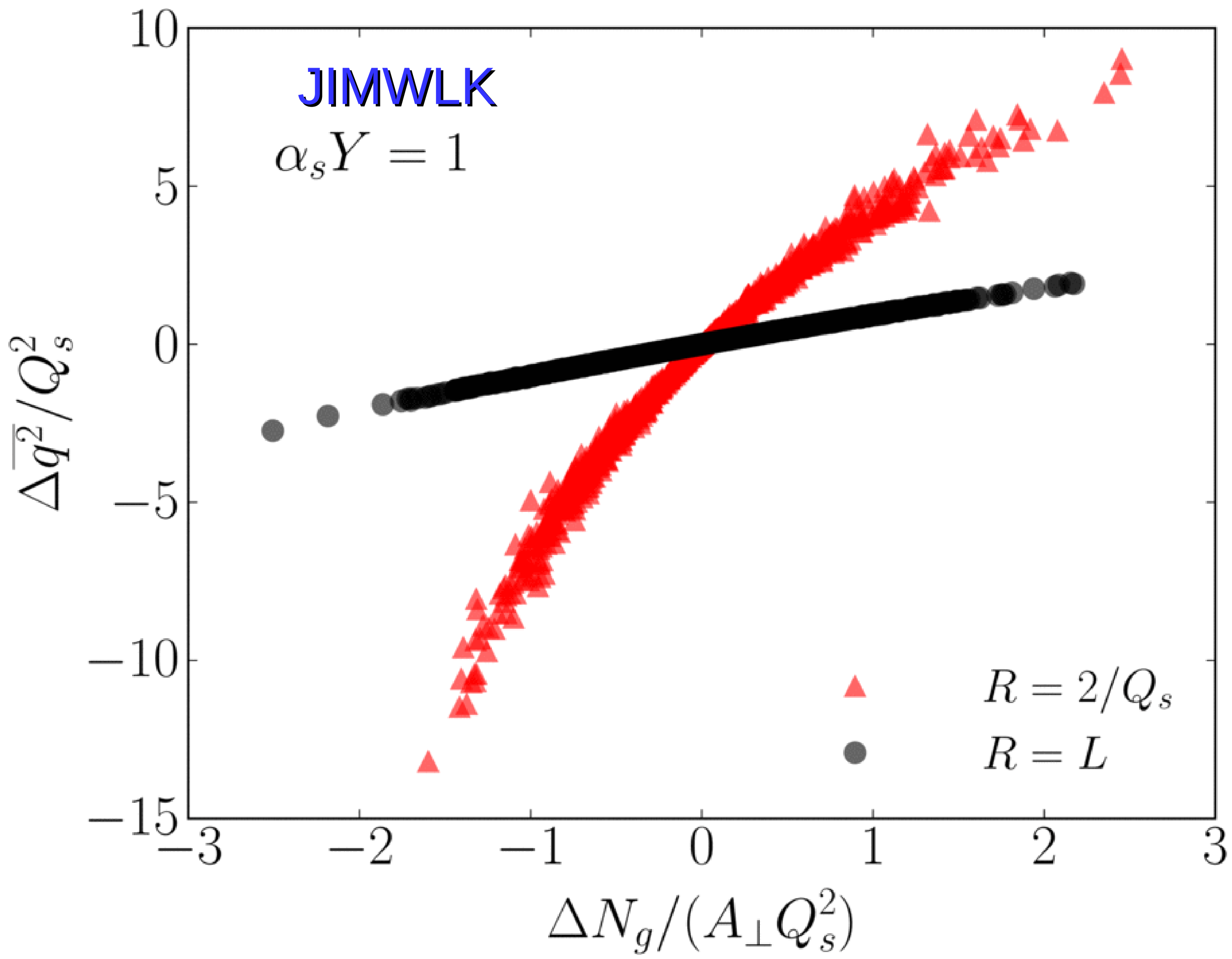
$$X_R(q,b) = \int d^2x \int d^2y\, e^{-iq(y-x)}\, e^{-\frac{(b-x)^2}{2R^2}}\, e^{-\frac{(b-y)^2}{2R^2}}\, \mathrm{tr}\left[V_Y(x)V_Y^\dagger(y)-1\right]$$

("local" distribution at b;
similar to "smeared" Wigner distr.)

