From light-cone wave function to NLO JIMWLK

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Alex Kovner, ML, and Yair Mulian
arXiv:1310.0378 (PRD), arXiv:1401.0374 (JHEP), arXiv:1405.0418 (JHEP)

ML and Yair Mulian, arXiv:1610.03453 (JHEP)

High Energy Scattering

Target (ρ^t) **Projectile (ρ^p)**

$$\langle \mathbf{T}| \quad \rightarrow \qquad\qquad \leftarrow \quad |\mathbf{P}\rangle$$

S-matrix:

$$\mathbf{S(Y)} \;=\; \langle \mathbf{T}\,\langle\,\mathbf{P}|\;\; \hat{\mathbf{S}}(\rho^{\mathbf{t}},\,\rho^{\mathbf{p}})\;\; |\mathbf{P}\rangle\,\mathbf{T}\rangle$$

or, more generally, any observable $\hat{\mathcal{O}}(\rho^{\mathbf{t}},\,\rho^{\mathbf{p}})$

$$\langle\hat{\mathcal{O}}\rangle_{\mathbf{Y}} \;=\; \langle \mathbf{T}\,\langle\,\mathbf{P}|\;\; \hat{\mathcal{O}}(\rho^{\mathbf{t}},\,\rho^{\mathbf{p}})\;\; |\mathbf{P}\rangle\,\mathbf{T}\rangle$$

The question we pose is how these averages change with increase in energy of the process

Projectile averaged $\mathbf{S}$-matrix:

$$\boldsymbol{\Sigma}(\mathbf{Y}) \equiv \langle \mathbf{P} | \, \hat{\mathbf{S}}(\rho^{\mathrm{t}}, \rho^{\mathrm{p}}) \, | \mathbf{P} \rangle = \int \mathbf{D}\rho^{\mathrm{p}} \; \mathbf{S}(\rho^{\mathrm{t}}, \rho^{\mathrm{p}}) \; \mathbf{W}^{\mathrm{p}}_{\mathbf{Y}}[\rho^{\mathrm{p}}]$$

evolve with rapidity as $\quad \mathbf{H} \rightarrow$ **the HE effective Hamiltonian**

$$\boldsymbol{\Sigma}(\mathbf{Y} + \delta\mathbf{Y}) = \int \mathbf{D}\rho^{\mathrm{p}} \; \mathbf{e}^{-\delta\mathbf{Y}\,\mathbf{H}} \; \mathbf{S}(\rho^{\mathrm{t}}, \rho^{\mathrm{p}}) \; \mathbf{W}^{\mathrm{p}}_{\mathbf{Y}}[\rho^{\mathrm{p}}]$$

Spectrum of $\mathbf{H}$ defines energy dependence of the observables.

$$\mathbf{e}^{-\delta\mathbf{Y}\,\mathbf{H}} \simeq \mathbf{1} - \delta\mathbf{Y}\,\mathbf{H} + \frac{\mathbf{1}}{\mathbf{2}}\,\delta\mathbf{Y}^{\mathbf{2}}\,\mathbf{H}^{\mathbf{2}} \ldots$$

$$\mathbf{H} = \mathbf{H}^{\mathrm{LO}}(\alpha_{\mathrm{s}}) + \mathbf{H}^{\mathrm{NLO}}(\alpha_{\mathrm{s}}^{\mathbf{2}}) + \ldots; \qquad \mathbf{H} = \mathbf{H}[\rho^{\mathrm{t}}, \, \delta/\delta\rho^{\mathrm{t}}]$$

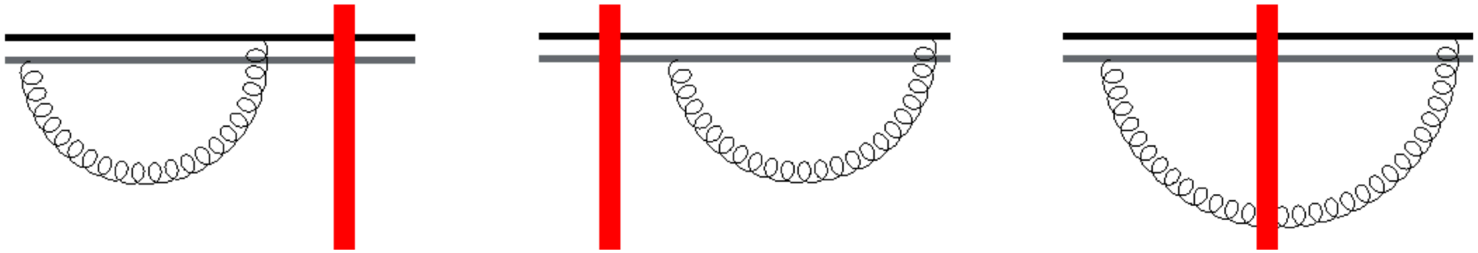
Charge densities ρ are important parameters (define dilute or dense limits)

LO JIMWLK Hamiltonian

Jalilian Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (1997-2002)

The JIMWLK Hamiltonian is a limit of H for dilute partonic system ($\rho^{\mathrm{p}} \to 0$) which scatters on a dense target. It accounts for linear gluon emission + multiple rescatterings.

$$H^{JIMWLK}_{LO} = \frac{\alpha_s}{2\pi^2}\int_{x,y,z}\frac{(x-z)_i(y-z)_i}{(x-z)^2(y-z)^2}\left\{J^a_L(x)J^a_L(y) + J^a_R(x)J^a_R(y) - 2J^a_L(x)S^{ab}_A(z)J^b_R(y)\right\}$$



Here $\rho^{\mathrm{p}} \to \mathrm{J_L}$ and $\hat{\mathrm{S}}\rho^{\mathrm{p}} \to \mathrm{J_R}$ are left and right SU(N) generators:

$$\mathbf{J^a_L(x)S^{ij}_A(z) = \left(T^aS_A(z)\right)^{ij}\,\delta^2(x-z)} \qquad \mathbf{J^a_R(x)S^{ij}_A(z) = \left(S_A(z)T^a\right)^{ij}\,\delta^2(x-z)}$$

- $\mathbf{H^{JIMWLK}}$ **contains all the LO BFKL / BKP / TPV physics**

JIMWLK Hamiltonian @ NLO

Alex Kovner, ML and Yair Mulian (2013)

$$H^{NLO\ JIMWLK} = \int_{x,y,z} K_{JSJ}(x,y;z) \left[J^a_L(x) J^a_L(y) + J^a_R(x) J^a_R(y) - 2 J^a_L(x) S^{ab}_A(z) J^b_R(y) \right]$$

$$+ \int_{x\,y\,z\,z'} K_{JSSJ}(x,y;z,z') \left[f^{abc} f^{def} J^a_L(x) S^{be}_A(z) S^{cf}_A(z') J^d_R(y) - N_c J^a_L(x) S^{ab}_A(z) J^b_R(y) \right]$$
$$+ \int_{x,y,z,z'} K_{q\bar q}(x,y;z,z') \left[2\, J^a_L(x)\, tr[S^\dagger_F(z)\, t^a\, S_F(z') t^b]\, J^b_R(y) \;-\; J^a_L(x)\, S^{ab}_A(z)\, J^b_R(y) \right]$$
$$+ \int_{w,x,y,z,z'} K_{JJSSJ}(w;x,y;z,z') f^{acb} \Big[J^d_L(x)\, J^e_L(y)\, S^{dc}_A(z)\, S^{eb}_A(z')\, J^a_R(w)$$
$$- J^a_L(w)\, S^{cd}_A(z)\, S^{be}_A(z')\, J^d_R(x)\, J^e_R(y) \Big]$$
$$+ \int_{w,x,y,z} K_{JJSJ}(w;x,y;z)\, f^{bde} \left[J^d_L(x)\, J^e_L(y)\, S^{ba}_A(z)\, J^a_R(w) - J^a_L(w)\, S^{ab}_A(z)\, J^d_R(x)\, J^e_R(y) \right]$$
$$+ \int_{w,x,y} K_{JJJ}(w;x,y) f^{deb} \left[J^d_L(x)\, J^e_L(y)\, J^b_L(w) \;-\; J^d_R(x)\, J^e_R(y)\, J^b_R(w) \right].$$

