Collectivity from the initial state:

Four-particle correlations in proton-nucleus collisions

A work in progress with

Mark Mace (Stony Brook / Brookhaven)
Raju Venugopalan (Brookhaven)

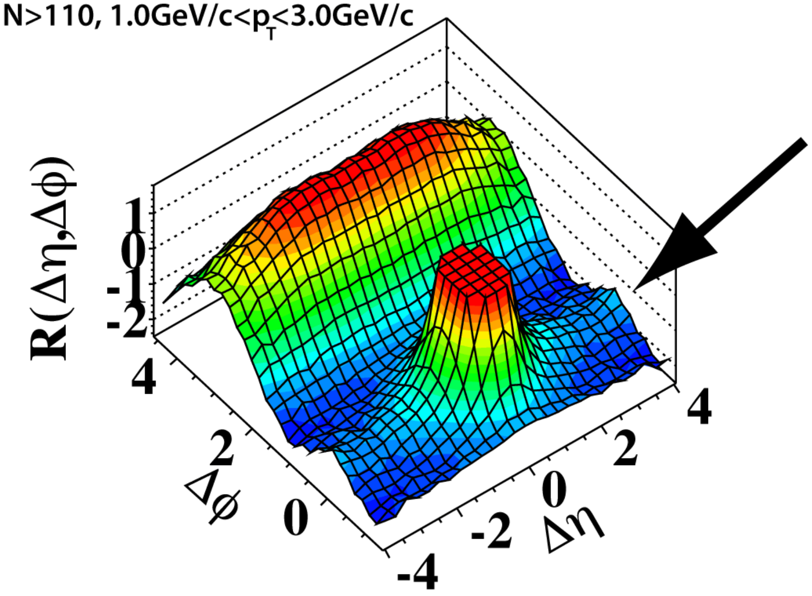
Contents

- Background / Motivation
- Particle production in proton-nucleus collisions
- Beyond the dipole: n-point correlators in the MV model
- Results