$$R = \frac{2}{Q_s}$$

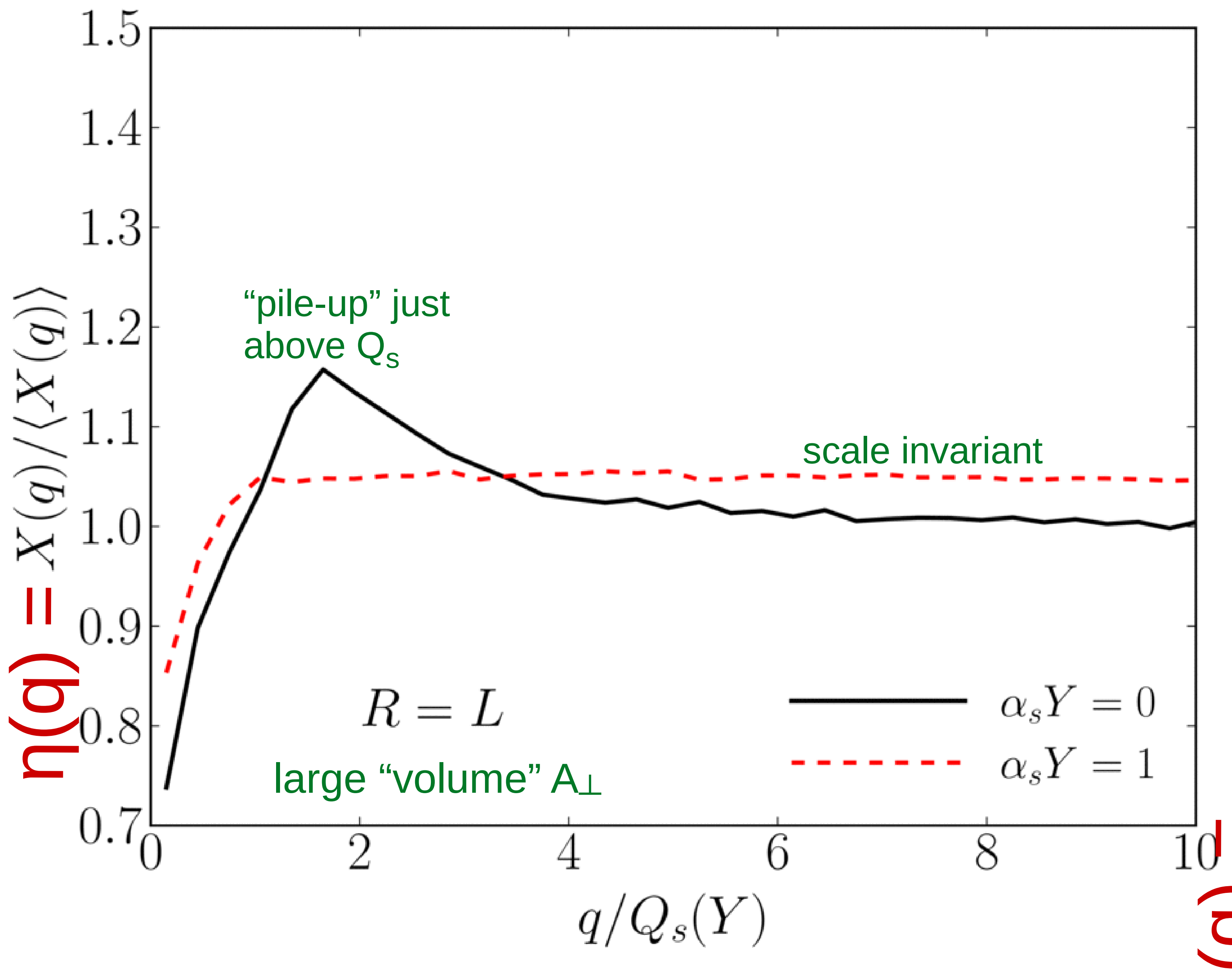
Correlation of transv. momentum and multiplicity fluctuations

