

RIKEN-BNL Research Center Workshop "Saturation: Recent Developments,
New Ideas, and Measurements", April 26-28, 2017

Deep Inelastic Scattering as a probe of entanglement

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Based on DK, E. Levin, arXiv:1702.03489



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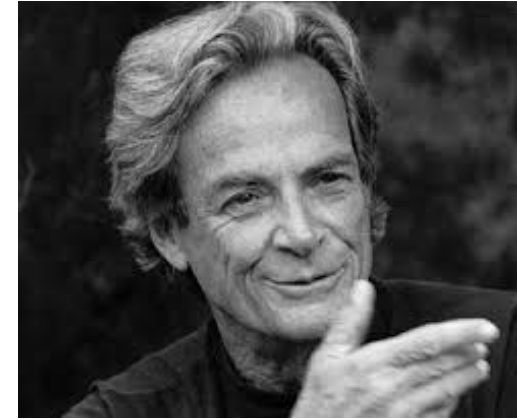
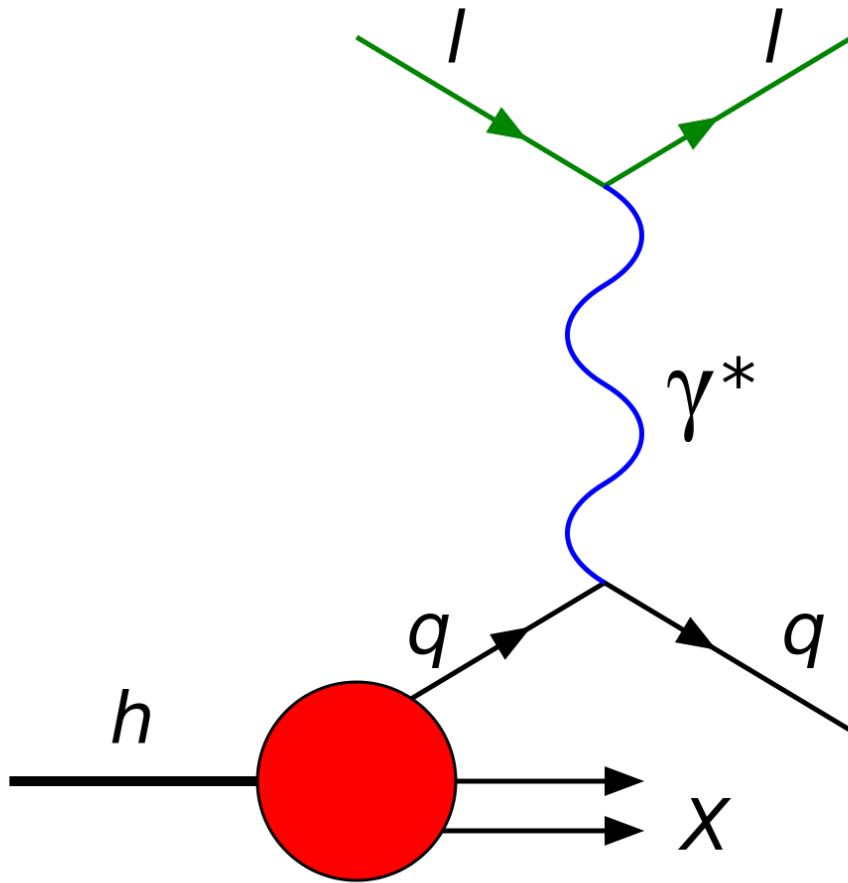
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Outline

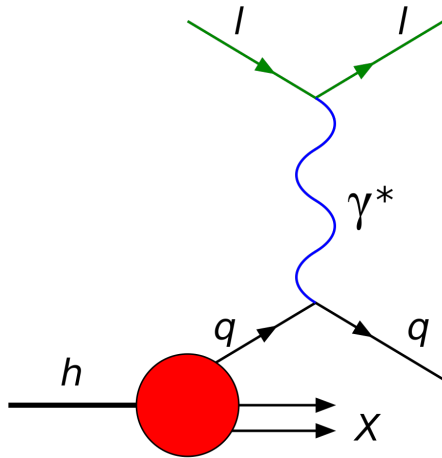
1. The puzzle of the parton model
2. Quantum mechanics of partons and entanglement
3. Entanglement entropy from QCD evolution
4. Proton at small x as a maximally entangled state
5. Relation between the entanglement entropy and the structure function
6. New experimental observables in DIS and pp collisions: from RHIC to EIC

The parton model: 50 years of success



In almost fifty years that have ensued after the birth of the parton model, it has become an indispensable building block of high energy physics

The parton model: basic assumptions



In parton model, the proton is pictured as a collection of point-like quasi-free partons that are frozen in the infinite momentum frame due to Lorentz dilation.

