

Exploring correlations in the CGC wave function.

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[with M. Lublinsky and V. Skokov, [arXiv:1612.07790 \[hep-ph\]](#)]

Correlations in pA: who is responsible?

Do they come from collective effects in the final state?

Is p-A collisions really hydro? Even up to $p_T \approx 10 \text{ GeV}$?

Or do they come from correlated structure of the initial wave function?

The Ridge in Double Inclusive Hadron Production in p-p.

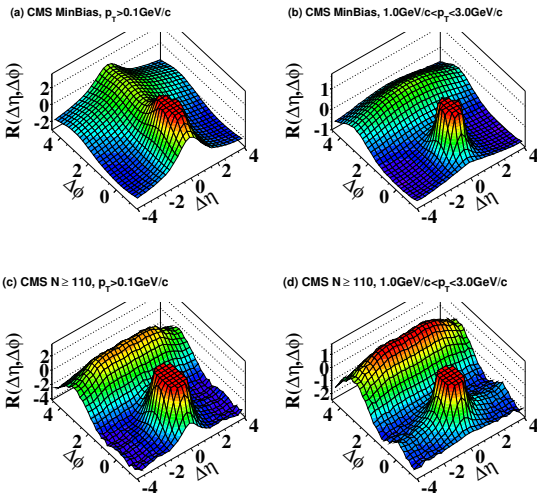


Figure: Ridge in p-p at CMS circa 2010, $\sim 10^{-6}$ events

Ridge in p-Pb.

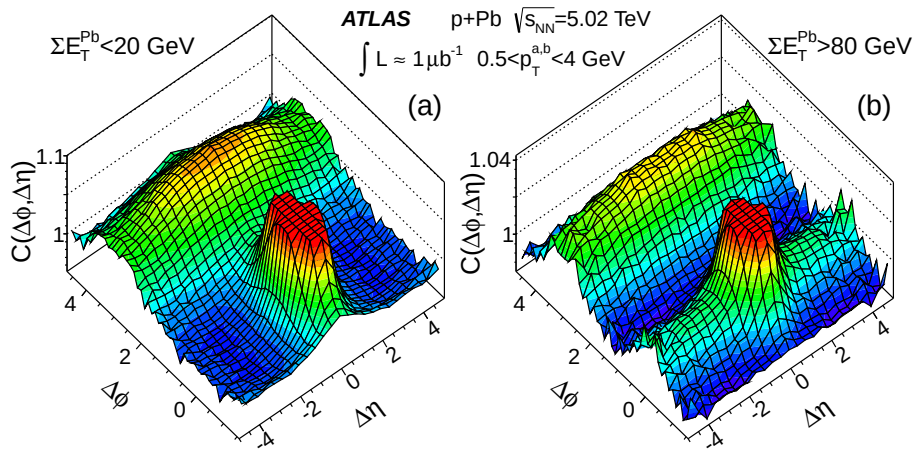
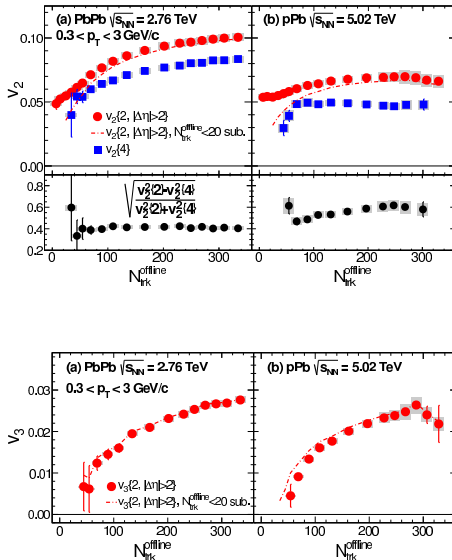


Figure: Ridge in p-Pb at ATLAS, $\sim 10^{-2}$ events

Things got more interesting.

The correlations point to collective, or at least quasi collective behavior.



"Flow coefficients" measure correlations between the emitted particles, and are believed to encode collectivity of the final state. For double inclusive spectrum

$$\frac{d^2 N}{d^2 p_1 d^2 p_2} \propto 1 + \sum_{n=1}^{\infty} 2V_n(\mathbf{p}_1, \mathbf{p}_2) \cos(n\Delta\phi)$$
$$v_n^2 = \frac{V_n(p_T, p_T^{\text{ref}})}{\sqrt{V_n(p_T^{\text{ref}}, p_T^{\text{ref}})}}; \quad n = 2, 3$$

Analogously for v_2^4 - from four particle inclusive spectrum.

