Towards higher-order accuracy in LCPT

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In collaboration with

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Work in progress

see also T. Lappi, RP **Annals of Physics** (2017) 379, 34-66
arXiv:1611.00497

Motivation

Hamiltonian perturbation theory in the light-cone form (LCPT)

- A standard tool for calculations of small-x dilute-dense scattering processes in the CGC picture
- LCPT calculations in CGC slowly approaching the NLO level
  - the inclusive DIS cross sections (G. Beuf arXiv:1606.00777)
  - small-x evolution equations (I. Balitsky, G. Chirilli arXiv:0710.4330)

Problem:

- Perturbative computation of wavefunctions is quite tedious (hard to automatize)

Solution:
We introduce a new helicity formulation for LCPT

- Perturbative (NLO) computation of wavefunctions is easy and can be fully automatized

LCPT in a nutshell

QCD Hamiltonian: $T^{\mu\nu}_{\text{QCD}}$, LC gauge $A^+_a = 0$ & EOMs

$$
\begin{aligned}
\mathcal{H}_{\text{QCD}} = & \frac{1}{2}\int \mathrm{d}x^- \mathrm{d}^2\mathbf{x} \left(\bar{\tilde{\Psi}}\gamma^+ \frac{m^2 + (i\nabla_\perp)^2}{i\partial^+}\tilde{\Psi} + \tilde{A}^\mu_a (i\nabla_\perp)^2 \tilde{A}^a_\mu\right) \\
& + g\int \mathrm{d}x^- \mathrm{d}^2\mathbf{x} \tilde{J}^\mu_a \tilde{A}^a_\mu \\
& + \frac{g^2}{2}\int \mathrm{d}x^- \mathrm{d}^2\mathbf{x} \left(\tilde{J}^+_a \frac{1}{(i\partial^+)^2}\tilde{J}^{+a} + \bar{\tilde{\Psi}}\gamma_\mu \tilde{A}^\mu_a t^a \frac{\gamma^+}{i\partial^+}\gamma_\nu \tilde{A}^\nu_b t^b \tilde{\Psi}\right) \\
& + \frac{g^2}{4}\int \mathrm{d}x^- \mathrm{d}^2\mathbf{x} \tilde{B}^{\mu\nu}_a \tilde{B}^a_{\mu\nu} = \mathcal{H}_0 + \mathcal{H}_{\text{I}}
\end{aligned}
$$

- $\tilde{J}^\mu_a = \bar{\tilde{\Psi}}\gamma^\mu t_a \tilde{\Psi} + f^{abc}\partial^\mu \tilde{A}^\nu_b \tilde{A}_{\nu c}$ & $\tilde{B}^{\mu\nu}_a = f^{abc}\tilde{A}^\mu_b \tilde{A}^\nu_c$

Note: in the $A^+_a = 0$ gauge

- positive: gluons have only two physical transverse degrees of freedom
- negative: breaking of manifest rotational invariance

Quantizing the $\mathcal{H}_{\rm QCD}$ by expanding field at $x^+ = 0$

$$\tilde{A}^\mu_c(x^-, \mathbf{x}) = \sum_{\lambda=\pm 1} \int \widetilde{\mathrm{d}k} \Big[a^c_\lambda \varepsilon^\mu_\lambda e^{-ik\cdot x} + \text{h.c.} \Big], \qquad \lambda \text{ gluon polarization}$$

$$\tilde{\Psi}(x^-, \mathbf{x}) = \sum_{h=\pm\frac{1}{2}} \int \widetilde{\mathrm{d}k} \Big[b_h u_h e^{-ik\cdot x} + \text{h.c.} \Big], \qquad h \text{ quark helicity}$$

- operators $(b_h), a^c_\lambda$... satisfy the standard (anti)commutation relations
- the integral measure in the LC coordinates is

$$\int \widetilde{\mathrm{d}k} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} (2\pi) \delta(k^2 - m^2) = \int \frac{\mathrm{d}k^+ \mathrm{d}^2\mathbf{k}}{(2\pi)^3 2k^+}, \quad k^+ > 0$$