The DIS cross section is given by the incoherent sum of cross sections of scattering off individual partons.

How to reconcile this with quantum mechanics?⁴

DIS and entanglement

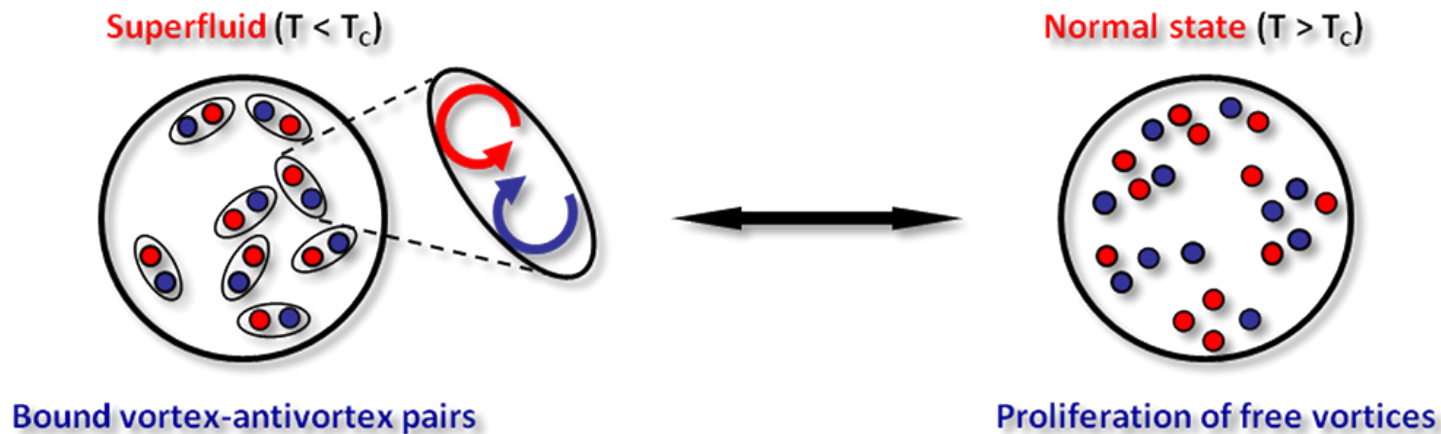
“...we never experiment with just one electron or atom or (small) molecule. In thought experiments, we sometimes assume that we do; this invariably entails ridiculous consequences”



Erwin Schrödinger, 1952

The puzzle of the parton model

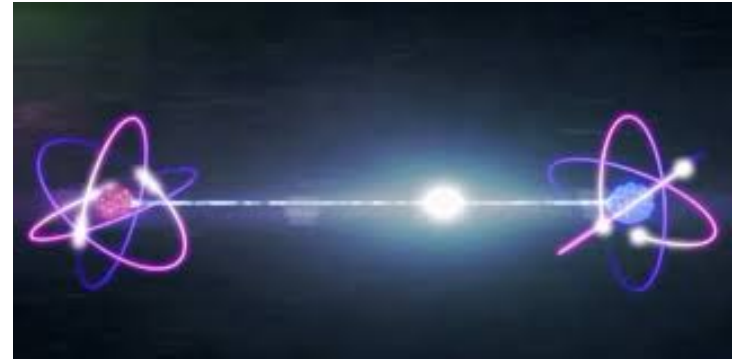
In quantum mechanics, the proton is a pure state with zero entropy. Yet, a collection of free partons does possess entropy... Boosting to the infinite momentum frame does not help, as a Lorentz boost cannot transform a pure state into a mixed one.