Symmetries: $\mathbf{SU_L(N)} \times \mathbf{SU_R(N)}$ **CPT,** **Unitarity**

Shortcuts to the Kernels

Step 1: Compute evolution of 3-quark Wilson loop operator in SU(3) (baryon)

$$\mathbf{B(u,v,w) = \epsilon^{ijk}\epsilon^{lmn} S_F^{il}(u) S_F^{jm}(v) S_F^{kn}(w)}$$

$$\mathbf{\partial_Y\, B(u,v,w) = -H^{NLO\,JIMWLK}\, B(u,v,w)}$$

and compare with Grabovsky (hep-ph/1307.5414) $\quad \rightarrow \quad \mathbf{K_{JJSSJ},\ K_{JJSJ}}$

Step 2: Compute evolution of quark dipole operator

$$\mathbf{s(u,v) = tr[S_F(u) S_F^{\dagger}(v)]/N_c}$$

$$\mathbf{\partial_Y\, s(u,v) = -H^{NLO\,JIMWLK}\, s(u,v)}$$

and compare with Balitsky and Chirilli (hep-ph/0710.4330) $\quad \rightarrow \quad \mathbf{K_{JSSJ},\ K_{JSJ},\ K_{qq}}$

NLO Kernels (for gauge invariant operators)

$$K_{JJSSJ}(w;x,y;z,z') = -i\frac{\alpha_s^2}{2\,\pi^4}\left(\frac{X_i Y'_j}{X^2 Y'^2}\right)$$
$$\times\Big(\frac{\delta_{ij}}{2(z-z')^2}+\frac{(z'-z)_i W'_j}{(z'-z)^2 W'^2}+\frac{(z-z')_j W_i}{(z-z')^2 W^2}-\frac{W_i W'_j}{W^2 W'^2}\Big)\ln\frac{W^2}{W'^2}$$

$$K_{JJSJ}(w;x,y;z) \;=\; -\,i\,\frac{\alpha_s^2}{4\,\pi^3}\Big[\frac{X\cdot W}{X^2\,W^2}\;-\;\frac{Y\cdot W}{Y^2\,W^2}\Big]\ln\frac{Y^2}{(x-y)^2}\,\ln\frac{X^2}{(x-y)^2},$$

$$K_{q\bar q}(x,y;z,z') = -\frac{\alpha_s^2\,n_f}{8\,\pi^4}\Big\{\frac{X'^2Y^2+Y'^2X^2-(x-y)^2(z-z')^2}{(z-z')^4(X^2Y'^2-X'^2Y^2)}\ln\frac{X^2Y'^2}{X'^2Y^2}-\frac{2}{(z-z')^4}\Big\}$$

$$X=x-z,\quad X'=x-z',\quad Y=y-z,\quad Y'=y-z',\quad W=w-z$$

$$K_{JSSJ}(x,y;z,z') = \frac{\alpha_s^2}{16\,\pi^4}\Bigg[-\frac{4}{(z-z')^4} + \Big\{2\frac{X^2Y'^2 + X'^2Y^2 - 4(x-y)^2(z-z')^2}{(z-z')^4[X^2Y'^2 - X'^2Y^2]}$$

$$+ \frac{(x-y)^4}{X^2Y'^2 - X'^2Y^2}\Big[\frac{1}{X^2Y'^2}+\frac{1}{Y^2X'^2}\Big]+\frac{(x-y)^2}{(z-z')^2}\Big[\frac{1}{X^2Y'^2}-\frac{1}{X'^2Y^2}\Big]\Big\}\ln\frac{X^2Y'^2}{X'^2Y^2}\Bigg] + \tilde{K}(x,y,z,z').$$

$$K_{JSJ}(x,y;z) = -\frac{\alpha_s^2}{16\pi^3}\frac{(x-y)^2}{X^2Y^2}\Big[b\ln(x-y)^2\mu^2 - b\frac{X^2-Y^2}{(x-y)^2}\ln\frac{X^2}{Y^2} + \big(\frac{67}{9}-\frac{\pi^2}{3}\big)N_c - \frac{10}{9}n_f\Big]$$

$$-\frac{N_c}{2}\int_{z'}\tilde{K}(x,y,z,z').$$

Here μ is the normalization point, $b = \frac{11}{3}N_c - \frac{2}{3}n_f$

$$\tilde{K}(x,y,z,z') \;=\frac{i}{2}\Big[K_{JJSSJ}(x;x,y;z,z') - K_{JJSSJ}(y;x,y;z,z') - K_{JJSSJ}(x;y,x;z,z')$$

$$+K_{JJSSJ}(y;y,x;z,z')\Big]$$

The kernels are not unique though...

NLO Kernels for color non-singlets

"By inspection" of Balitsky and Chirilli (arXiv:1309.7644 [hep-ph])

$$K_{JSJ}(x,y,z) \to \bar{K}_{JSJ}(x,y,z) \equiv K_{JSJ}(x,y,z) +$$
$$+\frac{\alpha_s^2}{16\pi^3}\left\{\left[\frac{1}{X^2}+\frac{1}{Y^2}\right]\left[\left(\frac{67}{9}-\frac{\pi^2}{3}\right)N_c-\frac{10}{9}n_f\right]+\frac{b}{X^2}\ln X^2\mu^2+\frac{b}{Y^2}\ln Y^2\mu^2\right\};$$

$$K_{JSSJ}(x,y;z,z') \to \bar{K}_{JSSJ}(x,y;z,z') +$$
$$\equiv K_{JSSJ}(x,y;z,z') + \frac{\alpha_s^2}{8\pi^4}\left[\frac{4}{(z-z')^4}-\frac{I(x,z,z')}{(z-z')^2}-\frac{I(y,z,z')}{(z-z')^2}\right];$$

$$K_{q\bar{q}}(x,y;z,z') \to \bar{K}_{q\bar{q}}(x,y;z,z') \equiv K_{q\bar{q}}(x,y;z,z') - \frac{\alpha_s^2 n_f}{8\pi^4}\left[\frac{I_f(x,z,z')}{(z-z')^2}+\frac{I_f(y,z,z')}{(z-z')^2}\right],$$

$$I(x,z,z') = \frac{1}{X^2-(X')^2}\ln\frac{X^2}{(X')^2}\left[\frac{X^2+(X')^2}{(z-z')^2}-\frac{X\cdot X'}{X^2}-\frac{X\cdot X'}{(X')^2}-2\right];$$
$$I_f(x,z,z') = \frac{2}{(z-z')^2}-\frac{2X\cdot X'}{(z-z')^2(X^2-(X')^2)}\ln\frac{X^2}{(X')^2}.$$

Comparing with Balitsky and Chirilli (arXiv:1309.7644 [hep-ph])

Compute evolution of Wilson lines with open color indices:

$$\partial_{\mathbf{Y}}\left[\mathbf{S}^{\mathbf{ab}}(\mathbf{x})\right] = -\mathbf{H}^{\mathbf{NLO\,JIMWLK}}\left[\mathbf{S}^{\mathbf{ab}}(\mathbf{x})\right]$$

$$\partial_{\mathbf{Y}}\left[\mathbf{S}^{\mathbf{ab}}(\mathbf{x})\mathbf{S}^{\mathbf{cd}}(\mathbf{y})\right] = -\mathbf{H}^{\mathbf{NLO\,JIMWLK}}\left[\mathbf{S}^{\mathbf{ab}}(\mathbf{x})\mathbf{S}^{\mathbf{cd}}(\mathbf{y})\right]$$