- In the LC gauge $\varepsilon^+ = 0$ and $k \cdot \varepsilon_\lambda(k) = 0$

$$\varepsilon^\mu_\lambda(k) = (0, \frac{\mathbf{k}\cdot\epsilon_\lambda}{k^+}, \epsilon_\lambda), \quad \text{physical 2d transverse pol.} \quad \epsilon_\lambda = \frac{1}{\sqrt{2}}(1, \pm i)$$

The x^+-ordered LCPT theory = familiar perturbative expansion in QM

- Express the full physical incoming particle state $|\Psi\rangle_{\rm int}$ as a perturbative Fock state decomposition (in momentum space)

$$|\Psi\rangle_{\rm int} = |\Psi\rangle + \sum_{n_1} \frac{|n_1\rangle\langle n_1|\mathcal{H}_{\rm I}|\Psi\rangle}{\Delta^-_{1\Psi}} + \sum_{n_1,n_2} \frac{|n_2\rangle\langle n_2|H_{\rm I}|n_1\rangle\langle n_1|\mathcal{H}_{\rm I}|\Psi\rangle}{\Delta^-_{2\Psi}\Delta^-_{1\Psi}} + \dots$$

- LC energy denominators

$$\Delta^-_{n\Psi} = \sum_{\Psi} p^-_{\Psi} - \sum_{n} p^-_{n} + i0_+$$

- Note: only $\vec{p} = (p^+, \mathbf{p})$ is conserved!

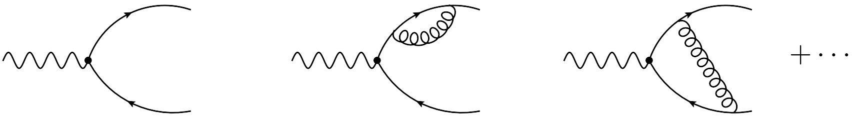
Consider now the DIS process $\gamma^* \to q\bar{q}$ (talk by G. Beuf)

$$|\gamma^*(\vec{q})\rangle_{\rm int} = \sqrt{Z_{\gamma^*}(q^+)}\Big[|\gamma^*(\vec{q})\rangle + \int \widetilde{\mathrm{d}p'}\widetilde{\mathrm{d}q}(2\pi)^3\delta^{(3)}(\vec{q}-\vec{p}-\vec{p}\,') \times \psi^{\gamma^*_{\rm T/L}\to q\bar{q}}|q(\vec{p})\bar{q}(\vec{p}\,')\rangle + \dots\Big]$$

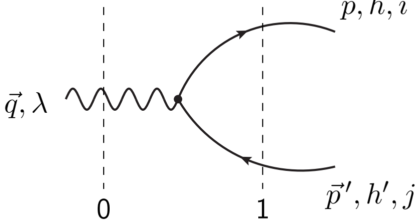
This expression **defines** the LC wavefunctions

$$\psi^{\gamma^*_{\rm T/L}\to q\bar{q}}, \quad \psi^{\gamma^*_{\rm T/L}\to q\bar{q}g}, \dots$$

e.g. in NLO: $\mathcal{O}(g^2)$ QCD virtual corrections



Leading-order $\psi_{\mathrm{LO}}^{\gamma_{\mathrm{T}}^{*}\to q\bar{q}}$ wavefunction (for transverse γ^{*})



- The WF (without overall momentum conservation $\vec{q} = \vec{p} + \vec{p}'$)

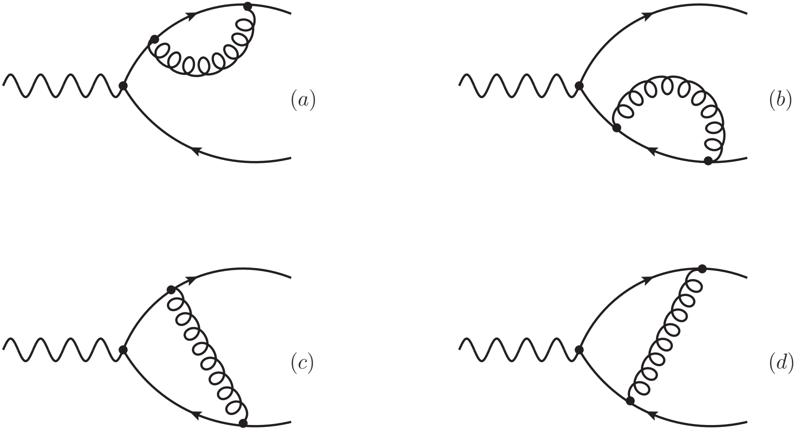
$$\psi_{\mathrm{LO}}^{\gamma_{\mathrm{T}}^{*}\to q\bar{q}} = (-ee_f)\delta_{ij}\frac{\bar{u}_h(p)\not{\epsilon}_\lambda(q)v_{h'}(p')}{\Delta_{01}^{-}}$$