The crucial importance of entropy in (2+1)D systems:
BKT phase transition (Nobel prize 2016)

Entanglement Entropy in quantum theory:

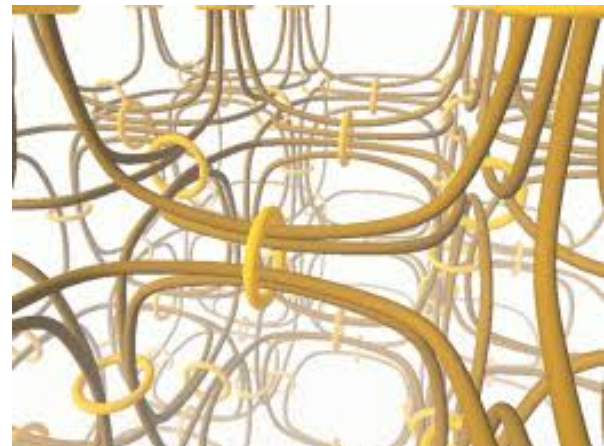
light



black holes



topological phases
of matter



The quantum mechanics of partons and entanglement

Our proposal: the key to solving this apparent paradox is entanglement.

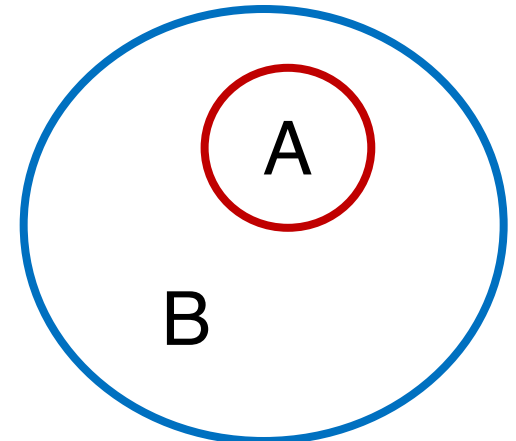
DK, E. Levin, arXiv:1702.03489

DIS probes only a part of the proton's wave function (region A). We sum over all hadronic final states; in quantum mechanics, this corresponds to accessing the density matrix of a mixed state

$$\hat{\rho}_A = \text{tr}_B \hat{\rho}$$

with a non-zero entanglement entropy

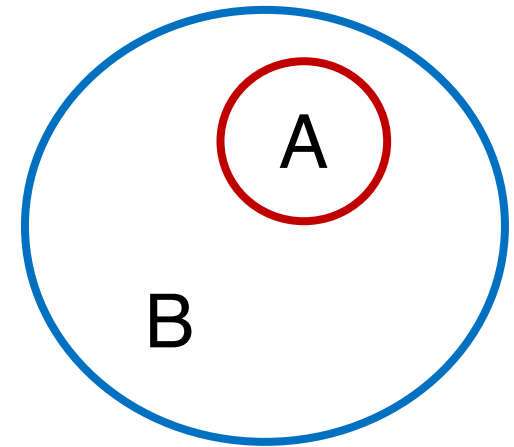
$$S_A = -\text{tr} [\hat{\rho}_A \ln \hat{\rho}_A]$$



The quantum mechanics of partons and entanglement

The proton is described by a vector

$$|\Psi_{AB}\rangle = \sum_{i,j} c_{ij} |\varphi_i^A\rangle \otimes |\varphi_j^B\rangle$$



in Hilbert space $\mathcal{H}_A \otimes \mathcal{H}_B$

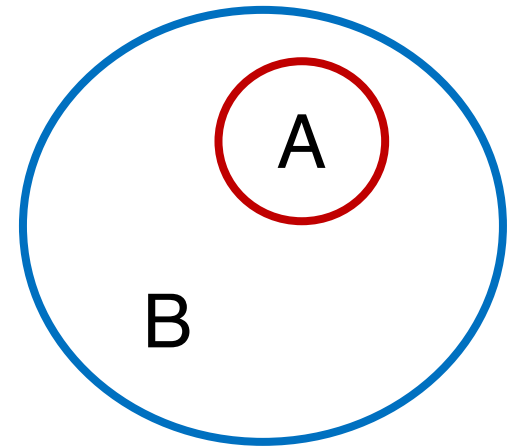
If $|\Psi_{AB}\rangle = |\varphi^A\rangle \otimes |\varphi^B\rangle$ only one term

contributes, then the state is separable (not our case!). Otherwise, the state is **entangled**.

