Quark Helicity Evolution at Small x

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with Yuri Kovchegov

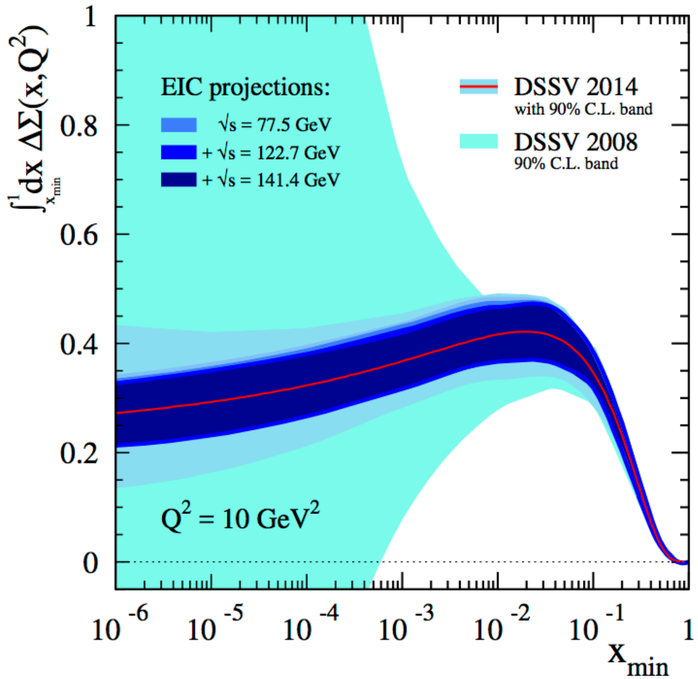
and Daniel Pitonyak

RBRC Workshop: Wed. April 26, 2017

Saturation: Recent Developments, New Ideas, and Measurements

Small-x Enhancement of Quark Polarization

Without Small-x Evolution

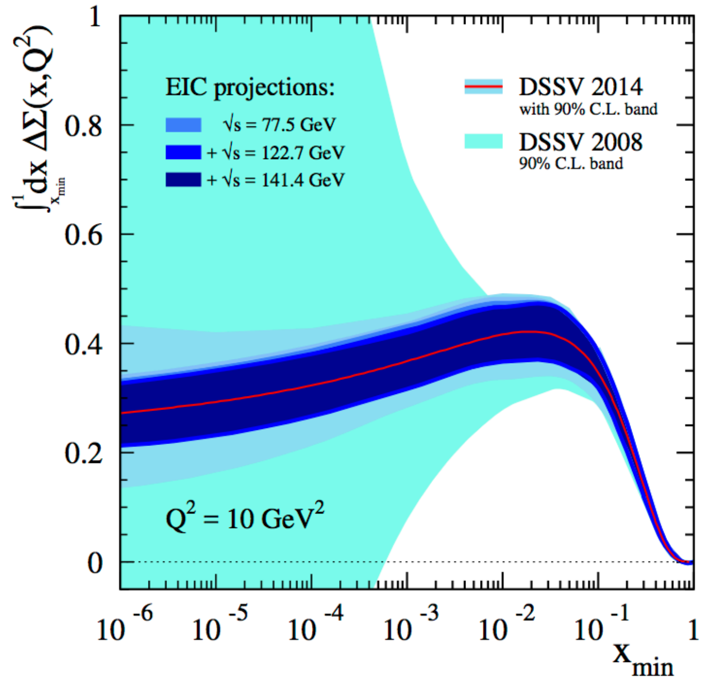


adapted from Aschenauer et al., Phys. Rev. ***D92*** *(2015) no.9 094030*

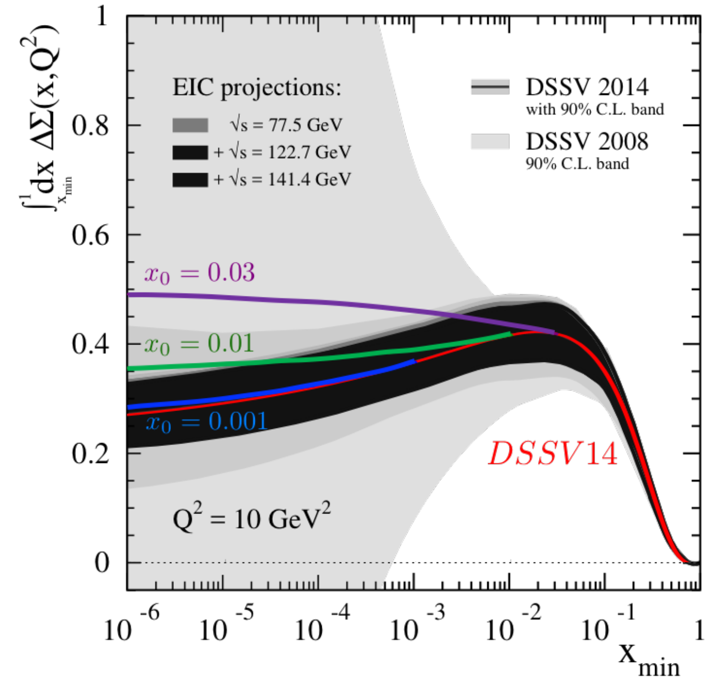
- Quark polarization not well constrained below $x \leq 10^{-2}$

Small-x Enhancement of Quark Polarization

Without Small-x Evolution



With Small-x Evolution



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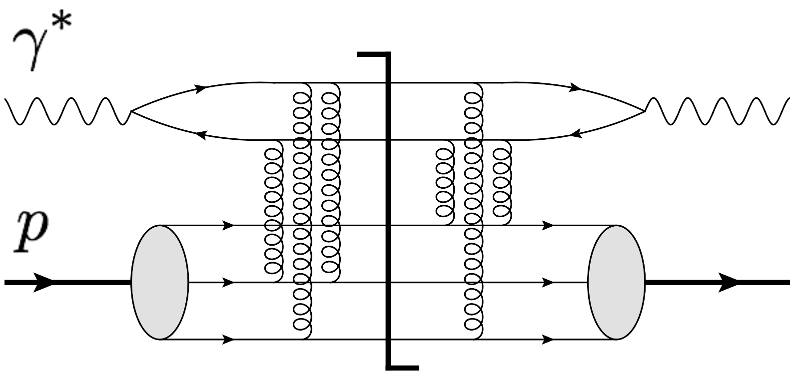
- Quark polarization not well constrained below $x \leq 10^{-2}$
- New small-x evolution can lead to significant enhancement

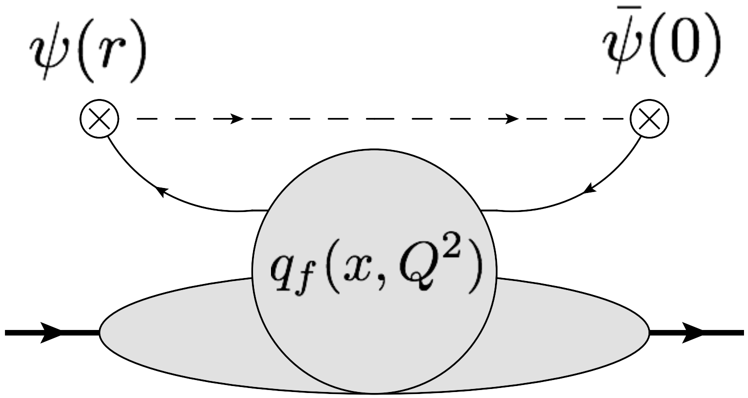
An Appetizer:

Small-x Evolution of the Unpolarized Quark Distribution

Unpolarized Cross Sections and PDFs

- **Factorization**: One-to-one correspondence between the DIS cross section and the parton distribution functions





$$\boxed{\frac{Q^2}{4\pi^2 \alpha_{EM}} \frac{d\sigma^{(\gamma^* p)}}{dx\, dQ^2} = F_2(x, Q^2) \overset{L.O.}{=} \sum_f e_f^2 \, x q_f(x, Q^2)}$$

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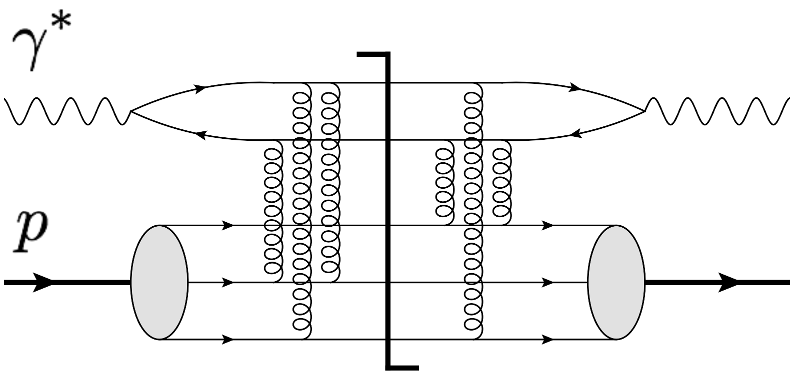

