Elliptic flow from color-dipole orientation in pp and pA collisions

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UNIVERSIDAD TECNICA FEDERICO SANTA MARIA

Centro Científico Tecnológico de Valparaíso

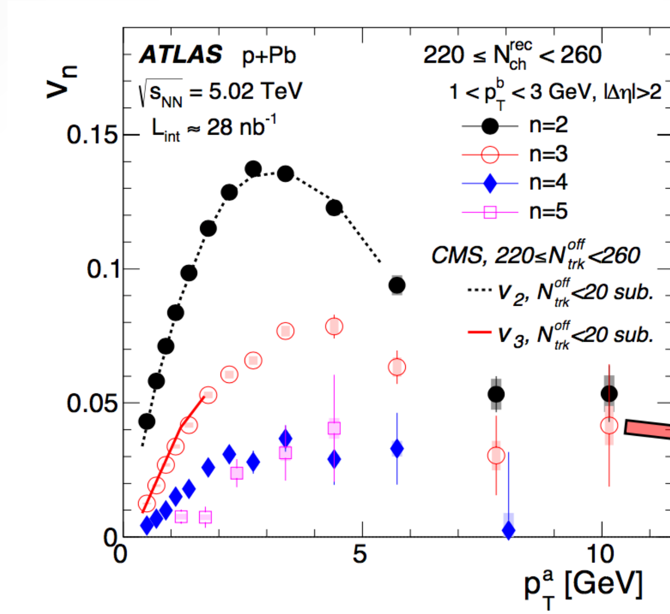
Outline:

- ❑ **Motivation**
- ❑ **Color dipole orientation in the MV model.**
- ❑ **Elliptic flow in pp collisions.**
- ❑ **Elliptic flow in pA collisions.**

***This talk is based on*: Iancu and Rezaeian, arXiv:1702.03943; in PRD press**

Can we understand this?

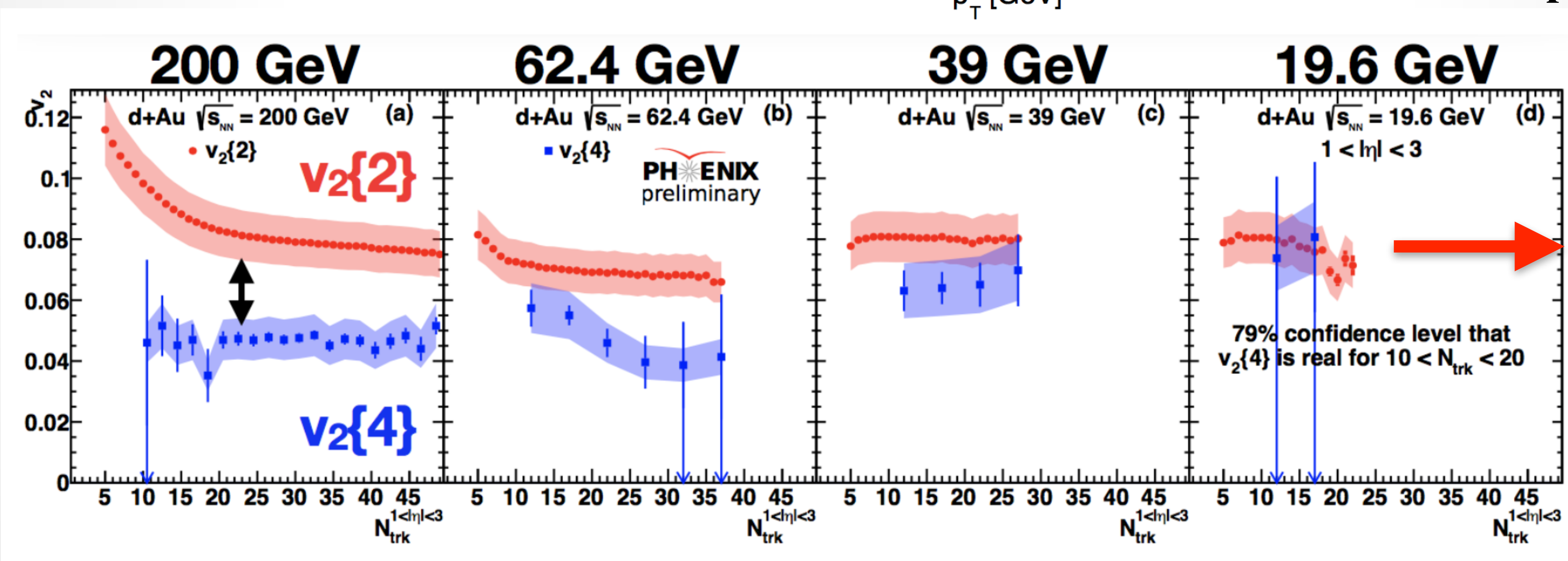
Motivation:



Jiangyong Jia's talk

$x \sim 10^{-4}$

Is this small-x physics?



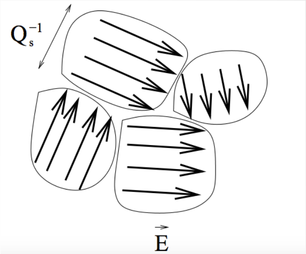
$x > 10^{-2}$

Different Saturation based mechanisms for elliptic flow

- **’Saturation domains’ inside a dense hadronic target**

Kovner, Lublinsky, PRD83, 034017 (2011).
Dumitru and Giannini, Nucl. Phys. A933, 212 (2015).
Dumitru and Skokov, Phys. Rev. D91, 074006 (2015).
Dumitru, McLerran, and Skokov, Phys. Lett. B743, 134 (2015).
Lappi, Schenke, Schlichting, and Venugopalan, JHEP 01, 061 (2016).

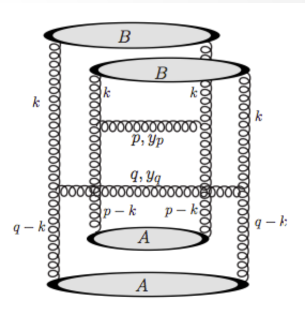
Alex’s talk



- **Glasma-like & ‘IP-Glasma’ scenarios**

Schenke, Tribedy, and Venugopalan, Phys. Rev. Lett. 108, 252301 (2012).
Schenke, Tribedy, and Venugopalan, Phys. Rev. C86, 034908 (2012).
Dusling and Venugopalan, Phys. Rev. Lett. 108, 262001 (2012).
Dusling and Venugopalan, Phys. Rev. D87, 094034 (2013).
Dusling, Tribedy, and Venugopalan, Phys. Rev. D93, 014034 (2016).
Kovchegov and Wertepny, Nucl. Phys. A925, 254 (2014).
Schenke and Schlichting, Phys. Rev. C94, 044907 (2016).
Schenke, Schlichting, Tribedy, and Venugopalan, Phys. Rev. Lett. 117, 162301 (2016)

Bjorn’s talk
Raju’s Talk



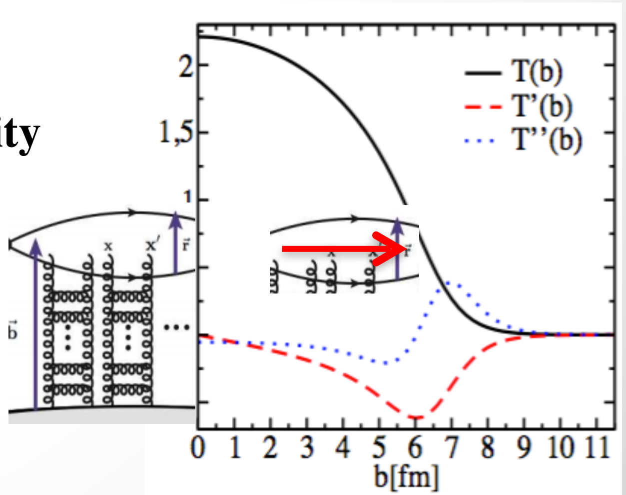
- **Spatial variation of partonic density**:

Levin and Rezaeian, Phys. Rev. D84, 034031 (2011).

✓ **‘Color-dipole orientation’ scenario as a probe of inhomogeneity**

Kopeliovich, Pirner, Rezaeian, and Schmidt, Phys. Rev. D77, 034011 (2008),
Zhou, Phys. Rev. D94, 114017 (2016).
Hagiwara, Hatta, Xiao, and Yuan (2017), 1701.04254.
Iancu and Rezaeian, arXiv:1702.03943; in PRD press. ← **In MV model**

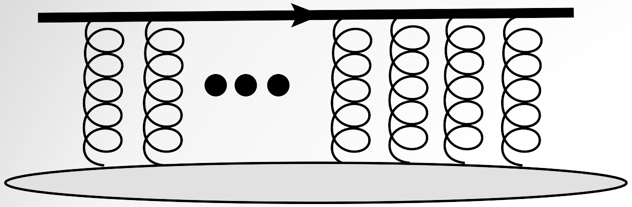
This talk



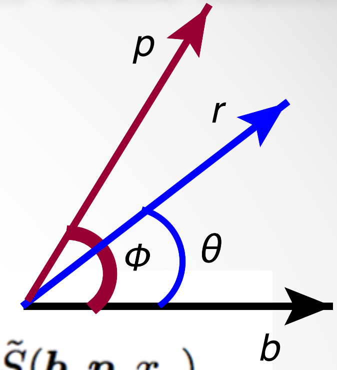
- **The effect is not $1/N_c$ suppressed!.**
- **Can generate sizable azimuthal asymmetries already in the absence of fluctuations (it is manifest for the single particle spectrum).**

Color-dipole as an origin of elliptic flow

Color-dipole orientation as an origin of elliptic flow in dilute-dense scatterings



See Edmond's talk

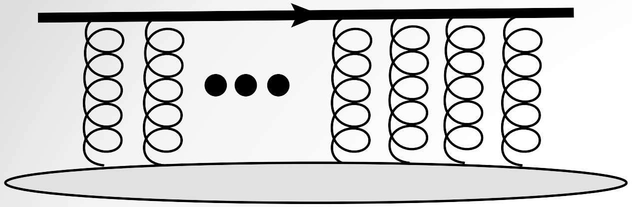


$$\frac{\mathrm{d}\sigma^q(qA\to qX)}{\mathrm{d}\eta\,\mathrm{d}^2\boldsymbol{p}\,\mathrm{d}^2\boldsymbol{b}} = x_p q(x_p)\int\frac{\mathrm{d}^2\boldsymbol{r}}{(2\pi)^2}\,\mathrm{e}^{-\mathrm{i}\boldsymbol{p}\cdot\boldsymbol{r}}S(\boldsymbol{b},\boldsymbol{r},x_g) = \frac{1}{(2\pi)^2}x_p q(x_p)\tilde{S}(\boldsymbol{b},\boldsymbol{p},x_g).$$

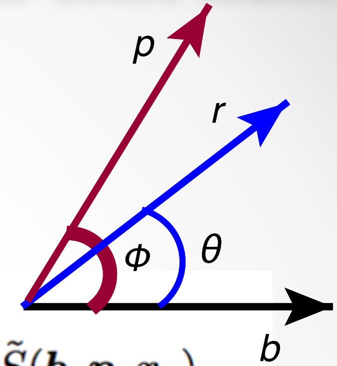
correlations btw (**p, r**) and btw (**r, b**) → correlations btw (**p, b**)

$$v_n(p,b) \equiv \frac{\int_0^{2\pi}\mathrm{d}\phi\cos(n\phi)\,\frac{\mathrm{d}\sigma^q(qA\to qX)}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{b}}}{\int_0^{2\pi}\mathrm{d}\phi\,\frac{\mathrm{d}\sigma^q(qA\to qX)}{\mathrm{d}\eta\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{b}}}$$

Color-dipole orientation as an origin of elliptic flow in dilute-dense scatterings



← *See Edmond's talk*



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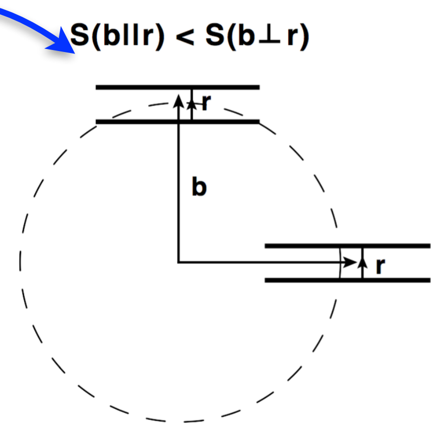
$cos(n\Phi) \longrightarrow cos(n\theta)$

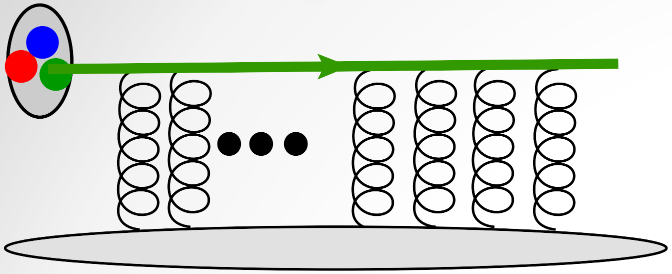
$$v_2(p,b) = -\frac{\int r\mathrm{d}r\mathrm{d}\theta\,\cos(2\theta)\,\mathrm{J}_2(pr)\,S(b,r,\theta)}{\int r\mathrm{d}r\mathrm{d}\theta\,\mathrm{J}_0(pr)\,S(b,r,\theta)},$$

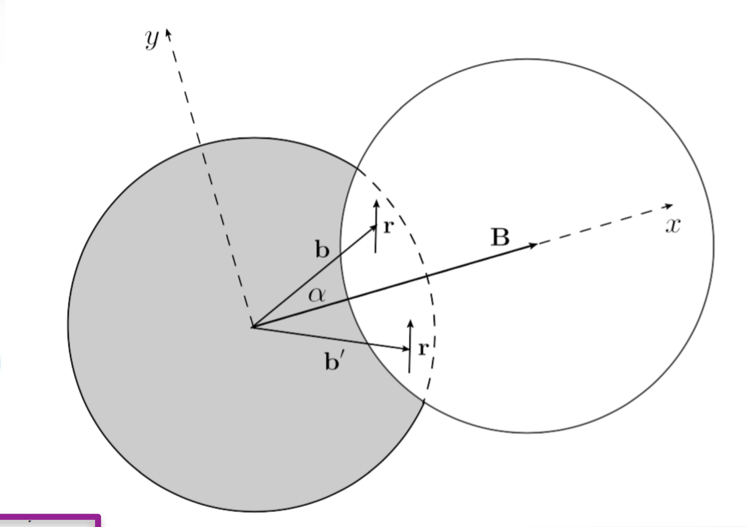
$$v_3(p,b) = -\mathrm{i}\,\frac{\int r\mathrm{d}r\mathrm{d}\theta\,\cos(3\theta)\,\mathrm{J}_3(pr)\,S(b,r,\theta)}{\int r\mathrm{d}r\mathrm{d}\theta\,\mathrm{J}_0(pr)\,S(b,r,\theta)}.$$



- **Spontaneous breaking of rotational symmetry in the transverse plane.**
- $S(\vec{b},\vec{r}) \to S(b,r) \longrightarrow v_n = 0$ **(Regardless of the shape of projectile)**
- $v_3 \neq 0 \longrightarrow$ non-zero odderon contribution

Color-dipole orientation: including finite-size effect of the projectile



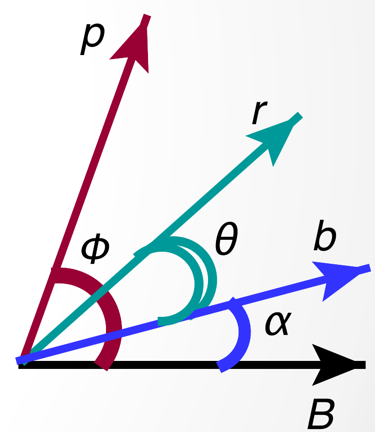


$$\frac{d\sigma^q(qA \to qX)}{d\eta\, d^2\boldsymbol{p}\, d^2\boldsymbol{B}} = \frac{1}{(2\pi)^2}\int d^2\boldsymbol{b}\; x_p q(x_p, \boldsymbol{b}-\boldsymbol{B})\, \tilde{S}(\boldsymbol{b},\boldsymbol{p},x_g)$$