The quantum mechanics of partons and entanglement

The Schmidt decomposition theorem:

$$|\Psi_{AB}\rangle = \sum_n \alpha_n |\Psi_n^A\rangle |\Psi_n^B\rangle$$



There exist the orthonormal states $|\Psi_n^A\rangle$ and $|\Psi_n^B\rangle$ for which the pure state can be represented as a single sum with real and positive coefficients α_n

If only one term (Schmidt rank one), then the state is separable. Otherwise, it is **entangled**; but no interference between different n's.

The quantum mechanics of partons and entanglement

We assume that the Schmidt basis $|\Psi_n^A\rangle|\Psi_n^B\rangle$ corresponds to the states with different numbers of partons n ; since it represents a relativistic quantum field theory (QCD), the Schmidt rank is in general infinite.

The density matrix is now given by

$$\rho_A = \text{tr}_B \rho_{AB} = \sum_n \alpha_n^2 |\Psi_n^A\rangle\langle\Psi_n^A|$$

where $\alpha_n^2 \equiv p_n$ is the probability of a state with n partons

The entanglement entropy

The entanglement entropy is now given by the von Neumann's formula

$$S = - \sum_n p_n \ln p_n$$


and can be evaluated by using the QCD evolution equations.

Let us start with a (1+1) case, followed by (3+1).

B

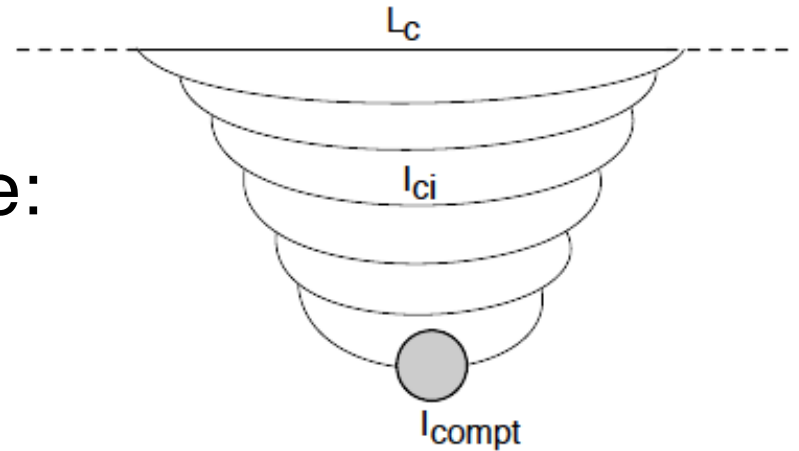
A

B


$$L = 1/(mx)$$

The entanglement entropy from QCD evolution

Space-time picture
in the proton's rest frame:



The evolution equation:

$$\frac{dP_n(Y)}{dY} = -\Delta n P_n(Y) + (n-1)\Delta P_{n-1}(Y)$$

The entanglement entropy from QCD evolution

$$\frac{dP_n(Y)}{dY} = -\Delta n P_n(Y) + (n-1)\Delta P_{n-1}(Y)$$

Solve by using the generating function method
(A.H.Mueller '94; E.Levin, M.Lublinsky '04):

