Particle production in pA collisions beyond leading order

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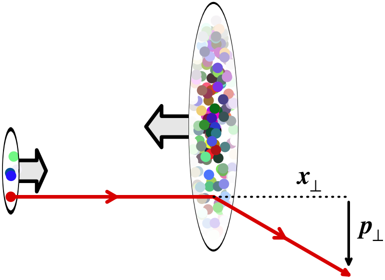
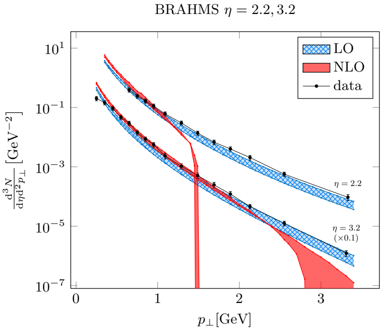
w/ A.H. Mueller and D.N. Triantafyllopoulos, arXiv:1608.05293



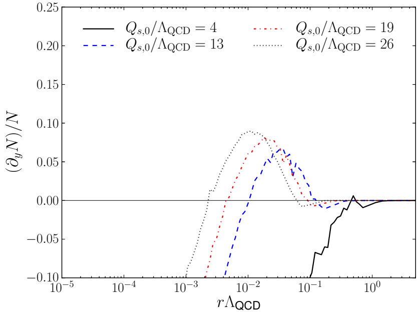


Introduction

- Particle production in pp and pA collisions at forward rapidities explores the physics of high gluon densities at small-x
  - non-linear phenomena: gluon saturation, multiple scattering
  - resummations based on the eikonal approximation (Wilson lines)
  - non-linear evolution equations: BK, B-JIMWLK
- Effective theory derived in pQCD: Color Glass Condensate
- The CGC formalism is now being promoted to NLO
  - NLO versions for the BK and B-JIMWLK equations *(Balitsky and Chirilli, 2008, 2013; Kovner, Lublinsky, and Mulian, 2013)*
  - NLO impact factor for particle production in pA collisions *(Chirilli, Xiao, and Yuan, 2012; Mueller and Munier, 2012)*
- But the strict NLO approximations turned out to be problematic

NLO BK evolution

- "Negative growth" of the dipole scattering amplitude



Lappi, Mäntysaari, arXiv:1502.02400

- Not really a surprise
  - similar problems for NLO BFKL
  - large transverse logarithms
  - collinear resummations
  - Mellin representation

(Salam, Ciafaloni, Colferai, Stasto, 98-03; Altarelli, Ball, Forte, 00-03)

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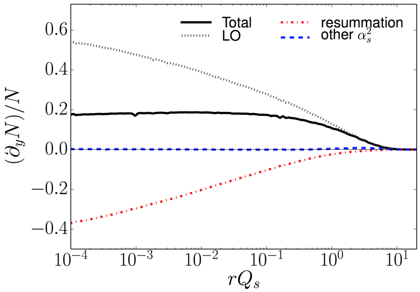

Lappi, Mäntysaari, arXiv:1601.06598

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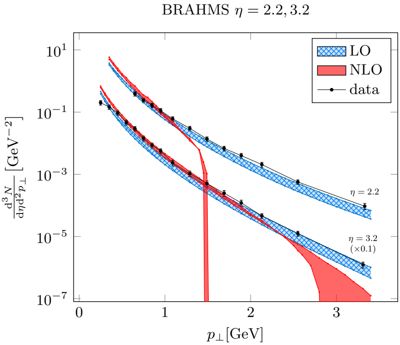
- Collinear improvement for NLO BK (transverse coordinates)

  (E.I., J. Madrigal, A. Mueller, G. Soyez, and D. Triantafyllopoulos, 2015)

- Evolution becomes stable with promising phenomenology
  - excellents fits to DIS *(Iancu et al, 2015; Albacete, 2015)*

Particle production in d+Au collisions (RHIC)

- Very good agreement at low $p_\perp$ ☺ ... but negative at larger $p_\perp$ ☹



Stasto, Xiao, and Zaslavsky, arXiv:1307.4057

- Is this a real problem ?
  - "small-x resummations do not apply at large $p_\perp$"
  - but $p_\perp \sim Q_s$ is not that large !
- Likely related to the rapidity subtraction in NLO impact factor

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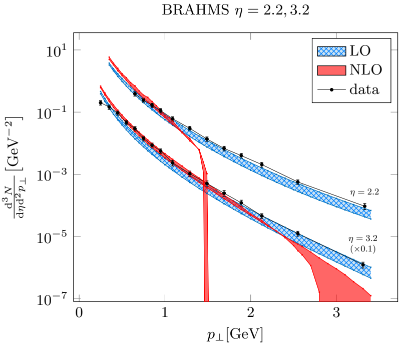

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- Various proposals which alleviate the problem (pushed to higher $p_\perp$)
  - *Kang, Vitev, and Xing, arXiv:1403.5221*
  - *Altinoluk, Armesto, Beuf, Kovner, and Lublinsky, arXiv:1411.2869*
  - *Ducloué, Lappi, and Zhu, arXiv:1604.00225*