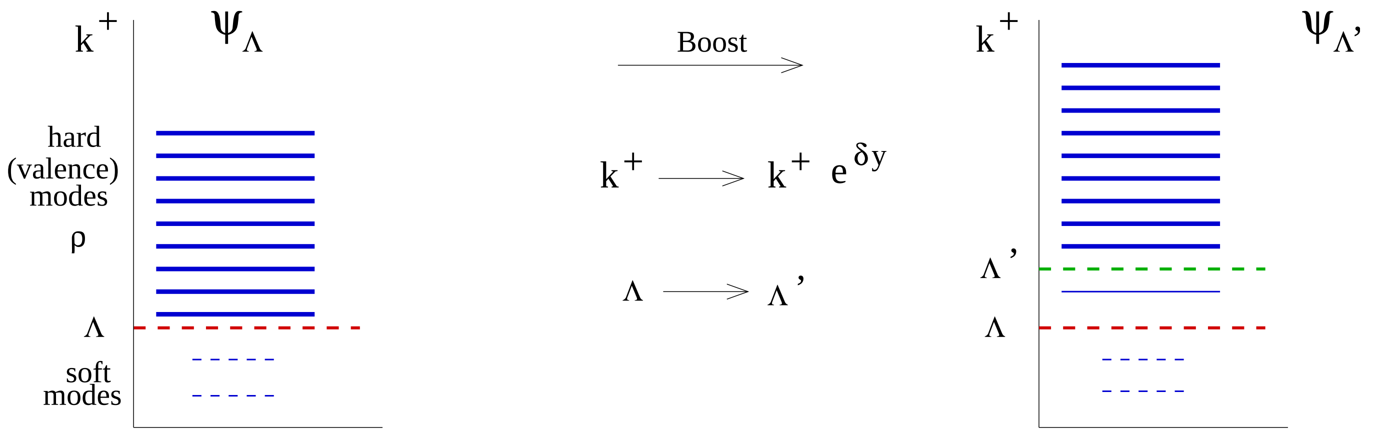
$$\partial_{\mathbf{Y}}\left[\mathbf{S}^{\mathbf{ab}}(\mathbf{x})\mathbf{S}^{\mathbf{cd}}(\mathbf{y})\mathbf{S}^{\mathbf{ef}}(\mathbf{z})\right] = -\mathbf{H}^{\mathbf{NLO\,JIMWLK}}\left[\mathbf{S}^{\mathbf{ab}}(\mathbf{x})\mathbf{S}^{\mathbf{cd}}(\mathbf{y})\mathbf{S}^{\mathbf{ef}}(\mathbf{z})\right]$$

100% agreement!

Light Cone Wave Function

$$\mathbf{H}_{\mathrm{LC\,QCD}}\,|\boldsymbol{\Psi}\rangle \;=\; \mathbf{E}\,|\boldsymbol{\Psi}\rangle$$

Born-Oppenheimer adiabatic approximation



Hard particles with $k^+ > \Lambda$ scatter of the target. Hard (valence) modes are described by the valence density $\rho(x_\perp)$ (shock wave).

The boost opens a window above Λ with the width $\sim \delta y$. The window is populated by soft modes, which became hard after the boost. These newly created hard modes do scatter off the target.

In the dilute limit $\rho \sim 1$; gluon emission $\sim \alpha_s\,\rho$, LO = one gluon, NLO = 2 gluons/quarks

Denote soft glue (quark) creation and annihilation operators as $\mathbf{a}$ and $\mathbf{a}^\dagger$.

$$\mathbf{H}_{\mathrm{LC\,QCD}} \;=\; \mathbf{H}[\rho,\,\mathbf{a},\,\mathbf{a}^\dagger] \;=\; \mathbf{H}_{\mathrm{V}}[\rho] \;+\; \mathbf{H}_{\mathrm{free}}[\mathbf{a},\,\mathbf{a}^\dagger] \;+\; \mathbf{H}_{\mathrm{int}}[\rho,\,\mathbf{a},\,\mathbf{a}^\dagger]$$

LCWF with no soft gluons

$$\mathbf{H}_{\mathrm{V}}\,|\mathbf{v},\,\mathbf{0}_{\mathrm{a}}\rangle \;=\; \mathbf{E}_0\,|\mathbf{v},\,\mathbf{0}_{\mathrm{a}}\rangle\,; \qquad \mathbf{a}\,|\mathbf{v},\,\mathbf{0}_{\mathrm{a}}\rangle \;=\; \mathbf{0}\,; \qquad \mathbf{E}_0 \;=\; \mathbf{0}$$

LCWF with soft gluon/quark dressing

$$|\mathbf{\Psi}\rangle \;=\; \mathbf{\Omega}(\rho,\,\mathbf{a},\,\mathbf{a}^\dagger)\,|\mathbf{v},\,\mathbf{0}_{\mathrm{a}}\rangle\,; \qquad \mathbf{\Omega}^\dagger\,\mathbf{H}_{\mathrm{LC\,QCD}}\,\mathbf{\Omega} \;=\; \mathbf{H}_{\mathrm{diagonal}}$$

Find Ω in perturbation theory

- **Dilute limit $(\rho \to 0)$ – usual perturbation theory in H_{int}**
- **Dense limit $(\rho \sim 1/\alpha_s)$ – all order resummation in a strong background field**

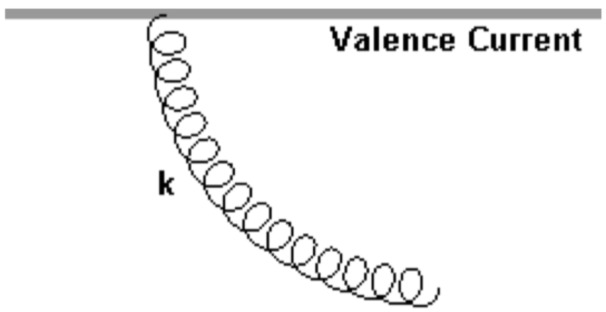
LCWF at LO

First order (g) perturbation theory

$$|\Psi_{LO}\rangle = \mathcal{N}\,|0_a\rangle - \sum_i |i\rangle \frac{\langle i|\,H_{int}\,|0_a\rangle}{E_i} \qquad\qquad \langle\Psi_{LO}|\Psi_{LO}\rangle = 1 \to \mathcal{N}$$

Eikonal coupling between valence and soft gluons due to separation of scales

$$H_{int} = -\int \frac{dk^+}{2\pi}\frac{d^2k_\perp}{(2\pi)^2}\frac{g\,k_i}{\sqrt{2}\,|k^+|^{3/2}}\left[a_i^{\dagger a}(k^+,\,k_\perp)\;\rho^a(-k_\perp) + a_i^a(k^+,\,-k_\perp)\;\rho^a(k_\perp)\right]$$



A cloud of WW gluons dressing the valence ones

ρ **are operators on the valence Hilbert space**

SU(N) algebra: $[\rho^a,\,\rho^b] = if^{abc}\,\rho^c$

$$\mathbf{\Omega_Y(\rho \to 0) \equiv C_Y = Exp\left\{i \int d^2z\, b_i^a(z) \int_{e^{Y_0}\Lambda}^{e^{Y}\Lambda} \frac{dk^+}{\pi^{1/2}|k^+|^{1/2}} \left[a_i^a(k^+, z) + a_i^{\dagger a}(k^+, z))\right]\right\}}$$

The classical Weizsaker-Williams field

$$\mathbf{b_i^a(z) = \frac{g}{2\pi} \int d^2x \frac{(z-x)_i}{(z-x)^2} \; \rho^a(x)}$$

The operator $\mathbf{C}$ dresses the valence charges by a cloud of the WW gluons

The LCWF is a starting point for numerous semi-inclusive calculations, such as single/double inclusive particle production.