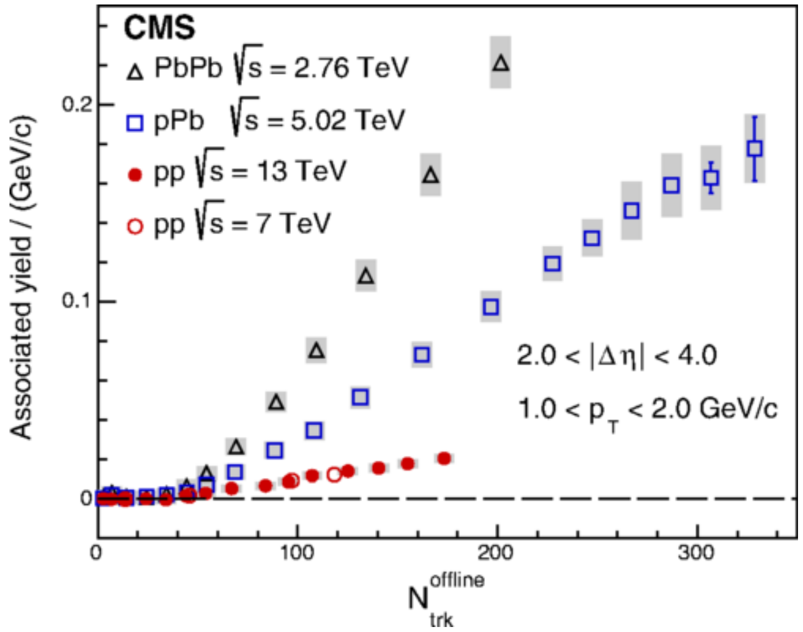
BACKGROUND / MOTIVATION

Background 1: Collectivity in small systems

V. Khachatryan et al. (CMS Collaboration)
JHEP, 1009:091, 2010.

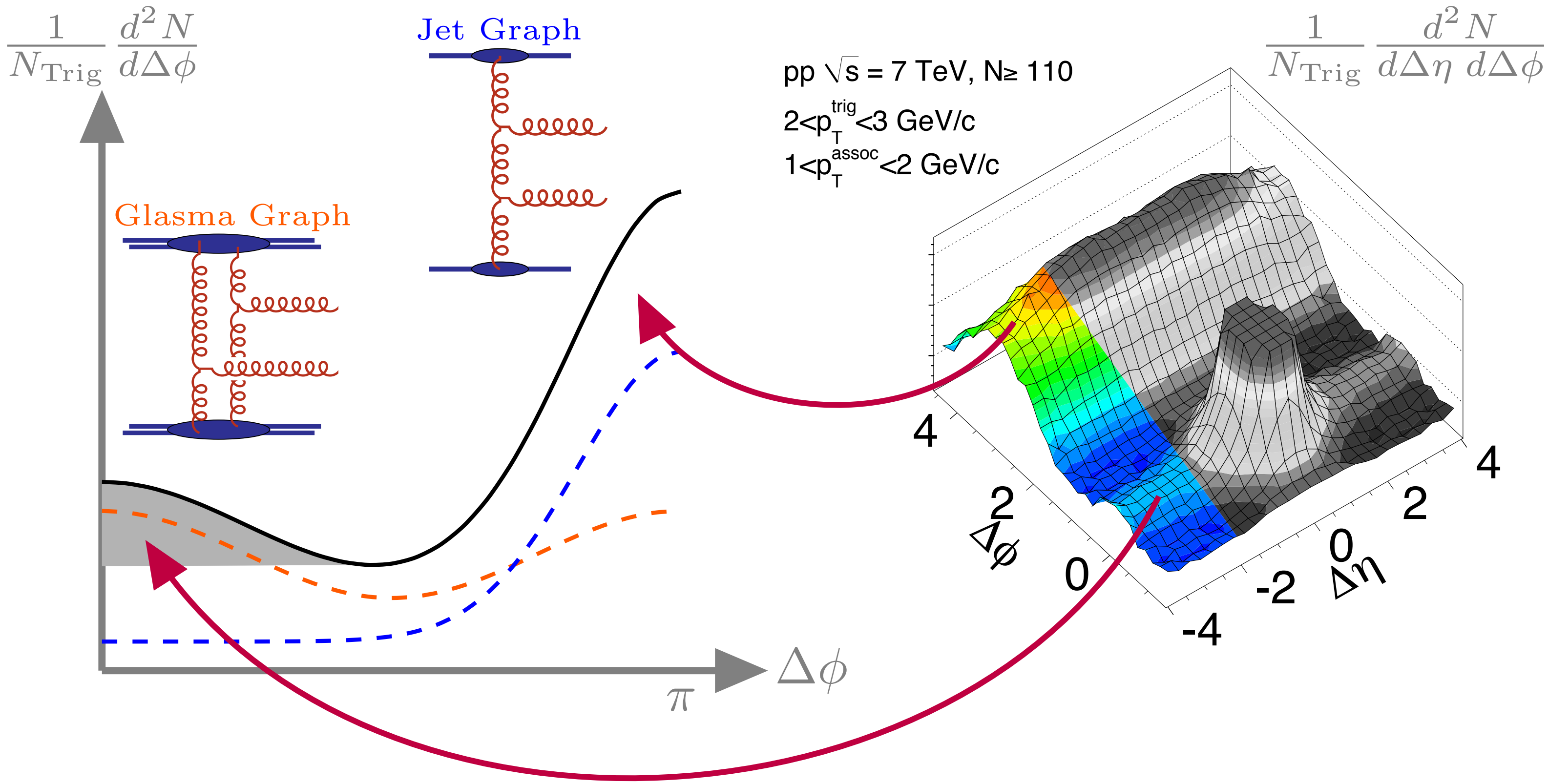


V. Khachatryan et al. (CMS Collaboration)
Phys. Rev. Lett., 116(17):172302, 2016.



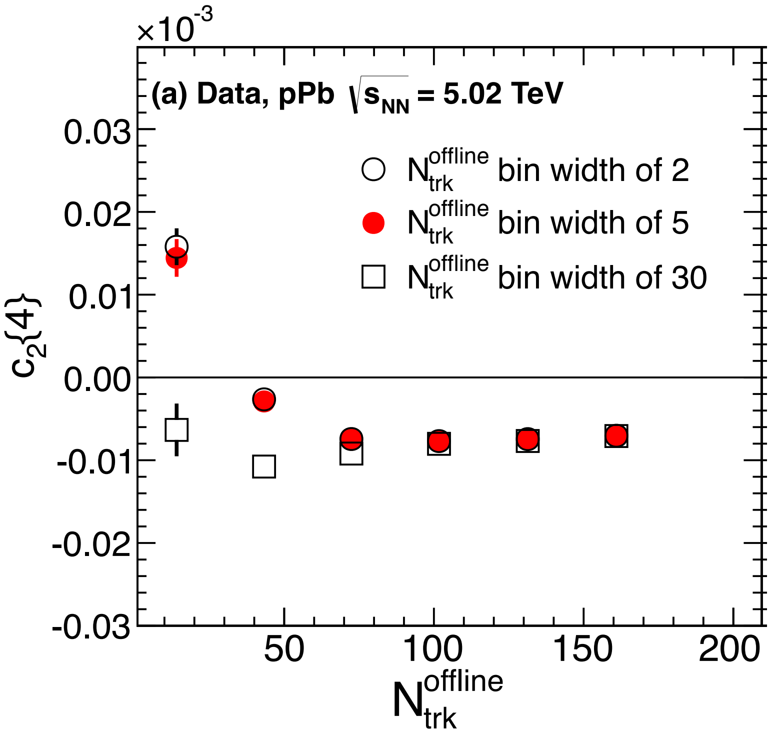
K. Dusling, W. Li, and B. Schenke
Novel collective phenomena in high-energy proton–proton and proton–nucleus collisions
Int. J. Mod. Phys., E25(01):1630002, 2016. [arXiv:1509.07939]

Background II: Glasma-graphs

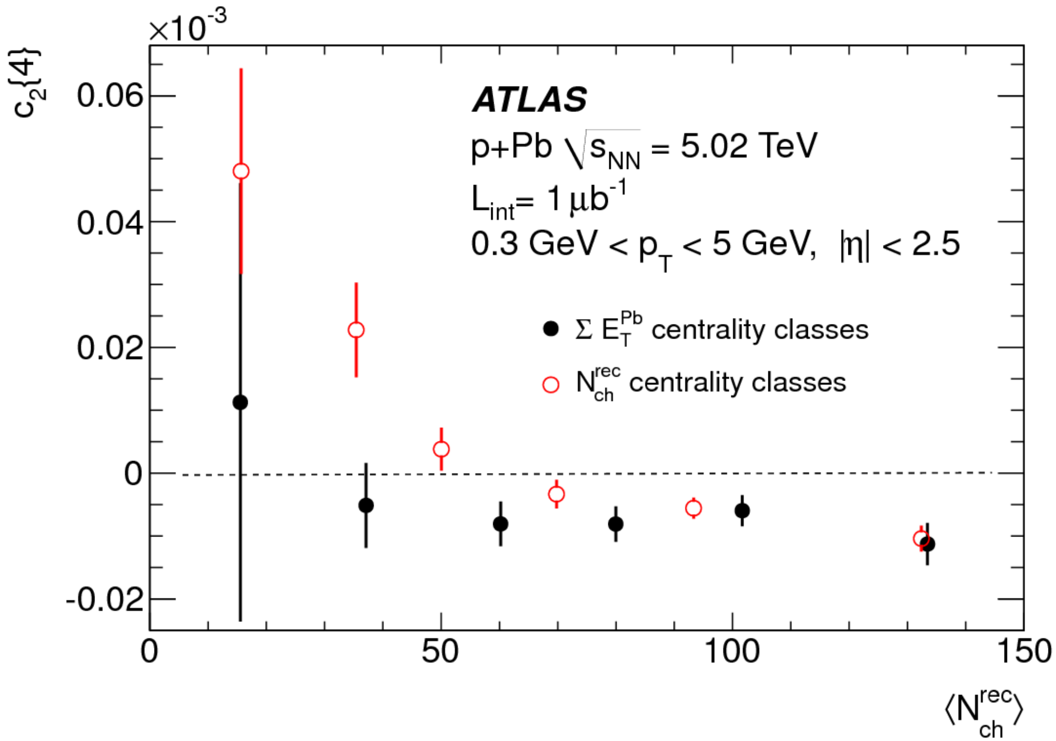


Dusling, Tribedy, Venugopalan, Phys.Rev. D93 (2016) no.1, 014034
Dusling, Venugopalan, PRD87 094034, PRD87 054014, PRD87 051502, PRL 108 262001
Dumitru, Dusling, Gelis, Jalilian-Marian, Lappi, Venugopalan, Phys.Lett. B697 (2011) 21-25

Motivation I: Four-particle cumulants



CMS Collaboration
Phys. Lett., B724 (2013) 213–240.



ATLAS Collaboration
Phys. Lett. B725 (2013) 60-78.

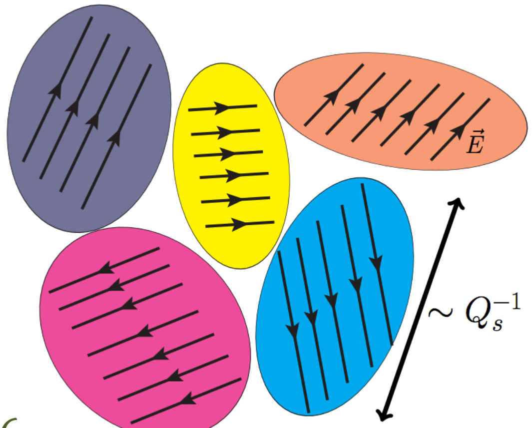
Motivation II: The color domain model

- Gaussian correlation of E-fields will produce particles isotropically

$$\frac{g^2}{N_c}\left\langle E_i^a(b_1)E_j^b(b_2)\right\rangle = \frac{1}{(N_c^2-1)}\delta^{ab}\delta_{ij}Q_s^2\Delta(b_1-b_2)$$

- Consider sub-class of events with a predefined orientation:

$$\frac{g^2}{N_c}\left\langle E_i^a(b_1)E_j^b(b_2)\right\rangle = \frac{1}{(N_c^2-1)}\delta^{ab}Q_s^2\Delta(b_1-b_2)\left(\delta_{ij}+2\mathcal{A}\left[\hat{a}_i\hat{a}_j-\frac{1}{2}\delta_{ij}\right]\right)$$



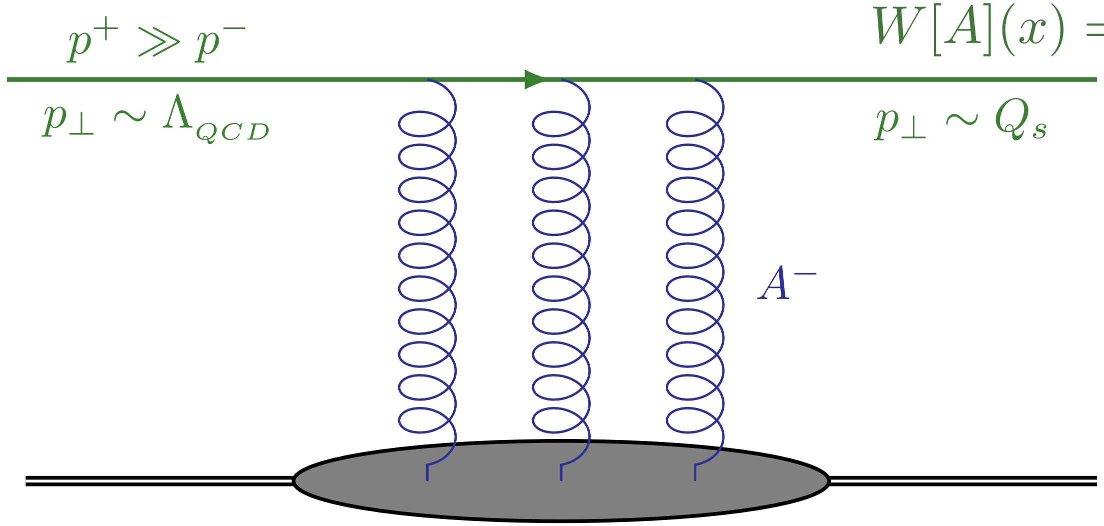
A. Kovner and M. Lublinsky,Phys.Rev., D84:094011, 2011.
A. Dumitru and A. V. Giannini, Nucl. Phys., A933:212–228, 2015.
A. Dumitru, L. McLerran, and V. Skokov, Phys. Lett., B743:134–137, 2015.
A. Dumitru and V. Skokov, Phys. Rev., D91(7):074006, 2015.
T. Lappi, B. Schenke, S. Schlichting, and R. Venugopalan. JHEP, 01:061, 2016.

