Finding small-x physics in final state jets

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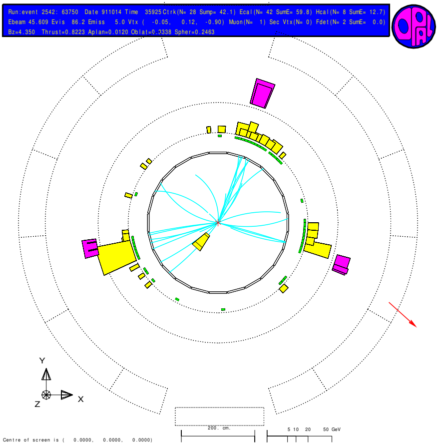
arXiv: 1501.04596, 1508.07568, 1609.04011, 1610.02031

BNL/RIKEN Saturation Workshop
April 26, 2017

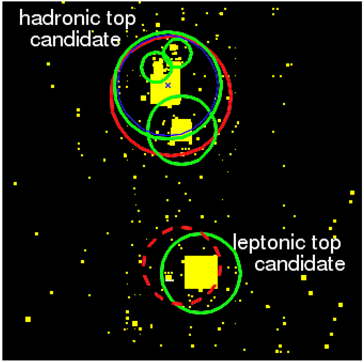
Outline

- Jet Physics
- What is an NGL?
- Banfi-Marchesini-Smye Equation.
- Conformal Mappings and the BK Equation.
- Asymptotics of NGLs and Saturation physics.
- Conclusions.

Question: Given a hard process at $t = 0$, what is the pattern of radiation at $t = \infty$ on S^2?



[ATLAS]



Is it a top quark or QCD gluon jet?

Scattering in QFT

- Hard interaction = scattering amplitude.
- Massive particles $\to$ small-space-time interaction region.
- Interactions stop for $\Delta x^2 \gg m_{gap}^{-2}$

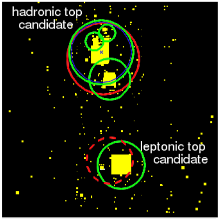
LSZ Recipe:

- Few particles emerge from scattering.
- Well defined mass poles or narrow resonances, definable perturbatively.
- Measure masses and angles and energies.

$$\begin{aligned}\text{Isolated Jet} &\approx \text{tight group of hadrons}\\ &\approx \text{off-shell parton+splittings}\\ &\approx \text{particle}\end{aligned}$$

LSZ ?

No. States are never (perturbatively) asymptotically isolated.



Continuous spectrum and long range-interactions.

On-shell Momentum Regions by power counting:

$$p = (\bar{n} \cdot p, n \cdot p, p_\perp)$$

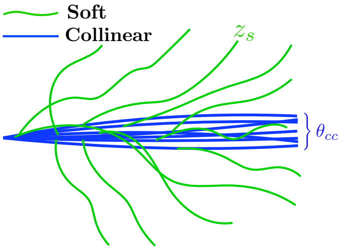
$$p_c \sim Q(1, \theta_{cc}^2, \theta_{cc}) \qquad\qquad p_s \sim Q(z_s, z_s, z_s)$$

Dispersion relations with homogeneous scaling:

$$\textbf{Collinear:}\ p_c^2 = \bar{n} \cdot p_c n \cdot p_c - p_{c\perp}^2 \sim Q^2 \theta_{cc}^2$$

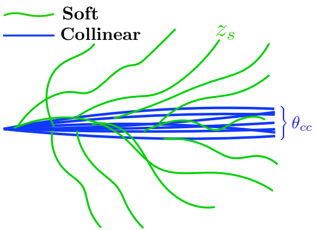
$$\textbf{Soft:}\ p_s^2 = \bar{n} \cdot p_s n \cdot p_s - p_{s\perp}^2 \sim Q^2 z_s^2$$

$\theta_{cc}, z_s \to 0$ recovers on-shell partons.



$$m_{ab}^2 = (p_a + p_b)^2 \sim Q^2 z_a z_b \theta_{ab}^2$$

The Soft and The Collinear: Factorization



These contributions factorize from each other:

- Collinears sensitive only to *total* momentum contributions from soft radiation.
- Softs sensitive only direction and total charge of collinear regions.
- This factorization is as close to LSZ we get.