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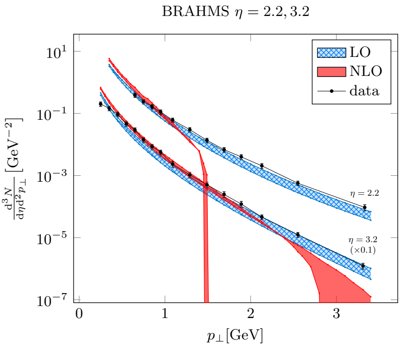

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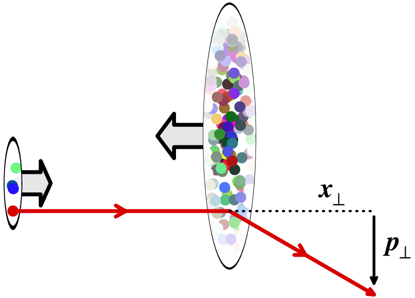
- A reorganization of the perturbative expansion which avoids the rapidity subtraction *(E.I., A. Mueller and D. Triantafyllopoulos, 2016)*
- Sensible numerical results (positive cross-section)... and a new puzzle *(Ducloué, Lappi, and Zhu, arXiv:1703.04962)*

Forward quark production in pA collisions

- A quark initially collinear with the proton acquires a transverse momentum $p_\perp$ via multiple scattering off the saturated gluons



$$x_p \equiv \frac{p^+}{q^+} = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{\eta}$$

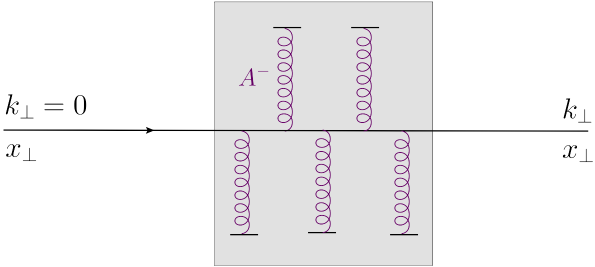
$$X_g \equiv \frac{p^-}{P^-} = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{-\eta}$$

$X_g \ll x_p$ when $\eta > 0$

  - η : quark rapidity in the COM frame
  - x_p : longitudinal fraction of the quark in the proton
  - X_g : longitudinal fraction of the gluon in the target

- Gluons in the nucleus have a typical transverse momentum $k_\perp \sim Q_s(X_g)$

- Eikonal approximation $\Longrightarrow$ the transverse coordinate representation

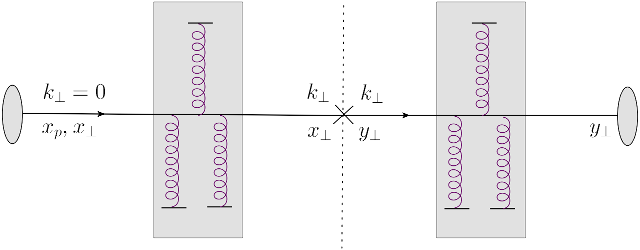


Amplitude: $$\mathcal{M}_{ij}(\boldsymbol{k}_\perp) \equiv \int \mathrm{d}^2\boldsymbol{x}_\perp\, \mathrm{e}^{-i\boldsymbol{x}_\perp\cdot\boldsymbol{k}_\perp}\, V_{ij}(\boldsymbol{x}_\perp)$$

Wilson line: $$V(\boldsymbol{x}_\perp) = \mathrm{P}\exp\left\{ ig\int \mathrm{d}x^+ A_a^-(x^+, \boldsymbol{x}_\perp)t^a \right\}$$

- A_a^- : color field representing small-x gluons in the nucleus

Multiple scattering



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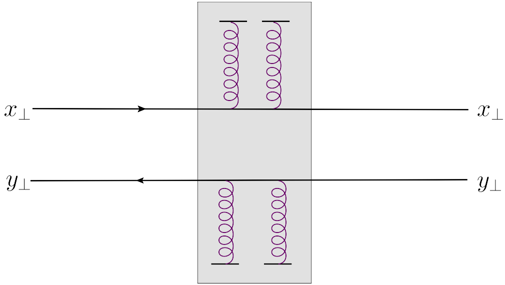
Cross-section: $$\frac{\mathrm{d}\sigma}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}_\perp} \simeq x_p q(x_p, Q^2)\, \frac{1}{N_c} \left\langle \sum_{ij} |\mathcal{M}_{ij}(\boldsymbol{k}_\perp)|^2 \right\rangle_{X_g}$$

- Average over the color fields A^- in the target (CGC)
- Two Wilson lines at different transverse coordinates, traced over color

Dipole picture

- Equivalently: the elastic S-matrix for a $q\bar{q}$ color dipole



$$S(\boldsymbol{x},\boldsymbol{y};X_g) \equiv \frac{1}{N_c}\left\langle \mathrm{tr}\left[V(\boldsymbol{x})V^\dagger(\boldsymbol{y})\right]\right\rangle_{X_g}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}} \simeq x_p q(x_p)\int_{\boldsymbol{x},\boldsymbol{y}} \mathrm{e}^{-\mathrm{i}(\boldsymbol{x}-\boldsymbol{y})\cdot\boldsymbol{k}}\, S(\boldsymbol{x},\boldsymbol{y};X_g)$$