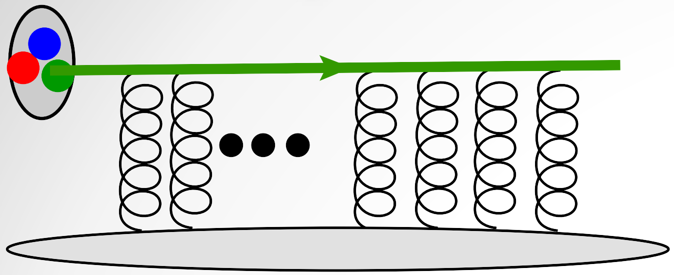
Proton GPD: $x_p q(x_p,\boldsymbol{b}) \simeq x_p q(x_p) f(\boldsymbol{b})$, with $\int d^2\boldsymbol{b}\, f(\boldsymbol{b}) = 1$

proton distribution is isotropic: $f(\boldsymbol{b}) = f(|\boldsymbol{b}|)$

$$v_n(p,B) \equiv \frac{\int_0^{2\pi} d\Phi\, \cos(n\Phi)\, \frac{d\sigma^q(qA\to qX)}{d\eta d^2\boldsymbol{p} d^2\boldsymbol{B}}}{\int_0^{2\pi} d\Phi\, \frac{d\sigma^q(qA\to qX)}{d\eta d^2\boldsymbol{p} d^2\boldsymbol{B}}}$$



Color-dipole orientation: including finite-size effect of the projectile



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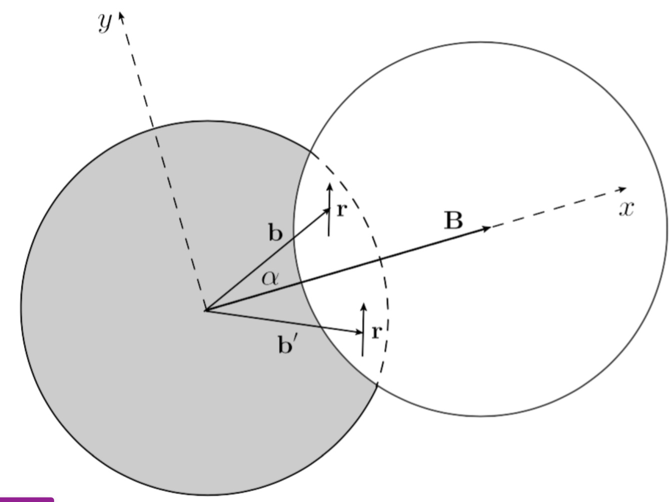

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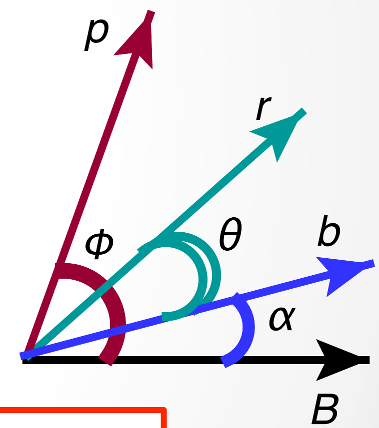

$$v_2(p,B) = -\frac{\int b db d\alpha\, \cos(2\alpha)\, f(|\boldsymbol{b}-\boldsymbol{B}|)\int r dr d\theta\, \cos(2\theta)\, J_2(pr)\, S(b,r,\theta)}{\int b db d\alpha\, f(|\boldsymbol{b}-\boldsymbol{B}|)\int r dr d\theta\, J_0(pr)\, S(b,r,\theta)}$$

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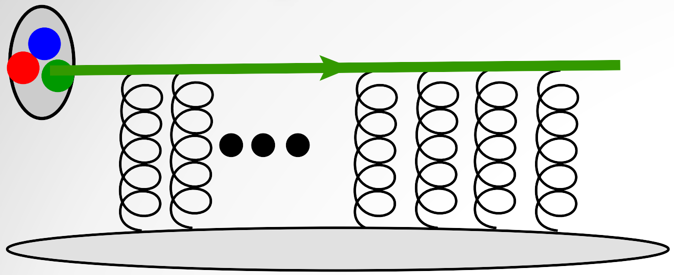
Color-dipole orientation: including finite-size effect of the projectile



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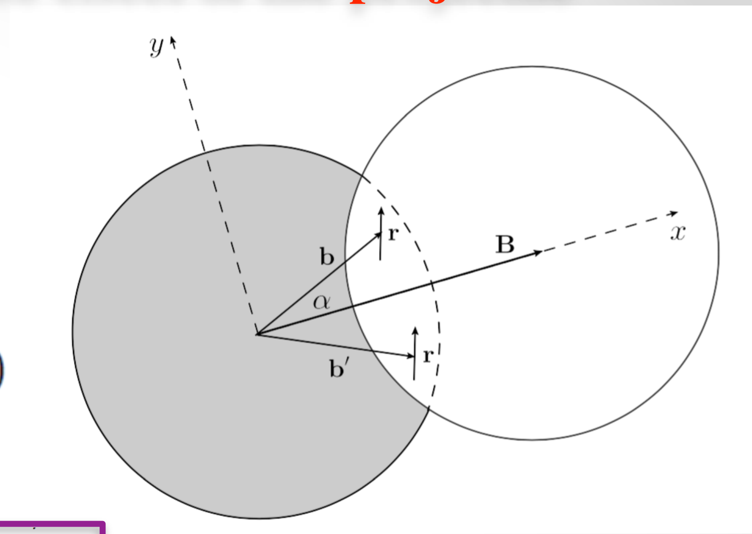

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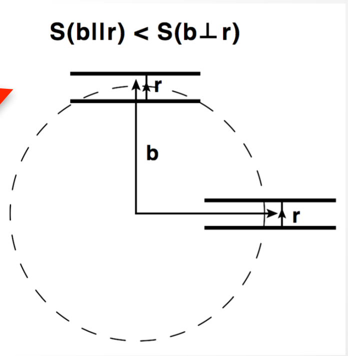
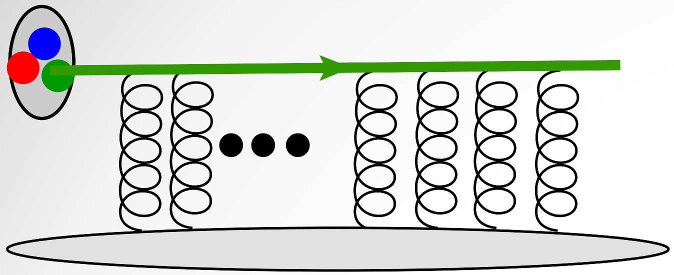
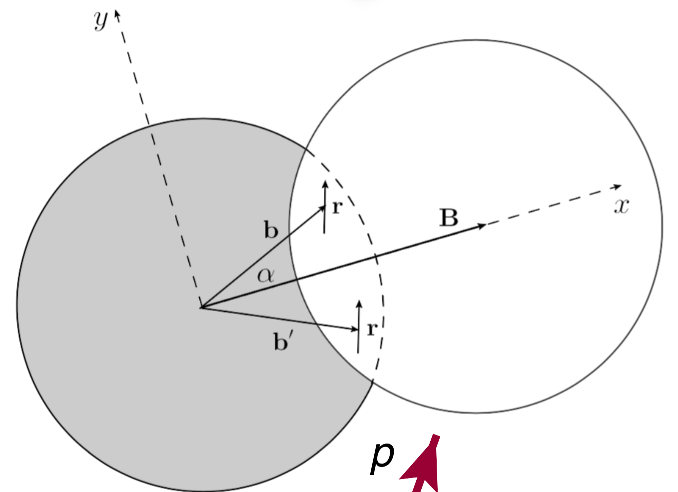


$$v_2(p,B) = -\frac{\int \mathrm{b}\mathrm{d}\mathrm{b}\mathrm{d}\alpha\, \cos(2\alpha)\, f(|\boldsymbol{b}-\boldsymbol{B}|) \int \mathrm{r}\mathrm{d}\mathrm{r}\mathrm{d}\theta\, \cos(2\theta)\, \mathrm{J}_2(pr)\, S(b,r,\theta)}{\int \mathrm{b}\mathrm{d}\mathrm{b}\mathrm{d}\alpha\, f(|\boldsymbol{b}-\boldsymbol{B}|) \int \mathrm{r}\mathrm{d}\mathrm{r}\mathrm{d}\theta\, \mathrm{J}_0(pr)\, S(b,r,\theta)}$$

- $S(\vec{b},\vec{r}) \to S(b,r) \longrightarrow v_n = 0$ **(Regardless of the shape of projectile and target)**

Elliptic flow and eccentricity in dilute-dense scatterings





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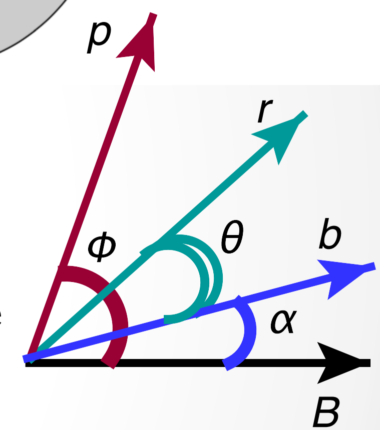

- **Eccentricity: is a measure of the projection of the impact parameters of the participants quarks along the direction of their average impact parameter**

$$\varepsilon_2(p,B) = \frac{\langle x^2-y^2\rangle}{\langle x^2+y^2\rangle} = \frac{\langle b^2\cos(2\alpha)\rangle}{\langle b^2\rangle},$$

$$= \frac{\int b\mathrm{d}b\mathrm{d}\alpha\,b^2\cos(2\alpha)\,f(|\boldsymbol{b}-\boldsymbol{B}|)\int r\mathrm{d}r\mathrm{d}\theta\,J_0(pr)\,S(b,r,\theta)}{\int b\mathrm{d}b\mathrm{d}\alpha\,b^2\,f(|\boldsymbol{b}-\boldsymbol{B}|)\int r\mathrm{d}r\mathrm{d}\theta\,J_0(pr)\,S(b,r,\theta)}$$

$\epsilon_2(B=0)=0,\;\; \epsilon_2(B\to\infty)=1$

$v_2(p,B)\propto\epsilon_2(p,B)$

Color-dipole orientation in the McLerran-Venugopalan model

$$\langle \rho^a(\boldsymbol{x})\rho^b(\boldsymbol{y})\rangle = \delta^{ab}\delta^{(2)}(\boldsymbol{x}-\boldsymbol{y})\mu(\boldsymbol{x})$$

In the original MV model:

$$\mu(\mathbf{b}) = \mu_0, \quad \tilde{\mu}(\mathbf{q}'+\mathbf{q}) = \delta^2(\mathbf{q}'+\mathbf{q})$$

$$\mu(b) = \mu_0\, \mathrm{e}^{-b^2/4R^2},$$

$$\tilde{\mu}(\Delta) = \int \mathrm{d}^2\boldsymbol{b}\, \mathrm{e}^{-\mathrm{i}\boldsymbol{b}\cdot\boldsymbol{\Delta}}\mu(b) = 4\pi R^2\mu_0\, \mathrm{e}^{-\Delta^2 R^2}$$

μ_0, R : fixed via a fit to the DIS & diffractive DIS data: **Using IP-Sat or bCGC**

$$\langle A_a^-(\boldsymbol{x})A_b^-(\boldsymbol{y})\rangle = \delta^{ab}\gamma(\boldsymbol{x},\boldsymbol{y})$$

$$\gamma(\boldsymbol{x},\boldsymbol{y}) \equiv \int \mathrm{d}^2\boldsymbol{z}\, G(\boldsymbol{x}-\boldsymbol{z})G(\boldsymbol{y}-\boldsymbol{z})\mu(\boldsymbol{z}) = \int \frac{\mathrm{d}^2\boldsymbol{q}}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{q}'}{(2\pi)^2}\mathrm{e}^{\mathrm{i}\boldsymbol{q}'\cdot\boldsymbol{x}+\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{y}}\frac{\tilde{\mu}(\boldsymbol{q}'+\boldsymbol{q})}{q'^2q^2}$$

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- **Single-scattering limit:**

Color-dipole orientation in the McLerran-Venugopalan model

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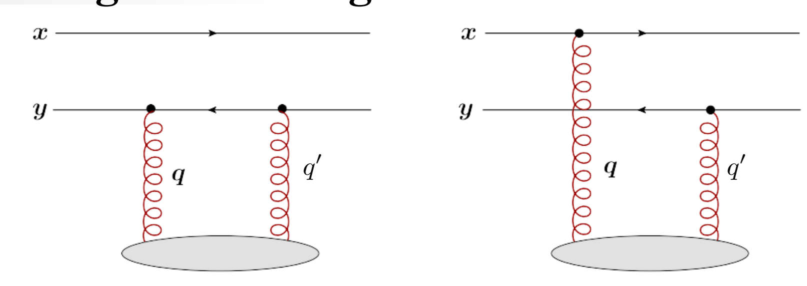
soft scale ~ 1/R

$$\boldsymbol{\Delta} = \mathbf{q}'+\mathbf{q} \qquad \mathbf{k} = (\mathbf{q}'-\mathbf{q})/2$$

$$\mathbf{r} = \mathbf{x}-\mathbf{y} \qquad \mathbf{b} = (\mathbf{x}+\mathbf{y})/2$$

- **Single-scattering limit:**

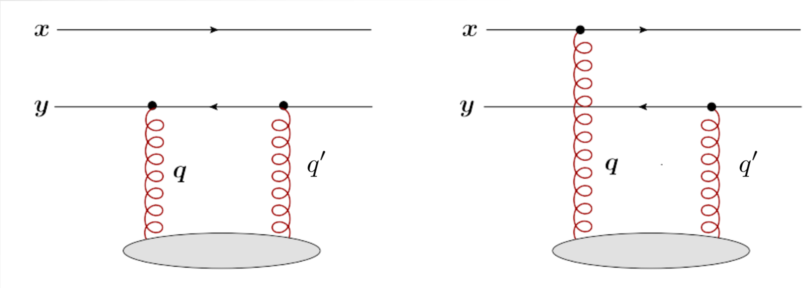


$$N_{2g}(\boldsymbol{b},\boldsymbol{r}) = \frac{g^2C_F}{2}\int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2}\frac{\tilde{\mu}(\boldsymbol{\Delta})}{(\boldsymbol{k}+\boldsymbol{\Delta}/2)^2(\boldsymbol{k}-\boldsymbol{\Delta}/2)^2}\,\mathrm{e}^{\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{b}}\left[\mathrm{e}^{\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{r}/2}+\mathrm{e}^{-\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{r}/2}-2\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\right]$$

$$\text{If } \tilde{\mu}(\boldsymbol{\Delta}) = \tilde{\mu}(|\boldsymbol{\Delta}|) \longrightarrow \mathbf{N_{2g}}(\mathbf{b},\mathbf{r}) \propto (\mathbf{b}.\mathbf{r})^{\mathbf{2n}} \longrightarrow \mathbf{v_{3,odd}} = \mathbf{0}$$