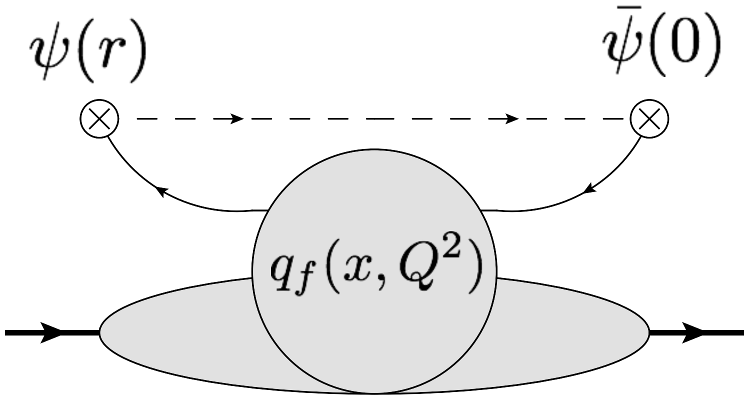

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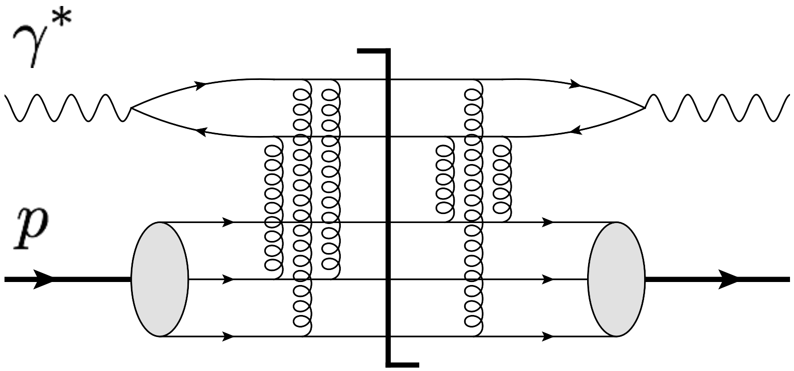
The Unpolarized Dipole Amplitude



Quark PDF

$$q(x,Q^2) = \int \frac{dr^-}{2\pi}\, e^{ixp^+r^-} \langle p|\, \bar\psi(0)\, \mathcal{U}[0,r^-]\, \frac{\gamma^+}{2}\, \psi(r^-)\, |p\rangle$$

- The DIS cross section / PDF is expressed in terms of a dipole scattering amplitude amplitude / cross section

$$x q_f(x,Q^2) \overset{L.O.}{=} \frac{Q^2\, N_c}{2\pi^2 \alpha_{EM}} \int \frac{d^2x_{10}\, dz}{4\pi\, z(1-z)} \left[\left|\Psi_{T,f}(x_{10}^2, z)\right|^2 + \left|\Psi_{L,f}(x_{10}^2, z)\right|^2 \right] \int d^2 b_{10}\, (1 - S_{10}(zs))$$

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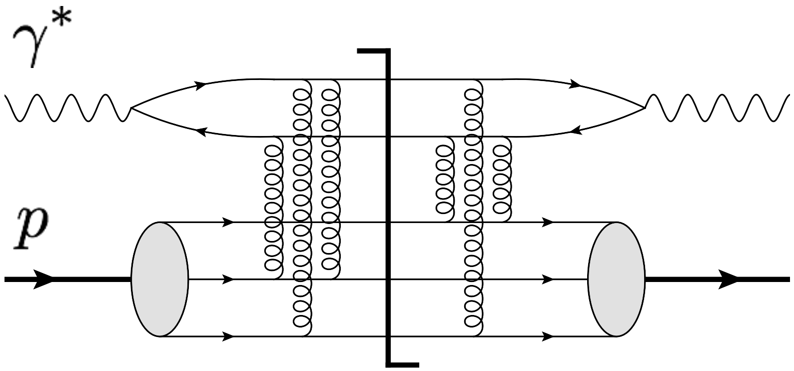

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$$\boxed{S_{10}(zs) \equiv \left\langle \frac{1}{N_c} \mathrm{tr}[V_{\underline{x}_0} V^\dagger_{\underline{x}_1}]_{(zs)} \right\rangle = 1 - \frac{1}{2} \frac{d\sigma^{(q^{unp}_{\underline{x}_0}\; \bar{q}^{unp}_{\underline{x}_1})}}{d^2 b_{10}}(zs)}$$

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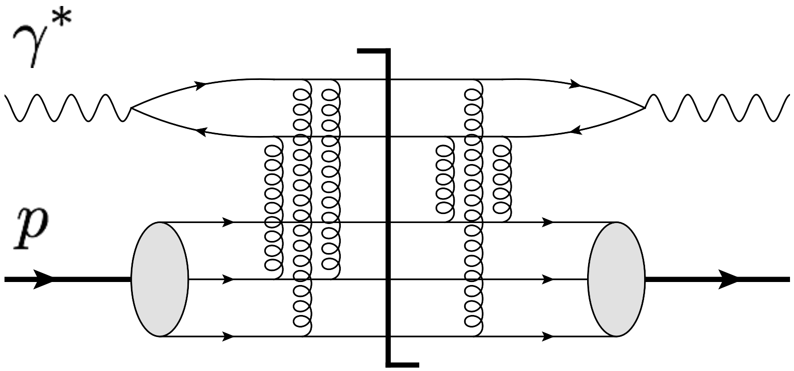

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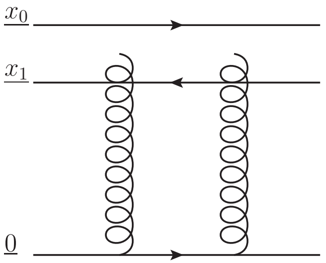
Wilson lines: $V_{\underline{x}} = \mathcal{P} \exp\left[ig \int dz^- \, \hat{A}^+(0^+, z^-, \underline{x}) \right]$

Origins of Unpolarized Evolution

- Initial conditions from the quark target model:

$$S_{10}^{(0)}(zs) = \frac{2\alpha_s^2 C_F}{N_c} \ln^2 \frac{x_{0T}}{x_{1T}}$$



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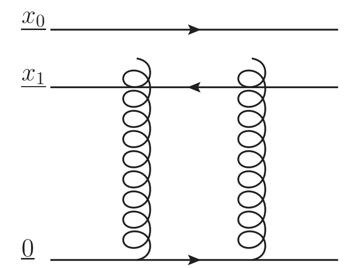
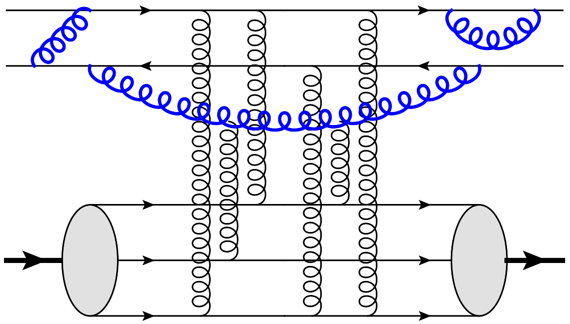


- Soft gluon emission spans the full rapidity interval $Y \sim \ln \frac{s}{\Lambda^2} \sim \ln \frac{1}{x}$

$$\frac{1}{N_c} \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_1}^\dagger]_{(zs)} \sim \alpha_s \underbrace{\int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'}}_{\ln \frac{zs}{\Lambda^2}} \int d^2 x_2 \, \mathcal{K}(\underline{x}_0, \underline{x}_1, \underline{x}_2) \, \mathcal{O}$$



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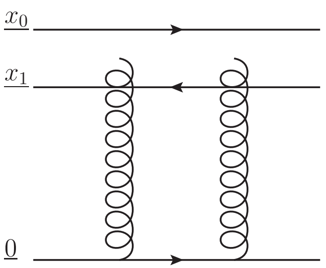

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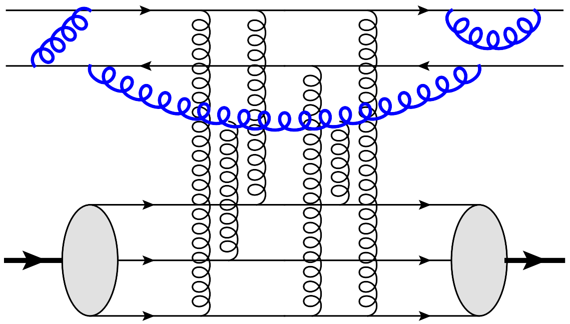


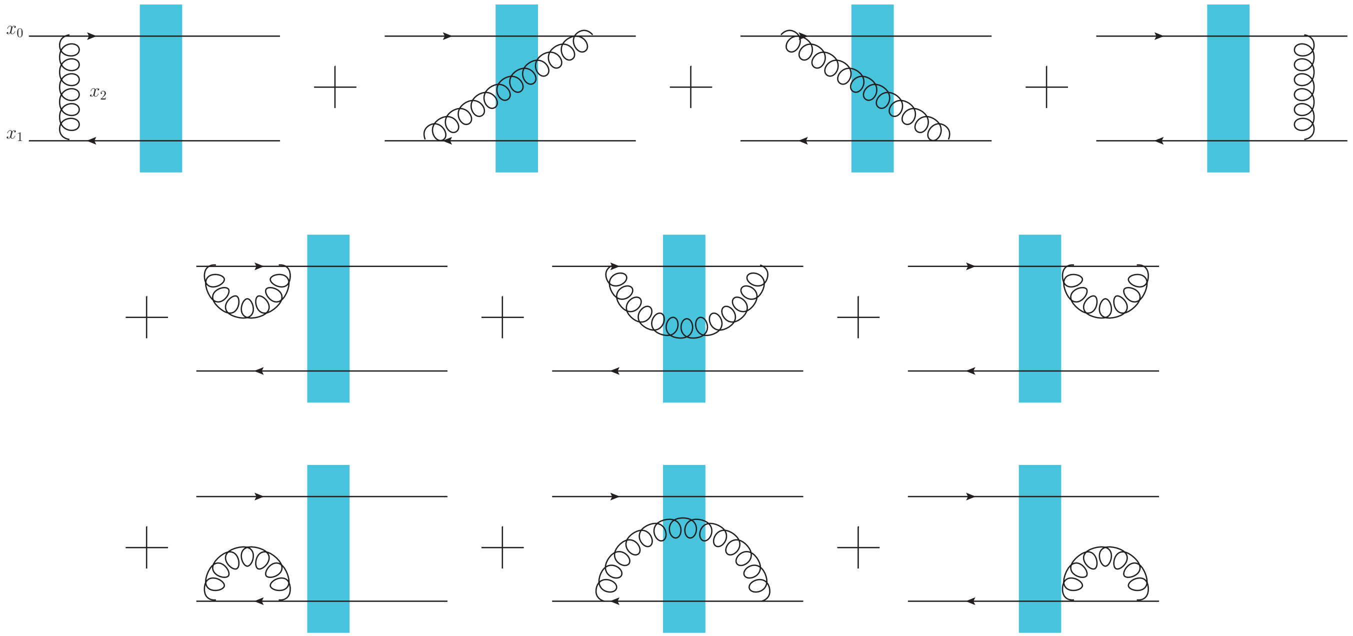
- Successive emissions continue to generate a logarithm of energy if they are ordered longitudinally