$$\mathbf{\Sigma^{LO} = \langle \Psi_{LO}|\, \hat{S}\, |\Psi_{LO}\rangle \;=\; \langle \rho,\, 0_a|\; C^\dagger\, \hat{S}\; C\, |\rho, 0_a\rangle \to \ LO\ JIMWLK}$$

LCWF at NLO

ML and Yair Mulian, arXiv:1610.03453

- g^3 + normalisation at g^4

$$|\Psi_{\rm NLO}\rangle = \mathcal{N}\,|0\rangle + \sum_{\rm i} |{\rm i}\rangle \left[-\frac{\langle {\rm i}|\,{\rm H_{int}}\,|0\rangle}{\rm E_i} + \frac{\langle {\rm i}|\,{\rm H_{int}}\,|{\rm j}\rangle\,\langle {\rm j}|\,{\rm H_{int}}\,|0\rangle}{\rm E_i\,E_j} + \right.$$

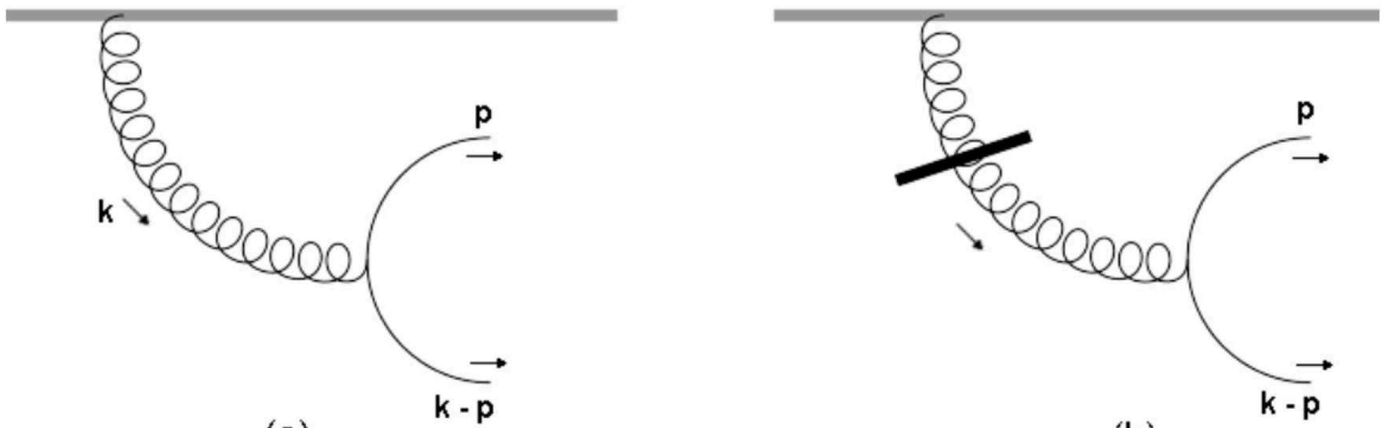
$$\left. + \frac{\langle {\rm i}|\,{\rm H_{int}}\,|0\rangle\,\langle {\rm j}|\,{\rm H_{int}}\,|0\rangle^2\,(2\,{\rm E_j} - {\rm E_i})}{2\,{\rm E_i^2}\,{\rm E_j^2}} - \frac{\langle {\rm i}|\,{\rm H_{int}}\,|{\rm j}\rangle\,\langle {\rm j}|\,{\rm H_{int}}\,|{\rm k}\rangle\,\langle {\rm k}|\,{\rm H_{int}}\,|0\rangle}{\rm E_i\,E_j\,E_k} \right]$$

i runs over one gluon, two gluons, and two quarks

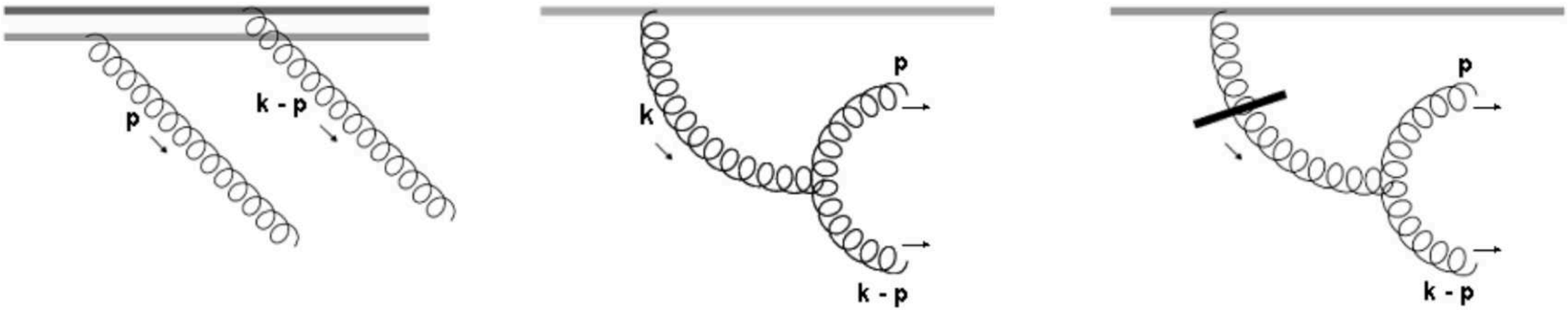
Operator valued matrix elements

Accounts for first non-linear/saturation effects in the projectile

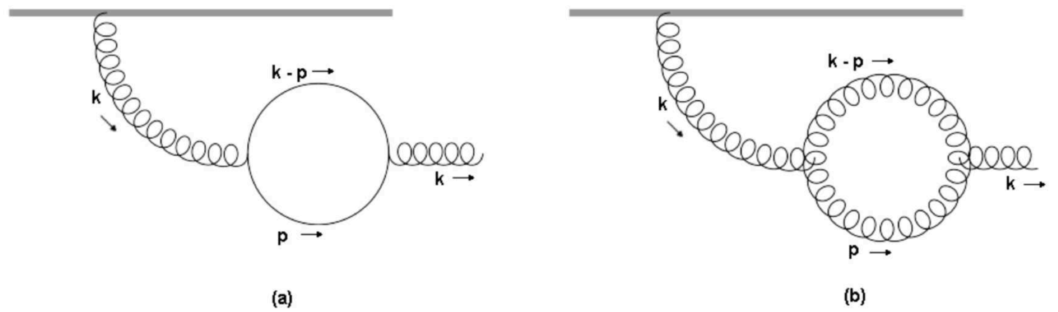
$$\langle \Psi_{\rm NLO}|\Psi_{\rm NLO}\rangle = 1 \to |\mathcal{N}|\,; \qquad\qquad \mathcal{N} = |\mathcal{N}|\,{\rm e}^{{\rm i}\phi}$$



(a) (b)