- The Fourier transform $\mathcal{S}(\boldsymbol{k},X_g)$: "unintegrated gluon distribution"

Dipole picture

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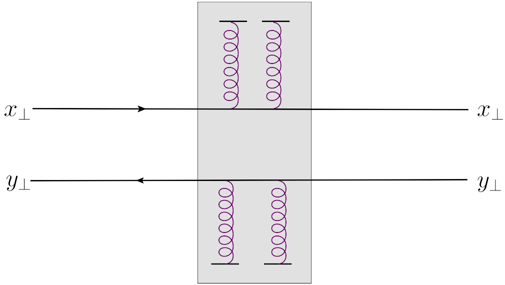

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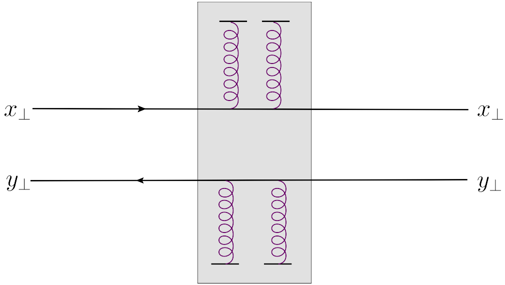
- 'Hybrid factorization': collinear fact. for p & CGC fact. for A *(Dumitru, Hayashigaki, and Jalilian-Marian, arXiv:hep-ph/0506308)*.

Dipole picture

- Equivalently: the elastic S-matrix for a $q\bar{q}$ color dipole



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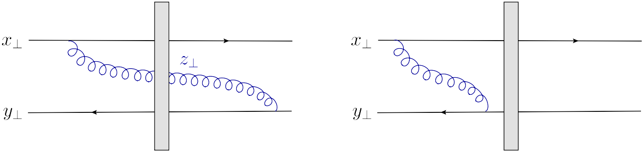
- The dipole picture is preserved by the high-energy evolution up to NLO *(Kovchegov and Tuchin, 2002; Mueller and Munier, 2012)*

BK equation *(leading order)*

- Probability $\sim \alpha_s \ln\frac{1}{x}$ to radiate a soft gluon with $x \equiv \frac{p^+}{k^+} \ll 1$



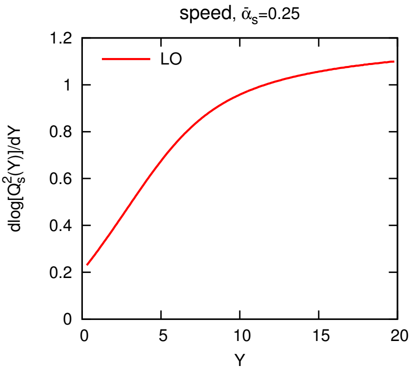
- When $\alpha_s \ln\frac{1}{x} \sim 1$: resummation to all orders (part of LO)
- Evolution equation for the dipole S–matrix $S_{\boldsymbol{xy}}(Y)$ with $Y \equiv \ln(1/x)$

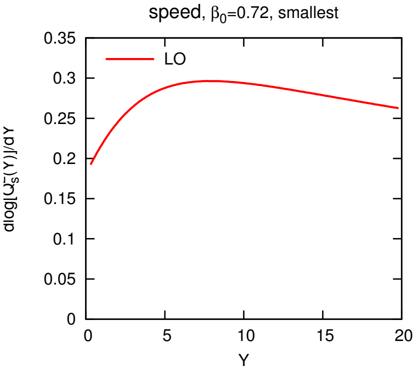
$$\frac{\partial S_{\boldsymbol{xy}}}{\partial Y} = \frac{\bar{\alpha}_s}{2\pi}\int \mathrm{d}^2\boldsymbol{z}\,\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2}\left[S_{\boldsymbol{xz}}S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}\right]$$

  - dipole kernel: probability for the dipole to emit a soft gluon at $\boldsymbol{z}$
  - large-N_c approximation to the Balitsky-JIMWLK hierarchy
  - saturation momentum $Q_s(Y)$: $S(r, Y) = 0.5$ when $r = 1/Q_s(Y)$

Adding running coupling: rcBK

- The evolution speed: saturation exponent $\lambda_s \equiv \mathrm{d}\ln Q_s^2/\mathrm{d}Y$
- At LO, $\lambda_s \sim 1$ is way too large: $\lambda_{\mathrm{HERA}} = 0.2 \div 0.3$

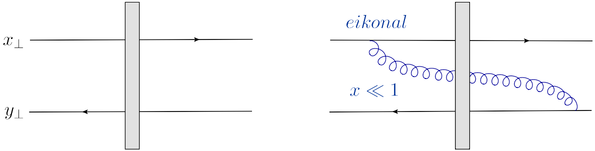




- Including running coupling dramatically slows down the evolution
- ... but there are other, equally important, NLO corrections !