Elliptic flow in the single scattering approximation

Single-scattering approximation:



$$\mathbf{\Delta} = \mathbf{q}' + \mathbf{q} \qquad \mathbf{k} = (\mathbf{q}' - \mathbf{q})/2$$

$$\mathbf{r} = \mathbf{x} - \mathbf{y} \qquad \mathbf{b} = (\mathbf{x} + \mathbf{y})/2$$

$$N_{2g}(\boldsymbol{b}, \boldsymbol{r}) = \frac{g^2 C_F}{2} \int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{(2\pi)^2} \frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2} \frac{\tilde{\mu}(\boldsymbol{\Delta})}{(\boldsymbol{k}+\boldsymbol{\Delta}/2)^2(\boldsymbol{k}-\boldsymbol{\Delta}/2)^2} \mathrm{e}^{\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{b}} \Big[\mathrm{e}^{\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{r}/2} + \mathrm{e}^{-\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{r}/2} - 2\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}$$

$pR \gg 1$ (while $\Delta \lesssim 1/R$)

$$\tilde{S}(\mathbf{b}, \mathbf{p}) \simeq -\tilde{\mathbf{N}}_{\mathbf{2g}}(\mathbf{b}, \mathbf{p}) \simeq \frac{\mathbf{g^2 C_F}}{\mathbf{p^4}} \mu(\mathbf{b}) \left[\mathbf{1} - \frac{\mathbf{b^2}}{\mathbf{8p^2R^4}} \cos(\mathbf{2}\phi)\right]$$

$$v_2(p,b) \simeq \frac{\int_{-\pi}^{\pi} \mathrm{d}\phi \cos(2\phi) \left[1 - \frac{b^2}{8p^2R^4}\cos(2\phi)\right]}{\int_{-\pi}^{\pi} \mathrm{d}\phi \left[1 - \frac{b^2}{8p^2R^4}\cos(2\phi)\right]} = -\frac{b^2}{16p^2R^4}$$

★ **v2=0 at b=0.**

★ **A quark produced via a single scattering has more chances to propagate along a direction which is perpendicular on its impact parameter rather than parallel to it.**

★ **Multiple scatterings will change the sign of v2.**

Color-dipole orientation in the MV model

Adding multiple scattering: → $S(\boldsymbol{b},\boldsymbol{r}) = \exp\{-N_{2g}(\boldsymbol{b},\boldsymbol{r})\}$

$$N_{2g}(\boldsymbol{b},\boldsymbol{r}) = \frac{g^2 C_F}{2}\int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2}\frac{\tilde{\mu}(\boldsymbol{\Delta})}{(\boldsymbol{k}+\boldsymbol{\Delta}/2)^2(\boldsymbol{k}-\boldsymbol{\Delta}/2)^2}\,\mathrm{e}^{\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{b}}\left[\mathrm{e}^{\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{r}/2}+\mathrm{e}^{-\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{r}/2}-2\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\right]$$

- In soft multiple scattering regime, $k << 1/r$ and $\Delta \leq 1/R$:

$$N_{2g}(\boldsymbol{b},\boldsymbol{r}) \simeq \frac{g^2 C_F}{2}\, r^i r^j \int \frac{\mathrm{d}^2\Delta}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2}\frac{(k^i k^j - \Delta^i\Delta^j/4)\tilde{\mu}(\Delta)}{[(\boldsymbol{k}+\boldsymbol{\Delta}/2)^2+m^2][(\boldsymbol{k}-\boldsymbol{\Delta}/2)^2+m^2]}\,\mathrm{e}^{\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{b}}$$

- **Infrared finite at m—> 0:**

‘gluon mass’ m is needed in order to restrict the phase-space allowed to very soft momenta $k \sim \Lambda_{QCD}$

Color-dipole orientation in the MV model

Adding multiple scattering: → $S(\boldsymbol{b},\boldsymbol{r}) = \exp\{-N_{2g}(\boldsymbol{b},\boldsymbol{r})\}$

$$N_{2g}(\boldsymbol{b},\boldsymbol{r}) = \frac{g^2 C_F}{2}\int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2}\frac{\tilde{\mu}(\boldsymbol{\Delta})}{(\boldsymbol{k}+\boldsymbol{\Delta}/2)^2(\boldsymbol{k}-\boldsymbol{\Delta}/2)^2}\,\mathrm{e}^{\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{b}}\left[\mathrm{e}^{\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{r}/2} + \mathrm{e}^{-\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{r}/2} - 2\,\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}}\right]$$

- In soft multiple scattering regime, $k << 1/r$ and $\Delta \leq 1/R$:

$$N_{2g}(\boldsymbol{b},\boldsymbol{r}) \simeq \frac{g^2 C_F}{2} r^i r^j \int \frac{\mathrm{d}^2\Delta}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2}\frac{(k^i k^j - \Delta^i\Delta^j/4)\tilde{\mu}(\Delta)}{[(\boldsymbol{k}+\boldsymbol{\Delta}/2)^2+m^2][(\boldsymbol{k}-\boldsymbol{\Delta}/2)^2+m^2]}\,\mathrm{e}^{\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{b}}$$

- **UV regularization :**

$$\frac{g^2 C_F}{4} r^2 \int \frac{\mathrm{d}^2\Delta}{(2\pi)^2}\frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2}\frac{k^2\tilde{\mu}(\Delta)}{(k^2+m^2)^2}\,\mathrm{e}^{\mathrm{i}\boldsymbol{\Delta}\cdot\boldsymbol{b}} \equiv \frac{Q_s^2(b) r^2}{4}\ln\left(\frac{1}{r^2 m^2} + \mathrm{e}\right)$$

$$Q_s^2(b) \equiv \alpha_s C_F \mu(b) = Q_{0s}^2\,\mathrm{e}^{-b^2/4R^2}$$

Color-dipole orientation in the MV model

Adding multiple scattering: → $S(\boldsymbol{b},\boldsymbol{r}) = \exp\{-N_{2g}(\boldsymbol{b},\boldsymbol{r})\}$

$$N_{2g}(b,r,\theta) = \mathcal{N}_0(b,r) + \mathcal{N}_\theta(b,r)\cos(2\theta)$$

$$\begin{aligned}\mathcal{N}_0(b,r) \;=\; & \frac{Q_s^2(b)r^2}{4}\ln\left(\frac{1}{r^2m^2}+\mathrm{e}\right) \\ & + \frac{g^2C_F}{4(2\pi)^2}r^2\int_0^\infty \mathrm{d}\Delta\,\Delta\,\tilde{\mu}(\Delta)\mathrm{J}_0(\Delta b) \\ & \times\int_0^\infty \mathrm{d}k\,k\left[\frac{k^2-\Delta^2/4}{(k^2+\Delta^2/4+m^2)\left((k^2+\Delta^2/4+m^2)^2-k^2\Delta^2\right)^{1/2}}-\frac{k^2}{(k^2+m^2)^2}\right]\end{aligned}$$

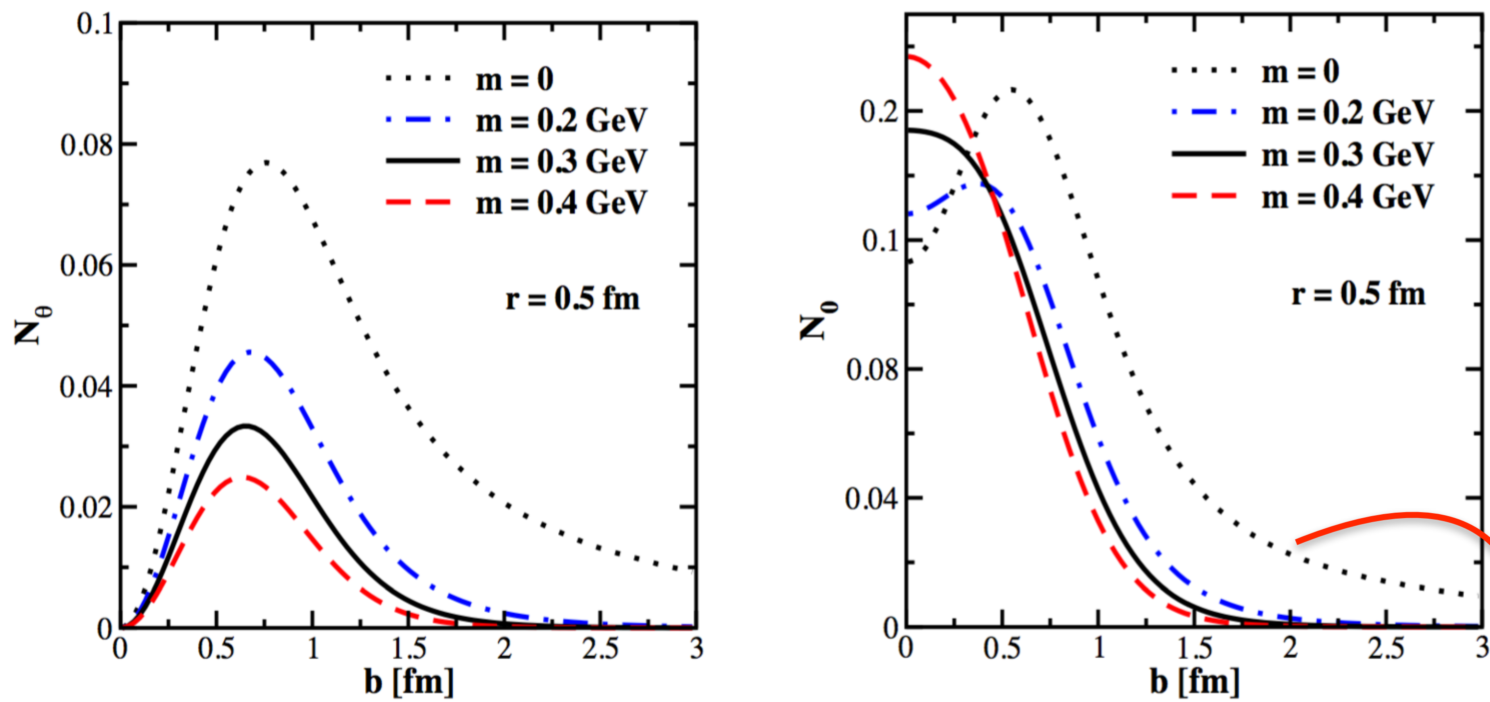
← **Original MV model**

$$\begin{aligned}\mathcal{N}_\theta(b,r) \;=\; & \frac{g^2C_F}{4(2\pi)^2}r^2\int_0^\infty \mathrm{d}\Delta\,\Delta\,\tilde{\mu}(\Delta)\mathrm{J}_2(\Delta b)\int_0^\infty \mathrm{d}k\,k\left[\frac{k^2+\Delta^2/4}{(k^2+\Delta^2/4+m^2)\left((k^2+\Delta^2/4+m^2)^2-k^2\Delta^2\right)^{1/2}}\right. \\ & \left.+\frac{2}{\Delta^2}-\frac{2\left(k^2+\Delta^2/4+m^2\right)}{\Delta^2\left((k^2+\Delta^2/4+m^2)^2-k^2\Delta^2\right)^{1/2}}\right].\end{aligned}$$

Color-dipole orientation in the MV model

Adding multiple scattering: $S(\boldsymbol{b}, \boldsymbol{r}) = \exp\{-N_{2g}(\boldsymbol{b}, \boldsymbol{r})\}$

$$N_{2g}(b, r, \theta) = \mathcal{N}_0(b, r) + \mathcal{N}_\theta(b, r)\cos(2\theta)$$

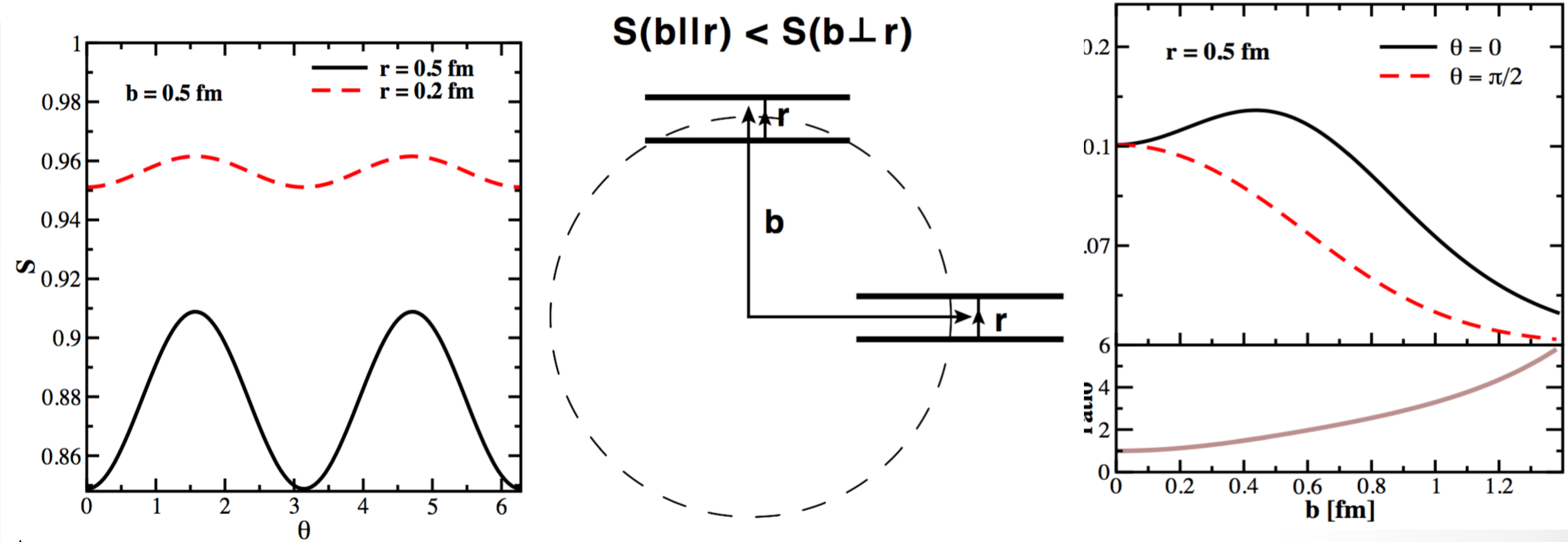


$$\mathcal{N}_\theta(b, r)\Big|_{m=0} = \frac{1}{2} \times \frac{g^2 C_F}{4(2\pi)^2} r^2 \int_0^\infty \mathrm{d}\Delta\, \Delta\, \tilde{\mu}(\Delta) \mathrm{J}_2(\Delta b) = Q_{0s}^2 r^2 \frac{R^2}{b^2}\left[1 - \left(1 + \frac{b^2}{4R^2}\right) \mathrm{e}^{-\frac{b^2}{4R^2}}\right]$$

no confinement in the limit $m \to 0$.