PARTICLE PRODUCTION IN PROTON-NUCLEUS COLLISIONS

The hybrid framework

- Consider eikonal scattering of quark from a dense target

$$W[A](x) = \mathcal{P}\exp\left[ig\int dz^+ A_a^-(z^+, x)\right]$$



- Differential cross-section for quark scattering on a nucleus

$$\frac{dN}{d^2p} \simeq \frac{1}{\pi B_p}\int_{x\bar{x}} e^{-(x^2+\bar{x}^2)/2B_p}\left\langle \frac{1}{N_c}\mathrm{Tr}\left[W(x)W^\dagger(\bar{x})\right]\right\rangle e^{ip\cdot(x-\bar{x})}$$

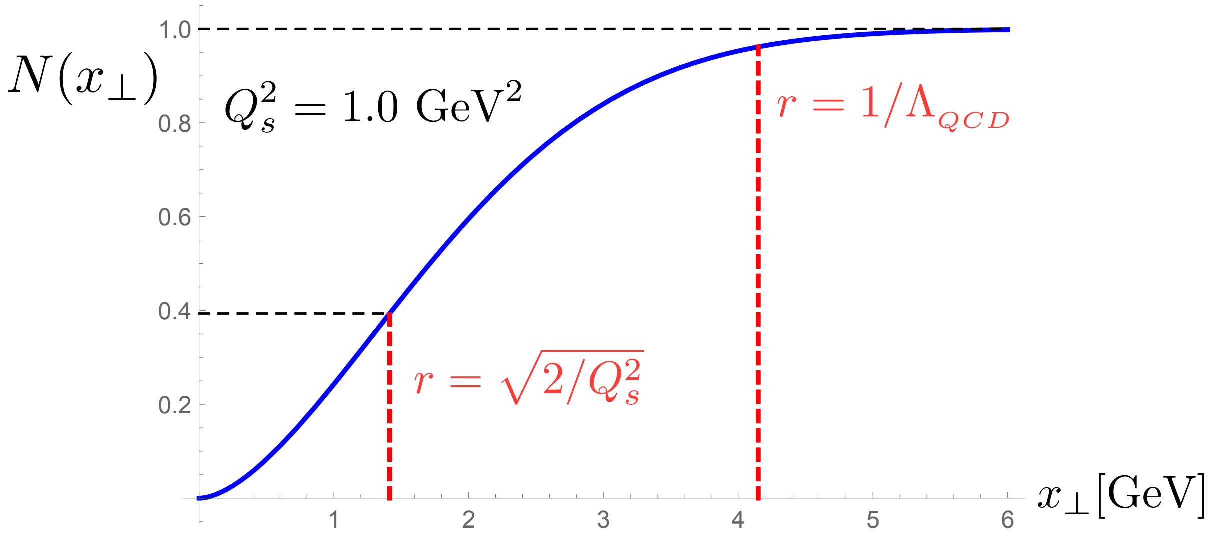
A. Dumitru and J. Jalilian-Marian, Phys. Rev. Lett., 89:022301, 2002.
J. D. Bjorken, J. B. Kogut, and D. E. Soper, Phys. Rev., D3:1382, 1971.

Multiple-scattering in the CGC

- Dipole correlator is evaluated in the MV model whereby averaging over the color field of the nucleus

$$g^2 \left\langle A_a^-(x) A_b^-(y) \right\rangle = \delta^{ab} L_{xy} \qquad L_{xy} = -\frac{g^4 \mu^2}{16\pi} |x-y|^2 \ln \frac{1}{\Lambda |x-y|}$$

- results in $D(x,\bar{x}) = \dfrac{1}{N_c} \mathrm{Tr}\left[W(x) W^\dagger(\bar{x})\right] = \exp(C_F L_{x\bar{x}})$



Multi-particle production

- Single quark scattering:

$$\frac{dN}{d^2p} \simeq \frac{1}{\pi B_p} \int_{x\bar{x}} e^{-(x^2+\bar{x}^2)/2B_p} \left\langle \frac{1}{N_c} \mathrm{Tr}\left[W(x)W^\dagger(\bar{x})\right] \right\rangle e^{ip\cdot(x-\bar{x})}$$

- Two quark scattering:

$$\frac{d^2N}{d^2p_1 d^2p_2} \simeq \frac{1}{(\pi B_p)^2} \int_{x\bar{x}y\bar{y}} e^{-(x^2+\bar{x}^2)/2B_p}\, e^{-(y^2+\bar{y}^2)/2B_p} e^{ip_1\cdot(x-\bar{x})} e^{ip_2\cdot(y-\bar{y})}$$
$$\times \left\langle \frac{1}{N_c} \mathrm{Tr}\left[W(x)W^\dagger(\bar{x})\right] \frac{1}{N_c} \mathrm{Tr}\left[W(y)W^\dagger(\bar{y})\right] \right\rangle$$

- Four quark scattering:

$$d^4N \sim \int \langle \mathrm{Tr}\left[W(w)W^\dagger(\bar{w})\right] \mathrm{Tr}\left[W(x)W^\dagger(\bar{x})\right] \mathrm{Tr}\left[W(y)W^\dagger(\bar{y})\right] \mathrm{Tr}\left[W(z)W^\dagger(\bar{z})\right] \rangle$$

BEYOND THE DIPOLE

n-point correlators in the MV model

Multiple-scattering in the CGC

- Start with the dipole scattering matrix: $\langle D(x,\bar{x})\rangle_W = \frac{1}{N_c}\left\langle \mathrm{Tr}\left[W(x)W^\dagger(\bar{x})\right]\right\rangle$

- Expand out the last slice in rapidity:

$$W(x) \equiv \mathcal{P}\exp\left[ig\int dz^+ A_a^-(z^+,x)\right] \simeq V(x)\left[1 + igA_a^-(\xi,x)T^a + \cdots\right]$$

- Take all two-point functions:

$$\langle D(x,\bar{x})\rangle_W \simeq \langle D(x,\bar{x})\rangle_V + \underbrace{g^2\left\langle A_a^-(x)A_b^-(\bar{x})\right\rangle}_{\delta^{ab}L_{x\bar{x}}}\frac{1}{N_c}\underbrace{\mathrm{Tr}\left[V(x)T^aT^bV^\dagger(\bar{x})\right]}_{\frac{\delta^{ab}}{2}\mathrm{Tr}\left[V(x)V^\dagger(\bar{x})\right]}$$

- And re-exponentiate:

$$\langle D(x,\bar{x})\rangle_W = \exp\left(C_F\, L_{x\bar{x}}\right)$$

- As before expand out all Wilson lines:

$$W(x) \equiv \mathcal{P}\exp\left[ig\int dz^+ A_a^-(z^+,x)\right] \simeq V(x)\left[1 + igA_a^-(\xi,x)T^a + \cdots\right]$$

- Four point function: $\langle D_{x\bar{x}} D_{y\bar{y}}\rangle_W \simeq \alpha_{x\bar{x}y\bar{y}}\langle D_{x\bar{x}} D_{y\bar{y}}\rangle_V + \beta_{xy\bar{x}\bar{y}}\langle Q_{x\bar{y}y\bar{x}}\rangle_V$

$$\begin{pmatrix} \langle D_{x\bar{x}} D_{y\bar{y}}\rangle \\ \langle Q_{x\bar{y}y\bar{x}}\rangle \end{pmatrix}_W = \begin{pmatrix} \alpha_{x\bar{x}y\bar{y}} & \beta_{xy\bar{x}\bar{y}} \\ \beta_{xy\bar{y}\bar{x}} & \alpha_{x\bar{y}y\bar{x}} \end{pmatrix} \begin{pmatrix} \langle D_{x\bar{x}} D_{y\bar{y}}\rangle \\ \langle Q_{x\bar{y}y\bar{x}}\rangle \end{pmatrix}_V$$