- The LO LC energy denominator (frame $\mathbf{q} = \mathbf{0}$)

$$\Delta_{01}^{-} = q^{-} - (p^{-} + p'^{-}) = \frac{\mathbf{q}^2 - Q^2}{2q^+} - \left(\frac{\mathbf{p}^2}{2p^+} + \frac{(\mathbf{q} - \mathbf{p})^2}{2p'^+}\right)$$

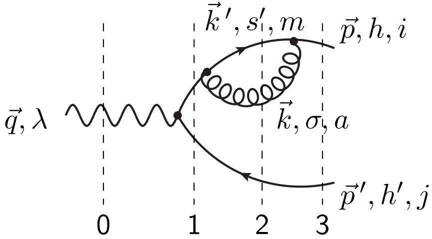
$$= \frac{-q^+}{2p^+p'^+}\left(\mathbf{p}^2 + \overline{Q}^2\right), \quad \text{where} \quad \overline{Q}^2 = \frac{p^+p'^+}{(q^+)^2}Q^2 > 0$$

One-loop $\mathcal{O}(g^2)$ QCD corrections to the $\psi^{\gamma^*_{\mathrm{T}} \to q\bar{q}}$



+ instantaneous diagrams (goes to zero in dim.reg)

One-loop quark self-energy correction (a) (for γ_{T}^*)



- The WF

$$\psi_{(a)}^{\gamma_{\mathrm{T}}^* \to q\bar{q}} = -ee_f g^2 C_{\mathrm{F}} \delta^{ij} \int_0^{q^+} \frac{\mathrm{d}k^+}{2k^+} \int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^3} \frac{1}{4p^+ k'^+} \frac{\mathrm{num}|_a}{\Delta_{01}^- \Delta_{02}^- \Delta_{03}^-}$$

- The LC energy denominators

$$\Delta_{01}^- = \Delta_{03}^- = \frac{-q^+}{2p^+ p'^+} \left(\mathbf{p}^2 + \overline{Q}^2 \right)$$

$$\Delta_{02}^- = -\frac{p^+}{2k^+ k'^+} \left[(\mathbf{k} - \frac{k^+}{p^+}\mathbf{p})^2 + \frac{k^+ k'^+ q^+}{(p^+)^2 p'^+} (\mathbf{p}^2 + \overline{Q}^2) \right]$$

- and the numerator

$$\mathrm{num}|_a = \sum_\sigma \varepsilon_\sigma^\mu(k) \varepsilon_\sigma^{*\nu}(k) \left[\bar{u}_h(p) \gamma_\mu \not{k}' \gamma_\nu \not{p} \not{\varepsilon}_\lambda(q) v_{h'}(p') \right]$$

To evaluate the (UV & col. IR divergent) transverse integrals and numerators we need some regularization framework

- **Conventional Dim. Reg.** (CDR) scheme: both observed and unobserved particles and momenta are continued to d dimension
- **Four Dimensional Helicity** (FDH) scheme: momenta of unobserved particles is continued to $d > 4$ and observed particles are kept in 4d
  - all unobserved internal states are treated as $d_s > d$ dim. and should be distinct from the dim. d
  - once the spin and Lorentz algebra is done the limit $d_s \to 4$ can be taken

$$\int \frac{\mathrm{d}^2\mathbf{k}}{(2\pi)^2} \frac{k^i k^j}{k^2 + M} \xrightarrow[\text{FDH}]{\text{CDR}} \frac{\delta^{ij}_{(d)}}{d} \int \frac{\mathrm{d}^{d-2}\mathbf{k}}{(2\pi)^{d-2}} \frac{\mathbf{k}^2}{\mathbf{k}^2 + M}$$

