Resilient Distribution Tool (RDT)

Optimal storm-hardening of distribution systems with microgrids and conventional assets

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Operate by Los Alamos National Security, LLC for the U.S. Department of Energy's NNSA

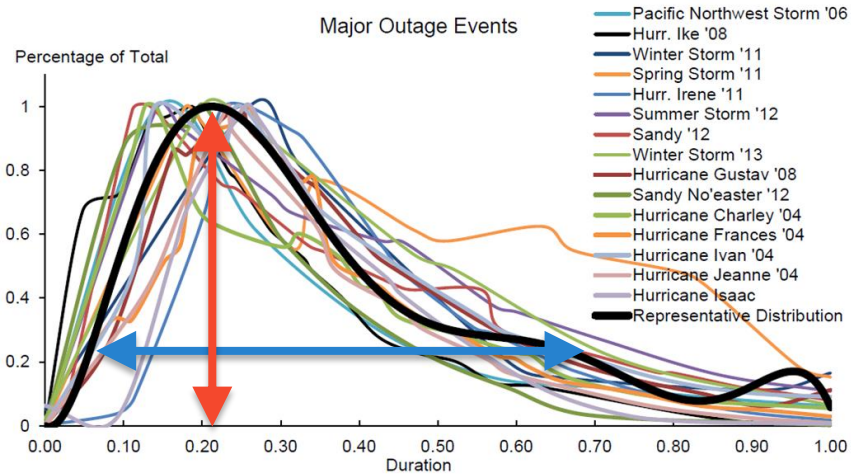
Goal: Develop new tools to design resilient distribution systems

Hardening/Resilience options

- **Asset hardening**
- **System design**
- **System operations**
- **Repair scheduling**
- **Emergency operations**

User-Specified Inputs

- Base network model
- Resilience metrics
- Suggested upgrades
- Costs
- Threats & scenarios