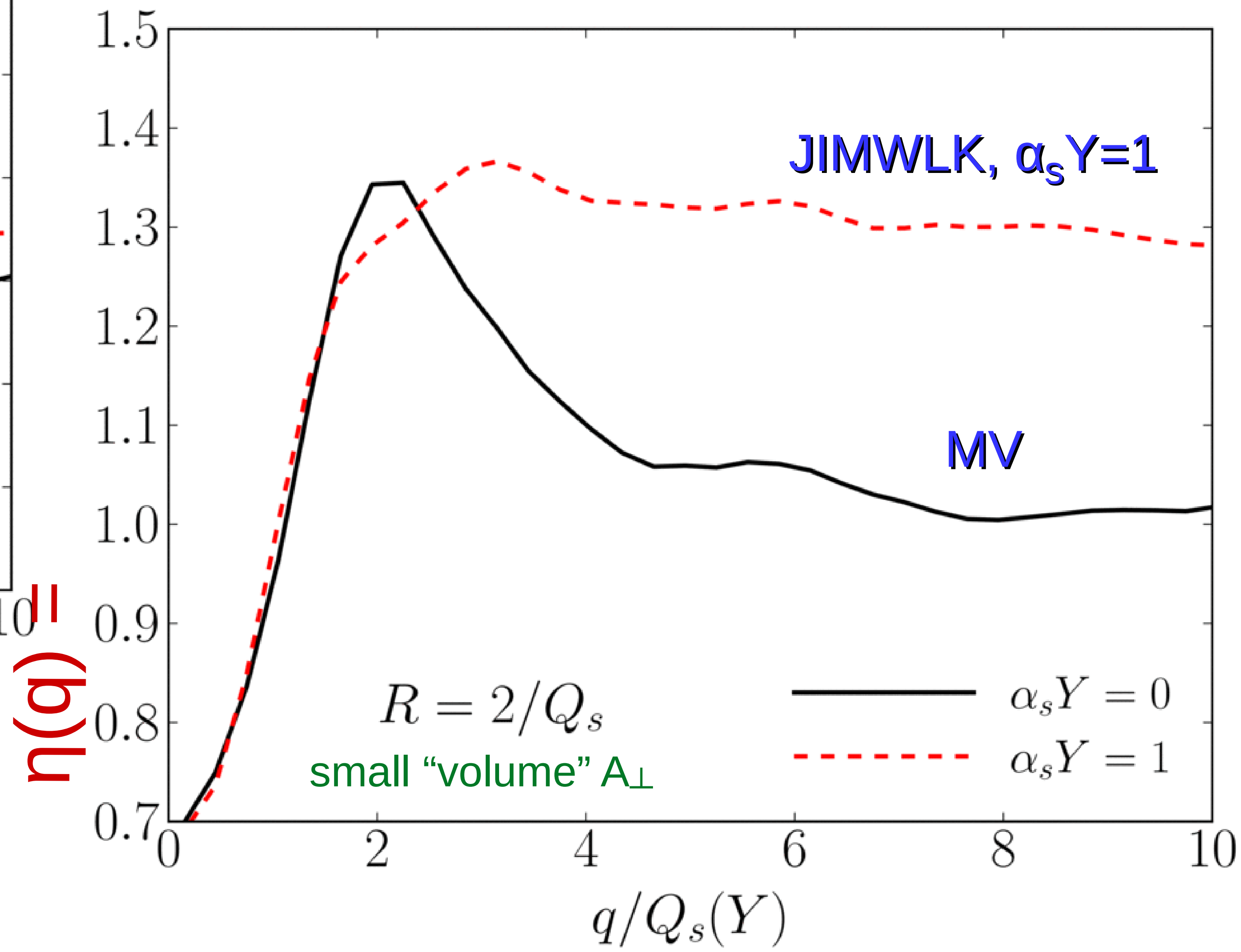




- $\Delta N_g \sim 10 - 100$ for $A = 0.1\ \mathrm{fm}^2$, $Q_s^2 = (1\text{-}2.5\ \mathrm{GeV})^2$
- tight correlation (~ single curve, few outliers)
- JIMWLK shows correlations in b-space
- strong increase of Δq^2 with ΔN_g in small area "patches"

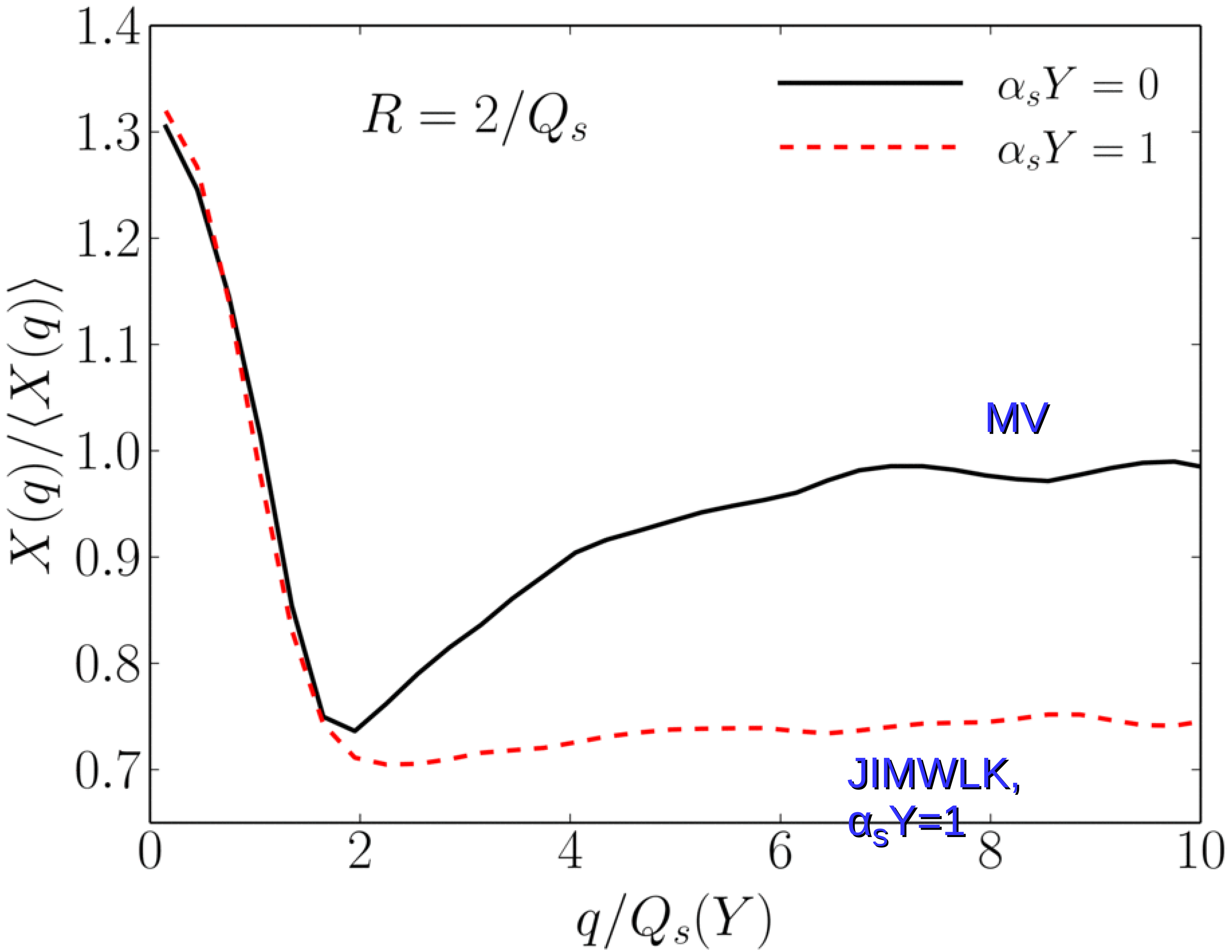
Spectral shape of high multiplicity fluctuations