The longitudinal k^+ (soft IR divergent) integrals are regulated with the momentum cut-off

$$k^+ > \alpha q^+, \quad \text{where} \quad \alpha > 0$$

CDR scheme result for (a) (G. Beuf Phys. Rev. D 94 (2016) 054016)

$$\psi_{(a)}^{\gamma_{\mathrm{T}}^* \to q\bar{q}}\Big|_{\mathrm{CDR}} = \psi_{\mathrm{LO}}^{\gamma_{\mathrm{T}}^* \to q\bar{q}}(\mathbf{p}, z) \left(\frac{g_{\mathrm{R}}^2 C_{\mathrm{F}}}{8\pi^2}\right) \left\{ \left[\frac{3}{2} + 2\log\left(\frac{\alpha}{z}\right)\right] C_a - \log^2\left(\frac{\alpha}{z}\right) - \frac{\pi^2}{3} + 3 + \frac{1}{2} \right\} + \mathcal{O}(\varepsilon)$$

where $z = p^+/q^+$ with $0 < z < 1$ and

$$C_a = \frac{1}{\varepsilon_{\overline{\mathrm{MS}}}} + \log\left(\frac{\mu^2}{\overline{Q}^2}\right) - \log\left(\frac{\mathbf{p}^2 + \overline{Q}^2}{\overline{Q}^2}\right) + \log(1 - z)$$

Note: the factor $\frac{1}{2}$ is scheme dependent

Full (= (a) + (b) + (c) + (d)) CDR scheme result (G. Beuf Phys. Rev. D 94 (2016) 054016)

$$\psi^{\gamma^*_{\mathrm{T}}\to q\bar{q}}_{(\mathrm{full})}\bigg|_{\mathrm{CDR}} = \psi^{\gamma^*_{\mathrm{T}}\to q\bar{q}}_{\mathrm{LO}}(\mathbf{p},z)\left(\frac{g_{\mathrm{R}}^2 C_{\mathrm{F}}}{8\pi^2}\right)\left\{\left[\frac{3}{2}+\log\left(\frac{\alpha}{z}\right)+\log\left(\frac{\alpha}{1-z}\right)\right]C_{\mathrm{full}} + \frac{1}{2}\log^2\left(\frac{z}{1-z}\right)-\frac{\pi^2}{3}+\frac{5}{2}+\frac{1}{2}\right\}+\mathcal{O}(\varepsilon)$$

where

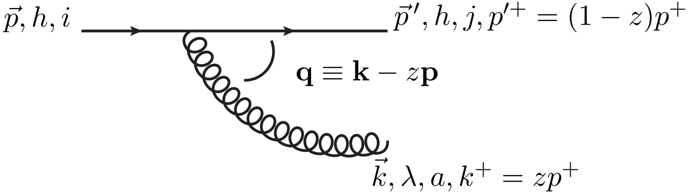
$$C_{\mathrm{full}} = \frac{1}{\varepsilon_{\overline{\mathrm{MS}}}}+\log\left(\frac{\mu^2}{\overline{Q}^2}\right)+\left(\frac{\overline{Q}^2-\mathbf{p}^2}{\mathbf{p}^2}\right)\log\left(\frac{\mathbf{p}^2+\overline{Q}^2}{\overline{Q}^2}\right)$$

Notes:

- complicated "brute force" calculation
  - in LCPT framework "covariant style" helicity algebra is difficult to automatize

New Helicity Approach (T. Lappi & RP in preparation)

The QCD light-cone vertices can be very compactly expressed in the helicity basis (**independent of spinor representation**)



$$-gt^{a}_{ji}\bar{u}_h(p')\not{\epsilon}^{*}_{\lambda}(k)u_h(p) = \frac{-2gt^{a}_{ji}}{z\sqrt{1-z}}\left[\left(1-\frac{z}{2}\right)\delta^{ij} + ih\frac{z}{2}\epsilon^{ij}\right]\mathbf{q}^{i}\epsilon^{*j}_{\lambda}$$
$$= V^{i;j,a}_{\lambda,h}(\mathbf{q}, z)$$