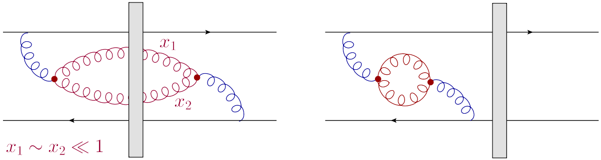
Particle production beyond leading order

- LO approximation: any number $n \geq 0$ of soft emissions $\Longrightarrow$ $(\alpha_s Y)^n$



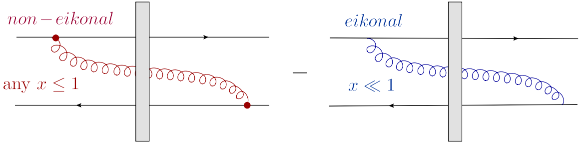
$= \mathcal{O}(\alpha_s Y)$

- NLO corrections to the evolution: 2 soft gluons, with similar values of x



$= \mathcal{O}(\alpha_s^2 Y)$

- NLO correction to impact factor: the first gluon can be hard

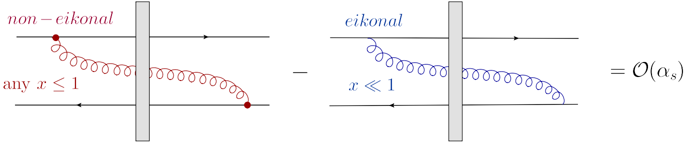


$= \mathcal{O}(\alpha_s)$

Towards NLO factorization in pA

- The first gluon contributes both to the evolution (when $x \ll 1$) and to the NLO impact factor (generic x) : How to avoid over counting ?

- $k_\perp$-factorization : use a 'rapidity subtraction'



  - the method used by *Chirilli, Xiao, and Yuan (arXiv:1203.6139)*
  - leads to a negative cross-section at semi-hard $k_\perp$

- Our proposal *(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)*
  - separate the first gluon emission from the evolution and compute it with the exact kinematics

- The integral representation of the BK equation is useful in that sense

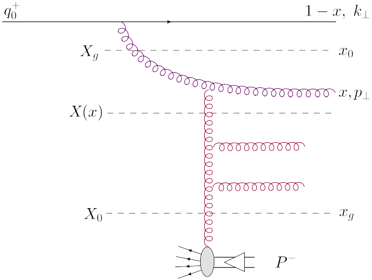
LO BK evolution in integral form

$$\left.\frac{\mathrm{d}N}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}}\right|_{\mathrm{LO}} = x_p q(x_p)\,\mathcal{S}(\boldsymbol{k},X_g)\,,\qquad \mathcal{S}(\boldsymbol{k},X_g) = \int \mathrm{d}^2\boldsymbol{r}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}S(\boldsymbol{r},X_g)$$

- $S(\boldsymbol{r},X_g)$ is the solution to the LO BK equation and can be written as

$$S_{\boldsymbol{xy}}(X_g) = S_{\boldsymbol{xy}}(X_0) + \bar{\alpha}_s\int_{X_g}^1\frac{\mathrm{d}x}{x}\int_{\boldsymbol{z}}\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2}\Big[S_{\boldsymbol{xz}}S_{\boldsymbol{zy}}-S_{\boldsymbol{xy}}\Big](X(x))$$

- Except for the first gluon, the evolution is associated with the nucleus



- $X(x)$: energy fraction in the target

$$X(x) \simeq \frac{k_\perp^2}{xs} = \frac{X_g}{x}$$

$$X \leq 1 \Longrightarrow x \geq X_g$$

LO BK evolution in integral form

$$\left.\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}}\right|_{\rm LO} = x_p q(x_p)\, \mathcal{S}(\boldsymbol{k}, X_g)\,, \qquad \mathcal{S}(\boldsymbol{k}, X_g) = \int \mathrm{d}^2\boldsymbol{r}\, \mathrm{e}^{-\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}} S(\boldsymbol{r}, X_g)$$

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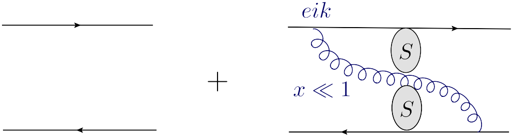
- In more compact, but formal, notations

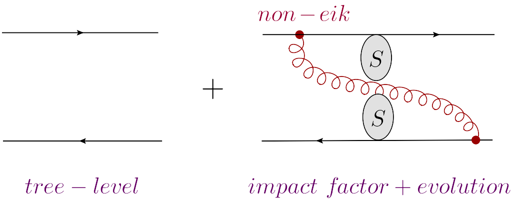
$$\mathcal{S}(\boldsymbol{k}, X_g) \;=\; \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s \int_{X_g}^1 \frac{\mathrm{d}x}{x}\, \mathcal{K}(\boldsymbol{k}; 0)\, \mathcal{S}\big(\boldsymbol{k}, X(x)\big)\,; \quad X(x) \equiv \frac{X_g}{x}$$