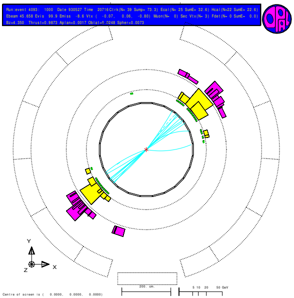
$$\mathcal{A}(i \to f) = C_H\left(\{p_j\}_{j=1}^N\right) \otimes \mathcal{A}_S(i_s \to f_s) \otimes \prod_{i=j}^{N} \mathcal{A}_j(i_{\|j} \to f_{\|j})$$

Global Event Shapes

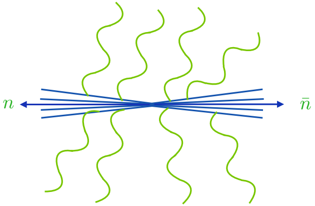
- Paradigmatic example: $e^+e^- \to hadrons$, measure thrust.
- One vetos the production of additional jets anywhere in phase space.

Find axis $\hat{n}$ maximizing momentum flow parallel.

$$T = \max_{\hat{n}} \sum_{i \in \text{event}} \frac{|\hat{n} \cdot \vec{p}_i|}{Q}, \quad \text{and} \quad \tau = 1 - T$$



$\tau \ll 1$



Global Logarithms

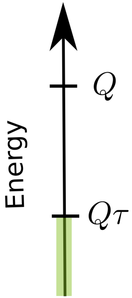
- How? Soft-Collinear Effective Field Theory: final state degrees of freedom precisely known.
- $\theta_{cc} \sim \sqrt{\tau}$
- $z_s \sim \tau$

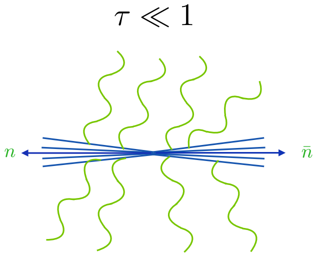
Global Event Shapes

Factorize cross-section into distinct objects:

$$\frac{d\sigma}{d\tau} = H\Big(Q,\mu\Big) J_n\Big(\tau,\mu\Big) \otimes J_{\bar{n}}\Big(\tau,\mu\Big) \otimes S_{n\bar{n}}\Big(\tau,\mu\Big) + \ldots$$

$$\tau = \tau_n + \tau_{\bar{n}} + \tau_s$$





- Nothing happens between Q and thrust value $Q\tau$.

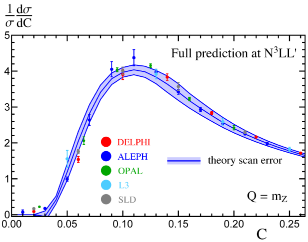
Global Logarithms

- Nothing happens (no on-shell final states emitted) between $q\bar{q}$ production at Q and thrust value $Q\tau$.

$\alpha_s \ln\tau$ resummed thru RG evolution:

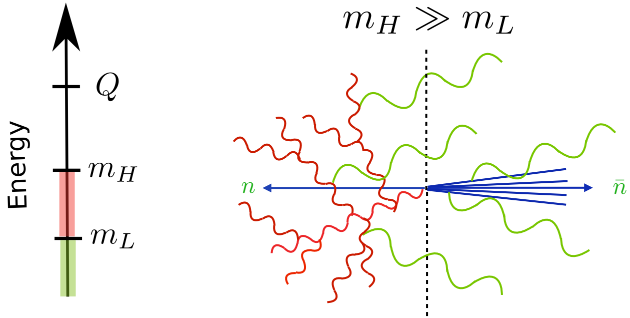
$$\mu \frac{d}{d\mu} F(\mu) = \gamma(\mu) \otimes F(\mu)$$

γ and F computable to high orders in fixed order perturbation theory.