Note: Our FDH formulation in (arXiv:1611.00497) was incorrect

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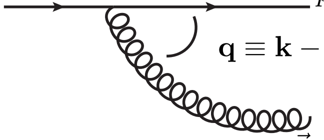

$$
\begin{aligned}
-g t^{a}_{ji} \bar{u}_h(p') \not{\epsilon}^{*}_{\lambda}(k) u_h(p) &= \frac{-2g t^{a}_{ji}}{z\sqrt{1-z}} \left[\left(1 - \frac{z}{2}\right) \delta^{ij} + ih \frac{z}{2} \epsilon^{ij} \right] \mathbf{q}^{i} \epsilon^{*j}_{\lambda} \\
&= V^{i;j,a}_{\lambda,h}(\mathbf{q}, z)
\end{aligned}
$$

- $+$ and transverse momentum conservation implemented automatically

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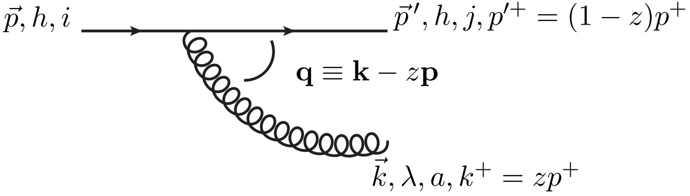

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\end{aligned}
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- $+$ and transverse momentum conservation implemented automatically
- all particles have exactly two helicity states $\Rightarrow$ FDH scheme

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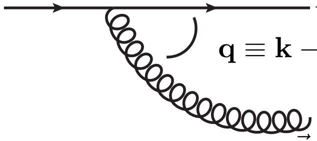
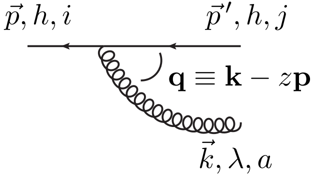


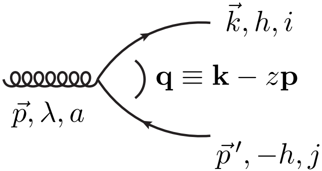
$$-gt^{a}_{ji}\bar{u}_h(p')\not\epsilon^{*}_{\lambda}(k)u_h(p) = \frac{-2gt^{a}_{ji}}{z\sqrt{1-z}}\left[\left(1-\frac{z}{2}\right)\delta^{ij} + ih\frac{z}{2}\epsilon^{ij}\right]\mathbf{q}^{i}\epsilon^{*j}_{\lambda}$$
$$= V^{i;j,a}_{\lambda,h}(\mathbf{q}, z)$$

- $+$ and transverse momentum conservation implemented automatically
- all particles have exactly two helicity states $\Rightarrow$ FDH scheme
- enables a very efficient computation of NLO WFs (easy to automatize)

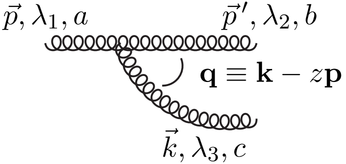
Note: Our FDH formulation in (arXiv:1611.00497) was incorrect



- $\overline{V}^{i;j,a}_{\lambda,h}(\mathbf{q}, z) =$
  $\frac{+2g t^a_{ji}}{z\sqrt{1-z}} \left[\left(1 - \frac{z}{2}\right) \delta^{ij} + ih\frac{z}{2}\epsilon^{ij} \right] \mathbf{q}^i \epsilon^{*j}_{\lambda}$



- $A^{a;j,i}_{\lambda,h}(\mathbf{q}, z) =$
  $\frac{-2g t^a_{ij}}{\sqrt{z(1-z)}} \left[\left(z - \frac{1}{2}\right) \delta^{ij} - ih\frac{1}{2}\epsilon^{ij} \right] \mathbf{q}^i \epsilon^{j}_{\lambda}$