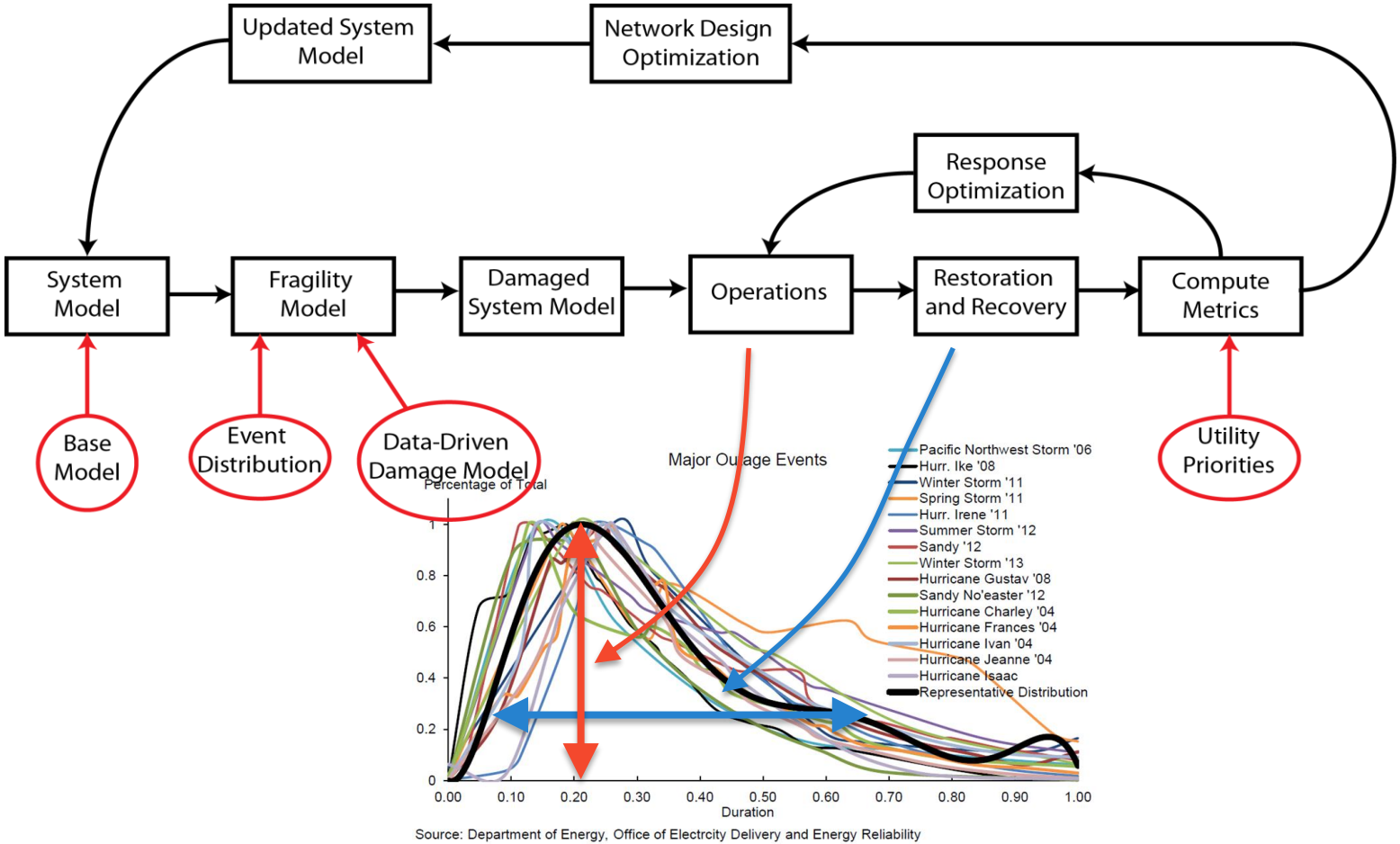


Source: Department of Energy, Office of Electrcity Delivery and Energy Reliability