Color-dipole orientation as an origin of elliptic flow

$$N_{2g}(b,r,\theta) = \mathcal{N}_0(b,r) + \mathcal{N}_\theta(b,r)\cos(2\theta) \qquad S(\boldsymbol{b},\boldsymbol{r}) = \exp\{-N_{2g}(\boldsymbol{b},\boldsymbol{r})\}$$

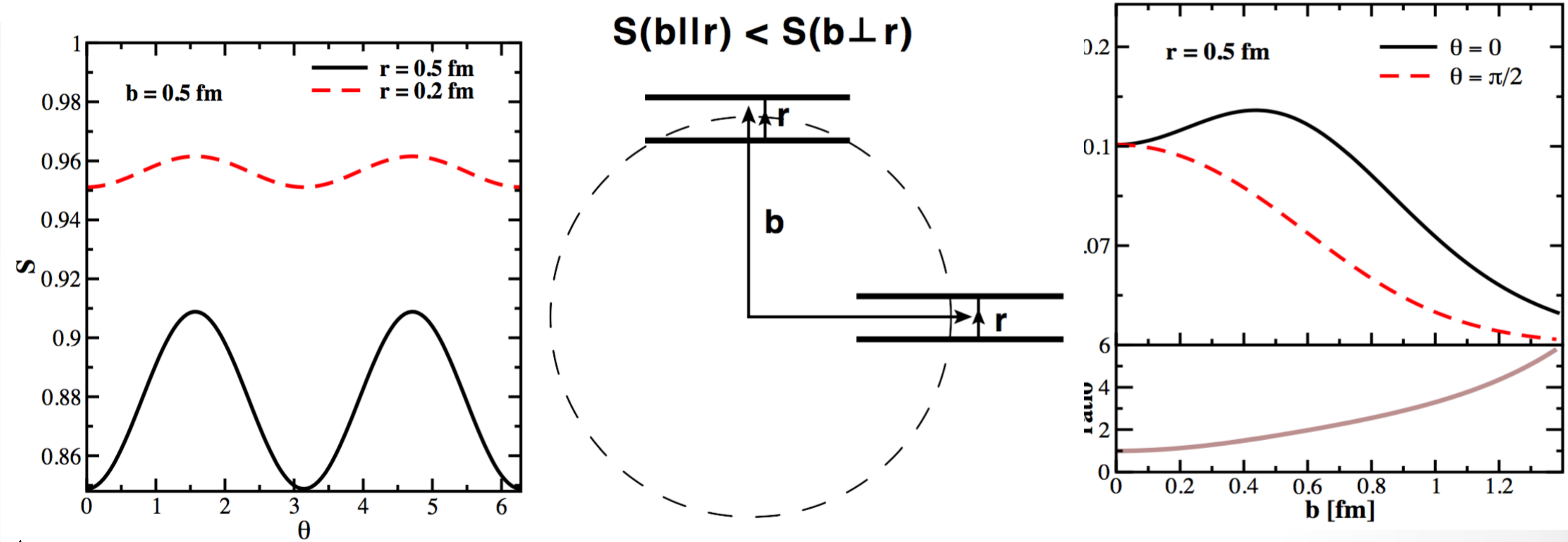


✦ The scattering is stronger when the dipole orientation is (anti)parallel to its impact parameter ($\theta = 0$ or $\theta = \pi$) than for a dipole perpendicular on b ($\theta = \pi/2$).

✦ The difference between 'parallel' and 'perpendicular' scattering increases with the dipole size r and also with the impact parameter b.

Color-dipole orientation as an origin of elliptic flow

$$N_{2g}(b,r,\theta) = \mathcal{N}_0(b,r) + \mathcal{N}_\theta(b,r)\cos(2\theta) \qquad S(\boldsymbol{b},\boldsymbol{r}) = \exp\{-N_{2g}(\boldsymbol{b},\boldsymbol{r})\}$$



- ✦ The scattering is stronger when the dipole orientation is (anti)parallel to its impact parameter ($\theta = 0$ or $\theta = \pi$) than for a dipole perpendicular on b ($\theta = \pi/2$).
- ✦ The difference between 'parallel' and 'perpendicular' scattering increases with the dipole size r and also with the impact parameter b.

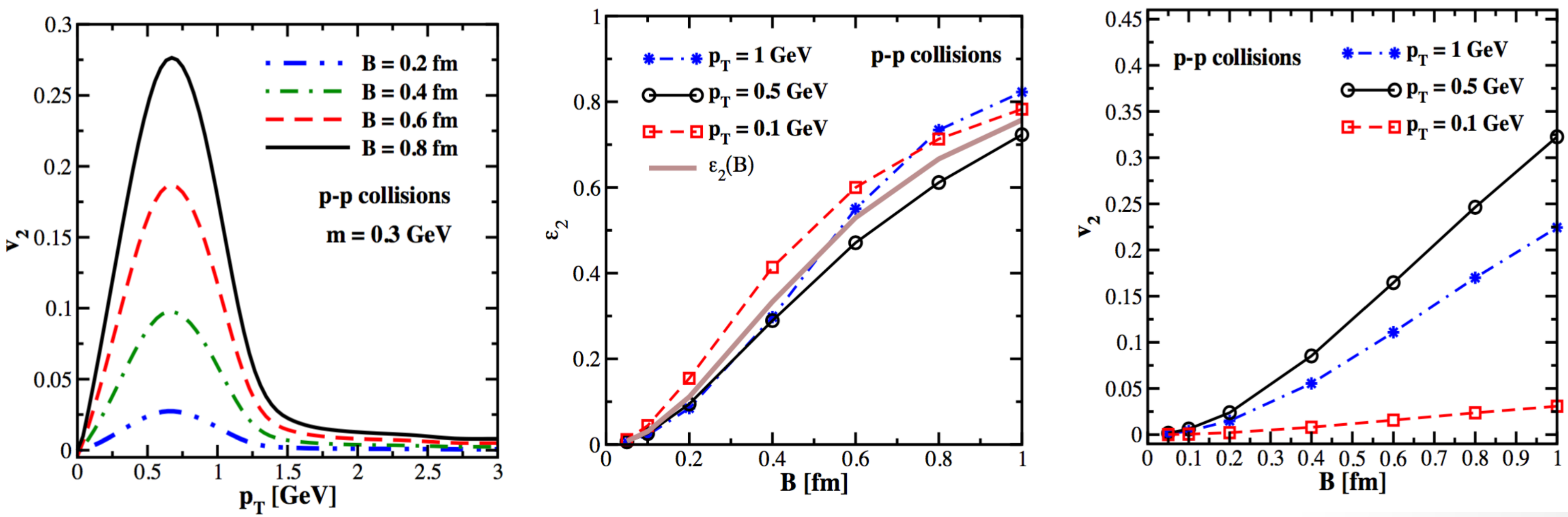
$$v_2(p,b) = -\frac{\int r\mathrm{d}r\, \mathrm{e}^{-\mathcal{N}_0(b,r)} \mathrm{J}_2(pr) \int \mathrm{d}\theta\, \mathrm{e}^{-\mathcal{N}_\theta(b,r)\cos(2\theta)} \cos(2\theta)}{\int r\mathrm{d}r\, \mathrm{e}^{-\mathcal{N}_0(b,r)} \mathrm{J}_0(pr) \int \mathrm{d}\theta\, \mathrm{e}^{-\mathcal{N}_\theta(b,r)\cos(2\theta)}}$$

$$= \frac{\int r\mathrm{d}r\, \mathrm{e}^{-\mathcal{N}_0(b,r)} \mathrm{J}_2(pr) \mathrm{I}_1(\mathcal{N}_\theta(b,r))}{\int r\mathrm{d}r\, \mathrm{e}^{-\mathcal{N}_0(b,r)} \mathrm{J}_0(pr) \mathrm{I}_0(\mathcal{N}_\theta(b,r))},$$

$$N(\mathbf{b}||\mathbf{r}) > \mathbf{N}(\mathbf{b} \perp \mathbf{r})$$

$$\mathcal{N}_\theta > 0 \longrightarrow v_2 > 0$$

Elliptic flow from color-dipole orientation in pp collisions

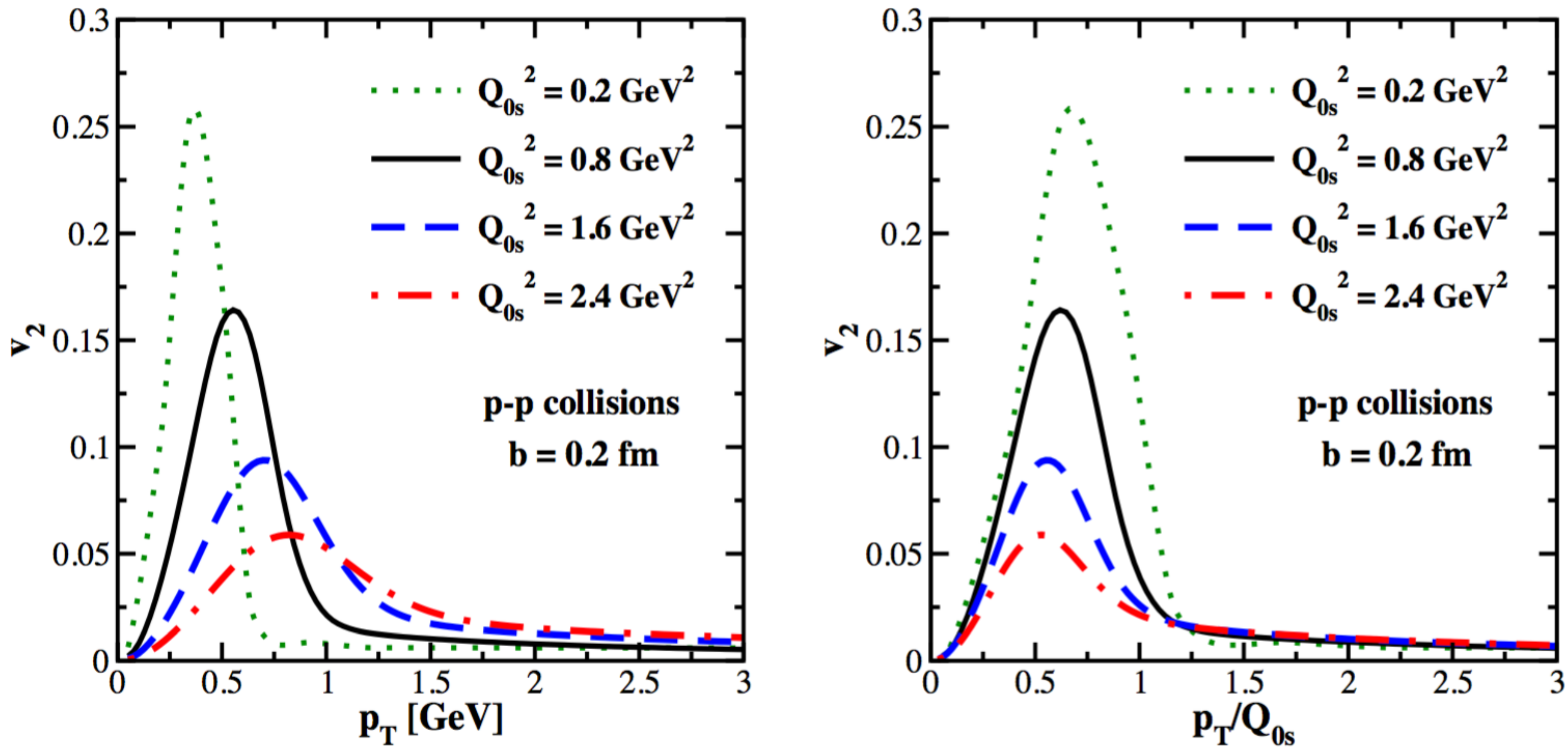


- The strength of v_2 increases with B (and b) since dipole orientation becomes important at large B (and b).
  The typical transverse size of the color charge distribution in the target is $2R \sim 0.6$ fm.
- v_2 and ϵ_2 show a similar trend with B, they monotonously increase with B:

$$v_2(p, B) \propto \epsilon_2(p, B)$$

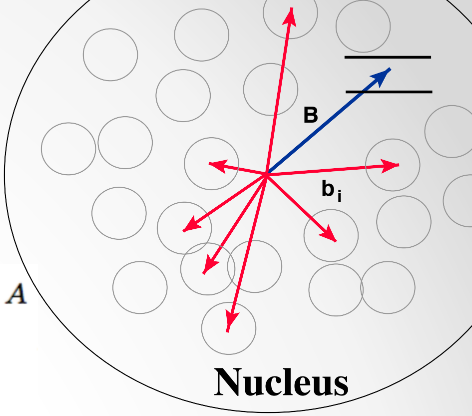
- $\varepsilon_2(p_T, B)$ is only weakly sensitive to the dipole scattering, since mostly controlled by the geometry.

Scaling property of v_2 with the saturation scale:



- v_2 develops a peak at a transverse momentum which scales with the saturation momentum in the target.

Dipole-nucleus scattering: the case of a lumpy target :



To include nucleon fluctuations in nucleus:

$$S_A(\boldsymbol{B},\boldsymbol{r}) = \int \prod_{i=1}^{A} \mathrm{d}^2\boldsymbol{b}_i\, T_A(\boldsymbol{b}_i)\, \mathrm{e}^{-\sum_{i=1}^{A} N_{2g}(\boldsymbol{B}-\boldsymbol{b}_i,\boldsymbol{r})} = \left(\int \mathrm{d}^2\boldsymbol{b}\, T_A(|\boldsymbol{B}-\boldsymbol{b}|)\, \mathrm{e}^{-N_{2g}(\boldsymbol{b},\boldsymbol{r})}\right)^A$$

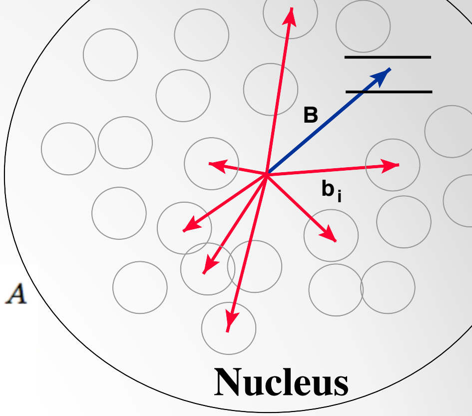
- **B** to denote the impact parameter of the dipole w.r.t. the center of the nucleus.
- **$\mathbf{b}_i$** is the position of the struck nucleon w.r.t. the center of the nucleus.

✦ Assuming scattering between the dipole and a *single* nucleon is weak $N_{2g}(\mathbf{b},\mathbf{r}) \ll 1$

$$S_A(\boldsymbol{B},\boldsymbol{r}) \simeq \left(1 - \int \mathrm{d}^2\boldsymbol{b}\, T_A(|\boldsymbol{B}-\boldsymbol{b}|)\, N_{2g}(\boldsymbol{b},\boldsymbol{r})\right)^A \simeq \mathrm{e}^{-AN_{2g}^A(\boldsymbol{B},\boldsymbol{r})}$$

$$N_{2g}^A(\boldsymbol{B},\boldsymbol{r}) = \int d^2\boldsymbol{b}\, N_{2g}(\boldsymbol{b},\boldsymbol{r})\, T_A(|\boldsymbol{B}-\boldsymbol{b}|)$$

Dipole-nucleus scattering: the case of a lumpy target :



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- **B** to denote the impact parameter of the dipole w.r.t. the center of the nucleus.
- $\mathbf{b_i}$ is the position of the struck nucleon w.r.t. the center of the nucleus.