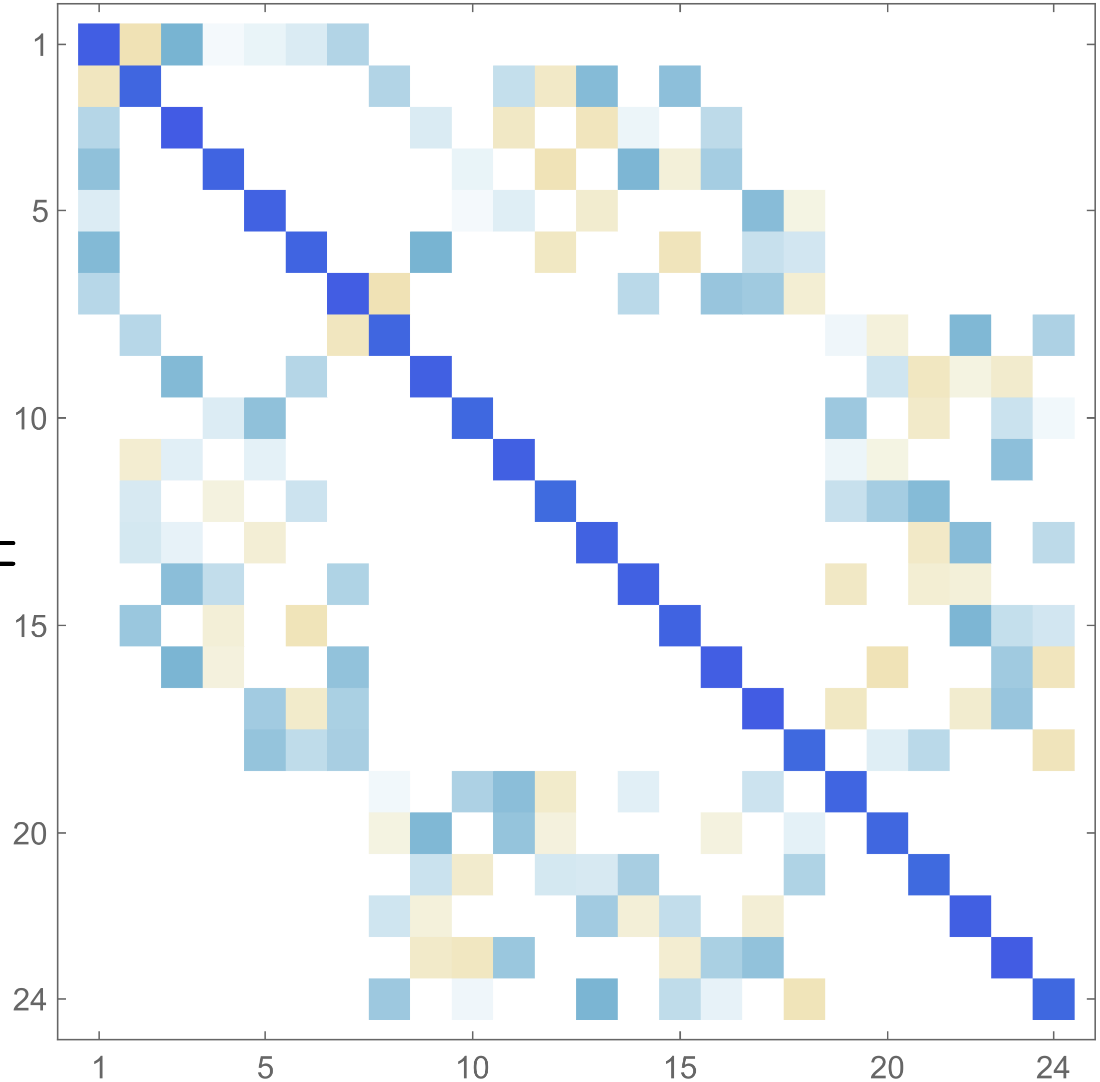
- Eight point function: $\langle D_{w\bar{w}} D_{x\bar{x}} D_{y\bar{y}} D_{z\bar{z}}\rangle_W \simeq \alpha_{w\bar{w}x\bar{x}y\bar{y}z\bar{z}}\langle D_{w\bar{w}} D_{x\bar{x}} D_{y\bar{y}} D_{z\bar{z}}\rangle_V$

$$\begin{aligned} &+\beta_{wx\bar{w}\bar{x}}\langle Q_{x\bar{w}w\bar{x}} D_{y\bar{y}} D_{z\bar{z}}\rangle_V + \beta_{wy\bar{w}\bar{y}}\langle Q_{y\bar{w}w\bar{y}} D_{x\bar{x}} D_{z\bar{z}}\rangle_V \\ &+\beta_{wz\bar{w}\bar{z}}\langle Q_{z\bar{w}w\bar{z}} D_{x\bar{x}} D_{y\bar{y}}\rangle_V + \beta_{xy\bar{x}\bar{y}}\langle Q_{y\bar{x}x\bar{y}} D_{w\bar{w}} D_{z\bar{z}}\rangle_V \\ &+\beta_{xz\bar{x}\bar{z}}\langle Q_{z\bar{x}x\bar{z}} D_{w\bar{w}} D_{y\bar{y}}\rangle_V + \beta_{yz\bar{y}\bar{z}}\langle Q_{z\bar{y}y\bar{z}} D_{w\bar{w}} D_{x\bar{x}}\rangle_V \end{aligned}$$

Kovner and U. A. Wiedemann. Phys. Rev., D64:114002, 2001.
H. Fujii.. Nucl. Phys., A709:236–250, 2002.
J. P. Blaizot, F. Gelis, and R. Venugopalan. Nucl. Phys., A743:57–91, 2004.
F. Dominguez, C. Marquet, and B. Wu, Nucl. Phys., A823:99–119, 2009.

Correlator of eight Wilson lines

$$\begin{bmatrix} \langle D_{w\bar{w}} D_{x\bar{x}} D_{y\bar{y}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{w}x\bar{x}} D_{y\bar{z}} D_{z\bar{y}} \rangle \\ \langle Q_{w\bar{w}x\bar{y}} D_{y\bar{x}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{w}x\bar{y}} D_{y\bar{z}} D_{z\bar{x}} \rangle \\ \langle Q_{w\bar{w}x\bar{z}} D_{y\bar{x}} D_{z\bar{y}} \rangle \\ \langle Q_{w\bar{w}x\bar{z}} D_{y\bar{y}} D_{z\bar{x}} \rangle \\ \langle Q_{w\bar{x}x\bar{w}} D_{y\bar{y}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{x}x\bar{w}} Q_{y\bar{z}z\bar{y}} \rangle \\ \langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{w}z\bar{z}} \rangle \\ \langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{z}z\bar{w}} \rangle \\ \langle S_{w\bar{x}x\bar{z}y\bar{w}} D_{z\bar{y}} \rangle \\ \langle S_{w\bar{x}x\bar{z}y\bar{y}} D_{z\bar{w}} \rangle \\ \langle S_{w\bar{y}x\bar{w}y\bar{x}} D_{z\bar{z}} \rangle \\ \langle S_{w\bar{y}x\bar{w}y\bar{z}} D_{z\bar{x}} \rangle \\ \langle S_{w\bar{y}x\bar{x}y\bar{w}} D_{z\bar{z}} \rangle \\ \langle S_{w\bar{y}x\bar{x}y\bar{z}} D_{z\bar{w}} \rangle \\ \langle S_{w\bar{y}x\bar{z}y\bar{w}} D_{z\bar{x}} \rangle \\ \langle S_{w\bar{y}x\bar{z}y\bar{x}} D_{z\bar{w}} \rangle \\ \langle O_{w\bar{z}x\bar{w}y\bar{x}z\bar{y}} \rangle \\ \langle O_{w\bar{z}x\bar{w}y\bar{y}z\bar{x}} \rangle \\ \langle O_{w\bar{z}x\bar{x}y\bar{w}z\bar{y}} \rangle \\ \langle O_{w\bar{z}x\bar{x}y\bar{y}z\bar{w}} \rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{w}z\bar{x}} \rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{x}z\bar{w}} \rangle \end{bmatrix}_W =$$