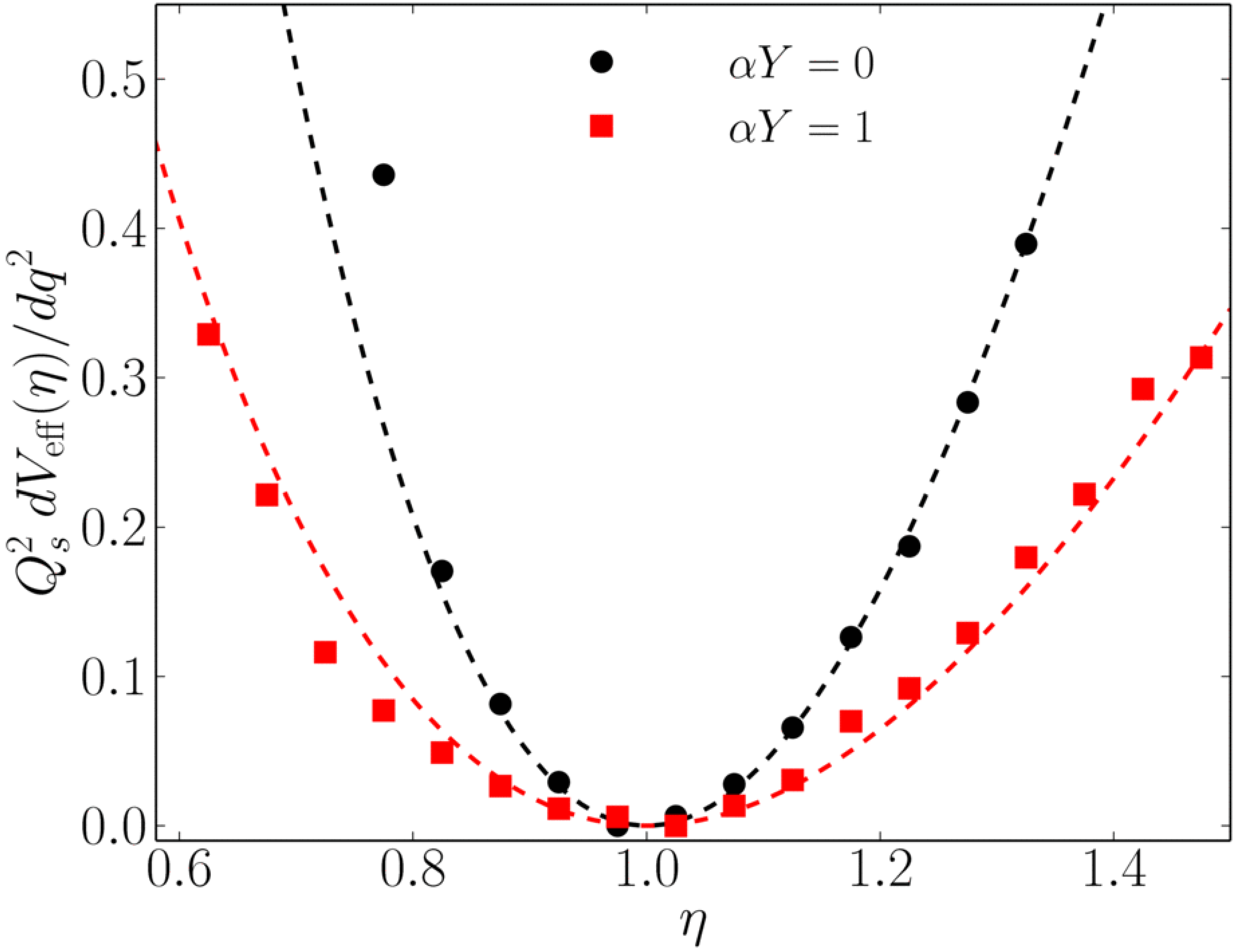
- evolution modifies spectrum of fluctuations
- MV model piles up addtl gluons around Qs
- perturbative evolution with scale invariant kernel → flat spectral shape above 'absorptive boundary' (canonical dimension of η(q) is 0)

Spectral shape of low multiplicity fluctuations



$V_{eff}[\eta]$ extracted from MC (histogram of fluctuations etc)

- linear minus log indeed appears to work ✓
- prefactor (curvature) ~ $N_c^2 A_\perp Q_s^2$, effective area of fluct. appears to decrease with Y (i.e. $A_\perp Q_s^2$ decreases with Y)