✦ Assuming scattering between the dipole and a *single* nucleon is weak $N_{2g}(\mathbf{b},\mathbf{r}) \ll 1$

$$S_A(\boldsymbol{B},\boldsymbol{r}) \simeq \left(1 - \int \mathrm{d}^2\boldsymbol{b}\, T_A\left(|\boldsymbol{B}-\boldsymbol{b}|\right)\, N_{2g}(\boldsymbol{b},\boldsymbol{r})\right)^A \simeq \mathrm{e}^{-A N_{2g}^A(\boldsymbol{B},\boldsymbol{r})}$$

$$N_{2g}^A(\boldsymbol{B},\boldsymbol{r}) = \int d^2\boldsymbol{b}\, N_{2g}(\boldsymbol{b},\boldsymbol{r})\, T_A\left(|\boldsymbol{B}-\boldsymbol{b}|\right)$$

Homogeneous target: $T_A(\mathrm{B}-\mathrm{b}) \simeq \mathrm{T_A}(\mathrm{B})$ → **No dipole orientation:** $v_n \approx 0$

In central collisions: $v_n \approx 0$

Dipole-nucleus scattering: the case of a lumpy target

$$N^A_{2g}(\boldsymbol{B},\boldsymbol{r}) = \int d^2\boldsymbol{b}\, N_{2g}(\boldsymbol{b},\boldsymbol{r})\, T_A\left(|\boldsymbol{B}-\boldsymbol{b}|\right)$$

B >> b:

$$\begin{aligned} T_A(|\boldsymbol{B}-\boldsymbol{b}|) &= \left(1 - b^i \frac{\partial}{\partial B^i} + \frac{1}{2} b^i b^j \frac{\partial^2}{\partial B^i \partial B^j} + \dots\right) T_A\left(|\boldsymbol{B}|\right), \\ &\simeq T_A(B) - \frac{\boldsymbol{b}\cdot\boldsymbol{B}}{B} T'_A(B) + \frac{b^i b^j}{2}\left\{\frac{B^i B^j}{B^2} T''_A(B) + \frac{1}{B}\left(\delta^{ij} - \frac{B^i B^j}{B^2}\right) T'_A(B)\right\} \end{aligned}$$

correlations (**B, b**) and (**b, r**) → correlations (**B, r**)

$$N^A_{2g}(B,r,\theta) = \mathcal{N}^A_0(B,r) + \mathcal{N}^A_\theta(B,r)\cos(2\theta)$$

$$\begin{aligned} \mathcal{N}^A_0(B,r) &= 2\pi \int \mathrm{d}b\, b\, \mathcal{N}_0(b,r)\left\{T_A(B) + \frac{b^2}{4}\left(T''_A(B) + \frac{1}{B} T'_A(B)\right)\right\} \\ \mathcal{N}^A_\theta(B,r) &= \frac{\pi}{4} \int \mathrm{d}b\, b^3\, \mathcal{N}_\theta(b,r)\left(T''_A(B) - \frac{1}{B} T'_A(B)\right). \end{aligned}$$

Dipole-nucleus scattering: the case of a lumpy target

$$N^A_{2g}(\boldsymbol{B},\boldsymbol{r}) = \int d^2\boldsymbol{b}\, N_{2g}(\boldsymbol{b},\boldsymbol{r})\, T_A\left(|\boldsymbol{B}-\boldsymbol{b}|\right)$$

B >> b:

$$T_A(|\boldsymbol{B}-\boldsymbol{b}|) = \left(1 - b^i\frac{\partial}{\partial B^i} + \frac{1}{2}b^ib^j\frac{\partial^2}{\partial B^i\partial B^j} + \dots\right)T_A\left(|\boldsymbol{B}|\right),$$
$$\simeq T_A(B) - \frac{\boldsymbol{b}\cdot\boldsymbol{B}}{B}T'_A(B) + \frac{b^ib^j}{2}\left\{\frac{B^iB^j}{B^2}T''_A(B) + \frac{1}{B}\left(\delta^{ij} - \frac{B^iB^j}{B^2}\right)T'_A(B)\right\}$$

correlations (**B, b**) and (**b, r**) → correlations (**B, r**)

$$N^A_{2g}(B,r,\theta) = \mathcal{N}^A_0(B,r) + \mathcal{N}^A_\theta(B,r)\cos(2\theta)$$

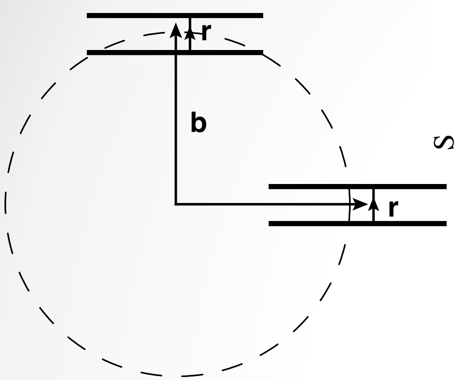
Original MV model ← $\pi R^2 Q^2_{0s} r^2 \ln\left(\frac{1}{r^2m^2}+\mathrm{e}\right)$

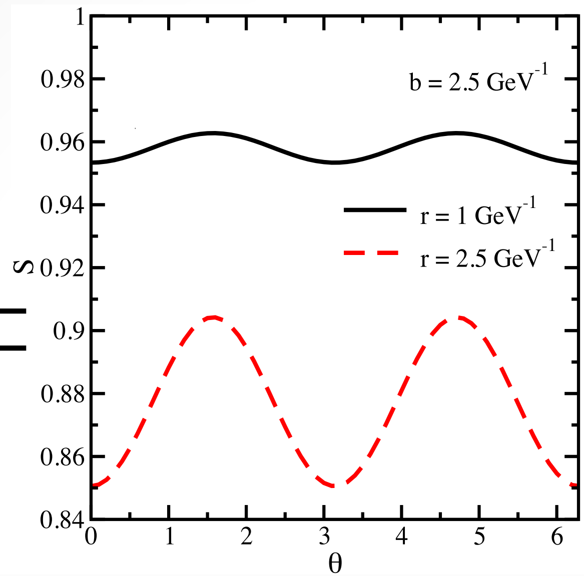
$$\mathcal{N}^A_0(B,r) = \pi R^2 Q^2_{0s} r^2 \ln\left(\frac{1}{r^2m^2}+\mathrm{e}\right)\left[T_A(B) + R^2\left(T''_A(B) + \frac{1}{B}T'_A(B)\right)\right]$$
$$+\frac{\pi}{3}\frac{R^2}{m^2}Q^2_{0s}r^2\left(T''_A(B) + \frac{1}{B}T'_A(B)\right),$$

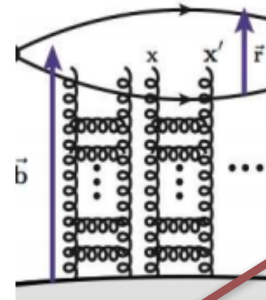
$$\mathcal{N}^A_\theta(B,r) = \frac{\pi}{6}\frac{R^2}{m^2}Q^2_{0s}r^2\left(T''_A(B) - \frac{1}{B}T'_A(B)\right).$$

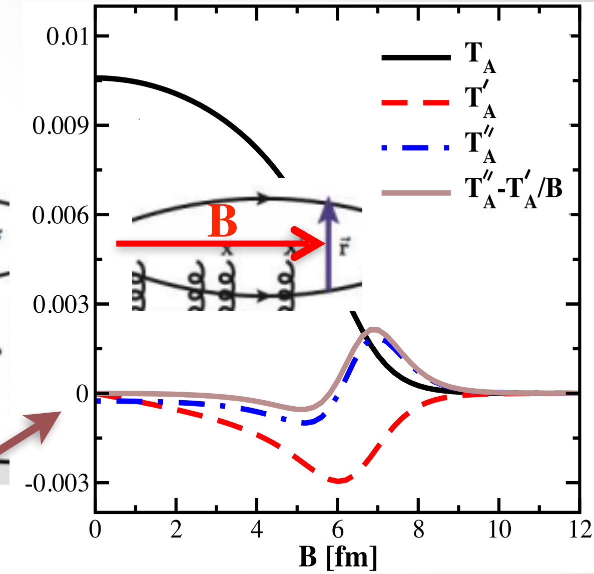
Elliptic flow as a good probe of the lumpiness of the target

S(b||r) < S(b⊥r)









The color-dipole orientation probes the inhomogeneity of the target in the transverse plane.

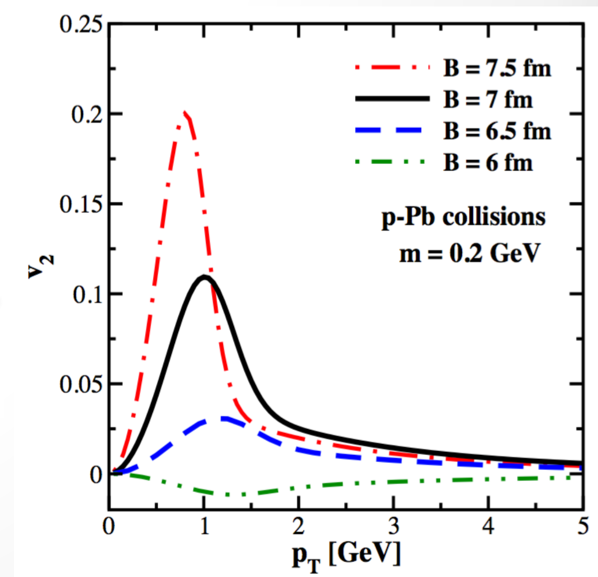
$$N_{2g}^{A}(B,r,\theta) = \mathcal{N}_0^A(B,r) + \mathcal{N}_\theta^A(B,r)\cos(2\theta),$$

Original MV model

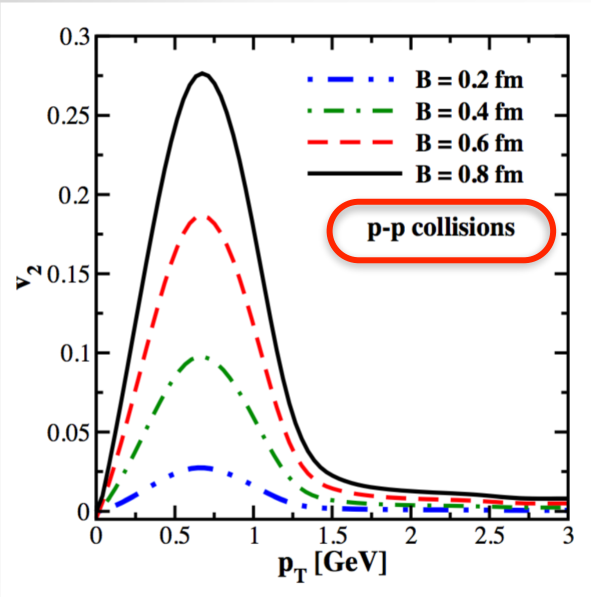
$$\mathcal{N}_0^A(B,r) = \pi R^2 Q_{0s}^2 r^2 \ln\left(\frac{1}{r^2 m^2} + \mathrm{e}\right)\left[T_A(B) + R^2\left(T_A''(B) + \frac{1}{B}T_A'(B)\right)\right] + \frac{\pi}{3}\frac{R^2}{m^2}Q_{0s}^2 r^2\left(T_A''(B) + \frac{1}{B}T_A'(B)\right),$$

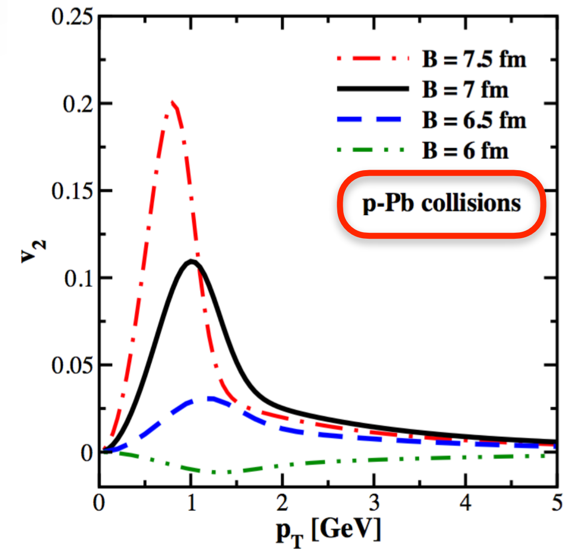
$$\mathcal{N}_\theta^A(B,r) = \frac{\pi}{6}\frac{R^2}{m^2}Q_{0s}^2 r^2\left(T_A''(B) - \frac{1}{B}T_A'(B)\right).$$

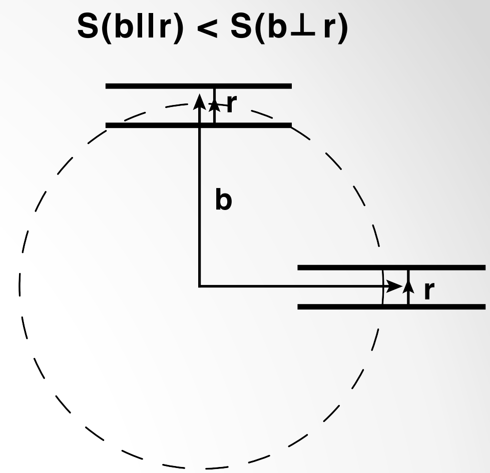
$$v_2(p,B) = \frac{\int r\mathrm{d}r\, \mathrm{e}^{-A\mathcal{N}_0^A(B,r)}\, \mathrm{J}_2(pr)\, \mathrm{I}_1\left(A\mathcal{N}_\theta^A(B,r)\right)}{\int r\mathrm{d}r\, \mathrm{e}^{-A\mathcal{N}_0^A(B,r)}\, \mathrm{J}_0(pr)\, \mathrm{I}_0\left(A\mathcal{N}_\theta^A(B,r)\right)}.$$

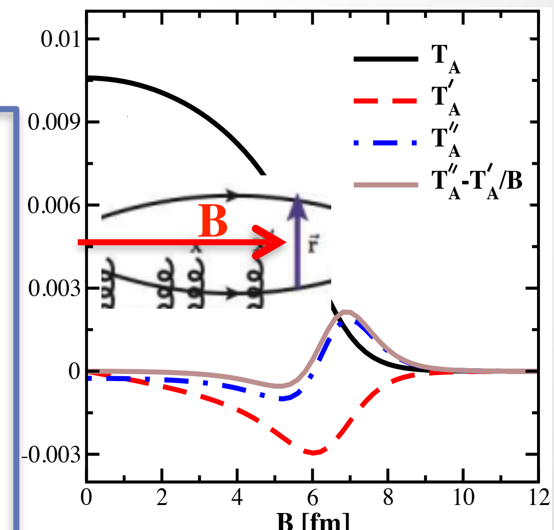


pp v. pA collisions





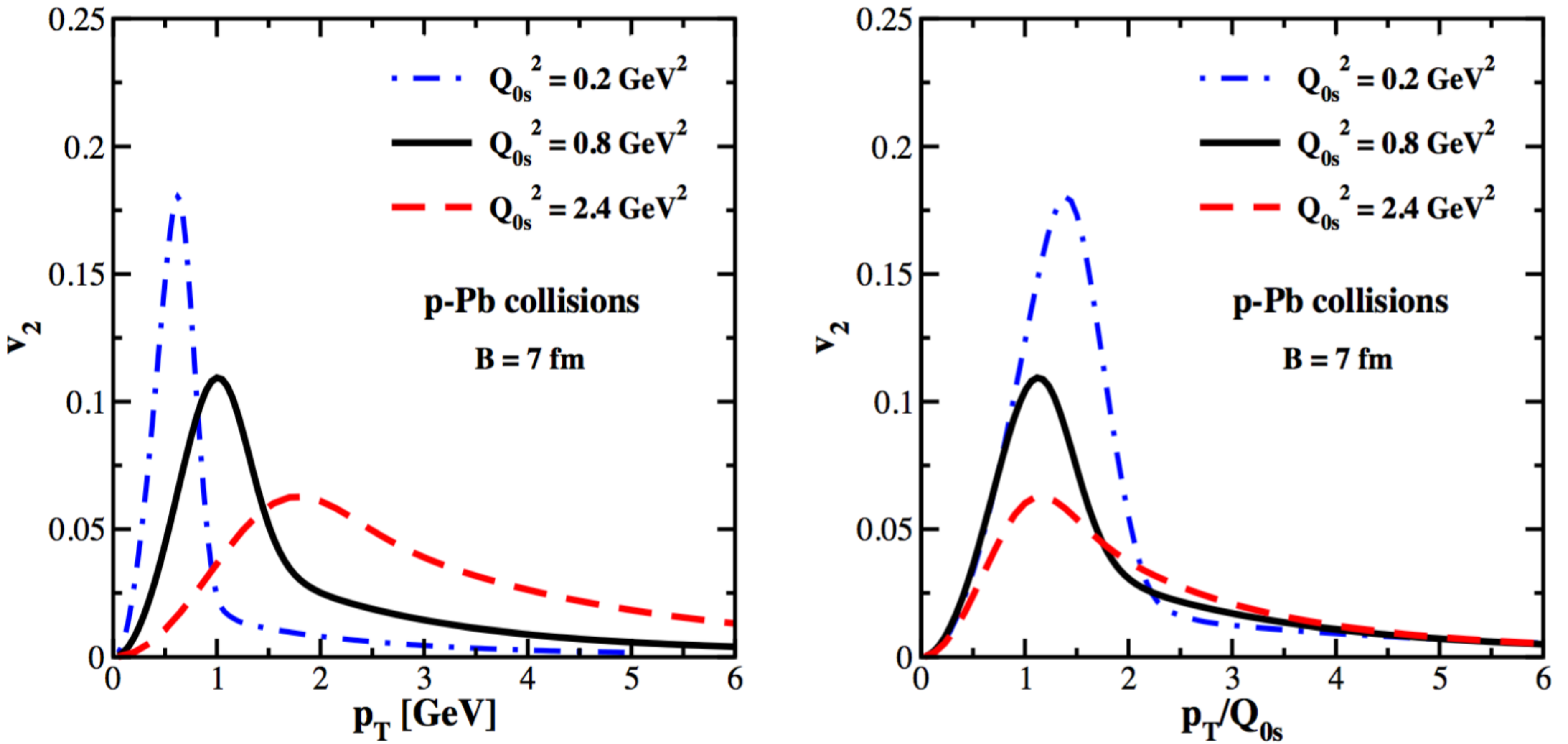




- $v_2 \to 0$ for $p_T \to 0$ or large p_T.
  The angular orientation cannot play any role when either the momentum p_T, or the dipole size r, are too small.
- $v_2 \to 0$ for $b, B \to 0$.
- $v_2(p_{\max}) \gtrsim 0.1$ when b or $B \sim$ the typical size for inhomogeneity in the target:
  $b \sim R \gtrsim 0.2$ fm for a proton and $B \sim R_A \gtrsim 6.5$ fm for a large nucleus.