$$\begin{bmatrix} \langle D_{w\bar{w}} D_{x\bar{x}} D_{y\bar{y}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{w}x\bar{x}} D_{y\bar{z}} D_{z\bar{y}} \rangle \\ \langle Q_{w\bar{w}x\bar{y}} D_{y\bar{x}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{w}x\bar{y}} D_{y\bar{z}} D_{z\bar{x}} \rangle \\ \langle Q_{w\bar{w}x\bar{z}} D_{y\bar{x}} D_{z\bar{y}} \rangle \\ \langle Q_{w\bar{w}x\bar{z}} D_{y\bar{y}} D_{z\bar{x}} \rangle \\ \langle Q_{w\bar{x}x\bar{w}} D_{y\bar{y}} D_{z\bar{z}} \rangle \\ \langle Q_{w\bar{x}x\bar{w}} Q_{y\bar{z}z\bar{y}} \rangle \\ \langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{w}z\bar{z}} \rangle \\ \langle Q_{w\bar{x}x\bar{y}} Q_{y\bar{z}z\bar{w}} \rangle \\ \langle S_{w\bar{x}x\bar{z}y\bar{w}} D_{z\bar{y}} \rangle \\ \langle S_{w\bar{x}x\bar{z}y\bar{y}} D_{z\bar{w}} \rangle \\ \langle S_{w\bar{y}x\bar{w}y\bar{x}} D_{z\bar{z}} \rangle \\ \langle S_{w\bar{y}x\bar{w}y\bar{z}} D_{z\bar{x}} \rangle \\ \langle S_{w\bar{y}x\bar{x}y\bar{w}} D_{z\bar{z}} \rangle \\ \langle S_{w\bar{y}x\bar{x}y\bar{z}} D_{z\bar{w}} \rangle \\ \langle S_{w\bar{y}x\bar{z}y\bar{w}} D_{z\bar{x}} \rangle \\ \langle S_{w\bar{y}x\bar{z}y\bar{x}} D_{z\bar{w}} \rangle \\ \langle O_{w\bar{z}x\bar{w}y\bar{x}z\bar{y}} \rangle \\ \langle O_{w\bar{z}x\bar{w}y\bar{y}z\bar{x}} \rangle \\ \langle O_{w\bar{z}x\bar{x}y\bar{w}z\bar{y}} \rangle \\ \langle O_{w\bar{z}x\bar{x}y\bar{y}z\bar{w}} \rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{w}z\bar{x}} \rangle \\ \langle O_{w\bar{z}x\bar{y}y\bar{x}z\bar{w}} \rangle \end{bmatrix}_V$$

RESULTS

Quantities of interest

- Particle spectra:

$$\frac{d^n N}{d^2 p \cdots} \sim \int \overbrace{e^{-(x^2+\bar{x}^2)/2B_p \cdots}}^{B_p = 4\ \mathrm{GeV}^{-2}} \left\langle \overbrace{\frac{1}{N_c} \mathrm{Tr}\left[W(x) W^\dagger(\bar{x})\right] \cdots}^{Q_s^2 \sim 2\ \mathrm{GeV}^2} \right\rangle e^{ip\cdot(x-\bar{x})\cdots}$$

- Two-particle cumulants:

$$c_n\{2\} = \frac{\kappa_n\{2\}}{\kappa_0\{2\}}, \quad v_n\{2\} = \sqrt{c_n\{2\}}$$

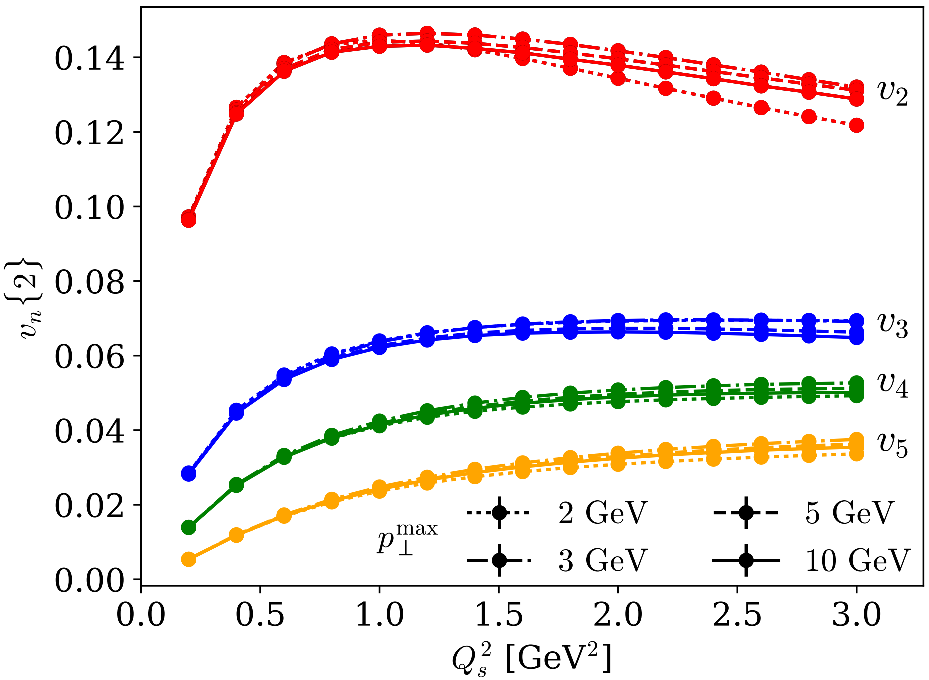
$$\kappa_n\{2\} \equiv \int d^2p_1 d^2p_2\ \cos\left[n\left(\phi_{p_1} - \phi_{p_2}\right)\right] \frac{d^2 N}{d^2p_1 d^2p_2}$$

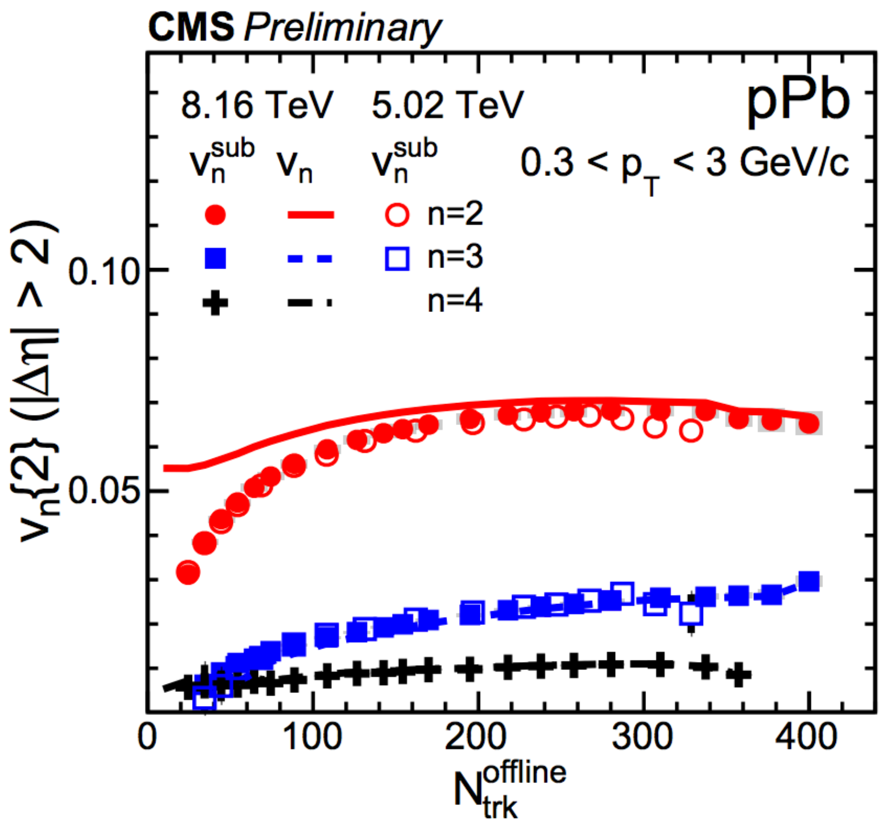
- Four-particle cumulants:

$$\kappa_n\{4\} \equiv \int d^2p_1 d^2p_2 d^2p_3 d^2p_4\ \cos\left[n\left(\phi_{p_1} + \phi_{p_2} - \phi_{p_3} - \phi_{p_4}\right)\right] \frac{d^4 N}{d^2p_1 d^2p_2 d^2p_3 d^2p_4}$$

$$c_n\{4\} = \frac{\kappa_n\{4\}}{\kappa_0\{4\}} - 2\left(\frac{\kappa_n\{2\}}{\kappa_0\{0\}}\right)^2, \qquad v_n\{4\} = \left(-c_n\{4\}\right)^{1/4}$$