(a) (b) (c)


k
k - p
k
p
(a)
k
k - p
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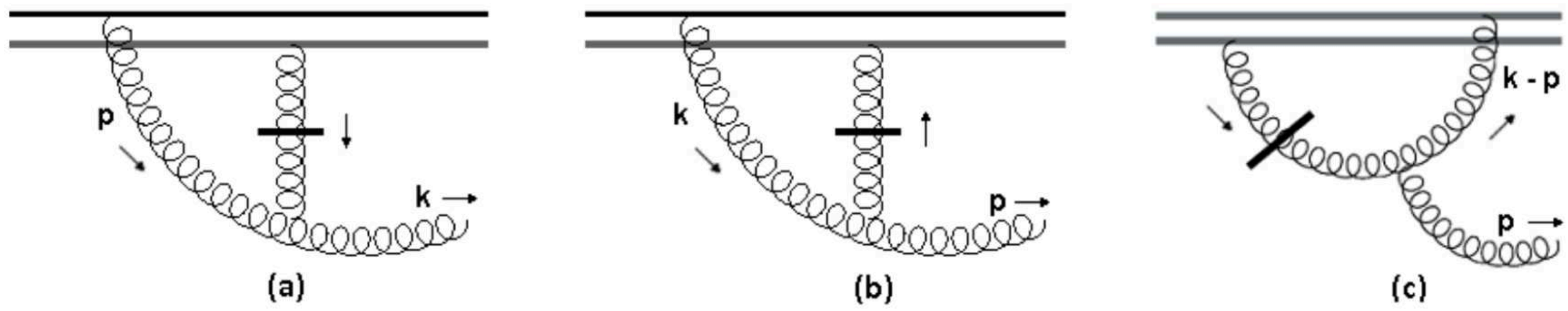
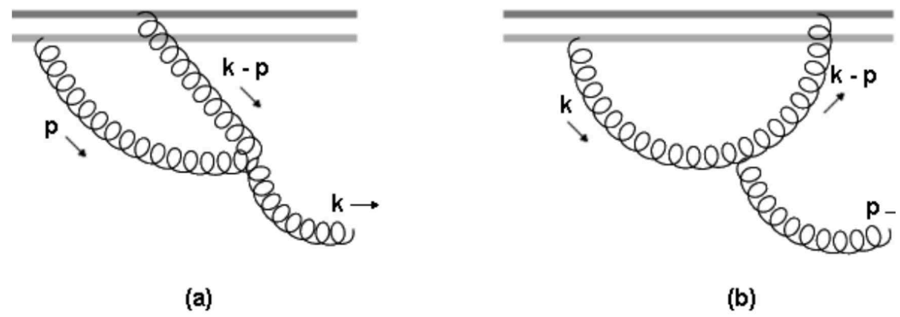
(b)


p
k
(a)
k
p
(b)
k - p
p
(c)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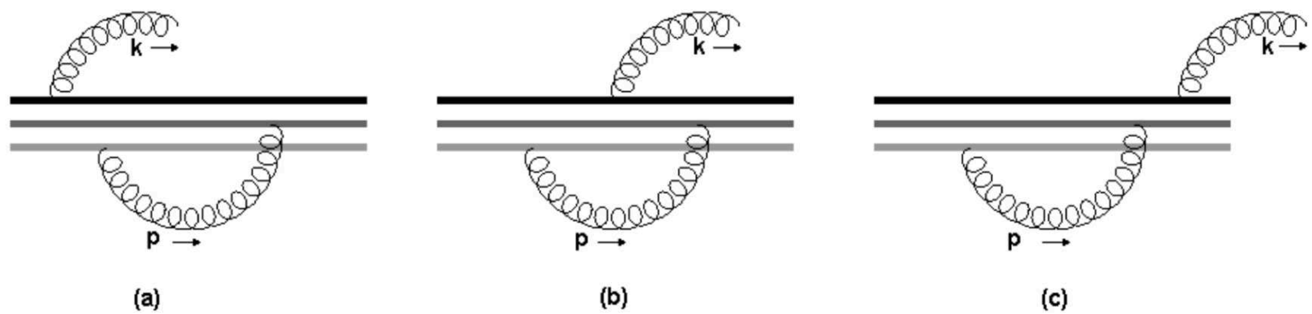
k
p
(a)
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p
(b)
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p
(c)

Phase of the LCWF @ NLO

$$\mathcal{N} \;=\; |\mathcal{N}|\, \mathbf{e}^{\mathbf{i}\phi}$$

Beyond perturbation theory: Born-Oppenheimer adiabatic approximation

$$\langle \mathbf{v}| \otimes \langle\psi|\, \mathbf{H_V}\, |\psi\rangle \otimes |\mathbf{v}\rangle \;\simeq\; \langle \mathbf{v}|\, \mathbf{H_V}\, |\mathbf{v}\rangle \qquad \textbf{or} \qquad \langle\psi|\; \mathbf{H_V}\; |\psi\rangle_{\mathbf{soft}} \;\simeq\; \mathbf{0}$$

the dynamics of the soft modes does not significantly affect that of the valence.

Berry connection

$$\langle\psi|\; \frac{\delta}{\delta\rho^{\mathrm{d}}(\mathbf{w})}\; |\psi\rangle \;=\; \mathbf{0} \qquad \rightarrow\; \phi$$

JIMWLK Hamiltonian @ NLO (again)

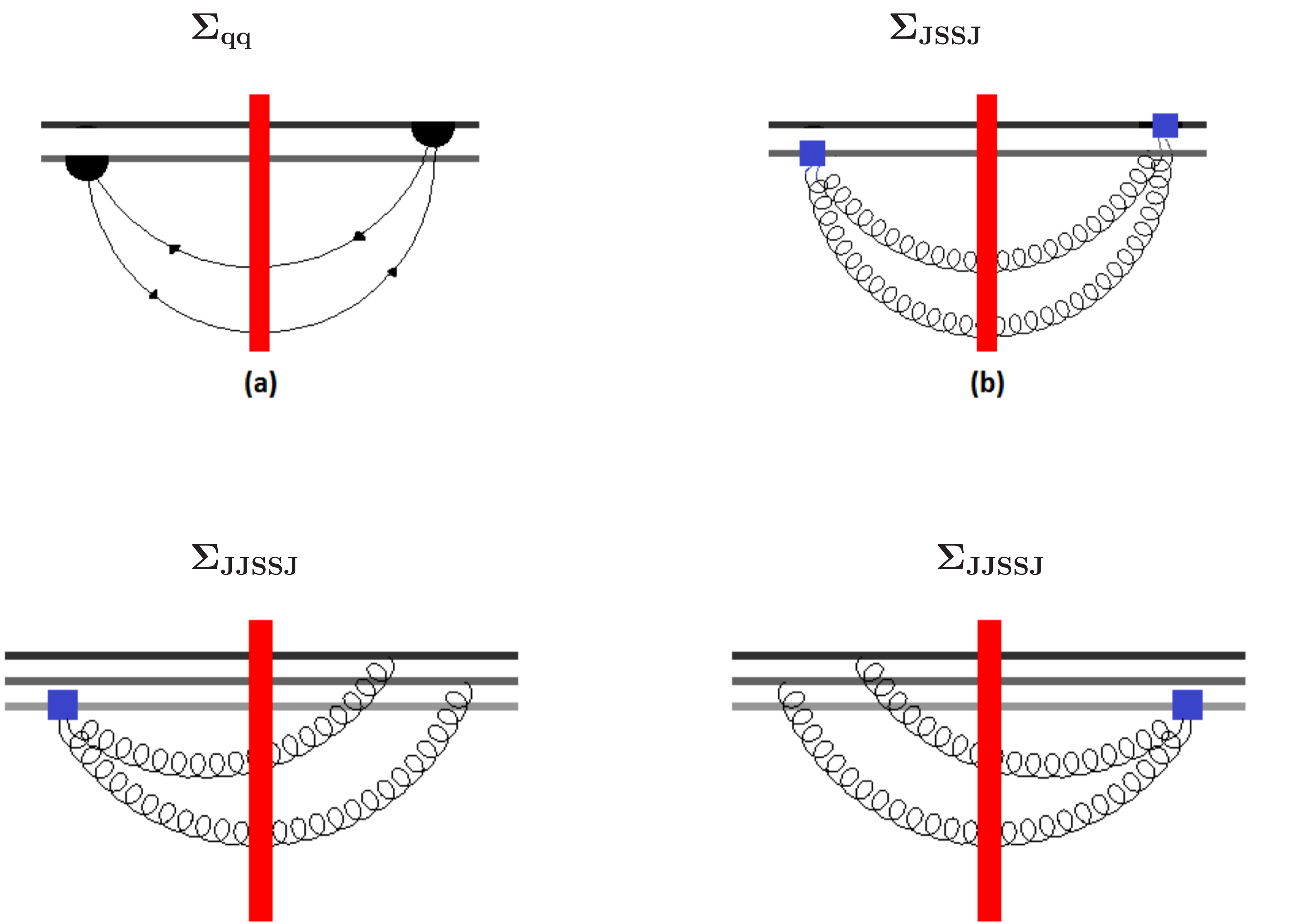
$$\boldsymbol{\Sigma} = \left\langle \psi^{\mathbf{NLO}} \right| \hat{\mathbf{S}} \left| \psi^{\mathbf{NLO}} \right\rangle = \langle \rho,\, \mathbf{0_a} |\; \boldsymbol{\Omega}^\dagger \, \hat{\mathbf{S}} \; \boldsymbol{\Omega} \, | \rho, \mathbf{0_a} \rangle$$