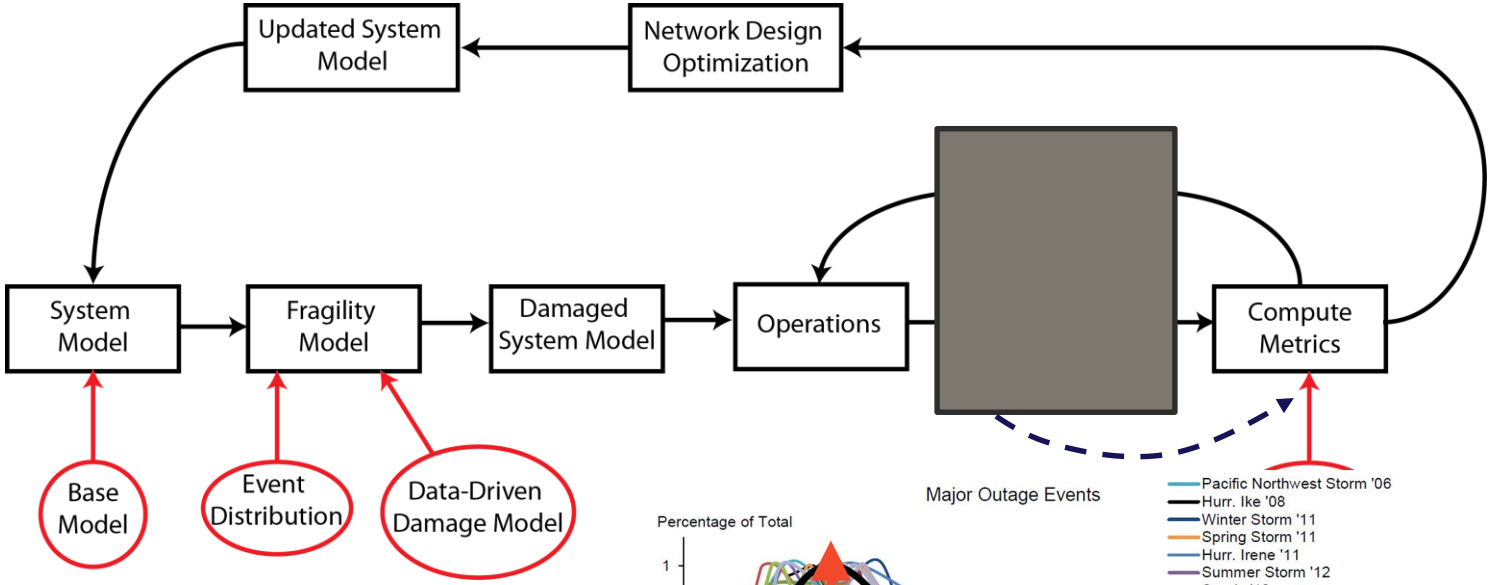
Capabilities

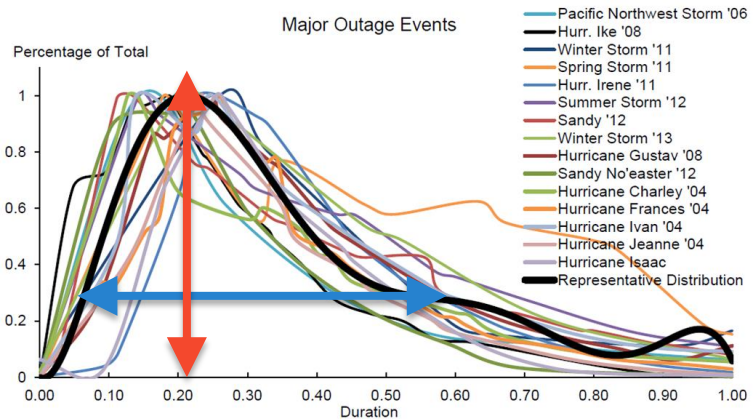
- Assess current resilience posture
- Optimize over user-suggested upgrades to improve resilience considering budget