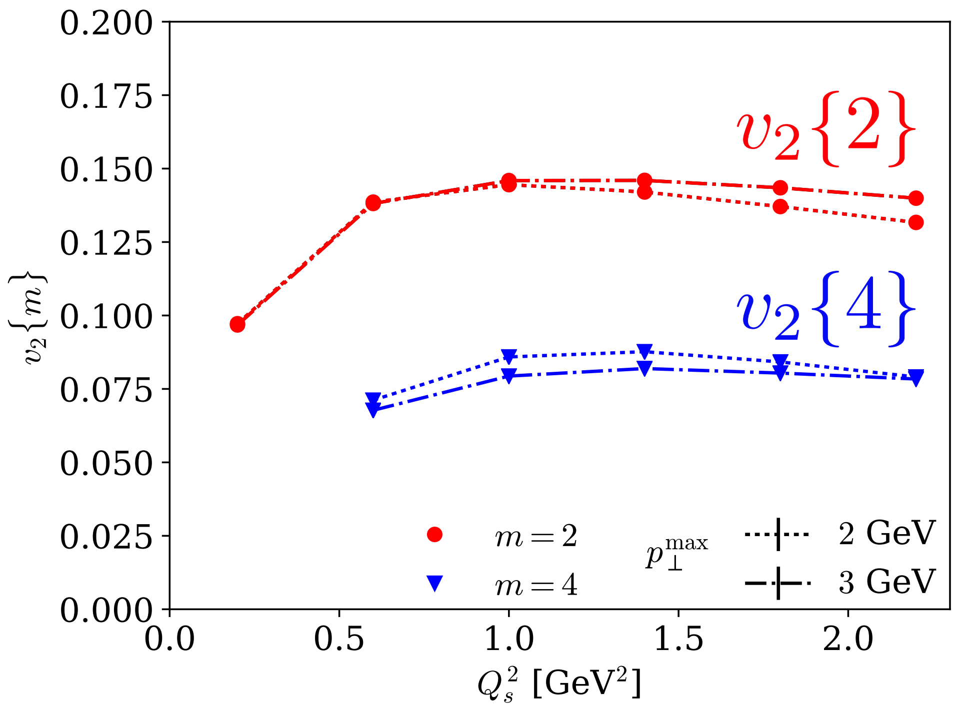
Integrated elliptic flow

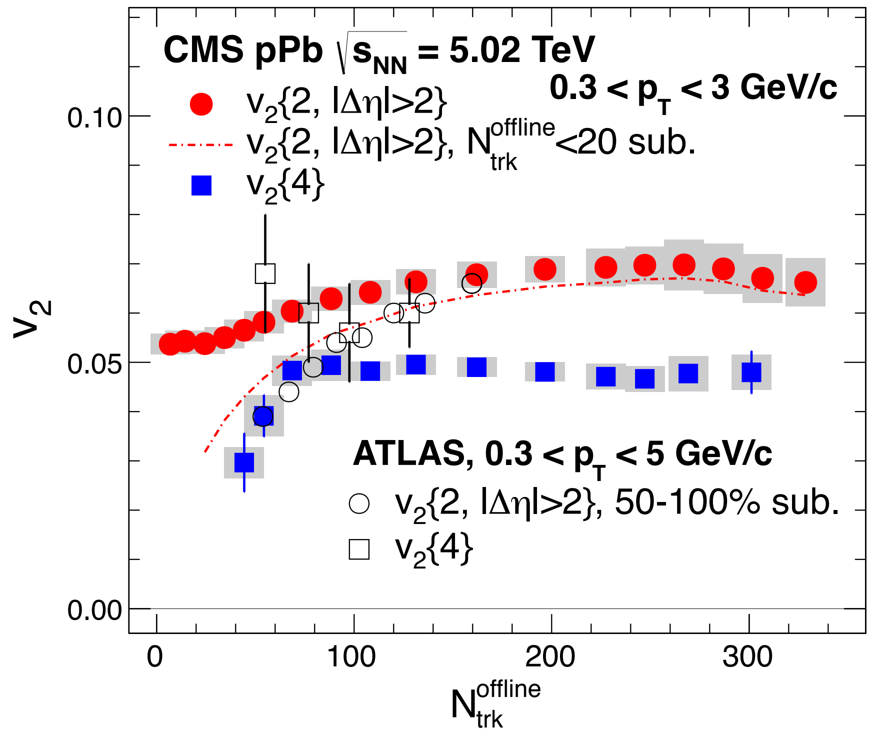




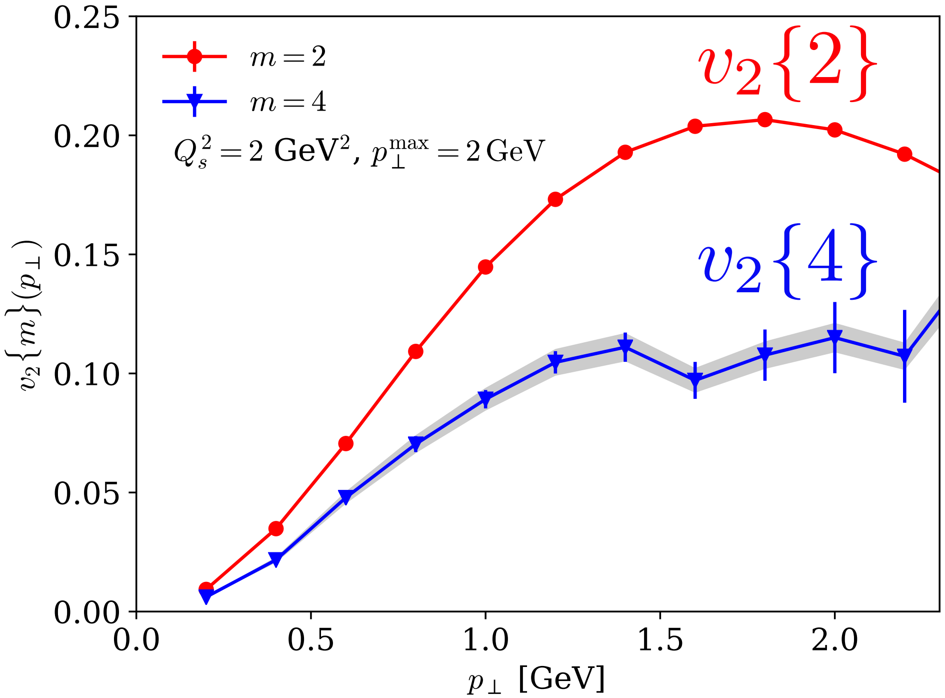
T. Lappi, Phys. Lett., B744:315–319, 2015.
T. Lappi, B. Schenke, S. Schlichting, and R. Venugopalan. JHEP, 01:061, 2016.