WW (light cone gauge) gluon distribution

$$g^2 \mathrm{tr}\, A^i(q) A^j(-q)$$

at order $(gA^+)^4$

$$\begin{aligned}
\delta^{ij}\, g^2 \mathrm{tr}\, A^i(q) A^j(-q) &= \frac{1}{2} q^2 g^2 A^{+a}(q) A^{+a}(-q) \\
&\quad - \frac{g^4}{8} f^{abe} f^{cde} \left(\delta^{lm} - \frac{q^l q^m}{q^2} \right) \int \frac{d^2k}{(2\pi)^2} \frac{d^2p}{(2\pi)^2} k^l p^m A^{+a}(q-k) A^{+b}(k) A^{+c}(-q-p) A^{+d}(p) \\
\left(2 \frac{q^i q^j}{q^2} - \delta^{ij} \right) g^2 \mathrm{tr}\, A^i(q) A^j(-q) &= \frac{1}{2} q^2 g^2 A^{+a}(q) A^{+a}(-q) \\
&\quad + \frac{g^4}{8} f^{abe} f^{cde} \left(\delta^{lm} - \frac{q^l q^m}{q^2} \right) \int \frac{d^2k}{(2\pi)^2} \frac{d^2p}{(2\pi)^2} k^l p^m A^{+a}(q-k) A^{+b}(k) A^{+c}(-q-p) A^{+d}(p)
\end{aligned}$$

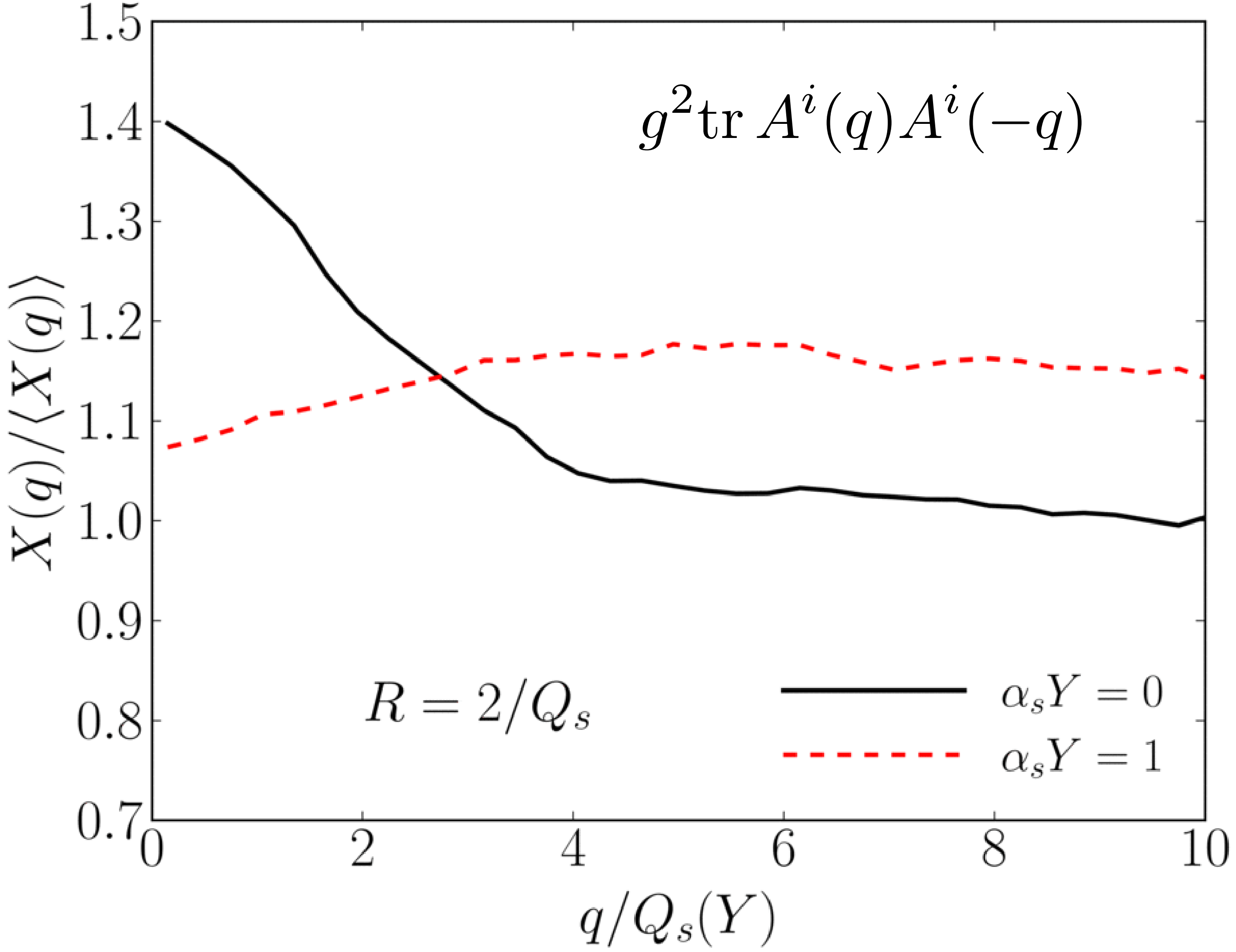
* not “diagonal” in q, analytic calculation of $V_{\mathrm{eff}}[X(q)]$ more complicated
* numerical MC computation is no problem though,
  (to all orders in gA^+: $A^i = (i/g)\, U[A^+]\, \partial^i\, U^\dagger[A^+]$)