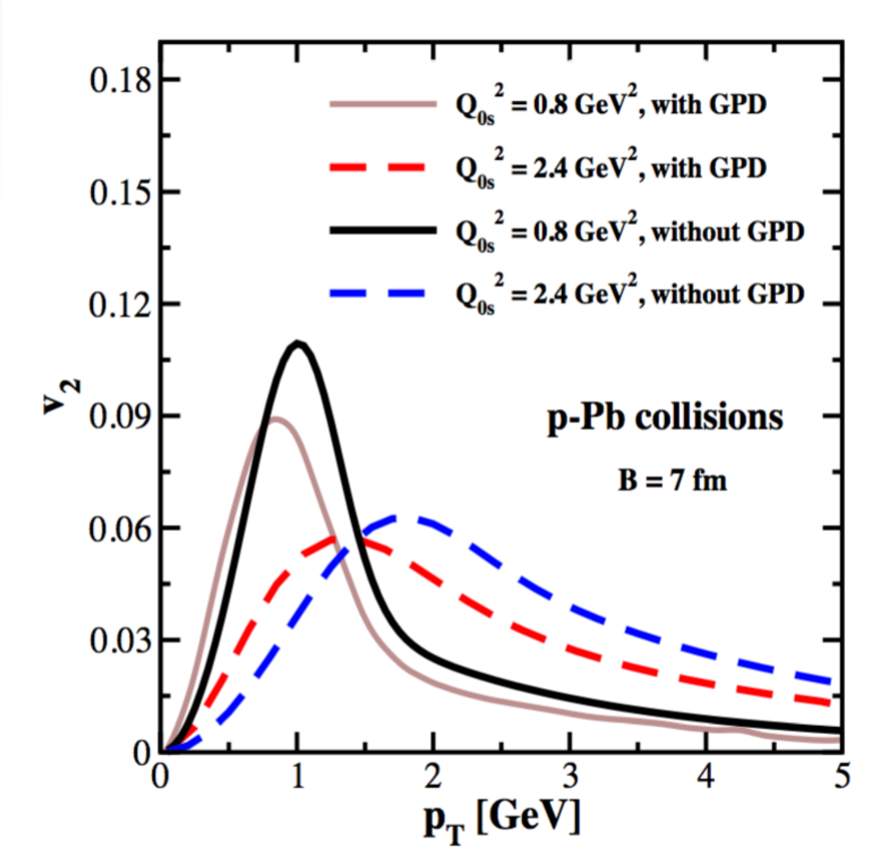
✓The color-dipole orientation (and v_2) probes the inhomogeneity of the target.

Scaling property of v_2



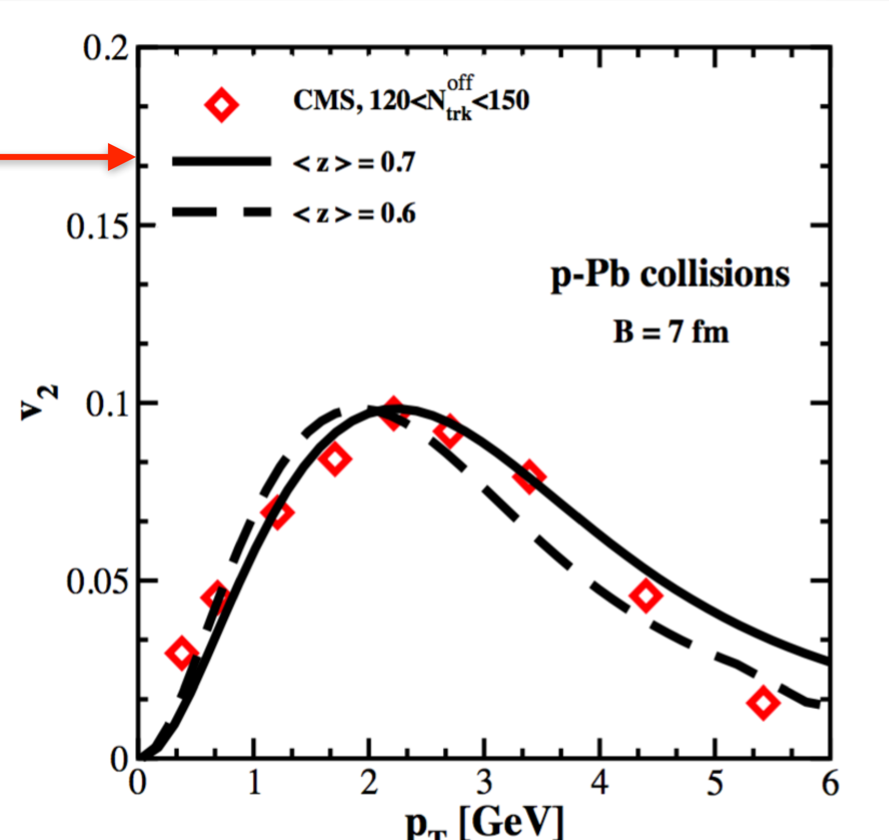
- Similar to pp collisions, v_2 develops a peak at a transverse momentum which scales with the saturation momentum in the target.

Finite-size effect of projectile proton (GPD):



✓ **The finite-size effect of proton is quite small — at most a change of 20% in the value of v_2 at its peak. This is due to the fact that the color-dipole orientation is only important for peripheral collisions.**

$p_{\mathrm{hadron}} = \langle z \rangle p_{\mathrm{quark}}$



- **Assuming that the final particles are correlated with each other only through the flow correlations with the reaction plane:** $v_2\{4\} = [-c_2\{4\}]^{1/4}$

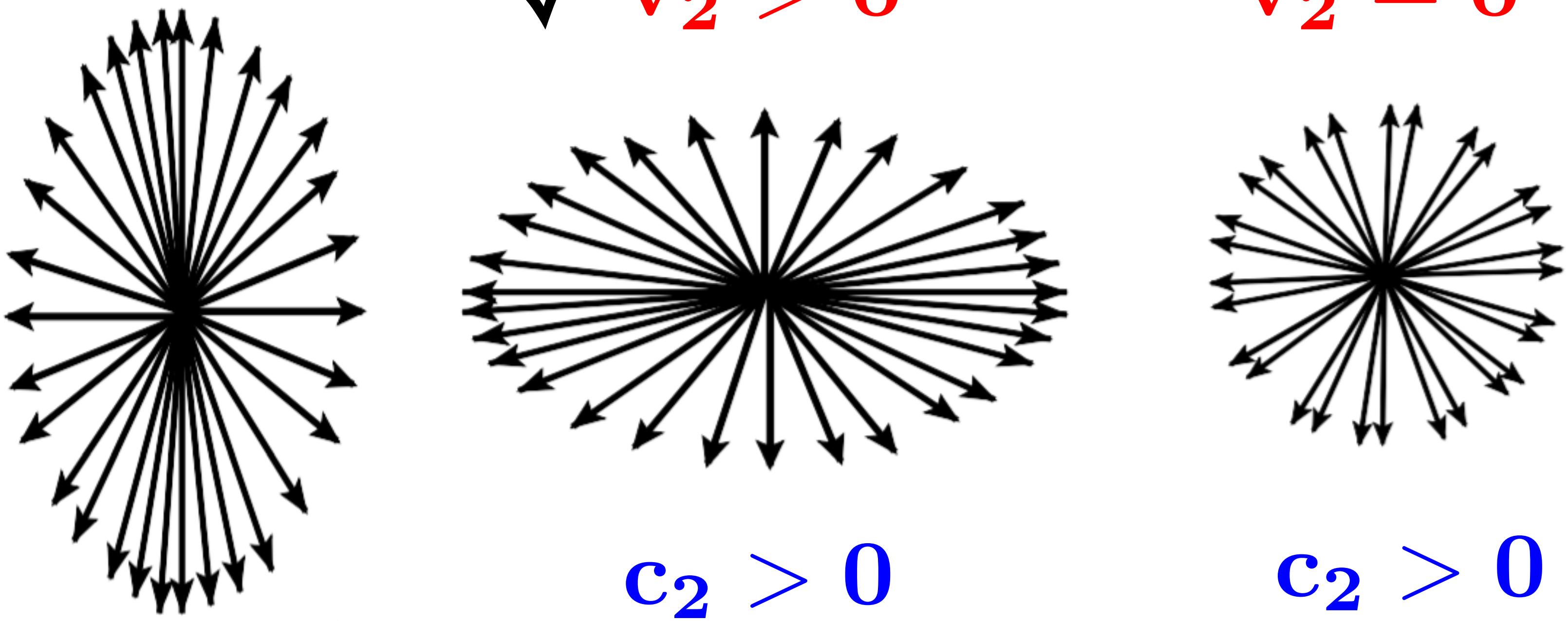
- **We have 3 free parameters:** $R^2 Q_{0s}^2,\ m,\ B$

From HERA: $R^2 = 2\ \mathrm{GeV}^{-2}$, **For the LHC:** $Q_{0s}^2 = 3\ \mathrm{GeV}^2$

This scenario is not excluded by the current data.

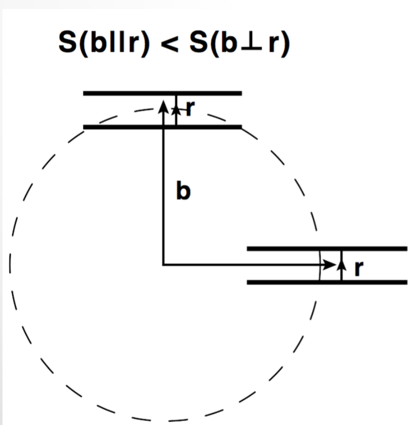
Geometrical picture of pp and pA collisions

$\mathbf{v_2 < 0}$ ✓ $\mathbf{v_2 > 0}$ $\mathbf{v_2 = 0}$



$\mathbf{c_2 > 0}$ $\mathbf{c_2 > 0}$ $\mathbf{c_2 > 0}$

(inspired by Snellings, arXiv:1102.3010)

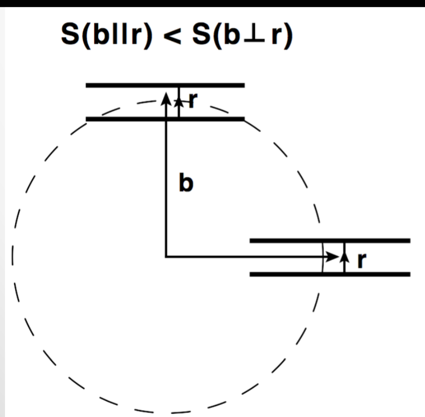


$N(\mathbf{b}||\mathbf{r}) > \mathbf{N}(\mathbf{b} \perp \mathbf{r})$ → $\mathcal{N}_\theta > 0 \longrightarrow v_2 > 0$

Such geometrical aspects are clearly reminiscent of the classical discussion of hydrodynamic flow in AA collisions:

✓ **AA collisions:** **the flow is driven by the ‘pressure gradient’ (the final state interactions) associated with the spatial asymmetry of the interaction region.**

✓ **pA (pp) collisions:** **the flow is rather a consequence of the angular dependence of the amplitude for dipole scattering.**



$$N(\mathbf{b}||\mathbf{r}) > \mathbf{N}(\mathbf{b} \perp \mathbf{r}) \longrightarrow \mathcal{N}_\theta > 0 \longrightarrow v_2 > 0$$

Geometrical picture of pp and pA collisions

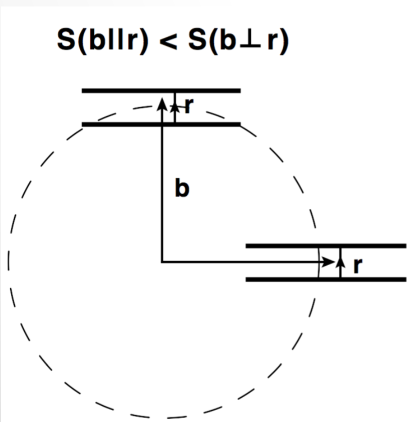
The anisotropy due to the color-dipole orientation mechanism is universal for different processes in dilute-dense scatterings.

There will be the analog of azimuthal anisotropy v_n in DIS and UPC.

Stay tuned!.

$\mathbf{c_2 > 0}$ $\mathbf{c_2 > 0}$

$\mathbf{c_2 > 0}$



$$N(\mathbf{b}||\mathbf{r}) > \mathbf{N}(\mathbf{b} \perp \mathbf{r}) \longrightarrow \mathcal{N}_\theta > 0 \longrightarrow v_2 > 0$$

Final remarks:

- *Typical sources of the inhomogeneity:*

1. The target geometry in our scenario (dipole orientation)

It can already be probed by the single inclusive particle spectrum and the effect is NOT suppressed at large N_c. The effect is important for peripheral collisions.

2. Fluctuations in the target gluon distribution (glasma scenario)

Can be probed by 2-particle correlations, and are suppressed at large N_c. The effect is important for central collisions.

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Typical sources of the inhomogeneity:

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2. Fluctuations in the target gluon distribution (glasma scenario)
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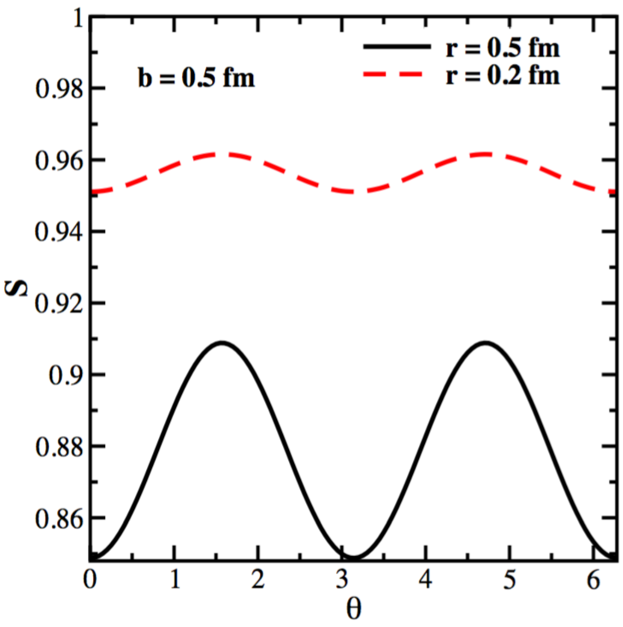
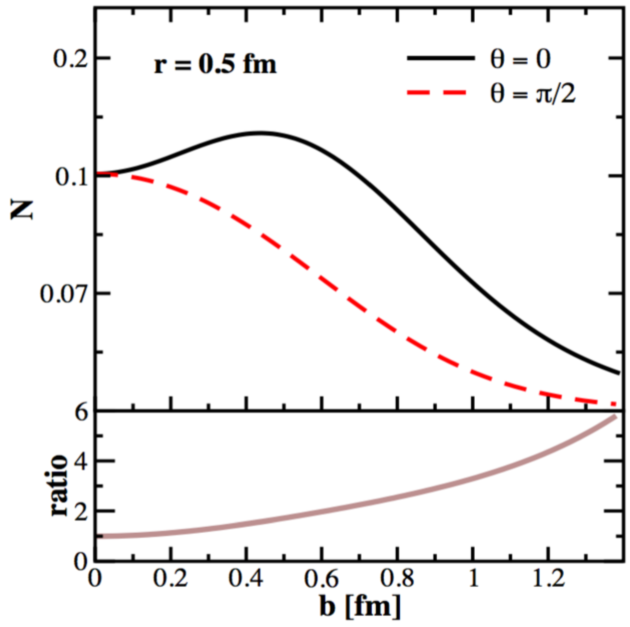
Remains to be studied:
#2 fluctuation can also be probed by the color-dipole orientation.