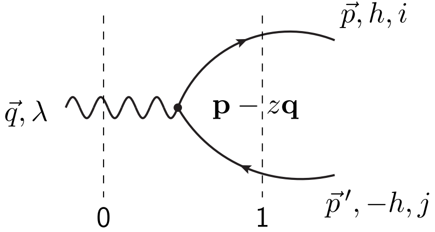
- $\Gamma^{a;b,c}_{\lambda_1,\lambda_2,\lambda_3}(\mathbf{q}, z) =$
  $-2igf^{abc} \left[\frac{\boldsymbol{\varepsilon}^{*j}_{\lambda_2} \boldsymbol{\varepsilon}^{*k}_{\lambda_3} \boldsymbol{\varepsilon}^{l}_{\lambda_1}}{1-z} + \frac{\boldsymbol{\varepsilon}^{*j}_{\lambda_3} \boldsymbol{\varepsilon}^{*k}_{\lambda_2} \boldsymbol{\varepsilon}^{l}_{\lambda_1}}{z} - \right.$
  $\left. \boldsymbol{\varepsilon}^{j}_{\lambda_1} \boldsymbol{\varepsilon}^{*k}_{\lambda_3} \boldsymbol{\varepsilon}^{*l}_{\lambda_2} \right] \delta^{ij} \delta^{kl} \mathbf{q}^i$

+complex conjugates & 4-gluon vertex & instantaneous vertices!

LO WF $\psi_{\rm LO}^{\gamma_{\rm T}^*\to q\bar{q}}$ (Helicity approach)

$p^+ = zq^+$

$p'^+ = (1-z)q^+$



- The LO WF (frame $\mathbf{q} = \mathbf{0}$)

$$\psi_{\rm LO}^{\gamma_{\rm T}^*\to q\bar{q}} = \delta_{ij}\frac{A_{\lambda,h}^{\gamma_{\rm T}^*}(\mathbf{p},z)}{\Delta_{01}^-},$$

where (QCD $\to$ QED vertex: $gt^a \to ee_f$)

$$A_{\lambda,h}^{\gamma_{\rm T}^*}(\mathbf{p},z) = \frac{-2ee_f}{\sqrt{z(1-z)}}\left[\left(z-\frac{1}{2}\right)\delta^{ij} - ih\frac{1}{2}\epsilon^{ij}\right]\mathbf{p}^i\epsilon_\lambda^j$$

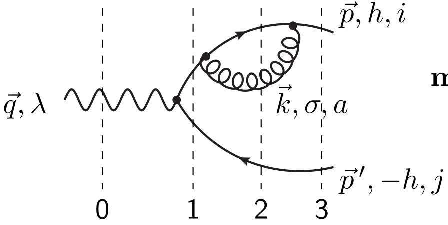
- The LO LC energy denominator

$$\Delta_{01}^- = \frac{1}{(-2q^+)z(1-z)}\left(\mathbf{p}^2 + \overline{Q}^2\right), \quad \text{where} \quad \overline{Q}^2 = z(1-z)Q^2 > 0$$

$$k^+ = z'q^+$$

$$p^+ = zq^+$$

$$p'^+ = (1-z)q^+$$



$$\mathbf{m} = \mathbf{k} - \tfrac{z'}{z}\mathbf{p}$$

- The WF (without overall momentum conservation $\vec{q} = \vec{p} + \vec{p}'$)

$$\psi_{(a)}^{\gamma_{\mathrm{T}}^* \to q\bar{q}} = \frac{1}{16\pi (q^+)^2} \int_0^z \frac{\mathrm{d}z'}{zz'(z-z')} \int \frac{\mathrm{d}^{d-2}\mathbf{m}}{(2\pi)^{d-2}} \frac{\mathrm{num}|_a}{\Delta_{01}^- \Delta_{02}^- \Delta_{03}^-}$$

- The LC energy denominators

$$\Delta_{01}^- = \Delta_{03}^- = \frac{1}{(-2q^+)z(1-z)} \left(\mathbf{p}^2 + \overline{Q}^2\right)$$

$$\Delta_{02}^- = \frac{z}{(-2q^+)z'(z-z')} \left[\mathbf{m}^2 + \frac{z'(z-z')}{z^2(1-z)}(\mathbf{p}^2 + \overline{Q}^2)\right]$$