$\mathcal{S}$ *(solution to LO BK equation)*

Adding the NLO impact factor

- Compute (only) the first gluon emission with the exact kinematics



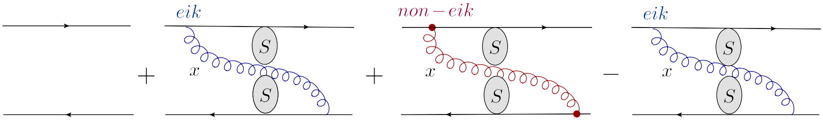
$$\frac{\mathrm{d}N}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x}\, \mathcal{K}(\boldsymbol{k};x)\, \mathcal{S}\big(\boldsymbol{k}, X(x)\big)\,; \quad X(x) \simeq \frac{X_g}{x}$$

- $\mathcal{K}(\boldsymbol{k};x)$: kernel for emitting a gluon with exact kinematics $(x \leq 1)$

  (Chirilli, Xiao, and Yuan, arXiv:1203.6139)

- This cross-section is (almost) manifestly positive definite
- LO evolution + NLO impact factor are mixed with each other
- To recover the LO result: $\mathcal{K}(\boldsymbol{k};x) \to \mathcal{K}(\boldsymbol{k};0)$ (eikonal limit)

- Add and subtract the LO result:



$\mathcal{S}$ (solution to $\mathcal{BK}$ equation) $\mathcal{NLO}$ correction to impact factor

$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = \mathcal{S}(\boldsymbol{k}, X_g) + \bar{\alpha}_s \int_{X_g}^{1} \frac{\mathrm{d}x}{x} \left[\mathcal{K}(x) - \mathcal{K}(0)\right] \mathcal{S}\big(\boldsymbol{k}, X(x)\big)$$

- To NLO accuracy, one can perform additional approximations:
  - replace $\mathcal{S}\big(X(x)\big) \simeq \mathcal{S}(X_g)$ (since integral dominated by $x \sim 1$)
  - ... and set $X_g \to 0$ in the lower limit ('plus prescription')
- Local in rapidity : $k_\perp$-factorization in the form presented by CXY

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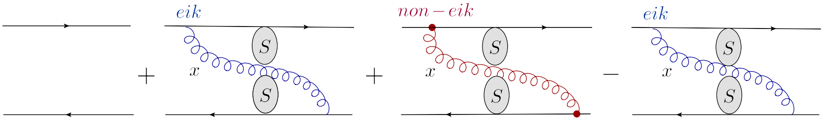

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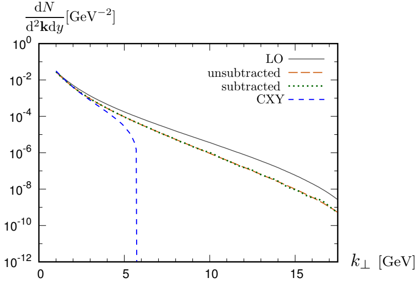
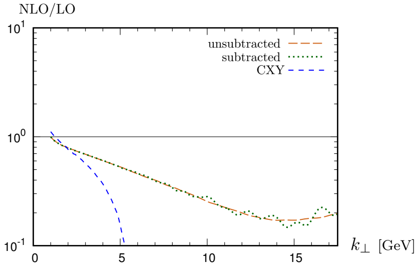
Numerical results: Fixed coupling

(Ducloué, Lappi, and Zhu, arXiv:1703.04962)

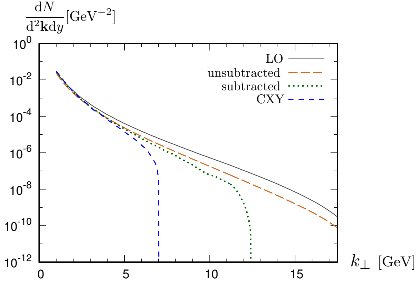


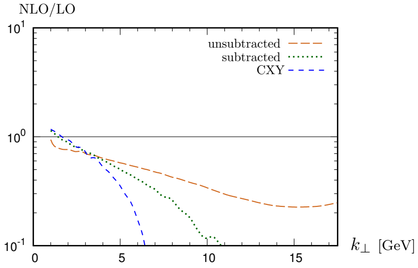


- Large NLO correction: $\gtrsim 50\,\%$ for $k_\perp \geq 5$ GeV
- The same results with and without subtraction (of the LO result)
- "A mathematical identity" ... sure, but tricky in practice !
  - one adds and subtracts a large, LO, contribution
  - small oscillations in "subtracted" due to numerical errors
- Strict $k_\perp$-factorization rapidly becomes negative : over-subtraction

Numerical results: Running coupling

(Ducloué, Lappi, and Zhu, arXiv:1703.04962)





- The running of the coupling renders the problem even more subtle:
  - already the "subtracted" result becomes negative
  - the "CXY" curve becomes negative even faster
- Mismatch between the running coupling prescriptions used ...
  - in coordinate space (for solving the BK equation)
  - ... and in momentum space (for computing the NLO impact factor)