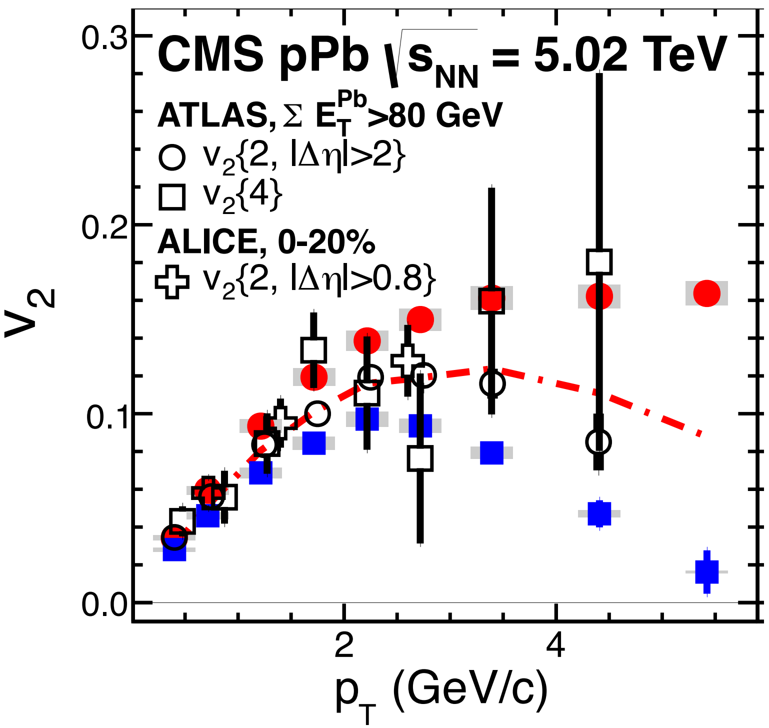
Integrated elliptic flow





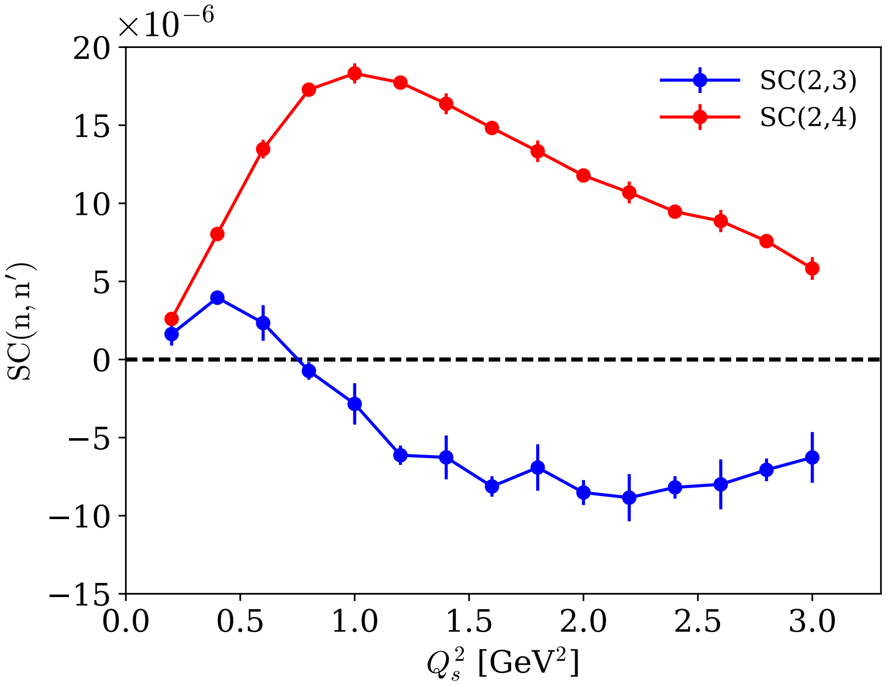
Differential flow

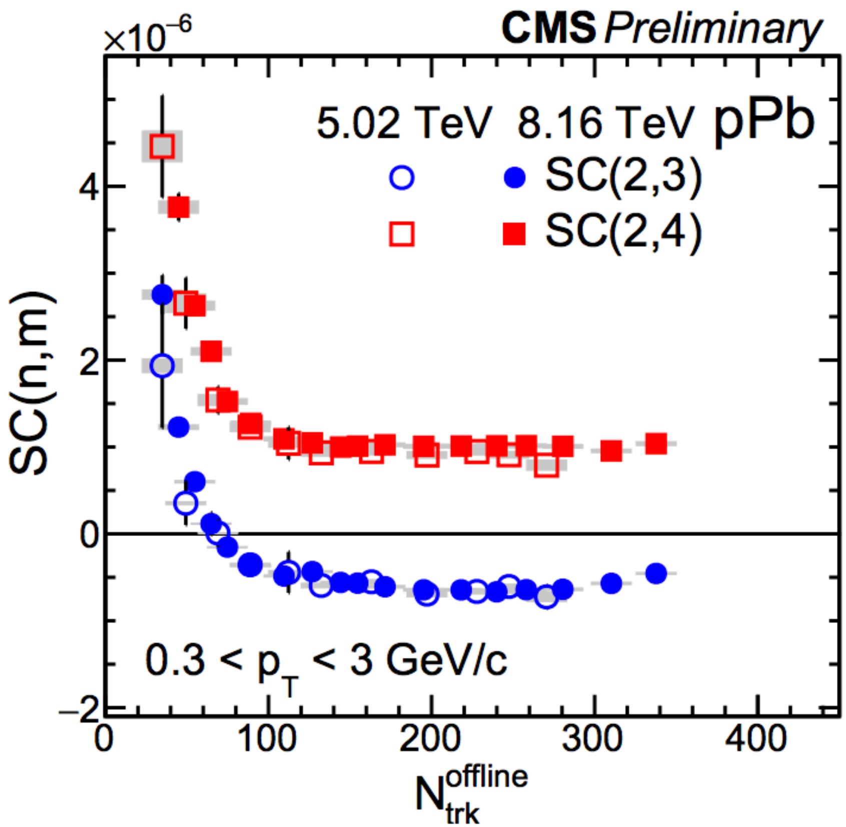




Symmetric cumulants

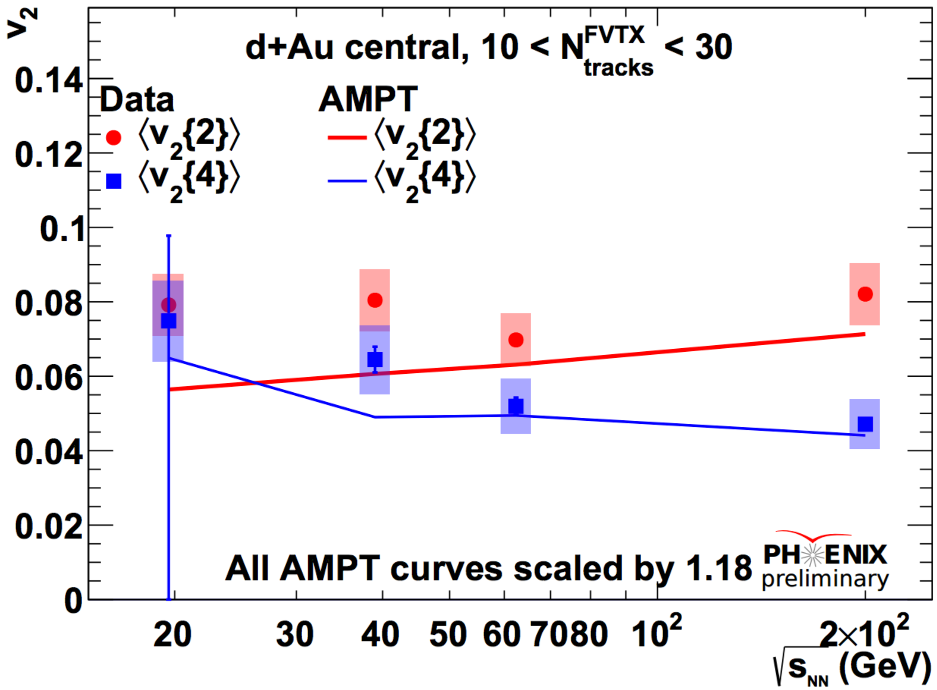
$$SC(m, n) = \langle v_m^2 v_n^2 \rangle - \langle v_m^2 \rangle \langle v_n^2 \rangle$$

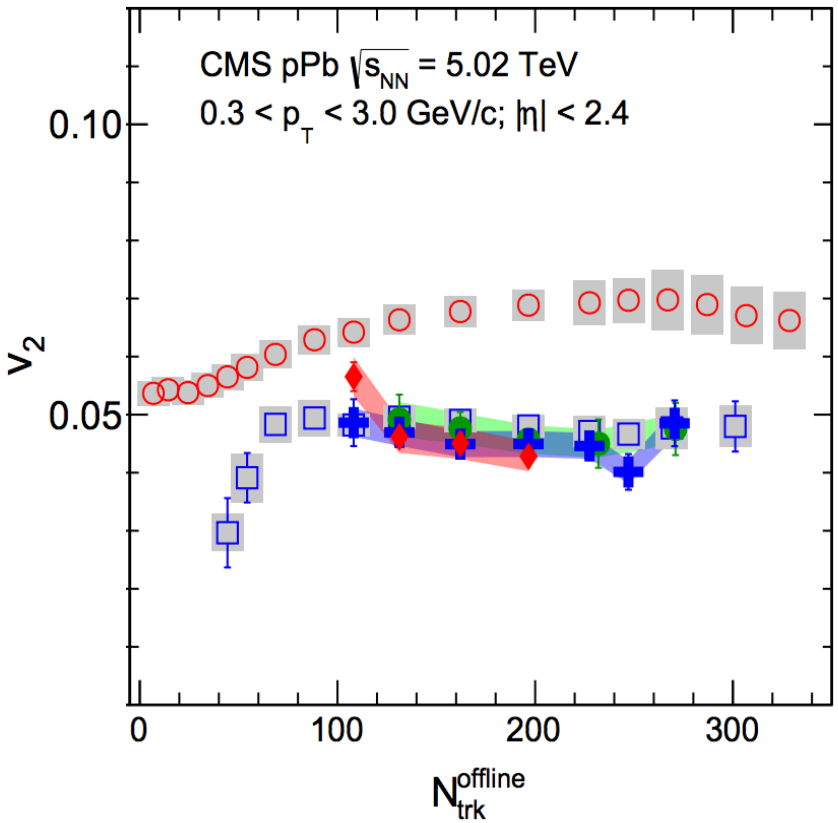




A random event having $v_2 > \langle v_2 \rangle$ will be more likely to have a $v_4 > \langle v_4 \rangle$.