- and the numerator

$$\mathrm{num}|_a = \sum_\sigma A_{\lambda,h}^{\gamma_{\mathrm{T}}^*}(\mathbf{p}, z) V_{\sigma,h}^{j;m,a}(\mathbf{m}, z'/z) V_{\sigma,h}^{m,a;i}(\mathbf{m}, z'/z)$$

now the algebra in the numerator is trivial

$$\mathrm{num}|_a = A^{\gamma^*_{\mathrm{T}}}_{\lambda,h}(\mathbf{p},z)\frac{4g_{\mathrm{R}}^2 C_{\mathrm{F}}\delta^{ij}}{z^2(1-z)}\sum_\sigma \epsilon^{*j}_\sigma \epsilon^\ell_\sigma \left[\left(1-\frac{z'}{2z}\right)\delta^{ij}_{(d_s)} + ih\frac{z'}{2z}\epsilon^{ij}_{(d_s)}\right] \times \left[\left(1-\frac{z'}{2z}\right)\delta^{k\ell}_{(d_s)} - ih\frac{z'}{2z}\epsilon^{k\ell}_{(d_s)}\right]\mathbf{m}^i\mathbf{m}^k$$

using (remember $d_s > d$)

$$\sum_\sigma \epsilon^{*j}_\sigma \epsilon^\ell_\sigma = \delta^{j\ell}_{(d_s)} \qquad \text{and} \qquad \mathbf{m}^i\mathbf{m}^k = \frac{\delta^{ik}_{(d)}}{d}\mathbf{m}^2$$

and

$$\epsilon^{jj}_{(d_s)} = 0, \qquad \epsilon^{ij}_{(d_s)}\epsilon^{k\ell}_{(d_s)} = \delta^{ik}_{(d_s)}\delta^{j\ell}_{(d_s)} - \delta^{i\ell}_{(d_s)}\delta^{jk}_{(d_s)} \qquad \text{(Fierz identity)}$$

we get (remember $h^2 = 1$)

$$\mathrm{num}|_a = A^{\gamma^*_{\mathrm{T}}}_{\lambda,h}(\mathbf{p},z)\frac{4g_{\mathrm{R}}^2 C_{\mathrm{F}}\delta^{ij}}{z^2(1-z)}\left[\left(1-\frac{z'}{2z}\right)^2 + \left(\frac{z'}{2z}\right)^2 (d_s-1)\right]\mathbf{m}^2$$

Taking limit $d_s \to 2$ & integrating over $\mathbf{m}$ and z' we find for (a)

$$\psi_{(a)}^{\gamma_{\mathrm{T}}^* \to q\bar{q}}\Big|_{\mathrm{FDH}} = \psi_{\mathrm{LO}}^{\gamma_{\mathrm{T}}^* \to q\bar{q}}(\mathbf{p}, z) \left(\frac{g_{\mathrm{R}}^2 C_{\mathrm{F}}}{8\pi^2}\right) \left\{ \left[\frac{3}{2} + 2\log\left(\frac{\alpha}{z}\right)\right] C_a - \log^2\left(\frac{\alpha}{z}\right) - \frac{\pi^2}{3} + 3 \right\} + \mathcal{O}(\varepsilon)$$

where $z = p^+/q^+$ with $0 < z < 1$ and

$$C_a = \frac{1}{\varepsilon_{\overline{\mathrm{MS}}}} + \log\left(\frac{\mu^2}{\overline{Q}^2}\right) - \log\left(\frac{\mathbf{p}^2 + \overline{Q}^2}{\overline{Q}^2}\right) + \log(1 - z)$$

For comparison the CDR scheme result was

$$\psi_{(a)}^{\gamma_{\mathrm{T}}^* \to q\bar{q}}\Big|_{\mathrm{CDR}} = \psi_{\mathrm{LO}}^{\gamma_{\mathrm{T}}^* \to q\bar{q}}(\mathbf{p}, z) \left(\frac{g_{\mathrm{R}}^2 C_{\mathrm{F}}}{8\pi^2}\right) \left\{ \left[\frac{3}{2} + 2\log\left(\frac{\alpha}{z}\right)\right] C_a - \log^2\left(\frac{\alpha}{z}\right) - \frac{\pi^2}{3} + 3 + \frac{1}{2} \right\} + \mathcal{O}(\varepsilon)$$