Resilience Design Process Flow—End Goal



Resilience Design Process Flow—Today



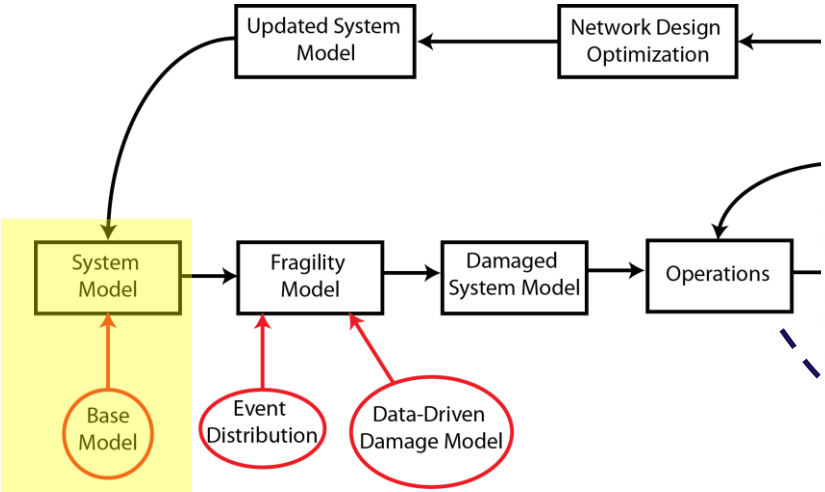


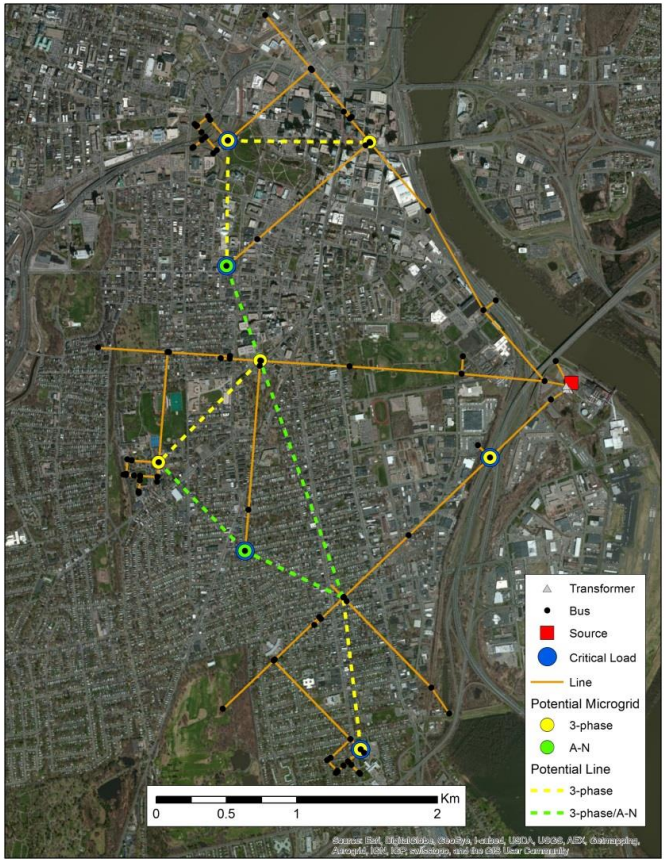
Source: Department of Energy, Office of Electrcity Delivery and Energy Reliability

Resilience Design Process—System Model

Flexibility for the user

- User's base network model
- User-defined metrics, e.g. critical load service
- User suggests upgrades
- User-defined costs
- User-defined threat and scenarios