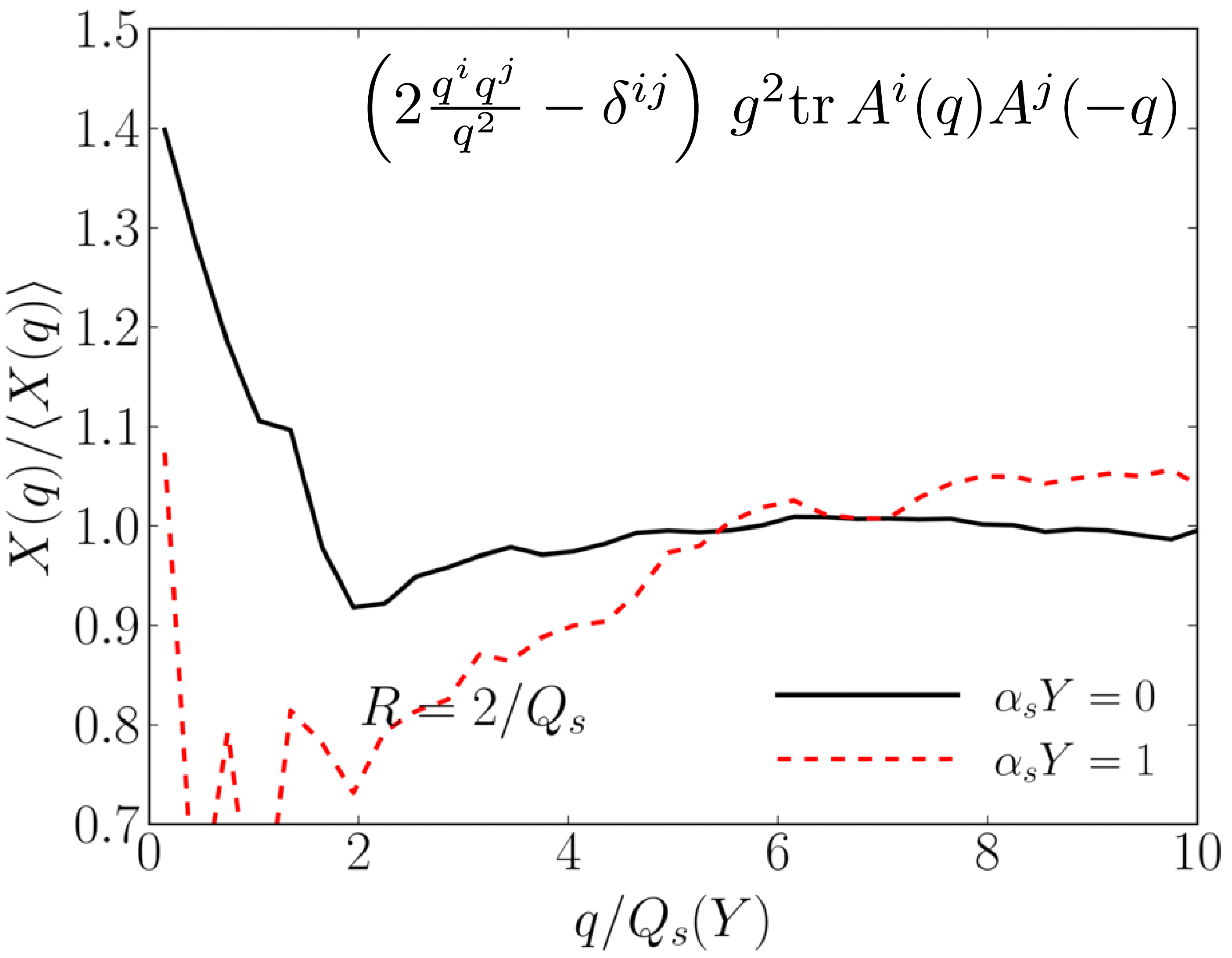
Summary

- effective potential for observable $O[A^+]$:
  integrate out "orthogonal" fluctuations of A^+ from MV / JIMWLK action
- for $O[A^+] = g^2 \,\mathrm{tr}\, |A^+(q)|^2$ we obtained $V_{\mathrm{eff}}[\eta]$ analytically & numerically: linear minus log
  $\eta(q) \equiv X(q) / X_s(q)$ describes fluctuations of cov. gauge gluon distribution
- tight correlation of gluon number and transverse momentum fluctuations
- strong increase of Δq^2 with ΔN_g, especially in small transv. area
- evolution modifies the spectrum of fluctuations:
  MV: gluon pile-up near Qs;
  JIMWLK: scale invariant fluctuation $\eta(q)$ (for fixed coupl. kernel !)
  above absorptive boundary