$$\boxed{z \gg z' \gg z'' \gg \cdots}$$

- Unpolarized evolution is leading-logarithmic

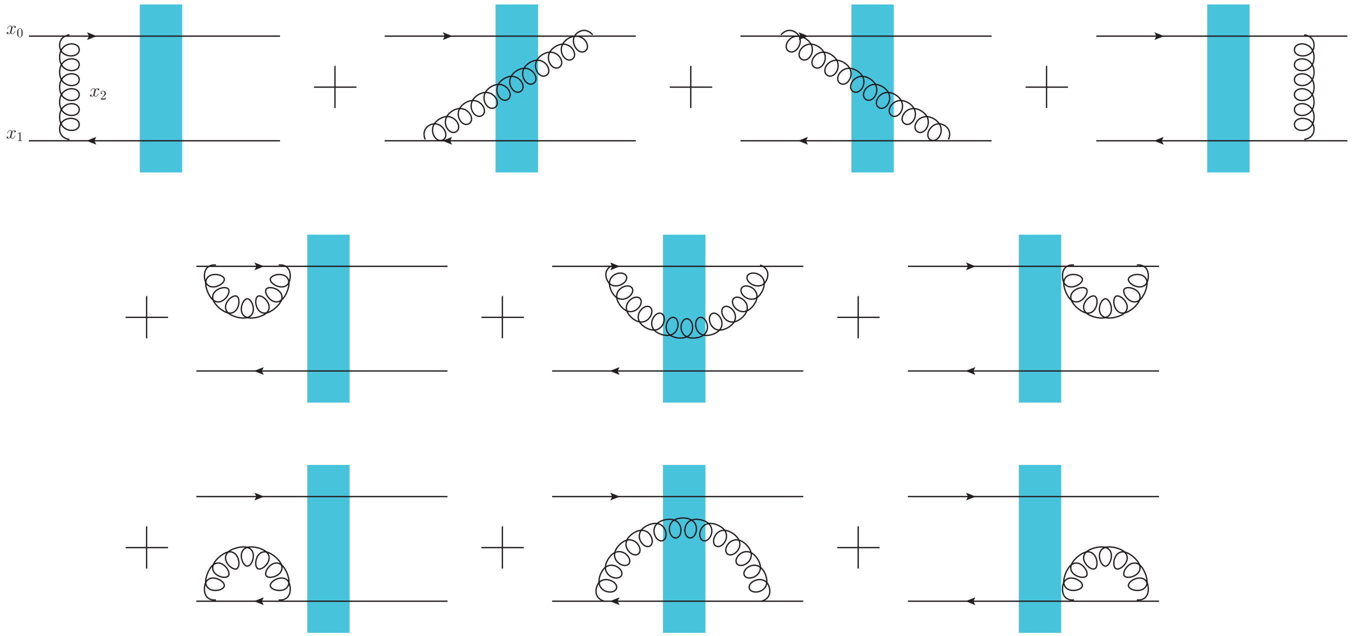
$$\boxed{\alpha_s \ln \frac{1}{x} \sim 1}$$

Unpolarized Small-x Evolution



$$S_{10}(zs) = S_{10}^{(0)}(zs) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2 x_2 \, \frac{x_{10}^2}{x_{12}^2 \, x_{20}^2} \left[\frac{1}{N_c^2} \left\langle \mathrm{tr}[V_{\underline{x}_2} V_{\underline{x}_1}^\dagger] \, \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^\dagger] \right\rangle_{(z's)} - S_{10}(z's) \right]$$

Unpolarized Small-x Evolution



Leading Log

$$S_{10}(zs) = S_{10}^{(0)}(zs) + \boxed{\frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'}} \int d^2 x_2 \, \frac{x_{10}^2}{x_{12}^2 \, x_{20}^2} \left[\frac{1}{N_c^2} \left\langle \mathrm{tr}[V_{\underline{x}_2} V_{\underline{x}_1}^\dagger] \, \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^\dagger] \right\rangle_{(z's)} - S_{10}(z's) \right]$$

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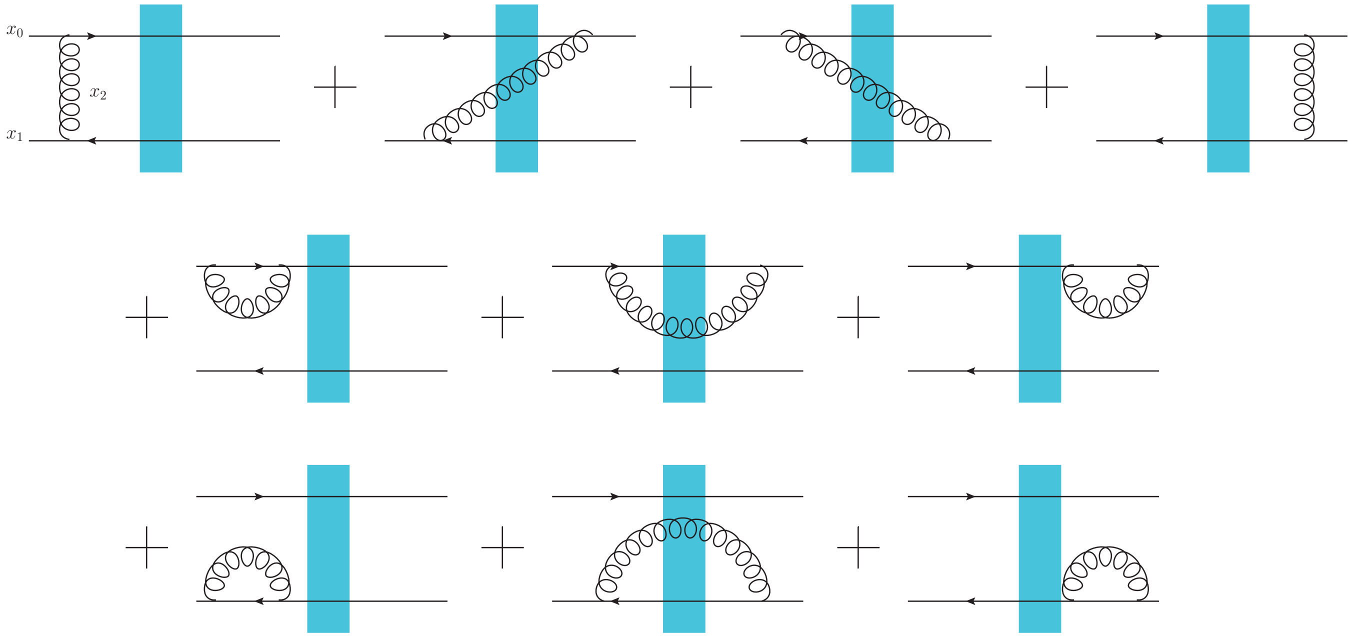

Leading Log **BFKL Kernel**

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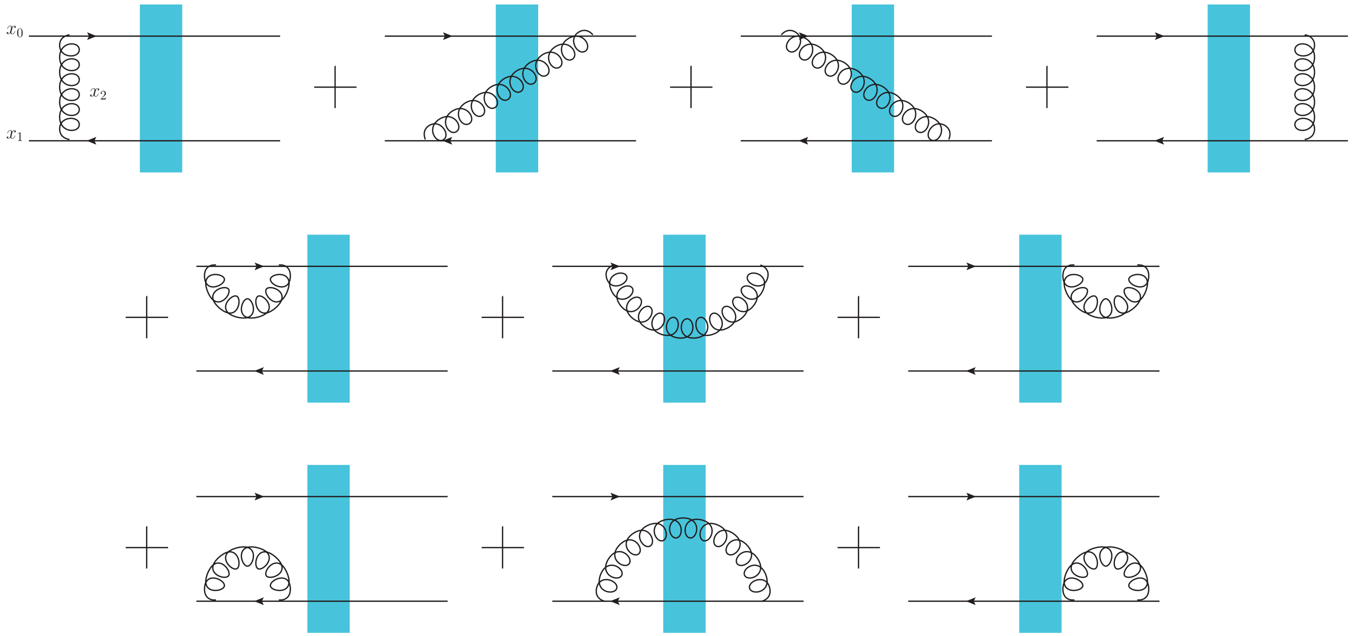