$$P_n(Y) = e^{-\Delta Y} (1 - e^{-\Delta Y})^{n-1}.$$

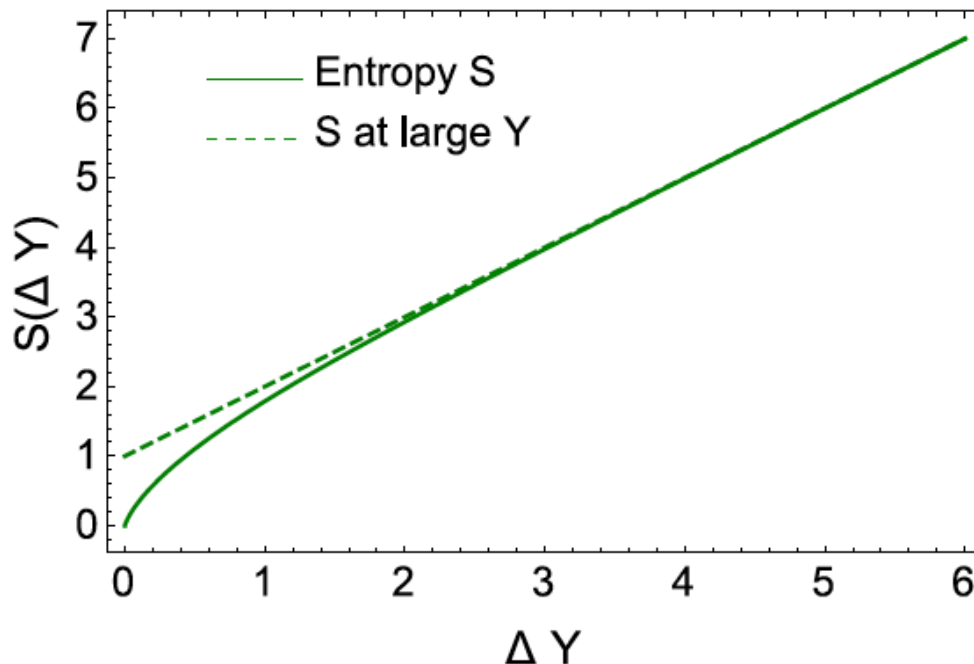
The resulting von Neumann entropy is

$$S(Y) = \ln(e^{\Delta Y} - 1) + e^{\Delta Y} \ln \left(\frac{1}{1 - e^{-\Delta Y}} \right)$$

The entanglement entropy from QCD evolution

At large ΔY , the entropy becomes

$$S(Y) \rightarrow \Delta Y$$



This “asymptotic”
regime starts rather
early, at

$$\Delta Y \simeq 2$$

The entanglement entropy from QCD evolution

At large ΔY ($x \sim 10^{-3}$) the relation between the entanglement entropy and the structure function

$$xG(x) = \langle n \rangle = \sum_n n P_n(Y) = \left(\frac{1}{x} \right)^\Delta$$

becomes very simple:

$$S = \ln[xG(x)]$$

The entanglement entropy from QCD evolution

The (3+1) case is cumbersome, but the result is the same, with $\Delta = \bar{\alpha}_s \ln(r^2 Q_s^2)$

What is the physics behind this relation?

$$S = \ln[xG(x)]$$

It signals that all $\exp(\Delta Y)$ partonic states have about equal probabilities $\exp(-\Delta Y)$ – in this case the **entanglement entropy is maximal**, and the proton is a **maximally entangled state** (a new look at the parton saturation and CGC?)

L. Boltzmann:

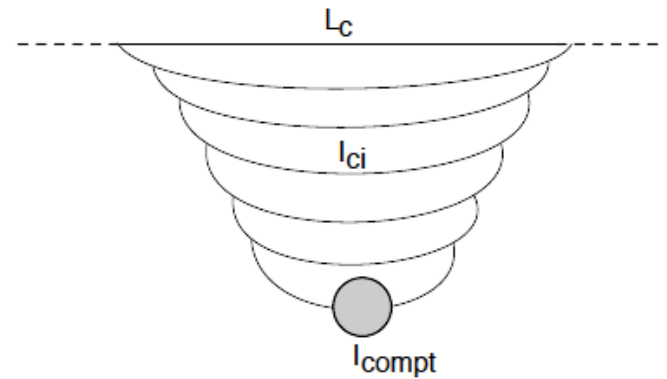


"Since a given system can never of its own accord go over into another equally probable state but into a more probable one, it is likewise impossible to construct a system of bodies that after traversing various states returns periodically to its original state, that is a perpetual motion machine."