Non-Global Measurements

Measure each jets mass independently:

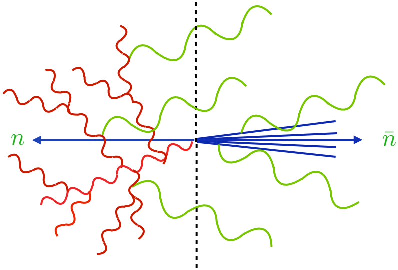


Between the scale m_H and m_L we can have active jet production. [Dasgupta, Salam]

- Distribution in m_L entangled with the "on-shell" branching history below m_H.

Non-Global Logarithms: The Global Factorization

$$m_H \gg m_L$$



$$\frac{d\sigma}{dm_H dm_L} = H\Big(Q,\mu\Big) J_n\Big(m_H,\mu\Big) J_{\bar{n}}\Big(m_L,\mu\Big) \otimes S_{n\bar{n}}\Big(m_H,m_L,\mu\Big) + \ldots$$

- Simulate arbitrarily complicated secondary branching.
- Linear evolution equations no longer sufficient for all logarithms. [Banfi, et. al.; Wiegert]

Non-Global Logarithms

- The final state degrees of freedom *not* precisely known.
- $\theta_{cc} \sim \sqrt{\frac{m_L}{Q}}$ or $\sqrt{\frac{m_H}{Q}}$?
- $z_s \leq \frac{m_H}{Q}$
- The heavier hemisphere → more jets at the light-jet "clustering" scale.
- Position of these jets are integrated over.

Banfi-Marchesini-Smye Equation

- Calculate the production of additional jets given jets already produced.
- Use full eikonal approximation for each step.
- Jets = Eikonal lines.
- Jet energies are ordered.

Note: This is at the heart of all modern parton-showers: ARIADNE, VINCIA, DEDUCTOR, DIRE, dipole versions of HERWIGG and SHERPA, etc.

BMS equation at large N_c

Soft function expressable in terms of color dipole functions:

Non-global Log:

$$L = \frac{C_A}{\pi}\int_{m_L}^{m_H}\frac{d\mu}{\mu}\alpha_s(\mu)$$

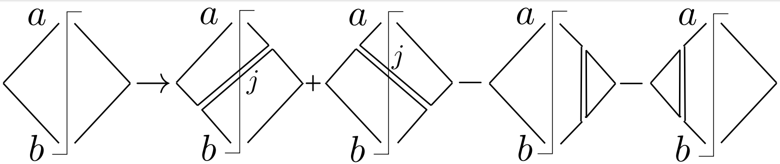
$$S_{ab}(m_H, m_L; \mu) \sim V_{ab}(\mu, m_H, m_L) g_{ab}(L)$$

- S_{ab}: soft function appearing in dijet factorization with eikonal lines $a = (1, \hat{n}_a)$ and $b = (1, \hat{n}_b)$.

  $$S_{ab}(m_H, m_L) \propto \mathrm{tr}\langle 0|T\{S_a S_b\}\theta(m_H - \hat{E}\theta_J)\theta(m_L - \hat{E}\theta_{S^2/J})\bar{T}\{S_a S_b\}|0\rangle$$

- g_{ab}: color dipole function with **global** or **Sudakov** evolution factored out.
- A measurable distribution.

BMS equation at large N_c



Evolution of Color Dipoles:

$$\partial_L g_{ab} = \int_J \frac{d\Omega_j}{4\pi} W_{ab}(j) \Big(U_{abj}(L) g_{aj} g_{jb} - g_{ab} \Big)$$

$$W_{ab}(j) = \frac{a \cdot b}{a \cdot j \, j \cdot b}\, ,\; j = (1, \hat{n}_j)$$

$$U_{abj}(L) = \exp\left(L \int_{S^2/J} \frac{d\Omega_q}{4\pi} W_{aj}(q) + W_{jb}(q) - W_{ab}(q) \right)$$

- Full color evolution $\rightarrow$ reduced density matrix [Wiegert, Caron-Huot, Nagy-Soper, Hatta et al.].
- EFT interpretation and small R[Becher et. al.].
- Evolution equation related to factorization for jet substructure [Larkoski, DN, Moult; DN].

BMS eqn. is (almost) the BK eqn.

If $J = S^2$ and the theory is conformal, and ignore initial conditions, then

- $U_{abj} \to 1$.