Leading Log **BFKL Kernel** **Operator Hierarchy**

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Solution: The Pomeron Intercept

- Operator hierarchy closes in the large-N_c limit (BK)

$$S_{10}(zs) = S_{10}^{(0)}(zs) + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int d^2x_2 \, \frac{x_{10}^2}{x_{12}^2 \, x_{20}^2} \left[\boxed{S_{12}(z's) \, S_{20}(z's)} - S_{10}(z's) \right]$$

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- Dilute / linearized regime (BFKL): $1 - S_{10} \ll 1$

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- Analytic solution by Laplace/Mellin transform (intercept):

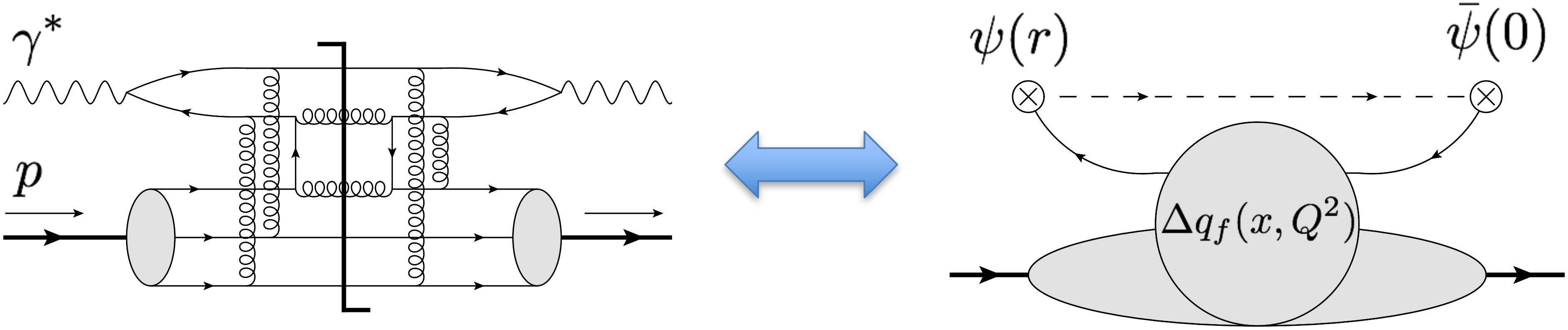
$$x \, q_f(x, Q^2) \sim S_{10}\left(s = \frac{Q^2}{x}\right) \sim \left(\frac{1}{x}\right)^{\alpha_P - 1} \qquad \boxed{\alpha_P - 1 = \frac{4\alpha_s N_c}{\pi} \ln 2}$$

The Main Course:

Small-x Evolution of the
Quark Helicity Distribution

Polarized Cross Sections and PDFs

- **Factorization:** One-to-one correspondence between the spin-dependent DIS cross-section and the quark helicity PDF



$$\boxed{\frac{Q^2}{4\pi^2 \alpha_{EM}} \frac{d\,\Delta\sigma^{(\gamma^* p)}}{dx\, dQ^2} = 2x\, g_1(x, Q^2) \stackrel{L.O.}{=} \sum_f e_f^2\, x \Delta q_f(x, Q^2)}$$

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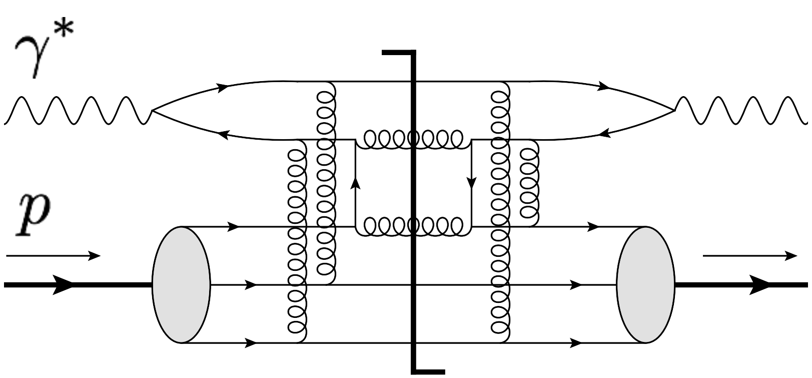

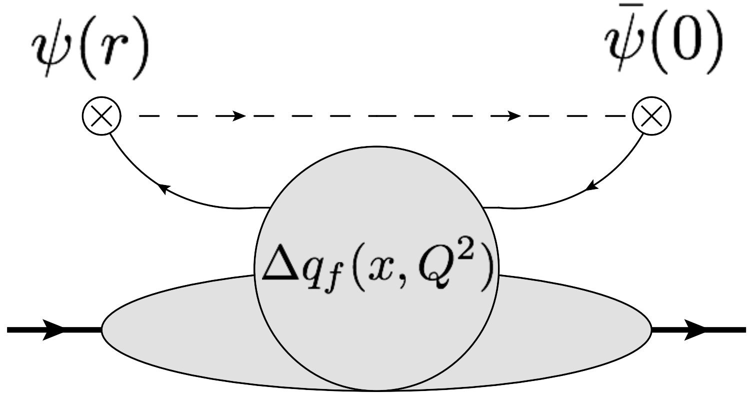

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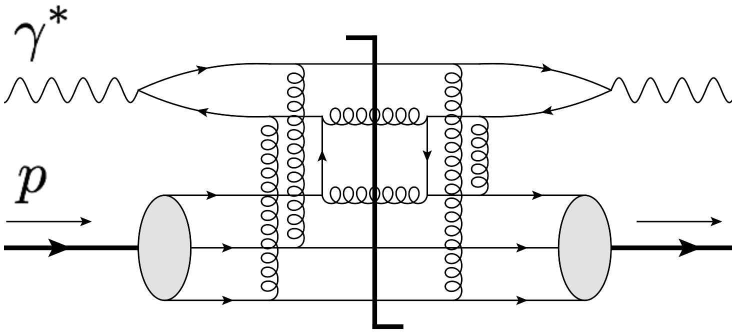
The Polarized Dipole Amplitude



Flavor-Singlet Quark Helicity PDF

$$\Delta q^S(x,Q^2) = \sum_f \int \frac{dr^-}{2\pi} \, e^{ixp^+r^-} \, \langle p | \, \bar\psi(0) \, \mathcal{U}[0,r^-] \, \frac{\gamma^+\gamma^5}{2} \, \psi(r^-) \, | p \rangle$$

- The spin-dependent DIS cross-section / hPDF is expressed in terms of a polarized dipole amplitude / cross section

$$x\Delta q^S(x,Q^2) \overset{L.O.}{=} \frac{Q^2 \, N_c}{2\pi^2 \alpha_{EM}} \sum_f \int \frac{d^2x_{10} \, dz}{4\pi \; z(1-z)} \left[\left| \Delta\Psi_{T,f}(x_{10}^2, z) \right|^2 + \left| \Delta\Psi_{L,f}(x_{10}^2, z) \right|^2 \right] \int d^2b_{10} \, \frac{1}{zs} G_{10}(zs)$$

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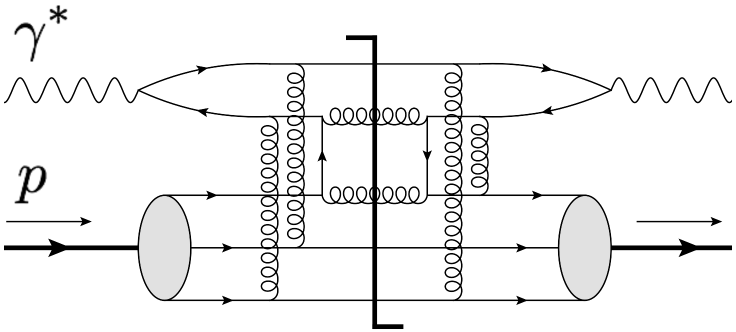

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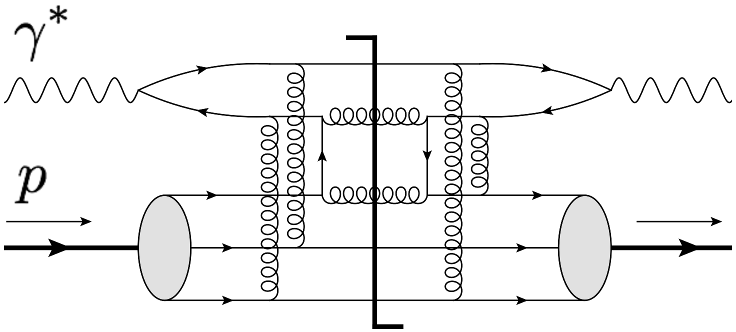