Hydro codes seem to describe the data on v_n .

But: the produced system is small, the momenta involved are quite large $\sim 8\text{ GeV}$, so that hydro is suspect.

Even more exciting: recent CMS and ATLAS analysis of p-p at LHC - ridge persists even in MIN. BIAS events, and below.

Does the ridge and v_n data necessarily require strong final state interactions?

Is it possible that nontrivial initial state correlations mimic collectivity (quasi collectivity)?

Saturation?

Initially ridge was found in small fraction, high multiplicity events:
"rare" proton configurations with high density. Perhaps saturation is at play?

Several possible mechanisms to generate correlations from initial state.

G. Levin and A. Rezaeian - density profile variation;

A.K and M. Lublinsky - local anisotropy of target fields;

The one explored phenomenologically:

"Glasma graphs" $\equiv$ gluon Bose enhancement Dumitru, Gelis, Jalilian-Marian, Lappi: Phys.Lett. B697 (2011) 21 (arXiv:1009.5295)

Followed by a quantitative effort to describe data: Dusling and Venugopalan Phys.Rev.Lett. 108 (2012) 262001 (arXiv:1201.2658); arXiv:1302.7018

In the calculation - no final state interactions. Correlations are "inherited" from the initial state.

Where is v_3 ?

All the approaches invariably lead to "symmetry"

$$\sigma(p_T, k_T) = \sigma(p_T, -k_T)$$

It is NOT a symmetry of QCD: it is "accidental".

E.G: It is broken by final state interactions: L.McLerran and V. Skokov :
arXiv:1611.09870; B.Schenke, S. Schlichting, R. Venugopalan Phys.Lett.
B747 (2015) 76-82,

Is the "dilute" CGC state we are using good enough?

Better approximation to the CGC state?

The CGC hadron wave function.

High energy factorization: the fast partons are dressed by the soft gluon cloud.

Fast partons: color charge density in the transverse plane $\rho^a(x_\perp)$.

Soft gluons: the Weizacker-Williams cloud.

Soft gluon wave function **in dilute limit:**

$$\Psi[A] = e^{i \int_{x_\perp} b_i[\rho] A_i(x_\perp)} |0\rangle$$

Solution of classical Yang-Mills equation:

$$\partial_i b_i^a(x_\perp) = g \rho^a(x_\perp)$$

ρ has to be averaged over with some weight functional, e.g. simplest Gaussian: McLerran-Venugopalan model.

Denser is better?

The inspiration: The Old Kharzeev-Levin-McLerran argument.

A single high p_T parton in the wave function is most likely accompanied by several lower p_T partons, who collectively balance the transverse momentum.

This is kinda like flow: many particles move along an axis, which is determined by a fluctuation.

But coherent state does not do that!

But it is also true that Coherent state is not the whole story: it is only dilute limit of the CGC wave function.

A better CGC state.

The first "dense" correction to the CGC wave function (T.Altinoluk,A.K., M. Lublinsky, J. Peressutti, JHEP 0903 (2009) 109)

$$\Psi_{\text{CGC}}[\phi] = \mathcal{N} e^{i\sqrt{2} \int_k b_{\alpha i}(-k) [a_{\alpha i}^\dagger(k) + a_{\alpha i}(-k)]} \times \\ e^{-\frac{1}{4} \int_{k,p} B_{\alpha\beta ij}^{-1}(k,p) [a_{\alpha i}^\dagger(k) + a_{\alpha i}(-k)] [a_{\beta j}^\dagger(p) + a_{\beta j}(-p)]}$$

The WW field $b_{\alpha i}$:

$$\partial_i b_{\alpha i}(x) = g \rho_\alpha(x)$$

The operator B :

$$B = (1 - I - L)^2 = 1 - I - L + [I, L]_+$$

where

$$I_{ij}^{\alpha\beta}(x, y) \equiv \delta^{\alpha\beta} \frac{\partial_i \partial_j}{\partial^2}(x, y); \quad L_{ij}^{\alpha\beta}(x, y) = U^{\alpha\gamma}(x) \frac{\partial_i \partial_j}{\partial^2}(x, y) U^{\dagger\gamma\beta}(y)$$

Gluon correlations in the wave function.