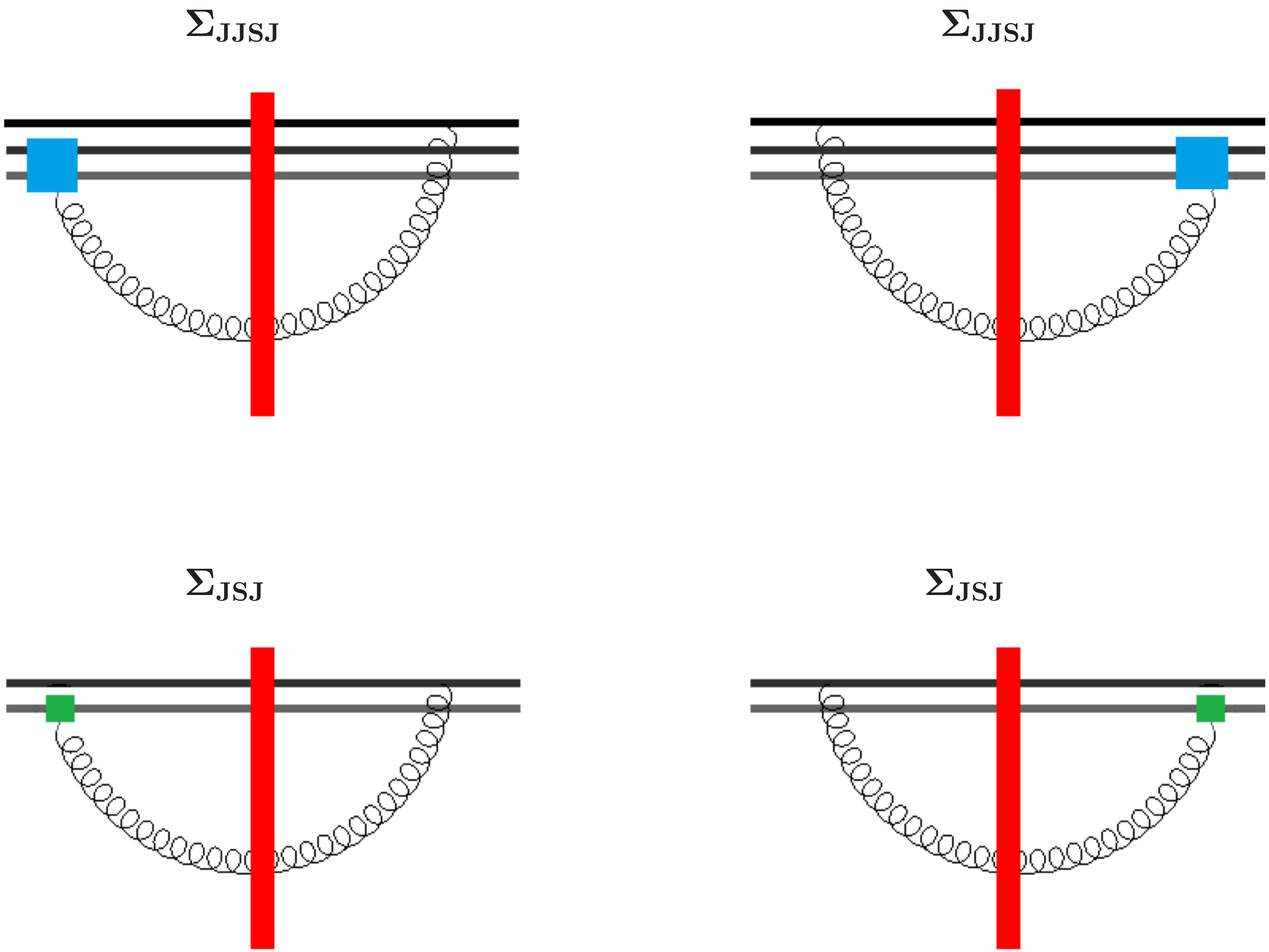
$$= \Sigma^{LO} + \Sigma_{q\bar{q}} + \Sigma_{JJSSJ} + \Sigma_{JSSJ} + \Sigma_{JJSJ} + \Sigma_{JSJ} + \Sigma_{JJSSJJ} + \Sigma_{JJJSJ} + \Sigma_{virtual}.$$

$$\boldsymbol{\Sigma}_{\dots} = \boldsymbol{\Sigma}^{\mathbf{NLO}}_{\dots}(\delta \mathsf{Y}) + \boldsymbol{\Sigma}^{(\delta \mathsf{Y})^{\mathbf{2}}}_{\dots}$$

$$\boldsymbol{\Sigma}^{(\delta \mathsf{Y})^{\mathbf{2}}} = \frac{\mathbf{1}}{\mathbf{2}} \left(\delta \mathbf{Y} \, \mathbf{H}^{\mathbf{LO}}_{\mathbf{JIMWLK}}\right)^{\mathbf{2}}$$

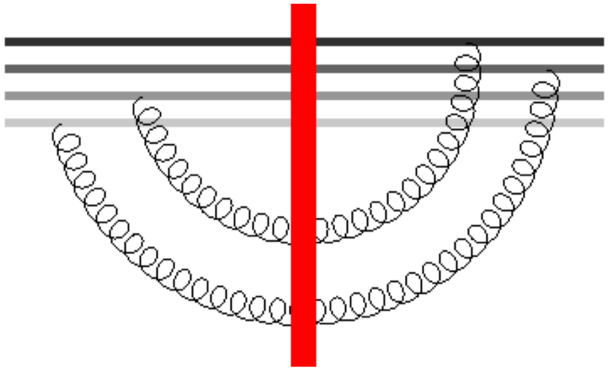
$$\boldsymbol{\Sigma}^{\mathbf{NLO}}(\delta \mathsf{Y}) \rightarrow \mathbf{H}^{\mathbf{NLO\,JIMWLK}}$$


$\Sigma_{\mathbf{qq}}$
$\Sigma_{\mathbf{JSSJ}}$
(a)
(b)
$\Sigma_{\mathbf{JJSSJ}}$
$\Sigma_{\mathbf{JJSSJ}}$

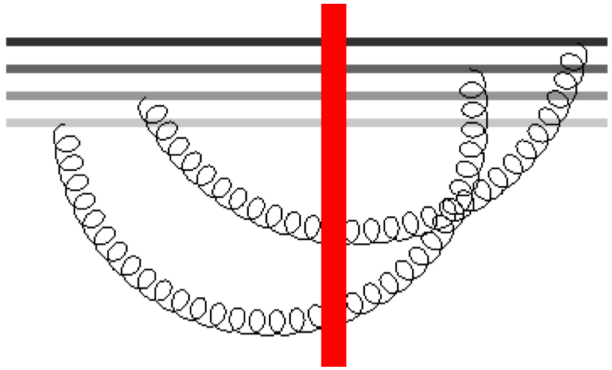


$\Sigma_{\mathbf{JJSJ}}$
$\Sigma_{\mathbf{JJSJ}}$
$\Sigma_{\mathbf{JSJ}}$
$\Sigma_{\mathbf{JSJ}}$