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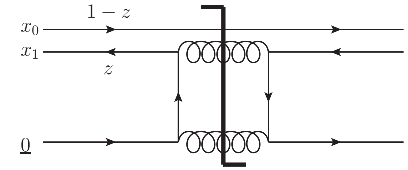
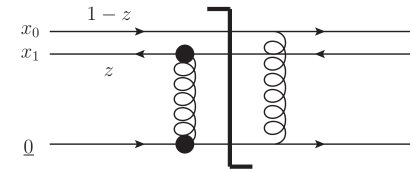
"Polarized Wilson line"

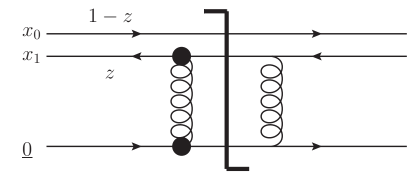
Helicity Evolution: Initial Conditions

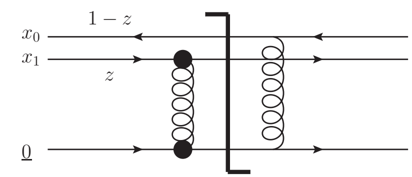
- Initial conditions from the quark target model
  - Quark and sub-eikonal gluon exchange

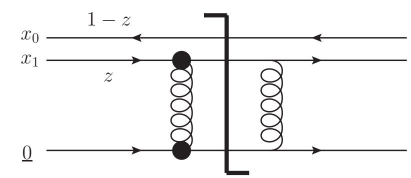
$$G_{10}^{(0)}(zs) = \frac{\alpha_s^2 C_F}{N_c} \left[\frac{C_F}{x_{1T}^2} - 2\pi \delta^2(\underline{x}_1) \ln(zs\, x_{10}^2) \right]$$











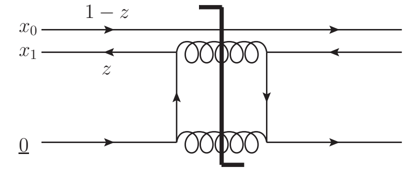
Helicity Evolution: Initial Conditions

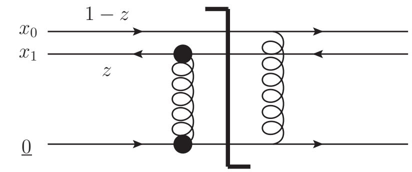
- Initial conditions from the quark target model
  - Quark and sub-eikonal gluon exchange

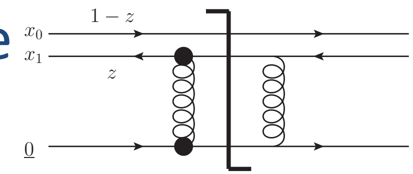
$$G_{10}^{(0)}(zs) = \frac{\alpha_s^2 C_F}{N_c}\left[\frac{C_F}{x_{1T}^2} - 2\pi\delta^2(\underline{x}_1)\,\ln(zs\,x_{10}^2)\right]$$

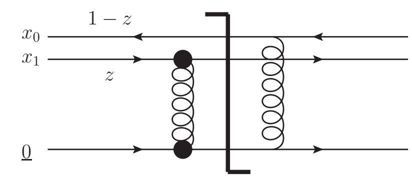
- Polarization transfer is suppressed at small x
  - Leading term has exactly 1 sub-eikonal exchange

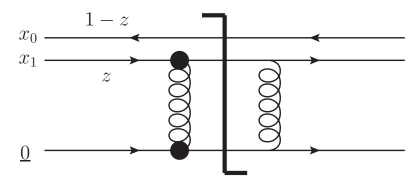
$$\frac{d\,\Delta\sigma^{(q^{unp}_{\underline{x}_0}\;\bar{q}^{pol}_{\underline{x}_1})}}{d^2 b_{10}}(zs) \propto \frac{1}{zs}$$

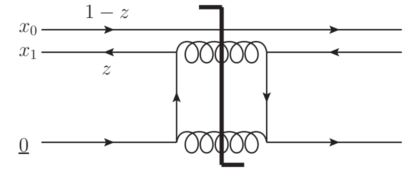












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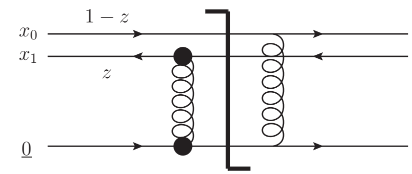

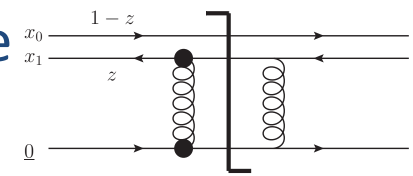

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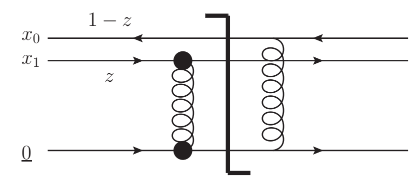
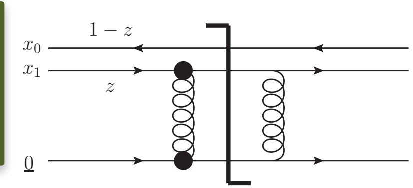


- Include this known scaling in the definition of the polarized dipole amplitude

$$\boxed{\frac{1}{zs}G_{10}(zs) \equiv \left\langle \frac{1}{2N_c}\mathrm{tr}[V_{\underline{x}_0} V^{pol\,\dagger}_{\underline{x}_1}] + c.c.\right\rangle_{(zs)} = -\frac{1}{4}\left(\frac{d\,\Delta\sigma^{(q^{unp}_{\underline{x}_0}\,\bar{q}^{pol}_{\underline{x}_1})}}{d^2 b_{10}}(zs) + ch.c.\right)}$$



Origins of Helicity Evolution

- Soft polarized quark and gluon emission spans the full rapidity interval:

$$\int \frac{dz'}{z'} \to \ln \frac{zs}{\Lambda^2}$$

- The polarized line is also sensitive to collinear / short-distance fluctuations:

$$\int \frac{dx_{21}^2}{x_{21}^2} \to \ln \frac{zs}{\Lambda^2}$$

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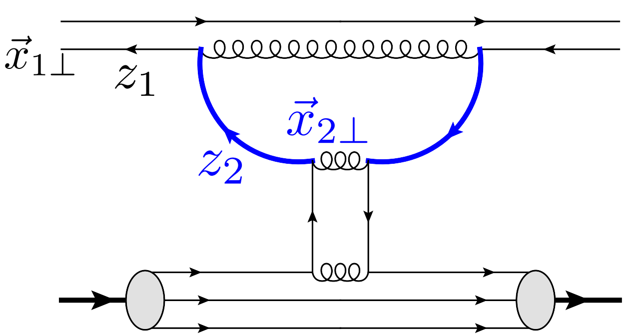

$$\underbrace{\left\langle V_1^{pol\,\dagger}(z_1) \right\rangle}_{\sim \frac{1}{z_1 s}} \sim \int \frac{dz_2}{z_2} \int d^2 x_2 \left(\frac{\alpha_s C_F}{2\pi^2} \frac{z_2}{z_1} \frac{1}{x_{21}^2} \right) \underbrace{\left\langle V_2^{pol\,\dagger}(z_2) \right\rangle}_{\sim \frac{1}{z_2 s}}$$

$$G_{10}(z_1) \sim \frac{\alpha_s C_F}{2\pi} \underbrace{\int \frac{dz_2}{z_2} \int \frac{dx_{21}^2}{x_{21}^2}}_{\ln^2 \frac{z_1 s}{\Lambda^2}} G_{21}(z_2)$$

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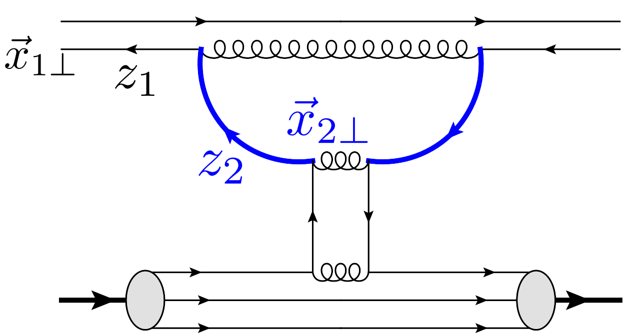

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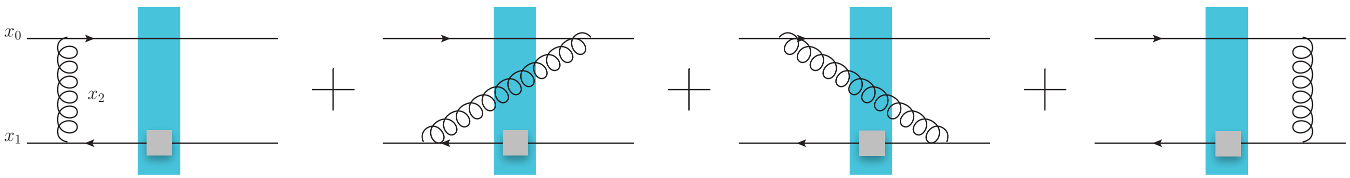
- Helicity evolution is double logarithmic
  - ➢ Stronger than unpolarized evolution!