The first question: is there "accidental" symmetry in the wave function?

Answer: No!

$$\begin{aligned} \frac{1}{2}(f(k, p) - f(k, -p)) &= \frac{1}{2} \left(b(k)\tilde{B}(-k, p)b(-p) + b(-k)\tilde{B}(k, -p)b(p) \right) \\ &\quad - \frac{1}{2} \left(b(k)\tilde{B}(-k, -p)b(p) + b(-k)\tilde{B}(k, p)b(-p) \right), \end{aligned}$$

Important thing for now: it does not vanish. More later.

Particle production.

Scatter this projectile wave function on a target eikonally. Things fundamentally do not change: the antisymmetric part of the production does not vanish.

$$\begin{aligned} \frac{1}{2}(\sigma(k, p) - \sigma(k, -p)) = & \frac{\mathbb{C}(k)}{2} \frac{\mathbb{A}\mathbb{A}^T(-k, p) - \delta(p - k)}{2} \frac{\mathbb{C}(-p)}{2} \\ & + \frac{\mathbb{C}(-k)}{2} \frac{\mathbb{A}\mathbb{A}^T(k, -p) - \delta(p - k)}{2} \frac{\mathbb{C}(p)}{2} \\ & - \frac{\mathbb{C}(k)}{2} \frac{\mathbb{A}\mathbb{A}^T(-k, -p) - \delta(p + k)}{2} \frac{\mathbb{C}(p)}{2} \\ & - \frac{\mathbb{C}(-k)}{2} \frac{\mathbb{A}\mathbb{A}^T(k, p) - \delta(p + k)}{2} \frac{\mathbb{C}(-p)}{2} \end{aligned}$$

$\mathbb{A}$ and $\mathbb{C}$ depend on the WW field b and the eikonal scattering matrix S .

The letters:

$$\begin{aligned}\mathbb{A} &\stackrel{\text{def}}{=} \bar{\Gamma}^{-1} S \Gamma, \\ \mathbb{C} &\stackrel{\text{def}}{=} 2 \bar{\Gamma}^T (S b - \bar{b}).\end{aligned}$$

with

$$\Gamma \equiv (\mathfrak{t} - \mathfrak{l})(1 - \mathfrak{l} - L); \quad \bar{b} \equiv b[S\rho]; \quad \bar{L} = L[\bar{b}]$$

Some drastic approximations.

The crucial question: **what is the sign of the coefficient of the third harmonic $V_3 \cos 3\phi$** ? $v_3 = \sqrt{V_3}$

Let's try to get an idea what our long expressions mean.

A. High transverse momentum:

$$p_T, k_T \gg Q_S^P \approx g^4 \mu^2; \quad p_T, k_T \gg Q_T^P \approx g^4 \lambda^2$$

B. McLerran-Venugopalan model for the projectile - expand to leading order in μ .

C. Operator product expansion on the target side.

Correlated production can be expressed in terms of "condensates" of the eikonal factors $\langle \partial S \partial S^\dagger \dots \rangle$

The Odderon rules.

At high k_T the leading term in the operator product expansion is the Odderon. At leading order in $1/k_T$:

$$\propto \frac{\mu^4}{p^4 k^4} \Lambda_{st}(k, p) \left[f^{acd} S^{de} \partial_t S^{\dagger eb} f^{cbf} S^{fg} \partial_s S^{\dagger ga} - (S \rightarrow S^{\dagger}) \right]$$

C- conjugation odd, i. e. Odderon.

We do not have a well motivated model for Odderon - so at this order the sign of the correlated contribution is not fixed.

But the Odderon is subleading at high energies.

Target averaging.

At short distances we can write without approximation:

$$S(x) = \exp\{iT^a E_i^a x_i\}$$

which leads to

$$\partial_s S(x) \rightarrow iT^a E_s^a; \quad \partial_r \partial_s S(x) \rightarrow -\frac{1}{2} \{T^a, T^b\} E_s^a E_t^b; \quad \text{etc.}$$

We then assume Wick factorization of averages with

$$\langle E_i^a E_j^b \rangle = \lambda^2 \delta^{ab} \delta_{ij}$$

The answer.