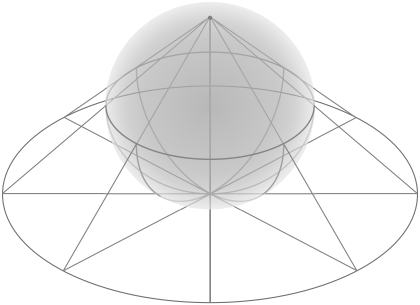
$$\partial_L g_{ab} = \int_{S^2} \frac{d\Omega_j}{4\pi} \frac{a \cdot b}{a \cdot j \, j \cdot b} \bigg(g_{aj} g_{jb} - g_{ab} \bigg)$$

$$\leftrightarrow$$

$$\partial_Y S_{\vec{a}\vec{b}} = \frac{\alpha_s C_A}{\pi} \int_{\mathbb{R}^2} \frac{d\Omega_j}{2\pi} \frac{x^2_{\vec{a}\vec{b}}}{x^2_{\vec{a}\vec{j}} x^2_{\vec{j}\vec{b}}} \bigg(S_{\vec{a}\vec{j}} S_{\vec{j}\vec{b}} - S_{\vec{a}\vec{b}} \bigg)$$

BMS eqn. is (almost) the BK eqn.

Why? Stereographic projection of $S^2 \to \mathbb{R}^2$



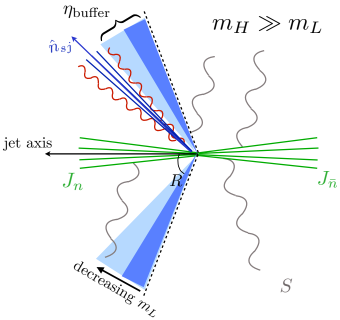
[Hatta et. al.]

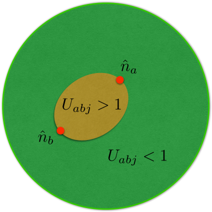
- Checked at NLO, used at NNLO to calculate BK in $N = 4$ SYM [Caron-Huot et. al.].

But Jet Physics is not Small-x

- Jet physics is manifestly dilute.
- At small $L \ll 1$ to moderate $L \sim 1$, the jet physics (J, U_{abj}) parts of BMS eqn. the most important.

The Buffer Region and Phenomonolgy of Soft jets





- Resummation of Sudakov Effects:
  $$U_{abj} \propto \left(1 - \frac{\tan^2\frac{\theta_{sj}}{2}}{\tan^2\frac{R}{2}}\right)^L$$
- Cross-section for production of a jet at the boundary vanishes!
- "Buffer Region" noticed in Monte Carlo by [Dasgupta,Salam].
- Existence implies finite region of convergence ($L = 1$) for LL series [DN et. al.].

The Importance of the Buffer Region

However, one can show that irrespective of the differences, BK and BMS equations imply the same asymptotics.
Key: The asymptotic behavior of the buffer region.
[DN]

Buffer Region Beyond $L = 1$

Intuition: suppression of soft gluon production at large angles is moderate for $L \leq 1$, driven by U_{abj}.

- Recall:

$$\partial_L g_{ab} = \int_J \frac{d\Omega_j}{4\pi} W_{ab}(j) \Big(U_{abj}(L) g_{aj} g_{jb} - g_{ab} \Big)$$

However, what happens to buffer region as $L \to \infty$?

- If g_{ab} dies off, then non-linear term dies off twice as fast.
- Truncate:

$$\lim_{L\to\infty} \partial_L g_{ab} =_? - \left(\int_J \frac{d\Omega_j}{4\pi} W_{ab}(j) \right) g_{ab}$$

Collinearly Regulated BMS

Add collinear regulator to make truncation possible:

- $$\partial_L g^{\delta}_{ab} = \int_J \frac{d\Omega_j}{4\pi} W^{\delta}_{ab}(j)\Big(U_{abj}(L) g^{\delta}_{aj} g^{\delta}_{jb} - g^{\delta}_{ab}\Big)$$
  $$W^{\delta}_{ab}(j) = \frac{a\cdot b}{(a\cdot j + \delta^2)(j\cdot b + \delta^2)}$$
- Comparison Theorem For Diff Eq.:
  $$\ln g^{\delta}_{ab} \geq -\int_0^L d\ell \gamma_{ab}\Big(\delta(\ell)\Big)$$
  $$\gamma_{ab}(\delta) = \int_J \frac{d\Omega_j}{4\pi} W^{\delta}_{ab}(j)$$

Idea: For large L, virtual correction dominates BMS equation except for small collinear region of size $\delta(L)$.