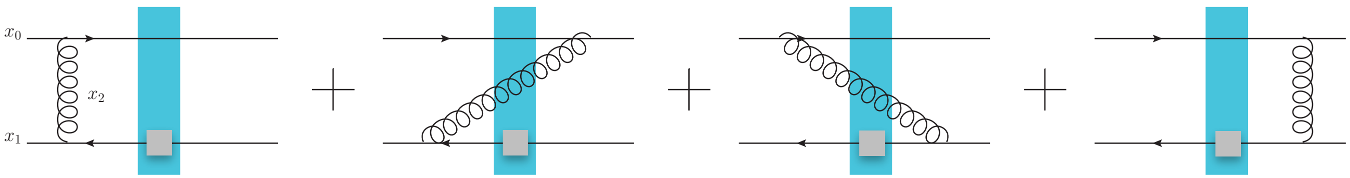
$$\alpha_s \ln^2 \frac{1}{x} \sim 1$$



$$\sim \int d^2x_2 \, \frac{(\underline{x}_{21} \cdot \underline{x}_{20})}{x_{21}^2 \, x_{20}^2}$$

- No collinear logarithms

An Example: The Collinear BFKL Sector



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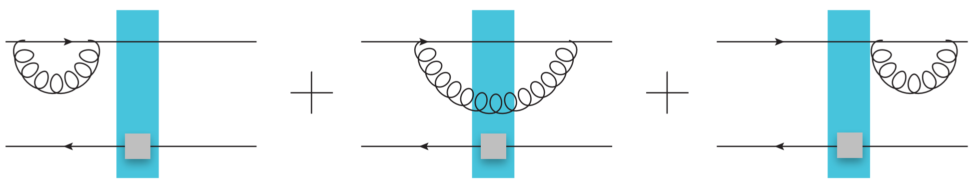

$$\sim \int d^2x_2 \frac{1}{x_{20}^2} \left[\frac{1}{N_c^2} \left\langle \mathrm{tr}[V_{\underline{x}_2} V_{\underline{x}_1}^{pol\,\dagger}]\, \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^{\dagger}] \right\rangle - \frac{1}{N_c} \left\langle \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_1}^{pol\,\dagger}] \right\rangle \right]$$

- Collinear enhancement as $\underline{x}_2 \rightarrow \underline{x}_0$, but vanishing support due to real-virtual cancellations

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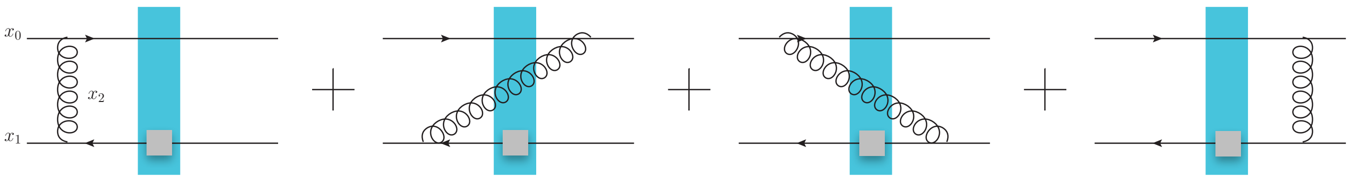

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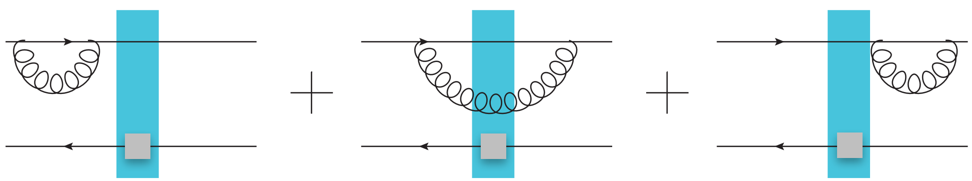

$$\sim \int d^2x_2 \, \frac{1}{x_{20}^2} \left[\frac{1}{N_c^2} \left\langle \mathrm{tr}[V_{\underline{x}_2} V_{\underline{x}_1}^{pol \, \dagger}] \, \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^{\dagger}] \right\rangle - \frac{1}{N_c} \left\langle \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_1}^{pol \, \dagger}] \right\rangle \right]$$

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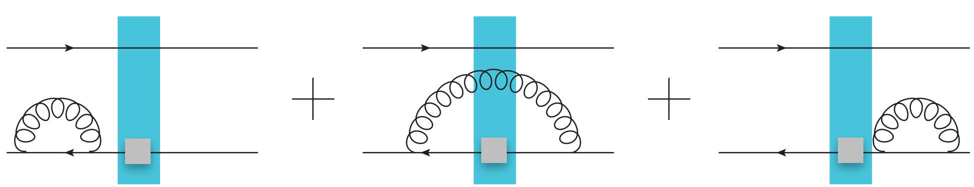

$$\sim \int d^2x_2 \, \frac{1}{x_{21}^2} \left[\frac{1}{N_c^2} \left\langle \mathrm{tr}[V_{\underline{x}_2} V_{\underline{x}_1}^{pol \, \dagger}] \, \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^{\dagger}] \right\rangle - \frac{1}{N_c} \left\langle \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_1}^{pol \, \dagger}] \right\rangle \right]$$

- Collinear enhancement as $\underline{x}_2 \to \underline{x}_1$, about the distinct polarized line

DLA Ordering for Helicity Evolution

- Because one line of the dipole is polarized, a subset of the BFKL kernel becomes double logarithmic (DLA) for $x_{21}^2 \ll x_{10}^2$

$$\delta G_{10}(zs) = \frac{\alpha_s N_c}{2\pi^2} \underbrace{\int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{d^2 x_{21}}{x_{21}^2}}_{\frac{1}{2}\ln^2(zs\, x_{10}^2)} \left[\frac{1}{N_c^2} \left\langle\!\!\left\langle \mathrm{tr}[V_{\underline{x}_2} V_{\underline{x}_1}^{pol\,\dagger}]\, \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^{\dagger}] \right\rangle\!\!\right\rangle_{(z's)} - \frac{1}{N_c} \left\langle\!\!\left\langle \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_1}^{pol\,\dagger}] \right\rangle\!\!\right\rangle_{(z's)} \right]$$

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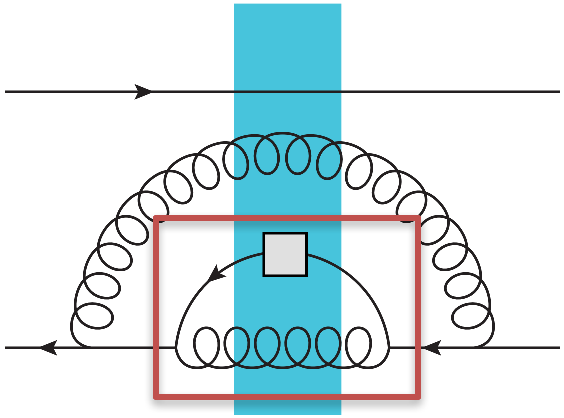


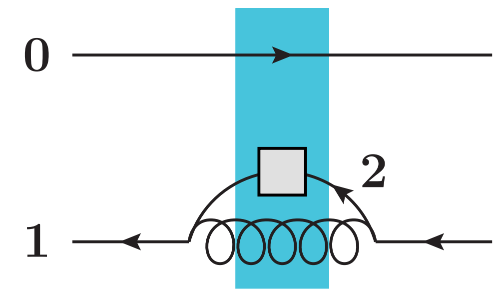
- To continue generating both soft and collinear logs, further evolution must be doubly ordered:

$$z \gg z' \gg z'' \gg \cdots$$

$$z\,\Delta x_T^2 \gg z'\,\Delta x_T'^2 \gg z''\,\Delta x_T''^2 \gg \cdots$$

New Contributions: Polarized Quarks

- Emission of a soft polarized (anti)quark is only possible from the polarized line
  - DLA contribution extends over the whole ordered phase space
  - No $x_{21}^2 \ll x_{10}^2$ restriction



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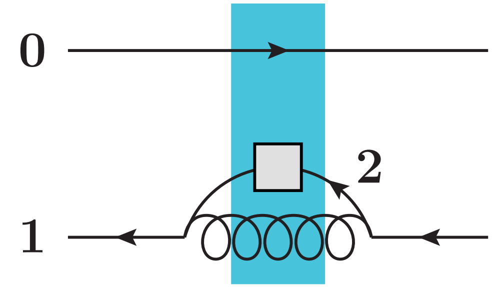
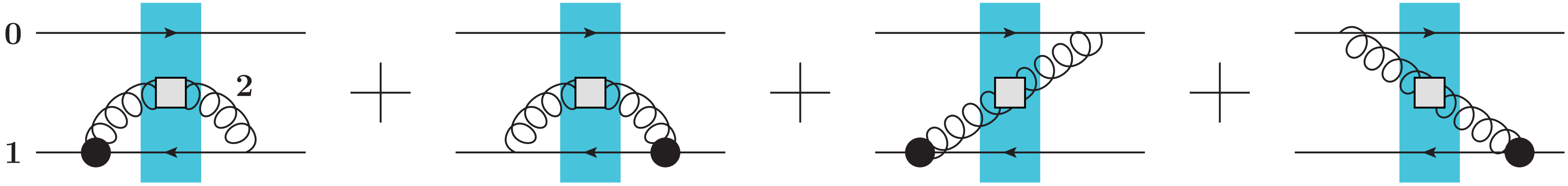