Further thoughts…



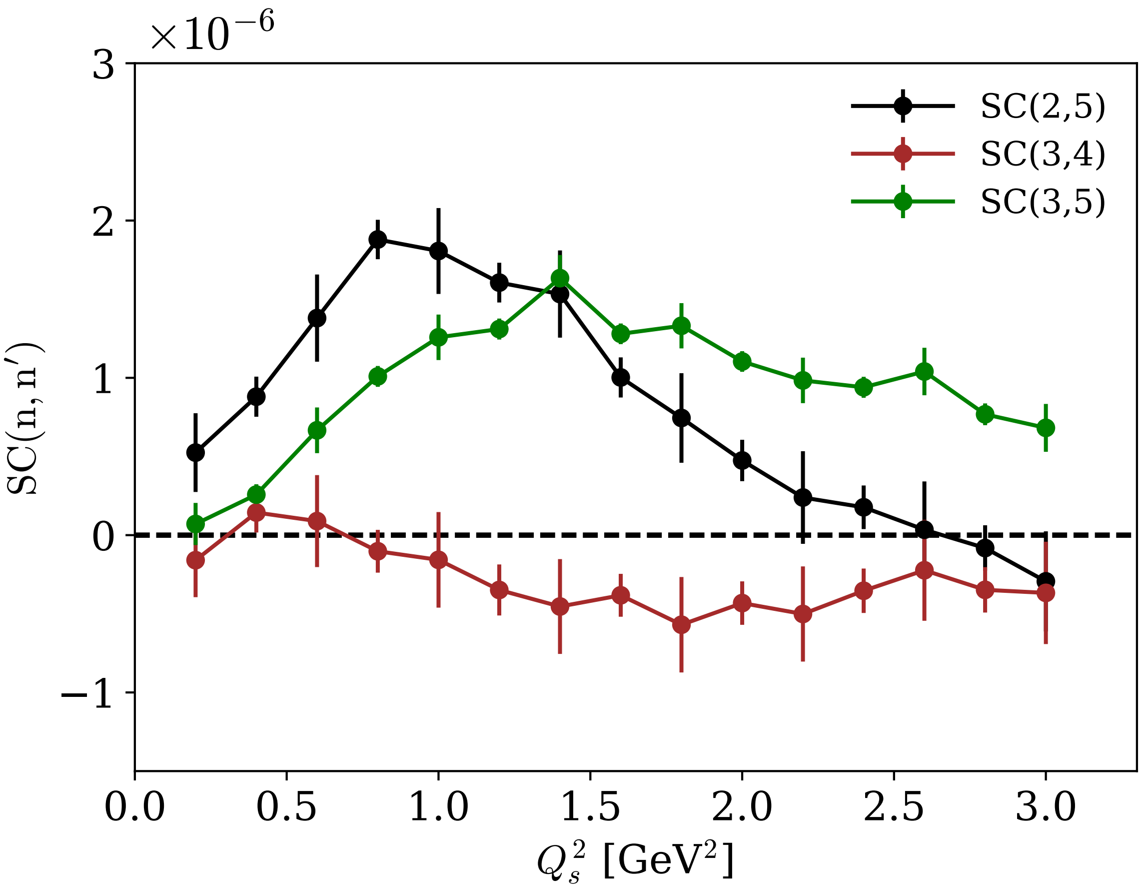


Conclusions

- Developed the machinery to compute correlators of six and eight fundamental Wilson lines at finite N_c
- Modulo many caveats qualitative features of the measured four-particle correlations are understood by non-linear classical fields

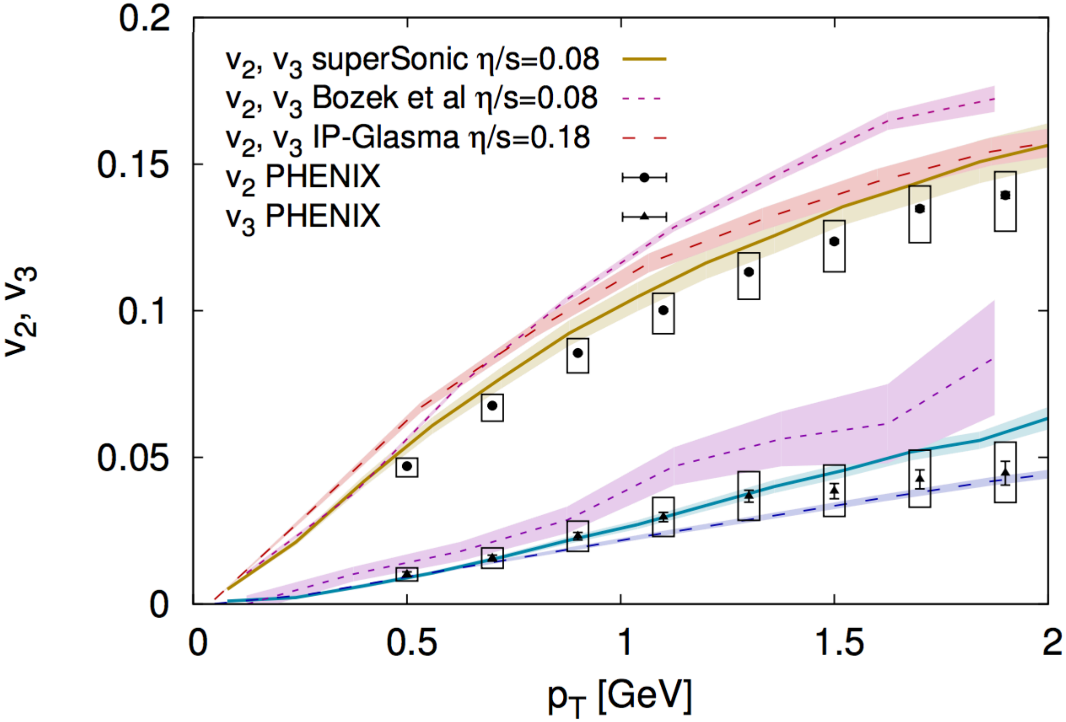
BACKUP

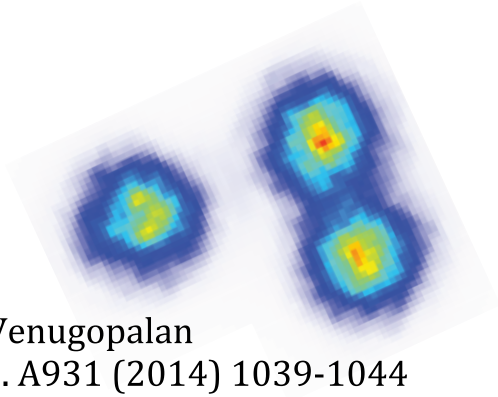
Symmetric cumulants



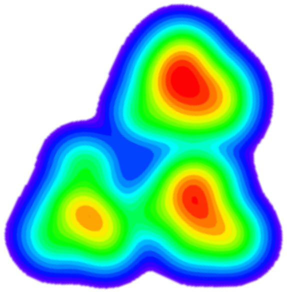
Background II: Collectivity

A. Adare et al. (PHENIX Collaboration)
Phys. Rev. Lett. 115, 142301 (2015)

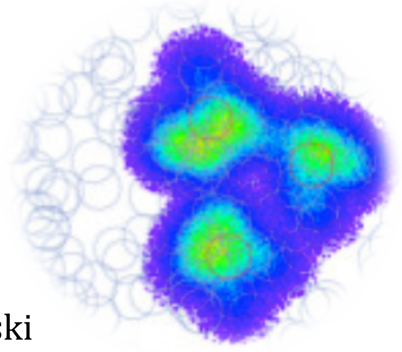




Schenke, Venugopalan
Nucl. Phys. A931 (2014) 1039-1044



J.L. Nagle, et al.
Phys. Rev. Lett. 113, 112301



Bożek, Broniowski
Physics Letters B 747 (2015) 135–138