Adding a running coupling

- The NLO impact factor is generally computed in momentum space
  - natural to use a running coupling $\bar{\alpha}_s(k_\perp^2)$ (at least for $k_\perp^2 \gtrsim Q_s^2$)

$$\frac{\mathrm{d}N}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s(k_\perp^2)\int_{X_g}^{1}\frac{\mathrm{d}x}{x}\,\mathcal{K}(\boldsymbol{k};x)\,\mathcal{S}\big(\boldsymbol{k},X(x)\big)$$

  - more generally: $\bar{\alpha}_s(k_{\max}^2)$

- Dipole S-matrix is computed by solving rcBK in coordinate space

$$S_{\boldsymbol{xy}}(X_g) = S_{\boldsymbol{xy}}(X_0) + \int_{X_g}^{1}\frac{\mathrm{d}x}{x}\int_{\boldsymbol{z}}\bar{\alpha}_s(r_{\min}^2)\frac{(\boldsymbol{x}-\boldsymbol{y})^2}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{y}-\boldsymbol{z})^2}\Big[S_{\boldsymbol{xz}}S_{\boldsymbol{zy}} - S_{\boldsymbol{xy}}\Big]$$

  - $r_{\min} \equiv \min\big\{|\boldsymbol{x}-\boldsymbol{y}|, |\boldsymbol{x}-\boldsymbol{z}|, |\boldsymbol{y}-\boldsymbol{z}|\big\}$

- Running coupling and Fourier transform do not "commute" with each other

Towards a new puzzle ?

- The FT transform $\mathcal{S}(\boldsymbol{k}, X)$ does not obey the expected integral equation in momentum space

$$\mathcal{S}(\boldsymbol{k}, X_g) \neq \mathcal{S}_0(\boldsymbol{k}) + \bar{\alpha}_s(k_\perp^2) \int_{X_g}^1 \frac{\mathrm{d}x}{x}\, \mathcal{K}(\boldsymbol{k}; 0)\, \mathcal{S}\big(\boldsymbol{k}, X(x)\big)$$

  - subtracting the LO result is not an identity anymore
  - mismatch between "subtracted" and "unsubtracted" results

- Our prescription *(E.I., Mueller, Triantafyllopoulos, arXiv:1608.05293)*
  - use the "unsubtracted" result with momentum-space RC $\bar{\alpha}_s(k_\perp^2)$
  - reasonable numerical results: positive definite

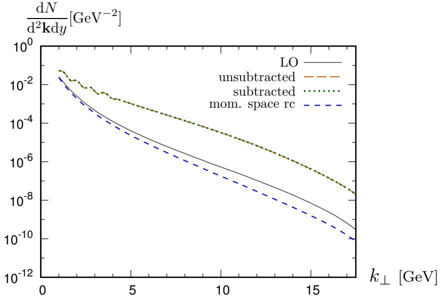
- But how sensitive are these results upon the choice of a scheme ?

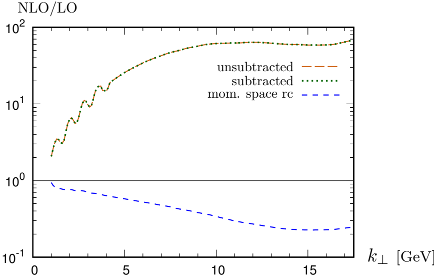
- Alternative scheme: compute the NLO impact factor fully in coordinate space and make the FT at the very end

  (Ducloué, Lappi, and Zhu, arXiv:1703.04962)

Numerical results: Coordinate space with RC

(Ducloué, Lappi, and Zhu, arXiv:1703.04962 – see the Appendix)



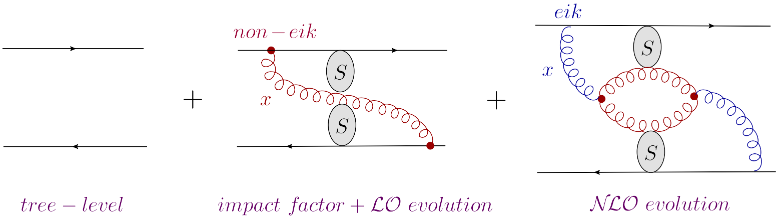


- "Unsubtracted" and "subtracted" results coincide with each other ☺
  - calculations systematically done in coordinate space
  - subtraction performed in coordinate space before the final FT
- ... but they are larger than the LO result by a factor ~ 100 !
- The mismatch with the "momentum-space scheme" is spectacular, but so far we do not understand its origin

Completing the NLO evolution

(E.I., A. Mueller and D. Triantafyllopoulos, arXiv:1608.05293)

- Recall: the NLO BK evolution also involves 2-loop graphs



$$\frac{\mathrm{d}N}{\mathrm{d}\eta \mathrm{d}^2\boldsymbol{k}} = \mathcal{S}_0 + \bar{\alpha}_s \int_{X_g}^1 \frac{\mathrm{d}x}{x}\, \mathcal{K}(x)\, \mathcal{S}\big(X(x)\big) + \bar{\alpha}_s^2 \int_{X_g}^1 \frac{\mathrm{d}x}{x}\, \mathcal{K}_2(0)\, \mathcal{S}\big(X(x)\big)$$