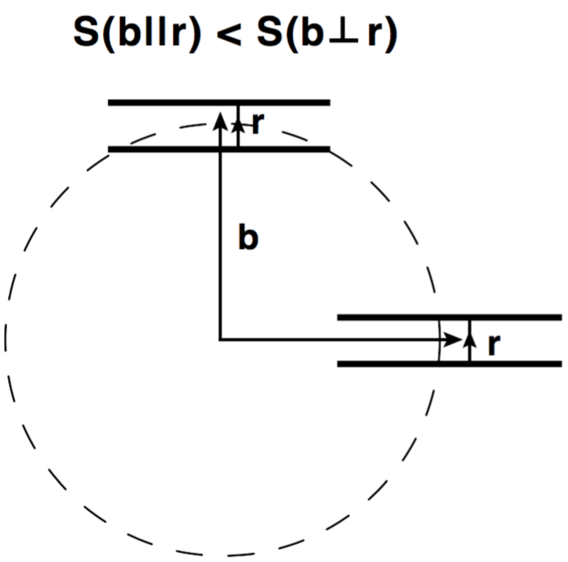
Backup

$$N_{2g}(b,r,\theta) = \mathcal{N}_0(b,r) + \mathcal{N}_\theta(b,r)\cos(2\theta)$$

$$S(\boldsymbol{b},\boldsymbol{r}) = \exp\{-N_{2g}(\boldsymbol{b},\boldsymbol{r})\}$$







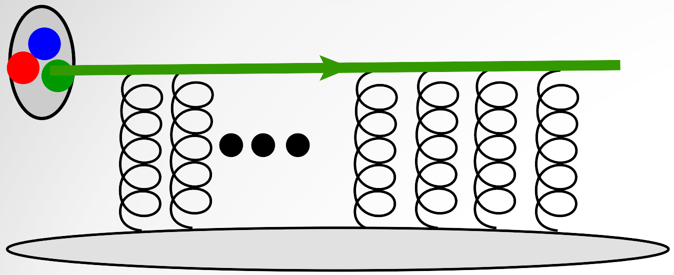
$$N(\mathbf{b}||\mathbf{r}) > \mathbf{N}(\mathbf{b}\perp\mathbf{r})$$

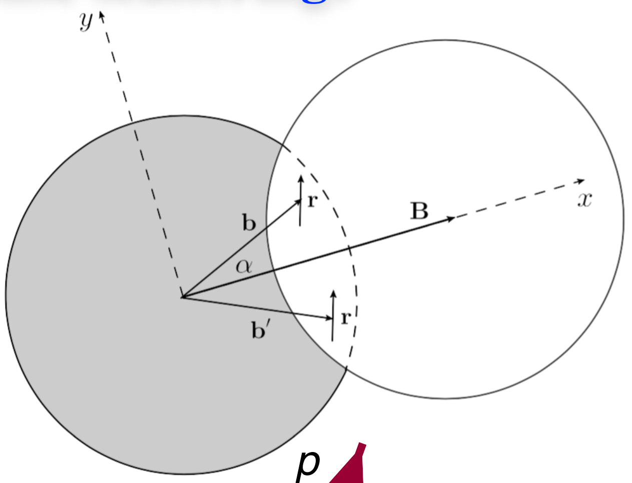
$$\mathcal{N}_\theta > 0 \longrightarrow v_2 > 0$$

Including finite-size effect for the projectile (GPD):

$$v_2(p,B) = \frac{\int b\mathrm{d}b\, \mathrm{e}^{-(b^2+B^2)/4R^2}\, \mathrm{I}_2\left(bB/2R^2\right) \int r\mathrm{d}r\, \mathrm{e}^{-\mathcal{N}_0(b,r)}\, \mathrm{J}_2(pr)\, \mathrm{I}_1\left(\mathcal{N}_\theta(b,r)\right)}{\int b\mathrm{d}b\, \mathrm{e}^{-(b^2+B^2)/4R^2}\, \mathrm{I}_0\left(bB/2R^2\right) \int r\mathrm{d}r\, \mathrm{e}^{-\mathcal{N}_0(b,r)}\, \mathrm{J}_0(pr)\, \mathrm{I}_0\left(\mathcal{N}_\theta(b,r)\right)}.$$

Elliptic flow and eccentricity in dilute-dense scatterings

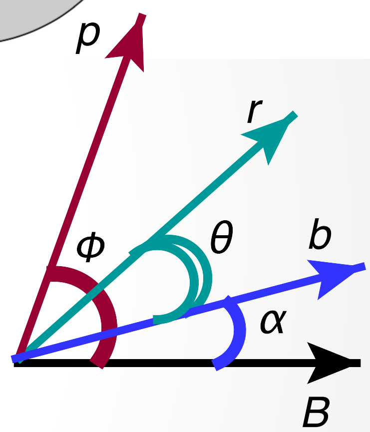




$$\frac{\mathrm{d}\sigma^q(qA \to qX)}{\mathrm{d}\eta\,\mathrm{d}^2\boldsymbol{p}\,\mathrm{d}^2\boldsymbol{B}} = \frac{1}{(2\pi)^2}\int \mathrm{d}^2\boldsymbol{b}\; x_p q(x_p, \boldsymbol{b}-\boldsymbol{B})\,\tilde{S}(\boldsymbol{b},\boldsymbol{p},x_g)$$

Proton GPD: $x_p q(x_p, \boldsymbol{b}) \simeq x_p q(x_p) f(\boldsymbol{b})$, with $\int \mathrm{d}^2\boldsymbol{b}\, f(\boldsymbol{b}) = 1$

$$\int \frac{\mathrm{d}^2\boldsymbol{p}}{(2\pi)^2}\,\tilde{S}(\boldsymbol{b},\boldsymbol{p}) = 1 \;\Longrightarrow\; \int p\mathrm{d}p \int r\mathrm{d}r\mathrm{d}\theta\, \mathrm{J}_0(pr)\, S(b,r,\theta) = 2\pi$$

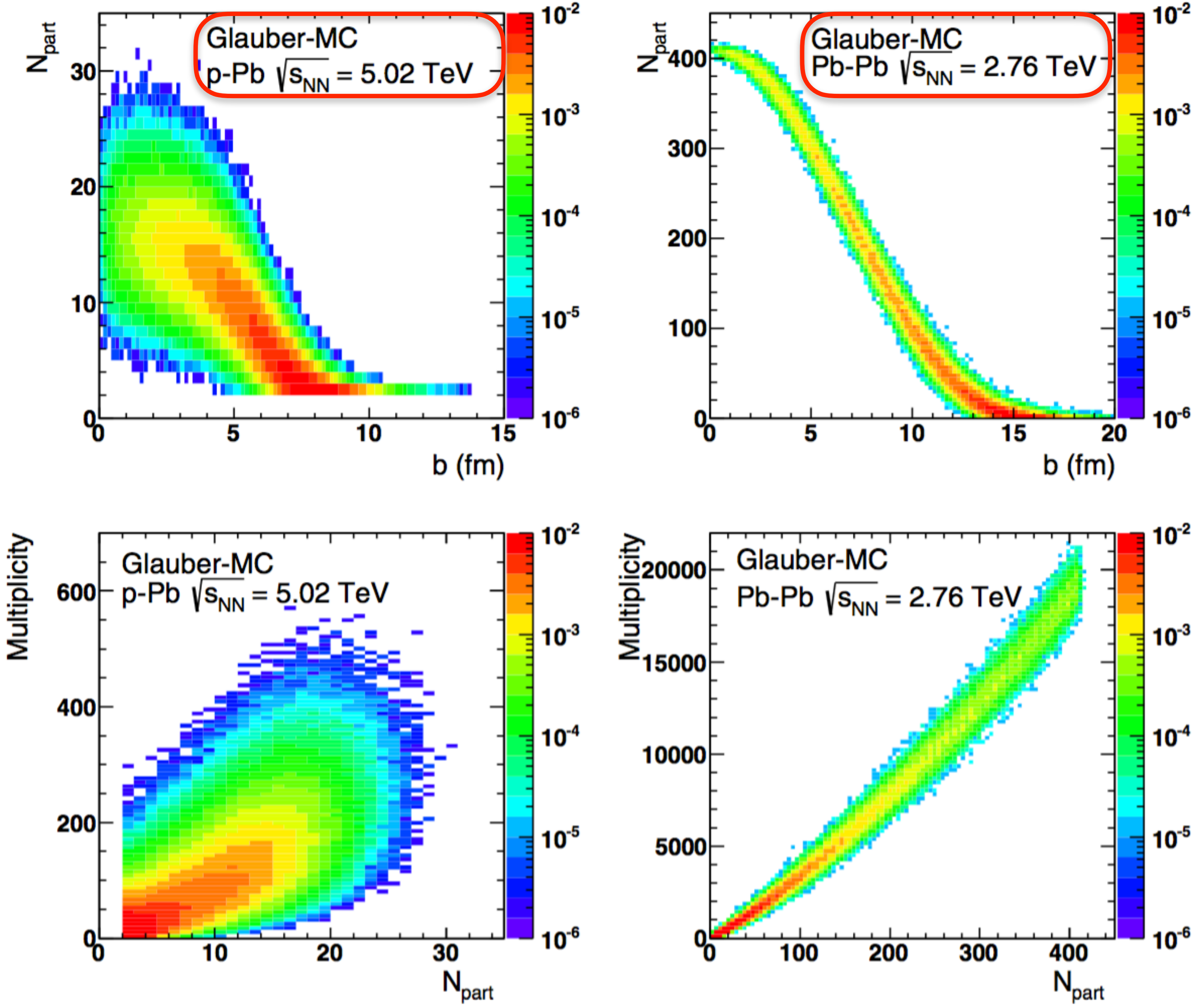


★ **Probability conservation: the total probability for the quark to emerge with any momentum must be equal to one.**

$$\varepsilon_2(B) = \frac{\int b\mathrm{d}b\mathrm{d}\alpha\, b^2 \cos(2\alpha)\, f(|\boldsymbol{b}-\boldsymbol{B}|)}{\int b\mathrm{d}b\mathrm{d}\alpha\, b^2\, f(|\boldsymbol{b}-\boldsymbol{B}|)}$$

★ **Purely geometrical quantity, without any information about the scattering of the dipole**

$$\epsilon_2(B=0)=0,\;\; \epsilon_2(B\to\infty)=1$$


Glauber-MC
p-Pb $\sqrt{s_{NN}}$ = 5.02 TeV
N_{part}
b (fm)
Glauber-MC
Pb-Pb $\sqrt{s_{NN}}$ = 2.76 TeV
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Multiplicity
N_{part}
Glauber-MC
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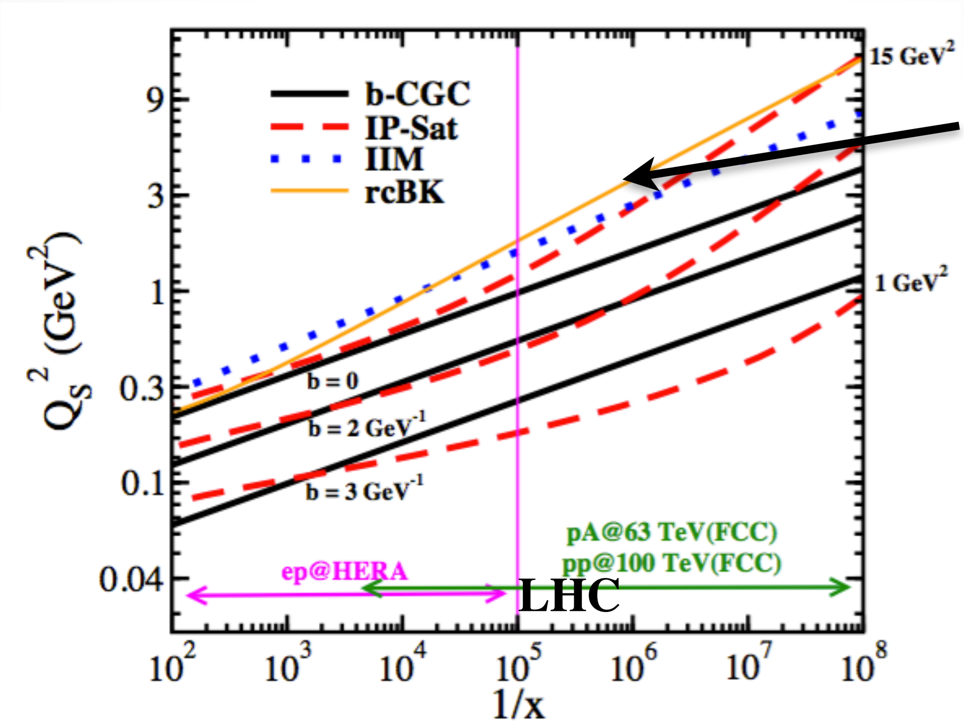
Multiplicity
N_{part}

b-dependence of Saturation scale & applicability of the CGC

Rezaeian and Schmidt, arXiv:1307.0825



- **b-independent saturation models significantly overestimate Q_s.**

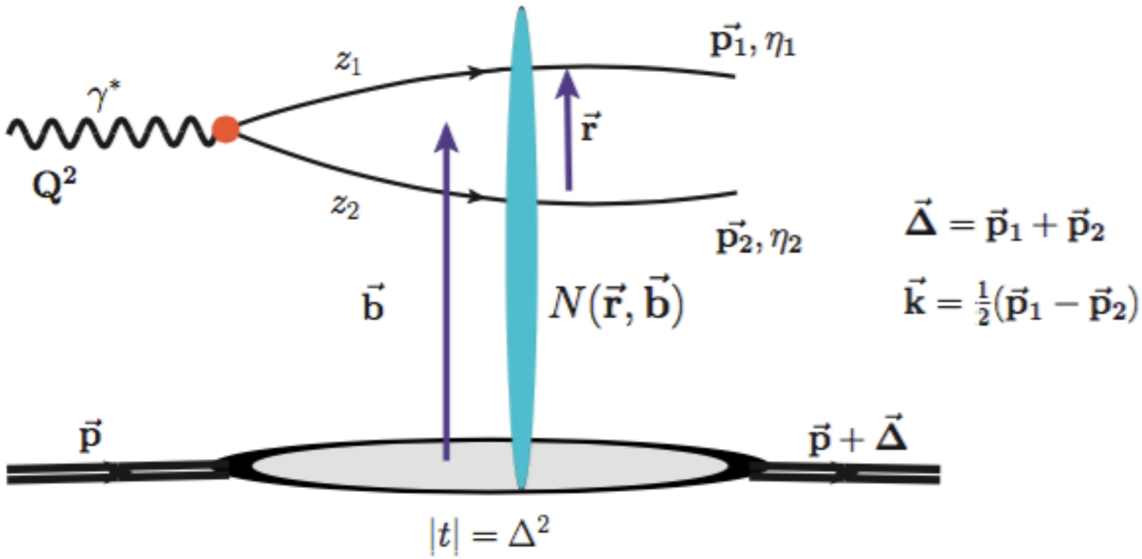
- **Proton saturation scale:**

$$\text{HERA} : Q_s < 1\,\text{GeV}$$
$$\text{LHC} : Q_s \leq 1-2\,\text{GeV}$$
$$\text{FCC} : Q_s \leq 2-4\,\text{GeV}$$