Full phase space

$$\delta G_{10}(zs) = \frac{\alpha_s N_c}{4\pi^2} \underbrace{\int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2 \frac{z}{z'}} \frac{d^2 x_{21}}{x_{21}^2}}_{\ln(zs\, x_{10}^2)\, \ln(zs/\Lambda^2)} \left[\frac{1}{N_c^2} \left\langle\!\!\left\langle \mathrm{tr}[V_{\underline{x}_1} V_{\underline{x}_2}^{pol\,\dagger}]\, \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_1}^{\dagger}] \right\rangle\!\!\right\rangle_{(z's)} - \frac{1}{N_c^3} \left\langle\!\!\left\langle \mathrm{tr}[V_{\underline{x}_0} V_{\underline{x}_2}^{pol\,\dagger}] \right\rangle\!\!\right\rangle_{(z's)} \right]$$

- Emission of a soft polarized gluon can couple to both lines



➢ **Ladder (from polarized line only): DLA everywhere**

$$\delta G_{10}(zs) = + \frac{\alpha_s N_c}{2\pi^2} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z's}}^{x_{10}^2 \frac{z}{z'}} \frac{d^2 x_{21}}{x_{21}^2} \left[\frac{1}{N_c^2} \left\langle\!\!\left\langle \mathrm{tr}[t^b V_{\underline{x}_0} t^a V_{\underline{x}_1}^\dagger] \left(U_{\underline{x}_2}^{pol} \right)^{ba} \right\rangle\!\!\right\rangle_{(z's)} \right]$$

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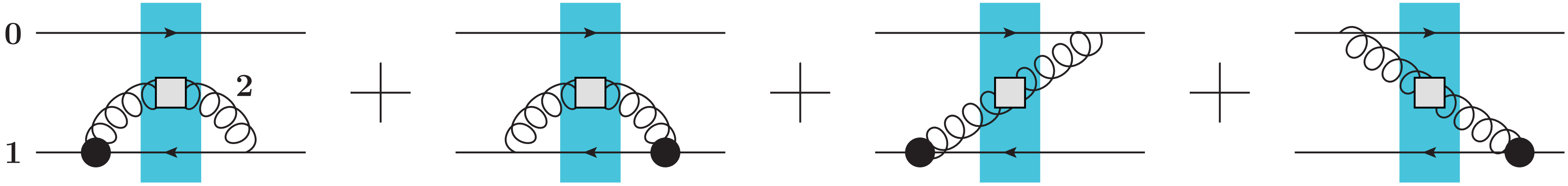

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➢ **Non-Ladder (across the dipole):** Cancel ladders for $x_{21}^2 \gg x_{10}^2$

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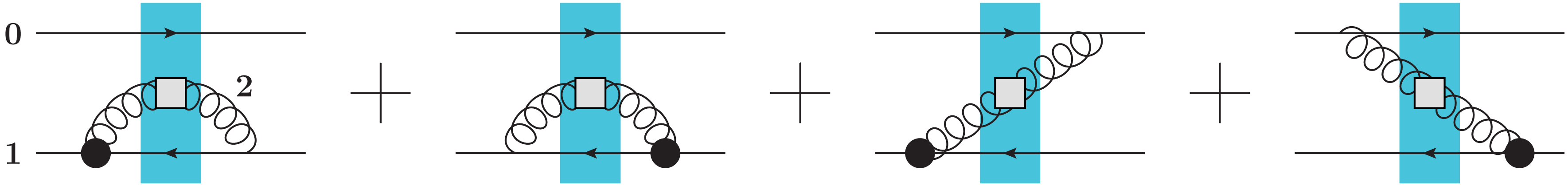

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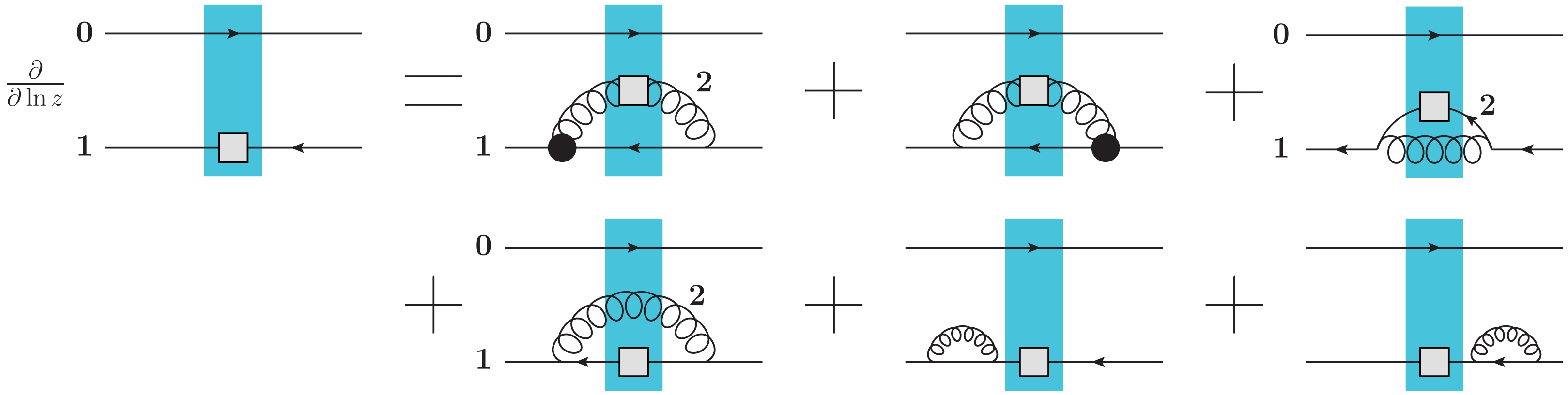
➢ **Limits the DLA phase space** for polarized ladder gluons: $x_{21}^2 \ll x_{10}^2$

- DLA evolution is only sensitive to fluctuations near the polarized line
  - For all gluon emissions, ladder / non-ladder cancellation limits the DLA to the strongly ordered phase space $x_{21}^2 \ll x_{10}^2$



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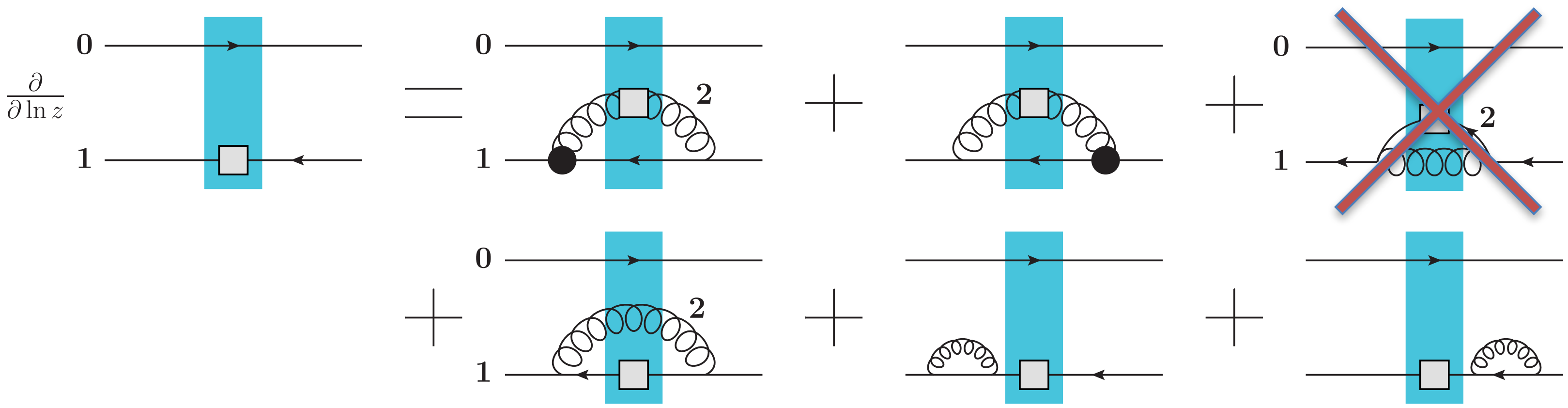