This is our correlated yield:

$$\begin{aligned} g^2 N_c^5 A \frac{\mu^4 \lambda^4}{k^4 p^4} \left\{ -\frac{29}{4} \frac{k \cdot p}{k^2 p^2} - 3 \frac{(k \cdot p)^3}{k^4 p^4} + \frac{15}{2} \frac{k \cdot (k-p) p \cdot (k-p)}{k^2 p^2 (k-p)^2} \right. \\ + \frac{k \cdot p (p \cdot (k-p))^2}{k^2 p^4 (k-p)^2} + \frac{k \cdot p (k \cdot (k-p))^2}{k^4 p^2 (k-p)^2} \\ + \frac{1}{4} \frac{(k \cdot (k-p))^2}{k^2 (k-p)^2} \left(\frac{5}{k^2} - \frac{7}{p^2} \right) + \frac{1}{4} \frac{(p \cdot (k-p))^2}{p^2 (k-p)^2} \left(\frac{5}{p^2} - \frac{7}{k^2} \right) \\ + \frac{7}{2} \left(\frac{1}{k^2} + \frac{1}{p^2} \right) \frac{k \cdot p k \cdot (k-p) p \cdot (k-p)}{k^2 p^2 (k-p)^2} \\ \left. + \frac{3}{8} \left[\frac{k \cdot (k-p)}{k^2 (k-p)^2} - \frac{p \cdot (k-p)}{p^2 (k-p)^2} \right] \right\} \end{aligned}$$

But what does it mean?

Correlation function.

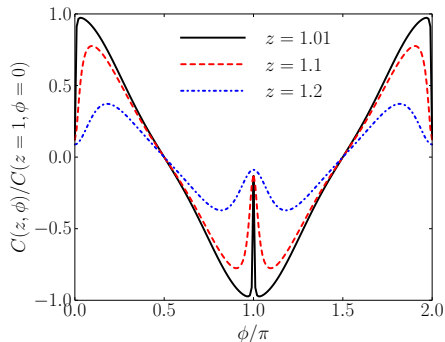
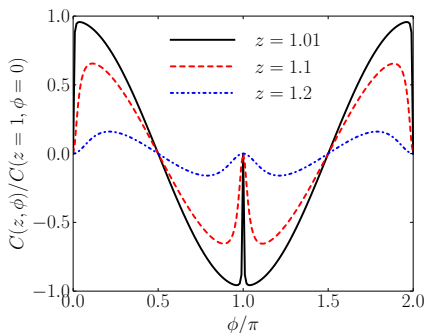


Figure: The correlation function as a function of the azimuthal angle, ϕ for different values of $z = p/k$. Left panel: in the projectile wave function. Right panel: double gluon inclusive production. The correlation functions are normalized by $C(z = 1, \phi = 0)$.

The odd harmonics.

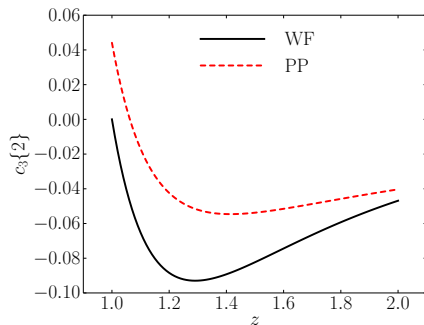
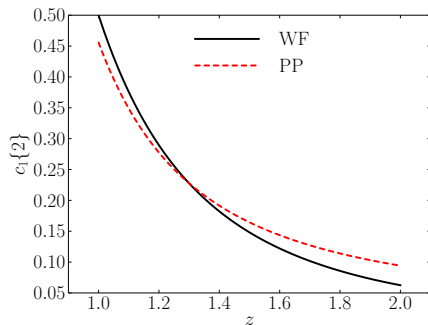


Figure: The first and the third cumulants as a function of $z = p/k$.

Conclusions.

1. The absence of odd harmonics in dilute-dense scattering is accidental: a denser projectile generates odd harmonics. The squeezed state has wider applicability parametrically: $N = O(1)$ rather than $N = O(g^2)$.
2. Relative to correlated piece from glasma graphs: our production cross section is $O(\alpha_s N_c)$ - so coupling suppressed but N_c enhanced.
3. The sign of V_3 is only positive for $1.1 > p/k > .9$. Keep momentum of trigger fixed, increase the momentum of associated particle, the v_3 should decrease pretty fast. Some sign of this in the data, although not clear that momenta large enough to trust our approximations.
4. Either way, our understanding of the proton wave function is quite rudimentary. We have a lot of work to do.