Backup Slides

MC results for WW (light cone gauge) gluon distribution





high multiplicity configurations via

$$\Delta N_g[\eta(q)] = \int \frac{d^2 q}{(2\pi)^2} \left[X(q) - X_s(q)\right]$$

← X(q)

Resummation of boost-invariant quantum fluctuations (JIMWLK):

classical ensemble at Y = log x_0/x = 0:

$$P[\rho] \sim e^{-S_{\rm cl}[\rho]} \ , \quad S_{\rm MV} \quad = \quad \int d^2x_\perp \, \frac{1}{2\mu^2} \rho^a \rho^a \ , \qquad V(x_\perp) = \mathcal{P} \exp \ ig^2 \int dx^- \frac{1}{\nabla_\perp^2} \rho(x_\perp)$$

quantum evolution to Y>0: random walk in space of Wilson lines

$$\partial_Y V(x_\perp) = V(x_\perp) \, it^a \left\{ \int d^2y_\perp \, \varepsilon_k^{ab}(x_\perp, y_\perp) \, \xi_k^b(y_\perp) + \sigma^a(x_\perp) \right\} \ .$$

$$\varepsilon_k^{ab} = \left(\frac{\alpha_s}{\pi} \right)^{1/2} \, \frac{(x_\perp - y_\perp)_k}{(x_\perp - y_\perp)^2} \ \left[1 - U^\dagger(x_\perp) U(y_\perp) \right]^{ab}$$

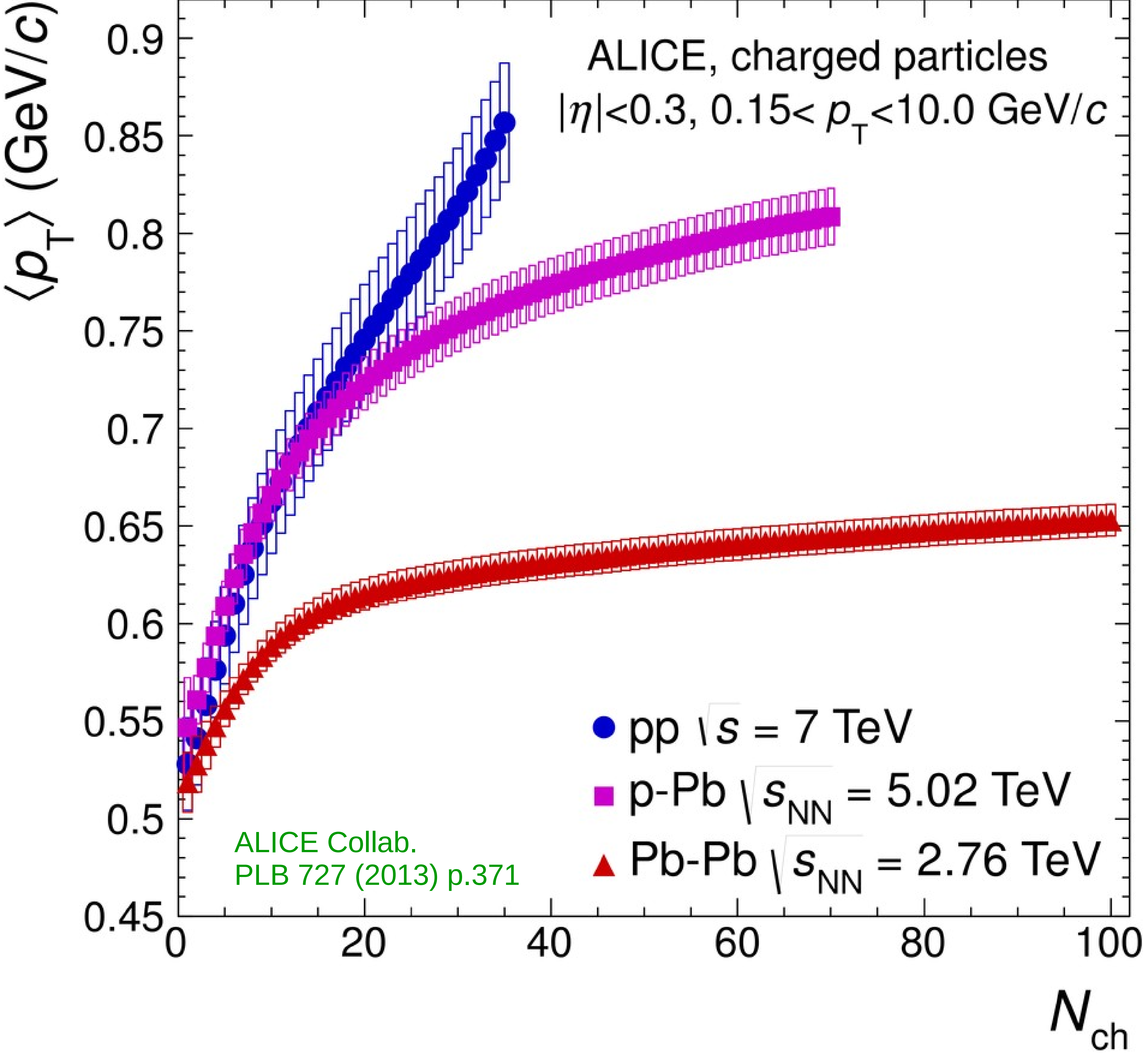
$$t^b U_{ab}(x_\perp) = \mathrm{tr} \ \left(V(x_\perp) t^a V^\dagger(x_\perp) \right)$$