- **Nuclear saturation scale:** $Q_{sA}^2 \approx A^{1/3} Q_s^2 \approx 6\, Q_s^2$

$$\text{EIC} : Q_{sA} < 2.5\,\text{GeV}$$

Diffractive Dijet production in the CGC: $\gamma^* + p(A) \to q\bar{q} + p(A)$



$$\sigma = \psi^{\gamma}_{q\bar{q}} \otimes \mathcal{N}(\vec{r}, \vec{b}) \otimes \mathcal{N}(\vec{r'}, \vec{b'}) \neq \psi^{\gamma}_{q\bar{q}} \otimes [\mathcal{N}(r, b)]^2$$

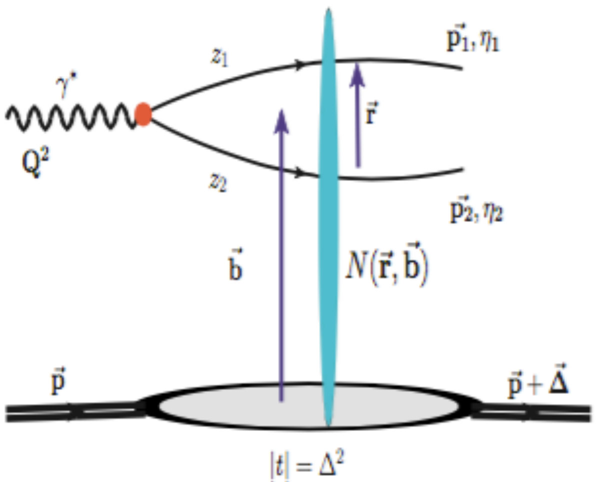
$$\left(\frac{d\sigma_T^{\text{dijet}}}{dz_1 dz_2 d^2\mathbf{p}_1 d^2\mathbf{p}_2}\right) = (2\pi)^2 \delta(z_1 + z_2 - 1) N_c \alpha_{em} \sum_f e_f^2 \int \frac{d^2\mathbf{r}}{(2\pi)^2} \int \frac{d^2\mathbf{r}'}{(2\pi)^2} \int \frac{d^2\mathbf{b}}{(2\pi)^2} \int \frac{d^2\mathbf{b}'}{(2\pi)^2}$$

$$\times\, \mathbf{e}^{-i(\mathbf{b}-\mathbf{b}')\cdot(\mathbf{p}_1+\mathbf{p}_2)} \mathbf{e}^{-i(\mathbf{r}-\mathbf{r}')\cdot(\mathbf{p}_1-\mathbf{p}_2)/2} \mathcal{N}(\mathbf{r}, \mathbf{b})\, \mathcal{N}(\mathbf{r}', \mathbf{b}')\, 2[z_1^2 + z_2^2]\, \frac{\mathbf{r}\cdot\mathbf{r}'}{\mathbf{r}^2 \mathbf{r}'^2}\, [\varepsilon|\mathbf{r}| K_1(\varepsilon|\mathbf{r}|)]\, \left[\varepsilon|\mathbf{r}'| K_1(\varepsilon|\mathbf{r}'|)\right]$$

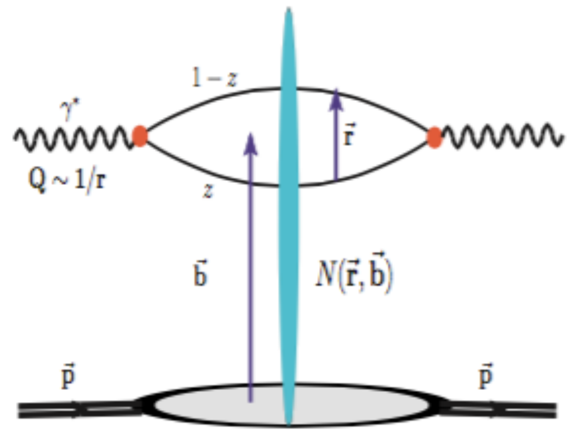
- **Diffractive dijet production is a sensitive probe of the color-dipole orientation.**

Diffractive dijet v. inclusive dijet in UPC & DIS

Diffractive (averaging over color at amplitude level): $\sigma \propto |\langle \mathcal{M} \rangle_\rho|^2$
Inclusive (averaging over color at cross-section level): $\sigma \propto \langle |\mathcal{M}|^2 \rangle_\rho$



Dominguez, Marquet, Xiao, Yuan, arXiv:1101.0715



Altinoluk, Armesto, Beuf, Rezaeian, arXiv:1511.07452

$$\sigma^{\text{Diffractive dijet}} \propto \psi^{\gamma}_{q\bar{q}} \otimes \mathcal{N}(\vec{r}, \vec{b}) \otimes \mathcal{N}(\vec{r'}, \vec{b'}) \neq \psi^{\gamma}_{q\bar{q}} \otimes [\mathcal{N}(r, b)]^2$$

$$\sigma^{\text{Inclusive dijet}} \propto \psi^{\gamma}_{q\bar{q}} \otimes [\mathcal{N}(\vec{r}, \vec{b}) + S^{\text{Quadrupole}}(\vec{r}, \vec{r'}, \vec{b}, \vec{b'})]$$

$$\sigma^{\text{Inclusive DIS}} \propto \psi^{\gamma}_{q\bar{q}} \otimes \mathcal{N}(\vec{r}, \vec{b})$$

- In contrast to inclusive dijet production, diffractive dijet production only depends on the dipole amplitude (not WW gluon distribution) at LO.

Diffractive dijet production as a probe of color-dipole orientation

Inspired by "saturation domain" picture of Kovner & Lublinsky (2011)

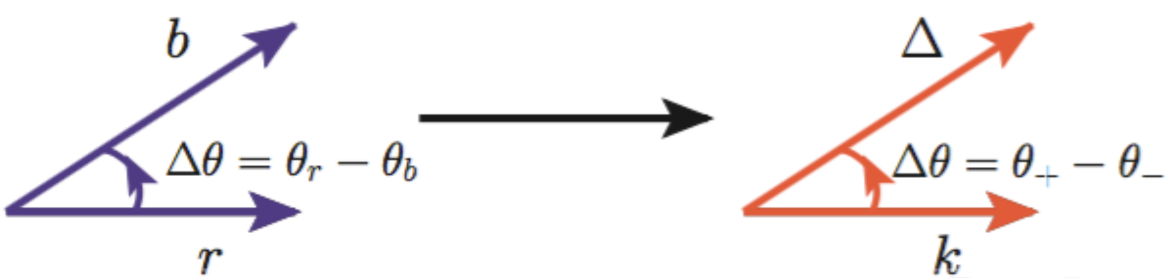
$$\mathcal{N}(\mathbf{r},\mathbf{b}) = \mathcal{N}(r,b,\theta_r-\theta_b) = 1 - e^{-\frac{Q_s^2(b)}{4} r^2\left(1+\mathbf{A}\cos^2(\theta_r-\theta_b)\right)}$$

θ_r, θ_b are the angles of vectors $\vec{r}$, $\vec{b}$ with respect to a reference vector, respectively. Assuming $Q_s^2 r^2 A/4 \ll 1$:

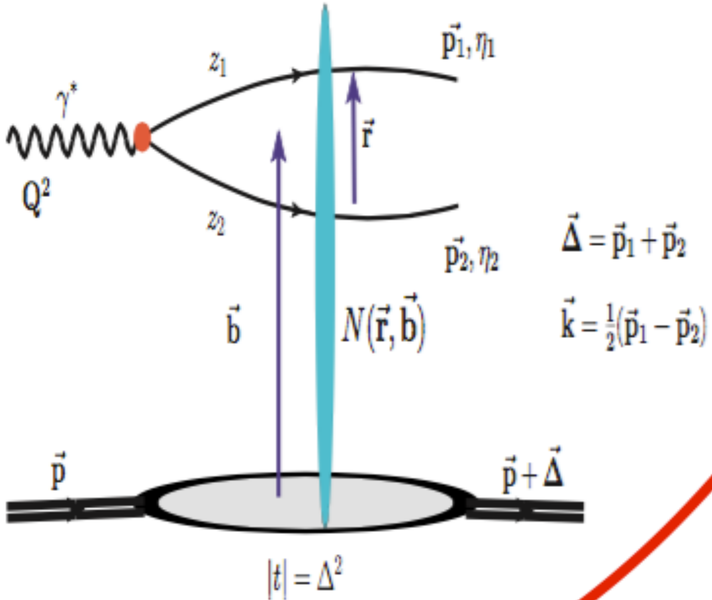
$$\int \frac{d^2\mathbf{r}}{(2\pi)^2} \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{b}\cdot(\mathbf{p}_0+\mathbf{p}_1)} e^{-i\mathbf{r}\cdot(\mathbf{p}_0-\mathbf{p}_1)/2} \mathcal{N}(\mathbf{r},\mathbf{b}) K_0(\varepsilon|\mathbf{r}|) \simeq \int_0^{+\infty} \frac{dr}{2\pi} r \int_0^{+\infty} \frac{db}{2\pi} b\, J_0\left(b|\mathbf{p}_0+\mathbf{p}_1|\right) \times J_0\left(r\frac{|\mathbf{p}_0-\mathbf{p}_1|}{2}\right) \mathcal{N}(r,b,\theta_- - \theta_+) K_0(\varepsilon r)$$

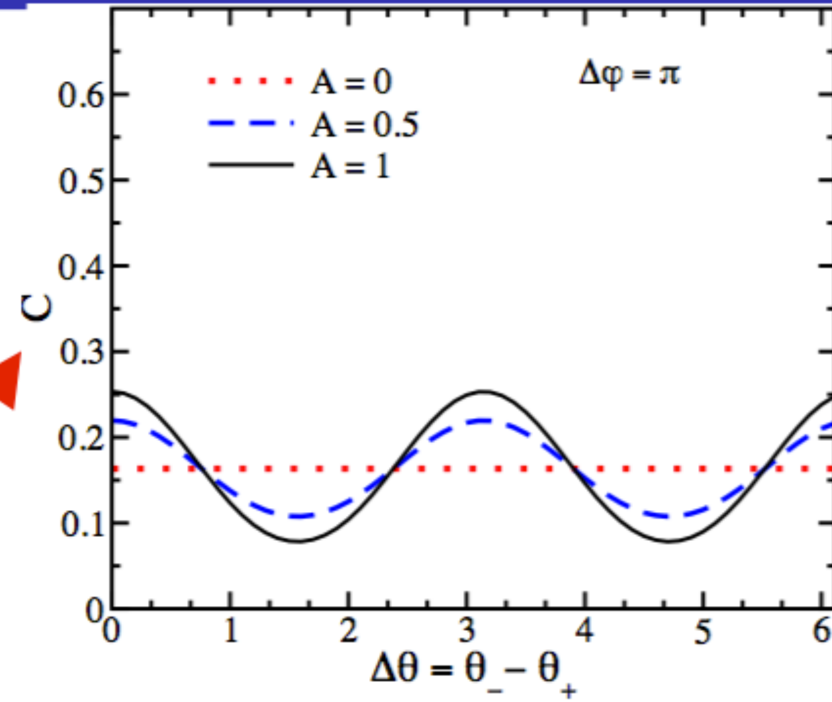
θ_+, θ_- denote the angles of vectors $\vec{\Delta} = \vec{p}_0 + \vec{p}_1$ and $\vec{k} = \frac{1}{2}(\vec{p}_0 - \vec{p}_1)$ with respect to a reference vector, respectively.

- **A nonzero A corresponding to the existence of $\vec{r} - \vec{b}$ correlations in the color dipole amplitude, induces azimuthal correlations between $\vec{\Delta}$ and $\vec{k}$.**



Diffractive dijet production as a probe of color-dipole orientation

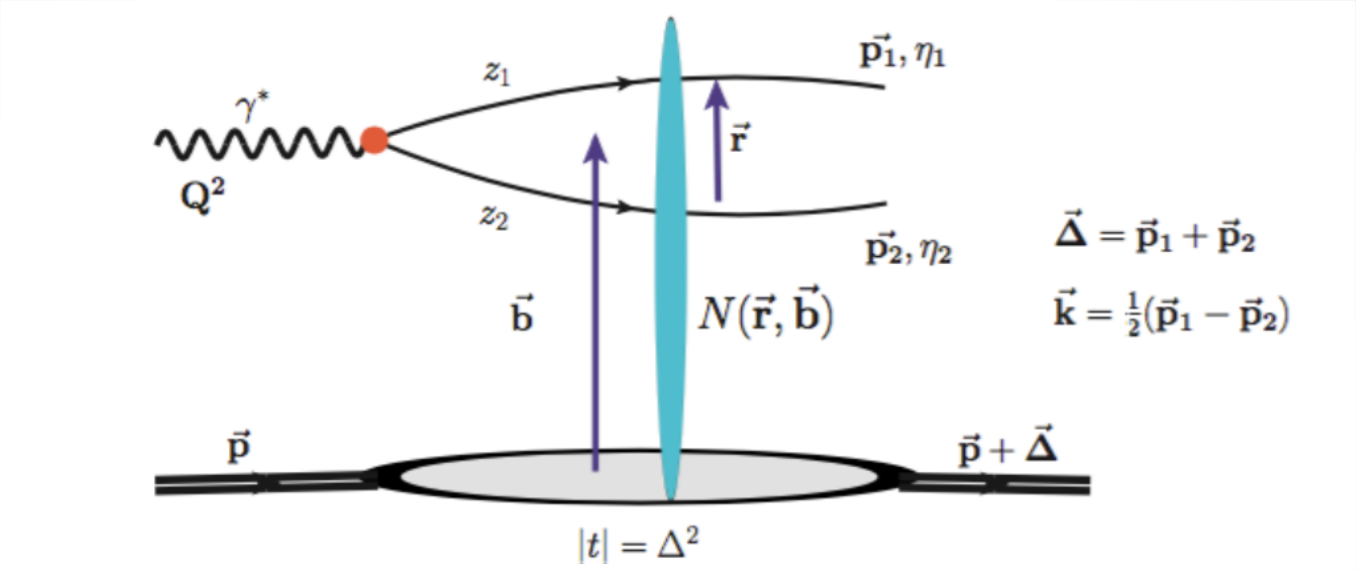




$$C(\Delta\theta) = \frac{d\sigma^{\gamma^* p \to q\bar{q}p}}{d\mathbf{p}_0\, \mathbf{p}_1 d\Delta\theta} \Bigg/ \int_0^{2\pi} d\Delta\theta \frac{d\sigma^{\gamma^* p \to q\bar{q}p}}{d\mathbf{p}_0\, \mathbf{p}_1 d\Delta\theta}$$

- **A nonzero A corresponding to the existence of $\vec{r} - \vec{b}$ correlations in the color dipole amplitude, induces sizeable azimuthal correlations for dijet between $\vec{\Delta}$ and $\vec{k}$.**

Main conclusion: Small-x gluon tomography in diffractive dijet in UPC



- ✦ **Correlations between** $\vec{p}_1$, $\vec{p}_2$ **probe the effective dipole size** $r \approx 1/Q_s(b)$.
- ✦ **Correlation between** $\vec{k}, \vec{\Delta}$ **probe the color-dipole orientation and correlations between** $\vec{r}, \vec{b}$.
- ✦ **t-distribution of the diffractive dijet photo-production probes the inhomogeneity of the target.**

Altinoluk, Armesto, Beuf, and Rezaeian, PLB 758 (2016) 373.
Hatta, Xiao and Yuan, PRL 116 (2016) 202301.