Asymptotic to Full Solution

Now choose $\delta(L)$ such that:

$$g_{ab}\Big(\gamma(\delta)\Big) = \text{Exp}\Big(- \int_0^L d\ell \gamma_{ab}(\delta(\ell))\Big)$$

Asymptotically solves *full* BMS:

$$\lim_{L\to\infty} \partial_L g_{ab}\big(\gamma(\delta)\big) = \int_J \frac{d\Omega_j}{4\pi} W_{ab}(j)\Bigg(U_{abj}(L) g_{aj}\big(\gamma(\delta)\big) g_{jb}\big(\gamma(\delta)\big) - g_{ab}\big(\gamma(\delta)\big)\Bigg)$$

This would be trivial, except for collinear regions!

Complication in Collinear Regions

Collinear safety of BMS equation demands:

$$g_{ab} \to 1, a \parallel b, \forall L$$

So expand BMS equation about limit $a \parallel b \parallel j$:

$$\begin{aligned} g_{ab} &= 1 + \Phi_{ab} + \ldots \\ \lim_{L\to\infty} \partial_L \Phi_{ab} &= \int_0^\infty d\theta_j \int_0^{2\pi} \frac{d\phi_j}{2\pi} \frac{\theta_{ab}^2}{\theta_{aj}^2 \theta_{jb}^2} \Big(\Phi_{aj} + \Phi_{jb} - \Phi_{ab} \Big) + O(\Phi^2) \end{aligned}$$

This is the position space BFKL equation for color dipoles. [Marchesini, Mueller]

NOTE: Jet region dependence is lost in collinear limits!
U_{abj} contributions are power suppressed.

Complication in Collinear Regions

Simply matter to write solutions in terms of eigenfunctions:

$$\Phi^{\nu}_{ab} = A\Big(\frac{\theta^2_{ab}}{\theta^2_c(L)}\Big)^{\nu}$$

$$\theta_c(L) = \mathrm{Exp}\Big(-\frac{\chi(\nu)}{2\nu}L\Big)$$

Note that the earlier anzats has small angle behavior:

$$g_{ab}\Big(\gamma(\delta)\Big) = 1 - \int_0^L d\ell \frac{\theta^2_{ab}}{4\delta^2(\ell)} + \ldots$$

$\nu = 2$ *is a pole! Must use BFKL solution in collinear regions.*

Returning to Asymptotic Condition

Suppose only a leg in active jet region:

$$\lim_{L\to\infty}\partial_L g_{ab}\big(\gamma(\delta)\big)=\int_J\frac{d\Omega_j}{4\pi}W_{ab}(j)\Big(U_{abj}(L)g_{aj}\big(\gamma(\delta)\big)g_{jb}\big(\gamma(\delta)\big)-g_{ab}\big(\gamma(\delta)\big)\Big)$$

$$=\int_{J,\theta_{aj}\leq\theta_c}\frac{d\Omega_j}{4\pi}W_{ab}(j)\Big(U_{abj}(L)g_{aj}\big(\gamma(\delta)\big)g_{jb}\big(\gamma(\delta)\big)-g_{ab}\big(\gamma(\delta)\big)\Big)$$

$$+\int_{J,\theta_{aj}\geq\theta_c}\frac{d\Omega_j}{4\pi}W_{ab}(j)\Big(U_{abj}(L)g_{aj}\big(\gamma(\delta)\big)g_{jb}\big(\gamma(\delta)\big)-g_{ab}\big(\gamma(\delta)\big)\Big)$$