$$\langle \xi_i^a(x_\perp) \, \xi_j^b(y_\perp) \rangle = \delta^{ab} \delta_{ij} \delta^{(2)}(x_\perp - y_\perp)$$ (Gaussian white noise)

$$\sigma^a(x_\perp) = -i \frac{\alpha_{\rm s}}{2\pi^2} \int d^2 z_\perp \, \frac{1}{(x_\perp - z_\perp)^2} \mathrm{tr} \ \left(T^a U^\dagger(x_\perp) \, U(z_\perp) \right)$$ (drag term)

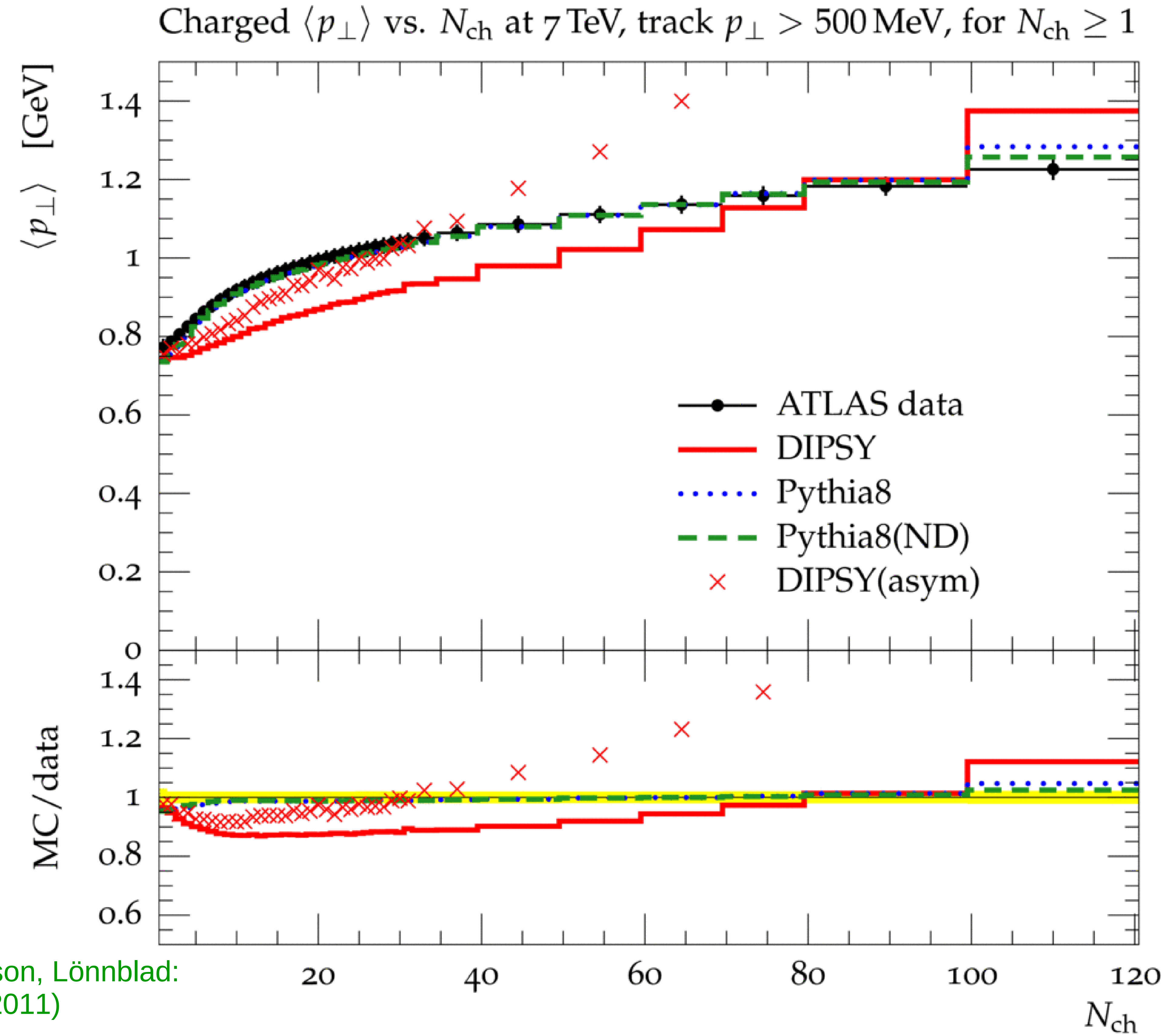
Increase of <pT>
in p+p, p+Pb collisions
with event multiplicity

?



$\langle p_T \rangle$ vs. N_{ch}:
ATLAS data vs. models

- PYTHIA8 virtually perfect
- DIPSY qualitatively ok
- can one relate this to B-JIMWLK $W[A^+]$?



Flensburg, Gustafson, Lönnblad:
JHEP 1108, 103 (2011)