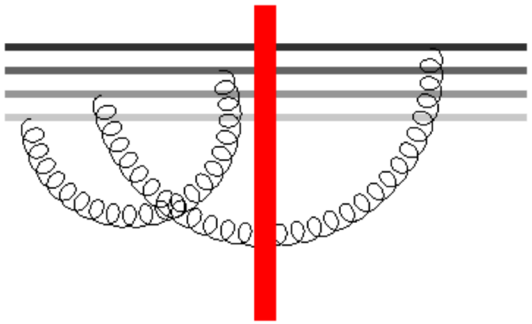
$\boldsymbol{\Sigma}_{\mathbf{JJSSJJ}}$

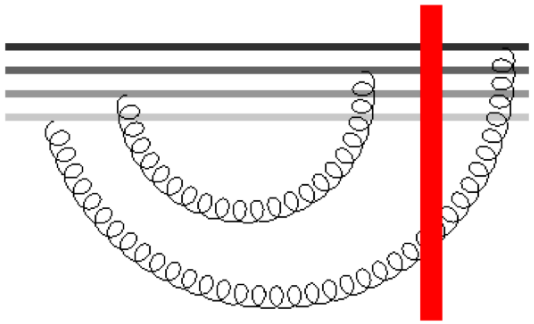


$\boldsymbol{\Sigma}_{\mathbf{JJSSJJ}}$

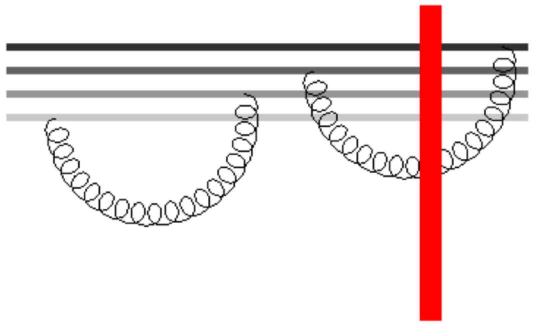


$\boldsymbol{\Sigma}_{\mathbf{JJJSJ}}$





$\boldsymbol{\Sigma}_{\mathbf{JJJSJ}}$



And also $\boldsymbol{\Sigma}_{\mathbf{virtual}}$ **;** $\quad \phi \;\to\; JJJ$

Is the JIMWLK Hamiltonian Conformally invariant?

Alex Kovner, ML and Yair Mulian (2014)

Scale invariance is trivial. Lets focus on inversion. Introduce $\mathbf{x_\pm = x_1 \pm i\, x_2}$

$$\text{Inversion transformation}: \quad x_+ \to 1/x_- \, ; \qquad\qquad x_- \to 1/x_+$$

A "naive" representation $\mathcal{I}_0$ of the inversion transformation is

$$\mathcal{I}_0 : \quad \mathbf{S(x_+, x_-) \to S(1/x_-, 1/x_+)} \, , \qquad \mathbf{J_{L,R}(x_+, x_-) \to \frac{1}{x_+ x_-} J_{L,R}(1/x_-, 1/x_+)} \, .$$

Conformal invariance (in the gauge invariant sector) @LO:

$$\mathcal{I}_0 \; \mathbf{H^{LO\,JIMWLK}} \; \mathcal{I}_0 \;=\; \mathbf{H^{LO\,JIMWLK}}$$

No (naive) Conformal invariance @NLO:

$$\mathcal{I}_0 \; \mathbf{H^{NLO\,JIMWLK}} \; \mathcal{I}_0 \;=\; \mathbf{H^{NLO\,JIMWLK}} \;+\; \mathcal{A}$$

QCD is not conformally invariant beyond tree level, but $\mathcal{N} = 4$ SUSY is.

JIMWLK Hamiltonian IS conformally invariant! (in $\mathcal{N}=4$)

S **forms a non-trivial representation of the conformal group:**

$$\mathcal{I} : S(x) \to S(1/x) + \delta S(x)\,, \qquad \mathcal{I} : H^{LO} \to H^{LO} - \mathcal{A}$$

Here δS is of order α_s. The condition is that the net anomaly cancels:

$$\mathcal{I}\,(\mathbf{H}^{\mathbf{LO}} + \mathbf{H}^{\mathbf{NLO}})\,\mathcal{I} \;=\; \mathbf{H}^{\mathbf{LO}} + \mathbf{H}^{\mathbf{NLO}}$$

We have constructed $\mathcal{I}$ perturbatively: $\qquad \mathcal{I} \;=\; (\mathbf{1} + \mathcal{C})\,\mathcal{I}_0\,.$

$$\mathcal{C} \;=\; -\,\frac{\mathbf{1}}{\mathbf{2}}\,\frac{\alpha_{\mathrm{s}}}{\mathbf{2}\pi^{\mathbf{2}}}\int_{\mathrm{x,y,z}} \ln\Big[\frac{(\mathbf{x}-\mathbf{y})^{\mathbf{2}}\mathbf{a}^{\mathbf{2}}}{(\mathbf{x}-\mathbf{z})^{\mathbf{2}}(\mathbf{y}-\mathbf{z})^{\mathbf{2}}}\Big]\times$$

$$\times\frac{(\mathbf{x}-\mathbf{y})^{\mathbf{2}}}{(\mathbf{x}-\mathbf{z})^{\mathbf{2}}(\mathbf{y}-\mathbf{z})^{\mathbf{2}}}\Big\{\mathbf{J}^{\mathrm{a}}_{\mathrm{L}}(\mathbf{x})\mathbf{J}^{\mathrm{a}}_{\mathrm{L}}(\mathbf{y}) + \mathbf{J}^{\mathrm{a}}_{\mathrm{R}}(\mathbf{x})\mathbf{J}^{\mathrm{a}}_{\mathrm{R}}(\mathbf{y}) - \mathbf{2}\mathbf{J}^{\mathrm{a}}_{\mathrm{L}}(\mathbf{x})\mathbf{S}^{\mathrm{ab}}_{\mathrm{A}}(\mathbf{z})\mathbf{J}^{\mathrm{b}}_{\mathrm{R}}(\mathbf{y})\Big\}$$

For an arbitrary operator $\mathcal{O}$ (s, B, H^{JIMWLK},...) we define its conformal extension:

$$\mathcal{O}^{conf} = \mathcal{O} + \frac{1}{2}\,[\mathcal{C},\,\mathcal{O}]\,; \qquad [s^{conf} \text{ by Balitsky and Chirilli (arXiv : 0903.5326)}]$$

CONCLUSIONS

- We have computed the LCWF@NLO. Apart of leading to JIMWLK equation, it can be used to compute semi-inclusive observables @NLO.

- We have first constructed and then independently derived the JIMWLK Hamiltonian at NLO. It fully reproduces and generalises (all?) previously known low x evolution equations at NLO, including Balitsky's hierarchy at NLO

- We have proven the conformal invariance of the NLO JIMWLK Hamiltonian (in $\mathcal{N}=4$). For any operator, we can construct its perturbative extension, such that the resulting operator evolves with conformal kernels.

- Once expanded in the dilute limit, the NLO JIMWLK makes it possible to study evolution of any multi-gluon BKP state and transition vertices at NLO.