- $\mathcal{K}_2(0)$: NLO correction to the BK kernel with collinear improvement

  (Balitsky and Chirilli, 2008; Iancu et al, 2015)

Conclusions

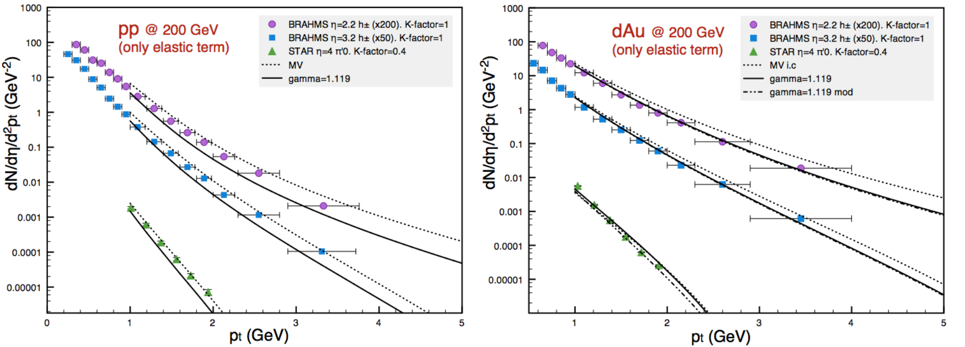
- The usual $k_\perp$-factorization at high energy (local in rapidity) can provide unphysical results at NLO
  - the strict separation between a 'LO result' and 'NLO corrections' involves a high degree of fine tuning, leading to instabilities in the presence of seemingly innocuous additional approximations
- A more general factorization has been proposed to circumvent this problem
  - no explicit separation between LO and NLO
  - non-local in rapidity
- Sensible physical results: positive cross-section, but smaller than at LO
  - at fixed coupling
  - with running coupling, but using a mixed scheme
- A fully coordinate-space calculation with RC leads to new difficulties
- Next step: attempt a fully momentum-space calculation with RC

Back-up Slides

LO phenomenology (rcBK)

(Albacete, Dumitru, Fujii, Nara, arXiv:1209:2001)

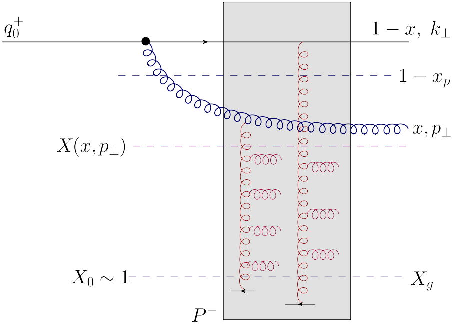
- Fit parameters: initial condition for the rcBK equation + K-factors



$$\left.\frac{\mathrm{d}N}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{k}}\right|_{\mathrm{LO}} = K^h \int_{x_p}^{1} \frac{\mathrm{d}z}{z^2}\, \frac{x_p}{z} q\left(\frac{x_p}{z}\right) \mathcal{S}\left(\frac{\boldsymbol{k}}{z}, X_g\right) D_{h/q}(z)$$

Exact kinematics for target evolution

- 'Real amplitude' : the gluon is produced in the final state



- LC energy conservation:

$$\frac{k_\perp^2}{2(1-x)q_0^+} + \frac{p_\perp^2}{2xq_0^+} = XP^-$$

- $\Longrightarrow \quad X = X(x, p_\perp)$
- simplifies when $k_\perp \simeq p_\perp \gg Q_s$

$$X(x) \simeq \frac{k_\perp^2}{xs} = \frac{X_g}{x}$$

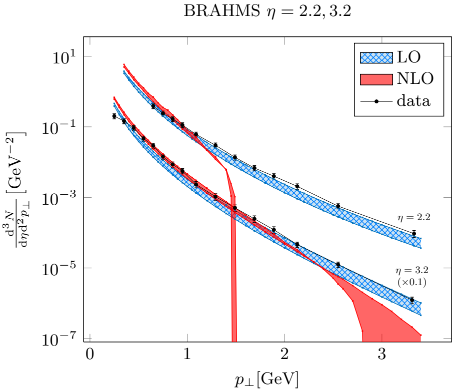
- $X \leq 1 \Longrightarrow x \geq X_g$

- Equivalently: gluon lifetime should be larger than the target width
- The same condition holds for the 'virtual' corrections
  - non-trivial cancellations required by probability conservation

The negativity problem

(Stasto, Xiao, and Zaslavsky, arXiv:1307.4057)

- Sudden drop in the numerical estimate at momenta $p_\perp$ of order Q_s



- "NLO evolution is notoriously unstable"
- Sure, but in this calculation $\mathcal{S} \approx \mathcal{S}_{\rm rcBK}$
  - rcBK evolution is well behaved
  - the actual "LO approx" in practice

$$\left.\frac{dN}{dy\, d^2\boldsymbol{k}}\right|_{\rm LO} = \mathcal{S}_{\rm rcBK}(\boldsymbol{k}, X_g)$$