For the full correction ((a) + (b) + (c) + (d)) we find

$$\psi^{\gamma^*_{\mathrm{T}}\to q\bar{q}}_{(\mathrm{full})}\Big|_{\mathrm{FDH}} = \psi^{\gamma^*_{\mathrm{T}}\to q\bar{q}}_{\mathrm{LO}}(\mathbf{p},z)\left(\frac{g_{\mathrm{R}}^2 C_{\mathrm{F}}}{8\pi^2}\right)\left\{\left[\frac{3}{2}+\log\left(\frac{\alpha}{z}\right)+\log\left(\frac{\alpha}{1-z}\right)\right]C_{\mathrm{full}} + \frac{1}{2}\log^2\left(\frac{z}{1-z}\right)-\frac{\pi^2}{3}+\frac{5}{2}-\log(z(1-z))\right\}+\left(\frac{g_{\mathrm{R}}^2 C_{\mathrm{F}}}{8\pi^2}\right)\times\Pi+\mathcal{O}(\varepsilon)$$

where

$$C_{\mathrm{full}} = \frac{1}{\varepsilon_{\overline{\mathrm{MS}}}}+\log\left(\frac{\mu^2}{\overline{Q}^2}\right)+\left(\frac{\overline{Q}^2-\mathbf{p}^2}{\mathbf{p}^2}\right)\log\left(\frac{\mathbf{p}^2+\overline{Q}^2}{\overline{Q}^2}\right)$$

$$\Pi = \delta^{ij}\frac{-2ee_f}{\Delta^-_{03}}\frac{(z-\frac{1}{2})}{\sqrt{z(1-z)}}(\mathbf{p}\cdot\epsilon_\lambda)\log(z(1-z))$$

For comparison the full CDR scheme result was

$$\psi^{\gamma^*_{\mathrm{T}}\to q\bar{q}}_{(\mathrm{full})}\Big|_{\mathrm{CDR}} = \psi^{\gamma^*_{\mathrm{T}}\to q\bar{q}}_{\mathrm{LO}}(\mathbf{p},z)\left(\frac{g_{\mathrm{R}}^2 C_{\mathrm{F}}}{8\pi^2}\right)\left\{\left[\frac{3}{2}+\log\left(\frac{\alpha}{z}\right)+\log\left(\frac{\alpha}{1-z}\right)\right]C_{\mathrm{full}} + \frac{1}{2}\log^2\left(\frac{z}{1-z}\right)-\frac{\pi^2}{3}+\frac{5}{2}+\frac{1}{2}\right\}+\mathcal{O}(\varepsilon)$$

Conclusion and Outlook

- Formulation of LCPT rules for massless QCD in helicity space independently of spinor representation
  - replace the complicated γ-algebra and spin sums by simple metric tensor algebra (easy to automatize - FORM)
  - full NLO corrections to WFs can be calculated by using the helicity approach
- To-Do List:
  - include the mass corrections
  - transition rules from FDH scheme to CDR scheme
  - full NLO computations to DIS and other processes
  - higher-order (NNLO) corrections (?)

BACKUP SLIDE:

T. Lappi, RP arXiv:1611.00497:

$$-gt^a_{ji}\bar{u}_h(p')\not{\epsilon}^*_\lambda(k)u_h(p) = \frac{-2gt^a_{ji}}{z\sqrt{1-z}}\Big[\delta_{\lambda,h} + (1-z)\delta_{\lambda,-h}\Big]\mathbf{q}\cdot\epsilon^*_\lambda$$

Rewriting this as

$$-gt^a_{ji}\bar{u}_h(p')\not{\epsilon}^*_\lambda(k)u_h(p) = \frac{-2gt^a_{ji}}{z\sqrt{1-z}}\left[\left(1-\frac{z}{2}\right) + \lambda h\frac{z}{2}\right]\mathbf{q}^i\epsilon^{*i}_\lambda$$

and using $\lambda\varepsilon^*_\lambda = i\epsilon \times \varepsilon^*_\lambda$ where ϵ is rank-2 levi-civita tensor,

$$-gt^a_{ji}\bar{u}_h(p')\not{\epsilon}^*_\lambda(k)u_h(p) = \frac{-2gt^a_{ji}}{z\sqrt{1-z}}\left[\left(1-\frac{z}{2}\right)\delta^{ij} + ih\frac{z}{2}\epsilon^{ij}\right]\mathbf{q}^i\epsilon^{*j}_\lambda$$