QCD

QCD Lagrangian

$$\mathcal{L} \;=\frac{1}{4}\, G_{\mu\nu}\, G^{\mu\nu} \;+\; \overline{\psi}\,(i\;\partial\!\!\!/ \;-\; g\, A\!\!\!/ - m)\;\psi$$

The field strength

$$G_a^{\mu\nu} \;=\; \partial^\mu\, A_a^\nu \;-\; \partial^\nu\, A_a^\mu \;-\; g\, f^{abc}\, A_b^\mu\, A_c^\nu$$

Equations of motion:

Maxwell equation:

$$\partial_\mu\, G^{\mu\nu} \;=\; g\; J^\nu\;; \qquad\qquad J_a^\nu \;=\; \bar{\psi}\,\gamma^\nu\,\tau^a\,\psi \;-\; f^{abc}\, G_b^{\nu\mu}\, A_c^\mu$$

Dirac equation

$$(i\,\gamma^\mu\, D_\mu \;-\; m)\,\psi \;=\; 0$$

Light Cone

LC time $x^+ = (t + z)/\sqrt{2}$ $\qquad x^- = (t - z)/\sqrt{2}$

LC gauge

$$A^+ = \frac{1}{\sqrt{2}}(A^0 + A^3) = 0$$

The Gauss law constraint

$$\partial_\mu G^{\mu +} = g\, J^+$$

is solved for the A^- field

$$-(\partial^+)^2 A_a^- + \partial^+ \partial_i A_a^i = g\, J_a^+$$

$$A_a^- = -\frac{\partial^i}{\partial^+} A_a^i + \frac{g}{(\partial^+)^2} J_a^+$$

Same story with quarks

Light Cone Hamiltonian

Canonical variables: $A^i, \quad \Pi^i = \frac{\delta L}{\delta(\partial^- A^i)} = G^{+i} = \partial^+ A^i$

Light Cone Hamiltonian:

$$H^{LC} = \int dx^- d^2 x_\perp \left[\Pi^i \, \partial^- A^i - L\right] = H_E + H_M$$

The electric and magnetic parts

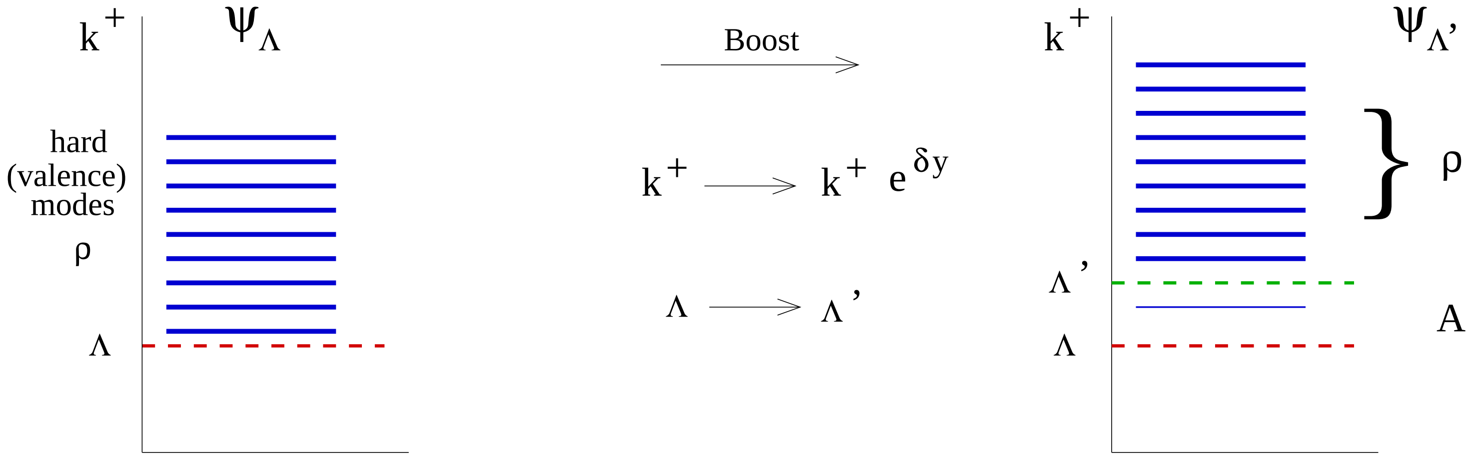
$$H_E = \frac{1}{2} \int \frac{dk^+}{(2\,\pi)} d^2 x \;\; \Pi^-_a(k^+, x)\, \Pi^-_a(-k^+, x)$$

$$H_M = \frac{1}{4} \int \frac{dk^+}{(2\,\pi)} d^2 x \;\; G^a_{ij}(k^+, x)\, G^a_{ij}(-k^+, x)$$

The chromoelectric field

$$\Pi^-_a(k^+, x) = \partial^+ A^- = -\,\partial^i A^a_i + \frac{g}{\partial^+} J^+_a$$

We split the modes into hard and soft: The hard modes act as an external current $j_a^+ = \delta(x^-)\,\rho^a$ **for the soft modes.** $J^+ = j^a + g\,A\,A + (quark\ current)$



$$H^{LC} = H_A^{LC} + H_\rho^{LC}\,; \qquad H_\rho^{LC}\,|\Psi_\Lambda\rangle = E\,|\Psi_\Lambda\rangle\,; \qquad H^{LC}\,|\Psi_{\Lambda'}\rangle = E'\,|\Psi_{\Lambda'}\rangle$$

$|\Psi_\Lambda\rangle$ **is a vacuum of the soft modes** A.

$$H_A^{LC} = H_0 + \delta H^\rho + g\,AAA + \ldots\,; \qquad \delta H^\rho \sim g\,\rho\,A$$

Quantization

$$A_i^a(x^-, \mathbf{x}_\perp) = \int_0^\infty \frac{dk^+}{2\pi} \frac{1}{\sqrt{2k^+}} \left\{ a_i^a(k^+, \mathbf{x})\, e^{-ik^+x^-} + a_i^{a\,\dagger}(k^+, \mathbf{x})\, e^{ik^+x^-} \right\}$$

$$\left[a_i^a(k^+, k), a_j^{b\,\dagger}(p^+, p) \right] = (2\pi)^3\, \delta^{ab}\, \delta_{ij}\, \delta^3(k - p)$$

The free part of the LCH

$$H_0 \;=\; \int_{k^+>0} \frac{dk^+}{2\pi} \frac{d^2k_\perp}{(2\pi)^2} \frac{k_\perp^2}{2k^+}\; a_i^{\dagger a}(k^+, k_\perp)\; a_i^a(k^+, k_\perp)$$

The vacuum of the LCH is simply the Fock space vacuum of the operators a

$$a_q|0\rangle = 0\,; \qquad\qquad E_0 \;=\; 0$$

The one particle state

$$|k, a, i\rangle \;=\; \frac{1}{(2\pi)^{3/2}}\, a_i^{a\,\dagger}(k^+, k)\, |0\rangle \qquad\qquad E_g \;=\; k^- \;=\; \frac{k_\perp^2}{2k^+}$$

Perturbation Theory

$$\delta H^{\rho} \;=\; - \int \frac{dk^{+}}{2\pi}\frac{d^{2}k_{\perp}}{(2\pi)^{2}}\frac{g\,k_{i}}{\sqrt{2}\,|k^{+}|^{3/2}}\;\left[a_{i}^{\dagger a}(k^{+},\,k_{\perp})\;\hat{\rho}^{a}(-k_{\perp})\;+\;a_{i}^{a}(k^{+},\,-k_{\perp})\;\;\hat{\rho}^{a}(k_{\perp})\right]$$

The first order perturbation theory

$$|\theta\rangle \;=\; \beta\,|0\rangle \;-\; \sum_{i}\,|i\rangle\,\frac{\langle i|\,\delta H^{\rho}\,|0\rangle}{E_{i}} \qquad\qquad \langle\theta|\theta\rangle \;=\; 1 \;\rightarrow\; \beta$$

This Hamiltonian creates only one particle state from the vacuum

$$\langle 1\,gluon|\,\delta H^{\rho}\,|0\rangle \;=\; \langle k_{\perp}, k^{+}, a, i|\,\delta H^{\rho}\,|0\rangle \;=\; \frac{g\,k_{i}}{4\,\pi^{3/2}\,|k^{+}|^{3/2}}\,\rho^{a}(-k_{\perp})$$

We can write the soft gluon vacuum state to the first order in the coupling as

$$|\theta\rangle \;=\; C_{\delta Y}\,|0\rangle\,; \qquad\qquad |\Psi_{\Lambda'}\rangle \;=\; C_{\delta Y}\,|\Psi_{\Lambda}\rangle$$