Ludwig Eduard Boltzmann

the system is driven to the most probable state with the largest entropy

Possible relation to CFT



The small x formula

$$S = \ln[xG(x)]$$

yields

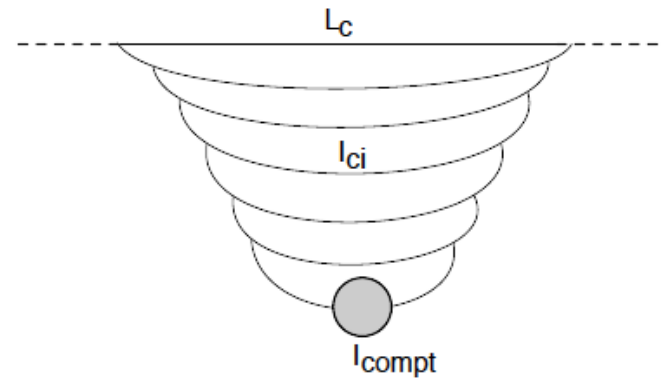
$$S(x) = \Delta \ln[1/x] = \Delta \ln \frac{L}{\epsilon}$$

where

$$L = (mx)^{-1}$$

is the longitudinal length probed in DIS
(in the target rest frame), and $\epsilon \equiv 1/m$
is the proton's Compton wavelength

Possible relation to CFT



This formula

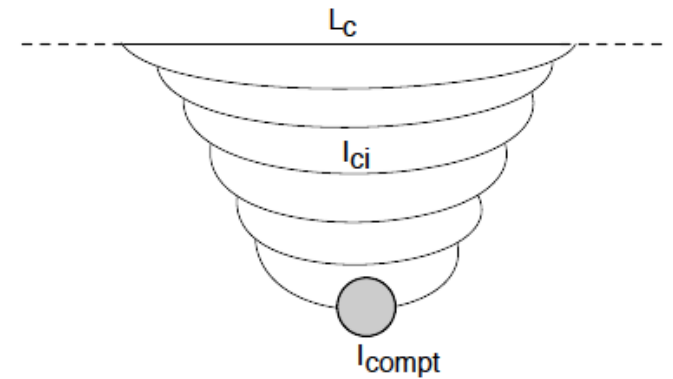
$$S(x) = \Delta \ln[1/x] = \Delta \ln \frac{L}{\epsilon}$$

looks very similar to the well-known (1+1) CFT result (Holzhey, Larsen, Wilczek '94; Calabrese, Cardy '05):

$$S_E = \frac{c}{3} \ln \frac{L}{\epsilon}$$

where c is the central charge of the CFT, and ϵ is the resolution scale

Possible relation to CFT



If this is not a mere coincidence:
the central charge $c \leq 1$:

$$c = 1 - \frac{6}{m(m+1)}, \quad m = 3, 4, \dots, \infty$$

$c=1$ corresponds to free bosonic theory.

This means that $\Delta \leq 1/3$;

a bound on the growth of the structure function!

$$xG(x) \leq \text{const} \frac{1}{x^{1/3}}$$

The Second Law for entanglement entropy?

If the Second Law applies to entanglement entropy (EE) (a number of indications, e.g. from black hole physics), then the entropy of hadronic final state in DIS has to be equal or larger than the EE of the initial state measured through the structure function:

$$S_{\text{hadrons}} \geq S_{EE}(x)$$

Indications from holography that the entropy does not increase at strong coupling; this leads to

$$S_{\text{hadrons}} \simeq S_{EE}(x) \quad \textbf{parton liberation ?}$$

Future directions

- The entanglement entropy (EE) can be defined and computed for any value of the coupling – generalization of parton distribution to strong coupling? holography
- The EE is a non-local order parameter in topologically ordered phases – relation of parton distributions to QCD topology?
- Elucidate connections to CFT
- Reformulate parton evolution in terms of EE (momentum space entanglement? – Kovner, Lublinsky '15)
- A new look at thermalization

Physics at EIC:

supplement the measurements of structure functions by the studies of hadronic final state (especially in the target fragmentation region).

Testing the Second Law for the Entanglement entropy is fundamentally important;

extracting the entropy of the final hadronic state will require event-by-event measurements.

This can be started at RHIC! Combine measurements of “hard” cross sections with the e-by-e measurements of associated hadron multiplicity

Summary

1. Entanglement entropy (EE) provides a viable solution to the apparent contradiction between the parton model and quantum mechanics.
2. The use of EE allows to generalize the notion of parton distribution to any value of the coupling, and possibly to elucidate the role of topology.
3. If the connection to CFT is confirmed, it will put a bound on the growth of parton distributions at small x (no other bound exists; unitarity does not provide a bound).
4. Measuring hadronic final states at EIC will allow to test the Second Law for the EE.