First term, use BFKL solution.
In second term use buffer region anzats.
Consistency requires:

$$\delta(L)=\theta_c(L)=\mathrm{Exp}\Big(-\frac{\chi(\nu)}{2\nu}L\Big)$$

Asymptotic NGL distribution

- At small cut-off $\gamma_{ab}(\delta)$ is dominated by universal collinear poles.
- Only geometry dependence is the number of radiators in the active jet!

$$\ln g_{ab}\big(\gamma(\delta)\big)\Big|_{\text{in-out}} = +\int_0^L d\ell\,\ln\delta(\ell) + \ldots = -\frac{\chi(\nu)}{4\nu}L^2 + \ldots$$

$$\ln g_{ab}\big(\gamma(\delta)\big)\Big|_{\text{in-in}} = +2\int_0^L d\ell\,\ln\delta(\ell) + \ldots = -\frac{\chi(\nu)}{2\nu}L^2 + \ldots$$

Slowest possible decay: $\frac{\chi(\nu_{\min})}{4\nu_{\min}} \approx 1.22$

In some sense this is "topological," since the location within and the shape of jet region is irrelevant.

Buffer Region = Saturation Region

Yet this may be obvious for the small-x acolytes:

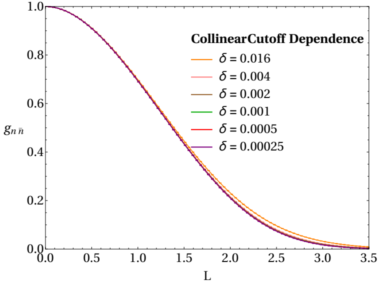
$$
\begin{aligned}
Q_s(Y) &= Q_0 \exp\Big(\frac{\chi(\nu_{\min})}{\nu_{\min}}\bar{\alpha}_s Y - \frac{3}{2\nu_{\min}}\ln\Big(\bar{\alpha}_s Y\Big) + ...\Big) \\
Q_s^{-1}(Y) &\leftrightarrow \delta(L) \\
g_{ab}\big(\gamma(\delta)\big) &\leftrightarrow \text{Levin-Tuchin Black Disc for BK}
\end{aligned}
$$

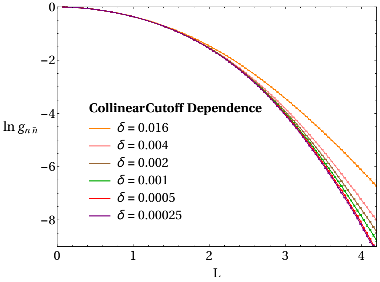
But the physical interpretation is distinct:
In the buffer region, there are no real emissions.
In the saturation region, there are lots of wee partons.

Numerical Check

Inspired by saturation, fit log of tail to Dasgupta-Salam MC:

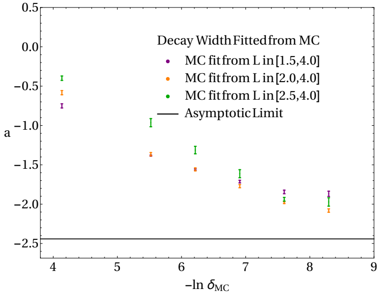
$$M(L) = aL^2 + bL\ln L + cL, \qquad 1.5 \geq L \geq 4$$

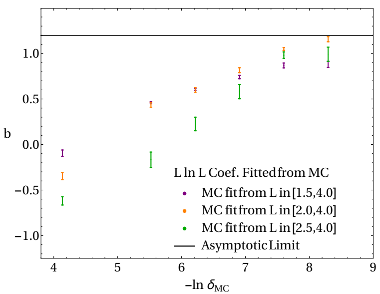




MC runs with collinear cutoff on all emmissions.
Must extrapolate to zero.

Extrapolate to zero cutoff.





Consistent with slowest possible decay: $\frac{\chi(\nu_{\min})}{4\nu_{\min}} \approx 1.22$

Conclusions

- *BFKL is the ultimate controlling equation for the asymptotics of pQCD, both in initial and final state distributions.*
- Asymptotics of NGLs is "topological": count number of initial hard emitters in active region.
- Angular size of buffer region directly maps to saturation scale.

Puzzles:

- BMS equation built upon soft branching only, but asymptotics governed by a *collinear* limit.
- Must we consider the full parton shower, even for purely soft-sensitive observables?