- The NLO correction to the impact factor is negative (not a real surprise) ... and dominates over the LO result at sufficiently large $k_\perp$

Some proposals to solve the problem

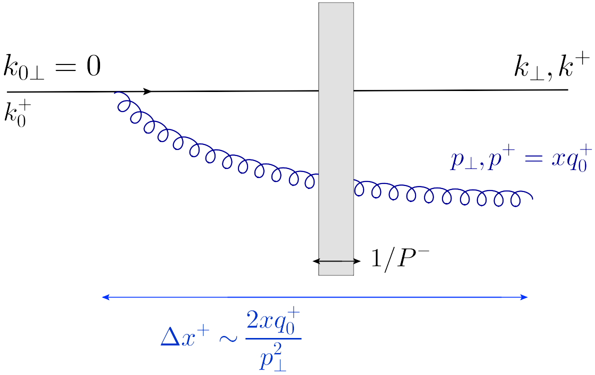
- General idea: the 'subtracted' term performs an ... over-subtraction
- Strategy: reduce the longitudinal (x) phase-space for the 'hard' gluon
  - factorization scale x_0 separating 'evolution' from 'impact factor' *(Kang, Vitev, and Xing, arXiv:1403.5221)*

$$\int_0^{1} \frac{\mathrm{d}x}{x}\,[\mathcal{K}(x)-\mathcal{K}(0)] \Longrightarrow \int_0^{x_0} \frac{\mathrm{d}x}{x}\,[\mathcal{K}(x)-\mathcal{K}(0)]$$

  - x_0 can depend upon $k_\perp$, say to account for 'time-ordering' *(Ducloué, Lappi, and Zhu, arXiv:1604.00225)*
- In principle, it shouldn't matter that much
  - the x_0–dependence must cancel in a complete calculation
- In practice, it only pushes the problem up to somewhat higher $k_\perp$
  - also, strongly dependent upon the precise implementation of x_0

(Altinoluk, Armesto, Beuf, Kovner, and Lublinsky, arXiv:1411.2869)

- x cannot be arbitrarily small since constrained by energy conservation



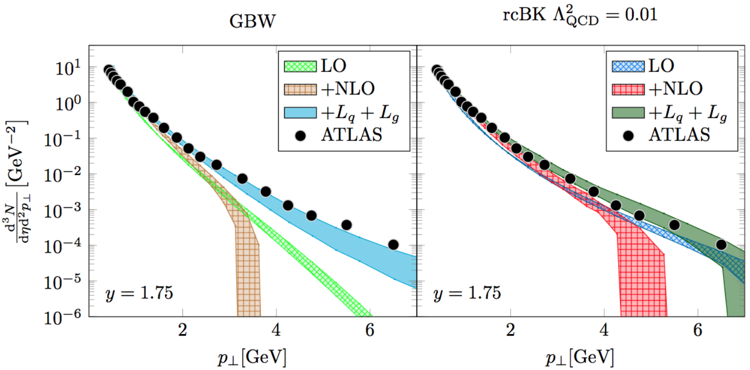
- Gluon lifetime should be larger than the target width

$$\frac{2xq_0^+}{p_\perp^2} > \frac{1}{P^-} \implies x > \frac{p_\perp^2}{s}$$

Implementing the constraint

(Watanabe, Xiao, Yuan, and Zaslavsky, arXiv:1505:05183)

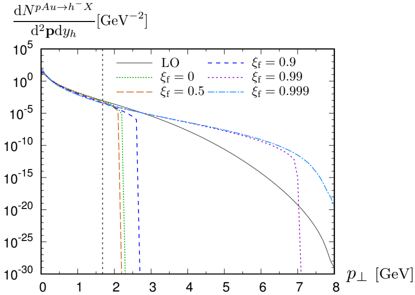
- It matters for the subtraction scheme only if $k_\perp \gg p_\perp$

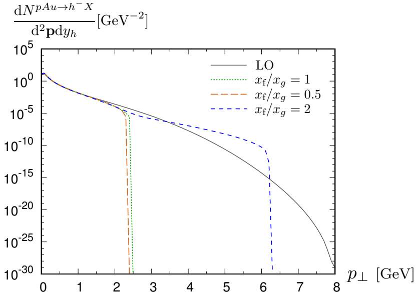


- Once again, it pushes the problem to higher $k_\perp$
  - ... and strongly dependent upon the model/evolution chosen for $\mathcal{S}$

Why is this a problem ?

- An extreme example: GBW saturation model $\mathcal{S}_{\rm GBW}(\boldsymbol{k}, X) \propto \mathrm{e}^{-\frac{k_\perp^2}{Q_s^2}}$
  - the 'added' piece is exponentially suppressed at $k_\perp \gg Q_s$
  - the 'subtracted' piece develops a power-law tail $\propto 1/k_\perp^4$
  - the overall result becomes negative at sufficiently large $k_\perp$





(Ducloué, Lappi, and Zhu, arXiv:1604.00225)