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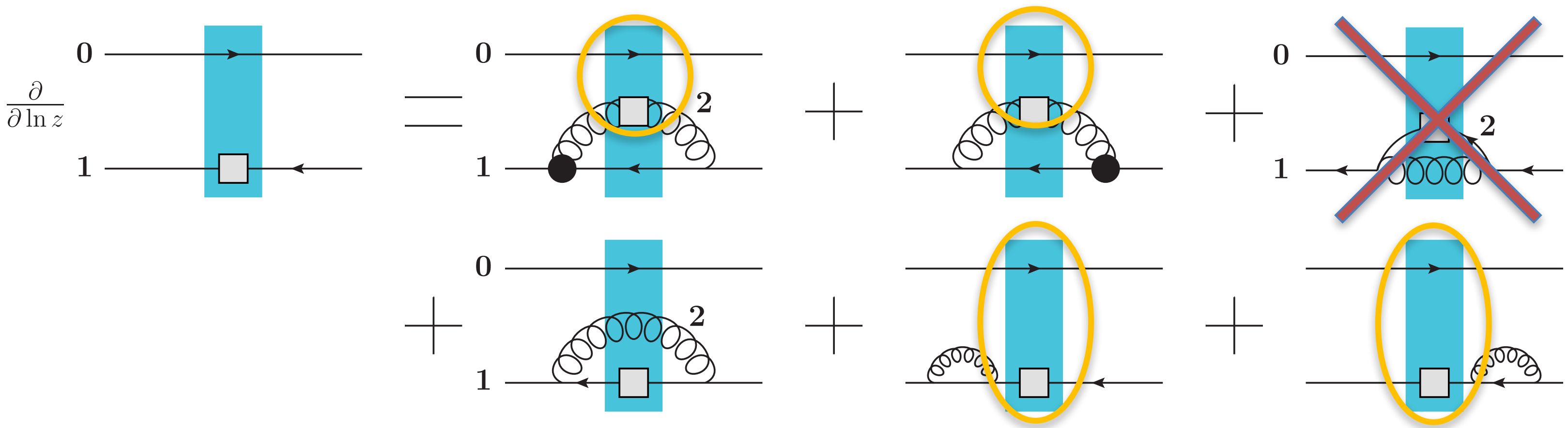
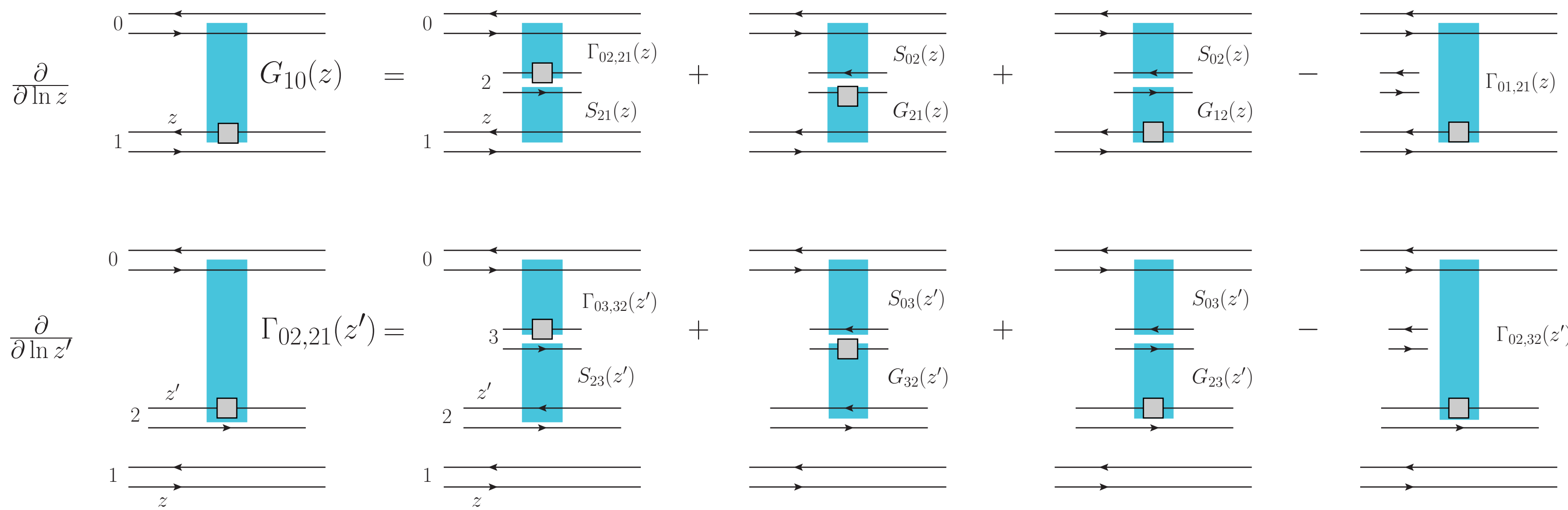


- The polarized analog of BFKL is the linearized, large-N_c limit
  - There is a nontrivial constraint on the lifetime of large polarized dipoles from their short-lived “neighbor” fluctuations!

Large-N_c Evolution Equations



- The impact-parameter integrated dipole amplitude evolves as:

$$\underline{G(x_{10}^2, z)} = G^{(0)}(x_{10}^2, z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\underline{\Gamma(x_{10}^2, x_{21}^2, z')} + 3\underline{G(x_{21}^2, z')} \right]$$

$$\underline{\Gamma(x_{10}^2, x_{21}^2, z')} = G^{(0)}(x_{10}^2, z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min\left[x_{10}^2, x_{21}^2 \frac{z'}{z''}\right]} \frac{dx_{32}^2}{x_{32}^2} \left[\underline{\Gamma(x_{10}^2, x_{32}^2, z'')} + 3\underline{G(x_{32}^2, z'')} \right]$$

Up Next: Dessert

Finding a Solution at Small x

The Task at Hand

$$G(x_{10}^2, z) = G^{(0)}(x_{10}^2, z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z') + 3G(x_{21}^2, z')\right]$$

$$\Gamma(x_{10}^2, x_{21}^2, z') = G^{(0)}(x_{10}^2, z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min[x_{10}^2, x_{21}^2 \frac{z'}{z''}]} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z'') + 3G(x_{32}^2, z'')\right]$$

- Must solve a set of coupled integro-differential equations
  - Neighbor dipole depends on two spatial arguments

The Task at Hand

$$G(x_{10}^2, z) = G^{(0)}(x_{10}^2, z) + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \left[\Gamma(x_{10}^2, x_{21}^2, z') + 3G(x_{21}^2, z')\right]$$

$$\Gamma(x_{10}^2, x_{21}^2, z') = G^{(0)}(x_{10}^2, z') + \frac{\alpha_s N_c}{2\pi} \int\limits_{\frac{1}{x_{10}^2 s}}^{z'} \frac{dz''}{z''} \int\limits_{\frac{1}{z'' s}}^{\min[x_{10}^2, x_{21}^2 \frac{z'}{z''}]} \frac{dx_{32}^2}{x_{32}^2} \left[\Gamma(x_{10}^2, x_{32}^2, z'') + 3G(x_{32}^2, z'')\right]$$

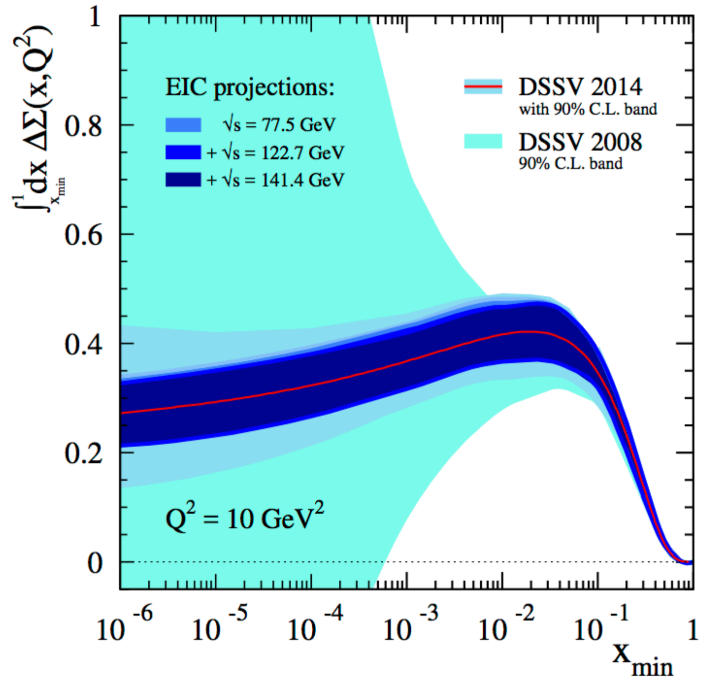
- Must solve a set of coupled integro-differential equations
  - Neighbor dipole depends on two spatial arguments
- Can it be done numerically? Can it be done analytically?
  - What is the helicity intercept at small x?

$$\frac{d\,\Delta\sigma^{(\gamma^* p)}}{dx\,dQ^2} \sim \left(\frac{1}{x}\right)^{\alpha_h - 1} \sim x\Delta q_f(x, Q^2)$$

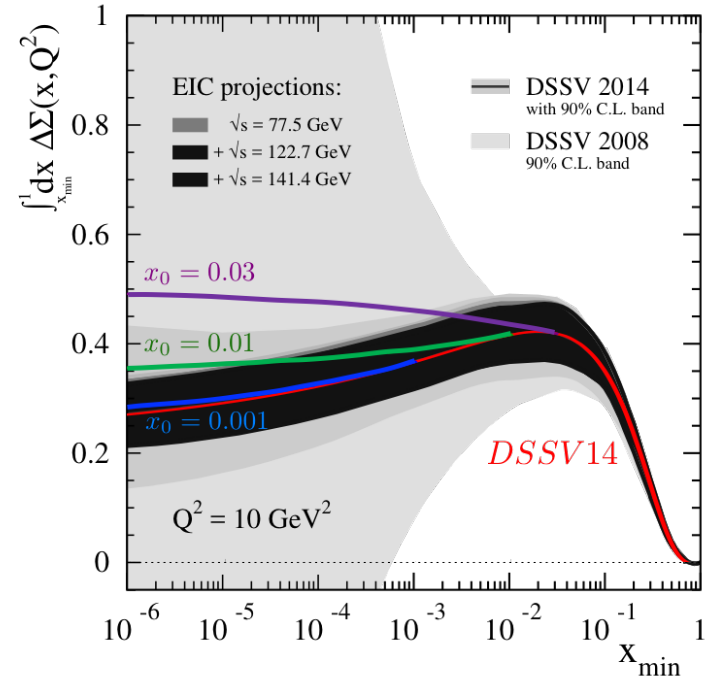
$$\boxed{\alpha_h = ???}$$

Conclusion: A Taste of the Answer

Without Small-x Evolution



With Small-x Evolution



adapted from Aschenauer et al., Phys. Rev. ***D92*** *(2015) no.9 094030*

- Quark helicity at small x receives **strong double-logarithmic enhancement** through the evolution near a polarized Wilson line