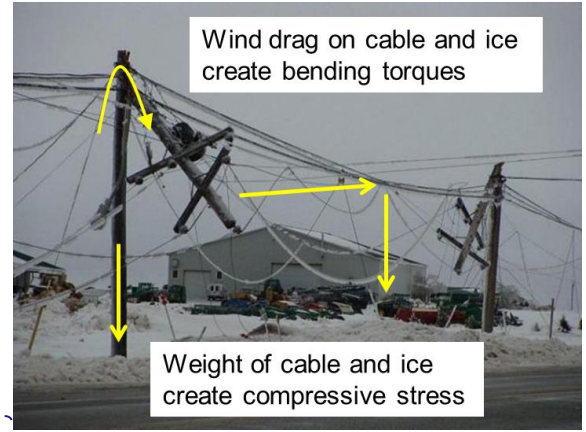


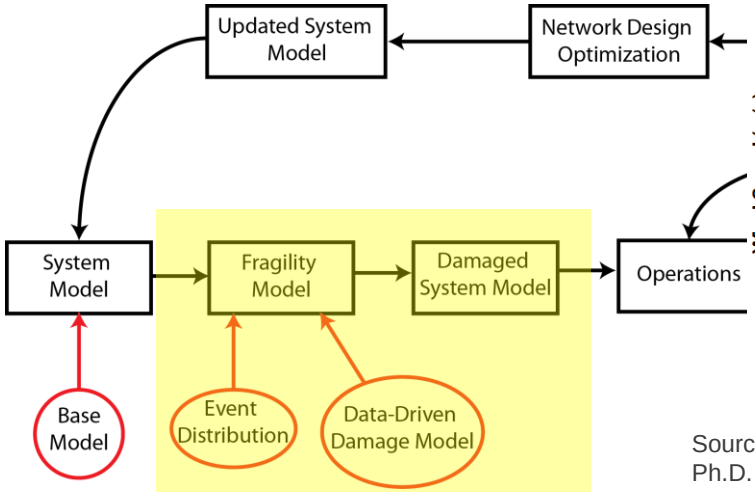


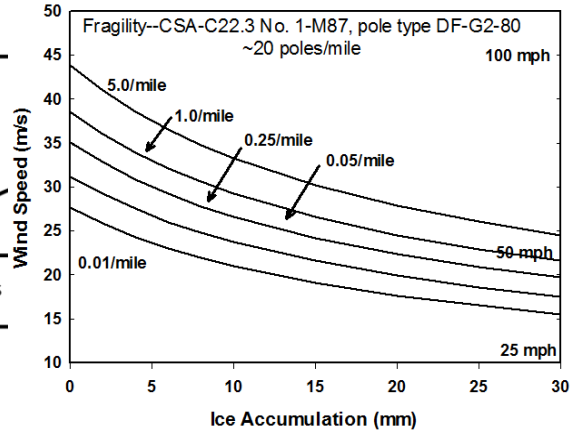
Resilience Design Process—Direct Impacts

Flexibility for the user

- User's base network model
- User-defined metrics, e.g. critical load service
- User suggests upgrades
- User-defined costs
- User-defined threat and scenarios







Source: Y. Sa, *Reliability analysis of electric distribution lines* Ph.D. dissertation, McGill University, Montreal, Canada, 2002

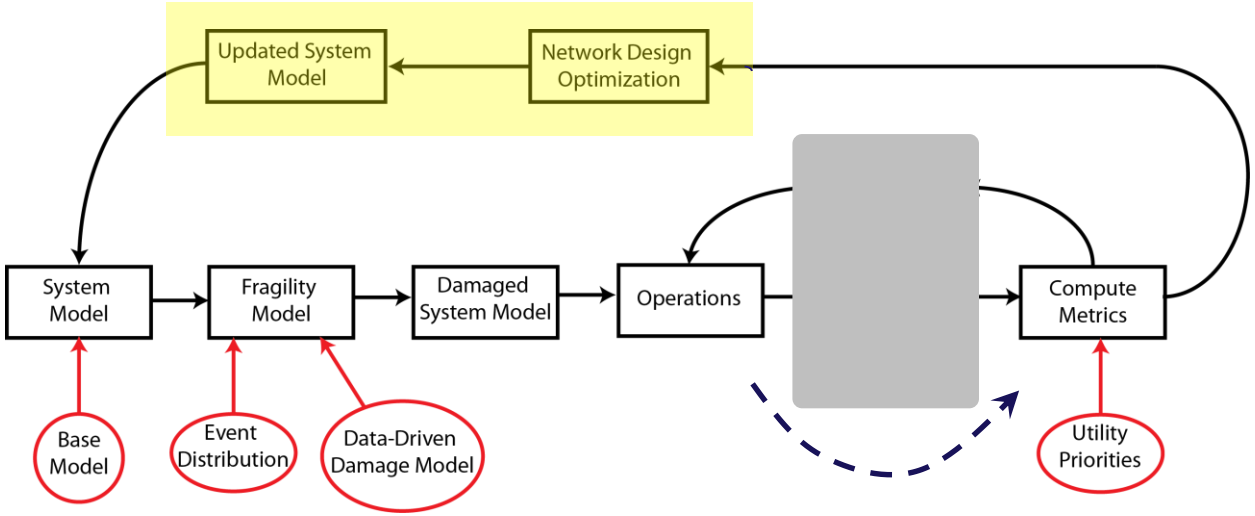
Resilience Design Process—Design Network

Hardening/Resilience options

- Asset hardening
- System design
- System operations
- Repair scheduling
- Emergency operations

Capabilities

- Assess current resilience posture
- Optimize over user-suggested upgrade to improve resilience considering budget



Implementation of "Withstand"—Scenario Based Decomposition

$$ResilientDesign(S)$$

Solve over all damage scenarios

$$s \leftarrow chooseScenario(S)$$

Select 1 scenario

$$\sigma \rightarrow solveMIP(s)$$

Design network for damage scenario 1

$$while\ \left(\sim Feasible(\sigma, S \backslash s)\right)$$

Is solution feasible for remaining scenarios

$$s \rightarrow s \cup chooseScenario(S \backslash s)$$

$$\sigma \rightarrow solveMIP(s)$$

If NOT, add an infeasible scenario to the set under consideration

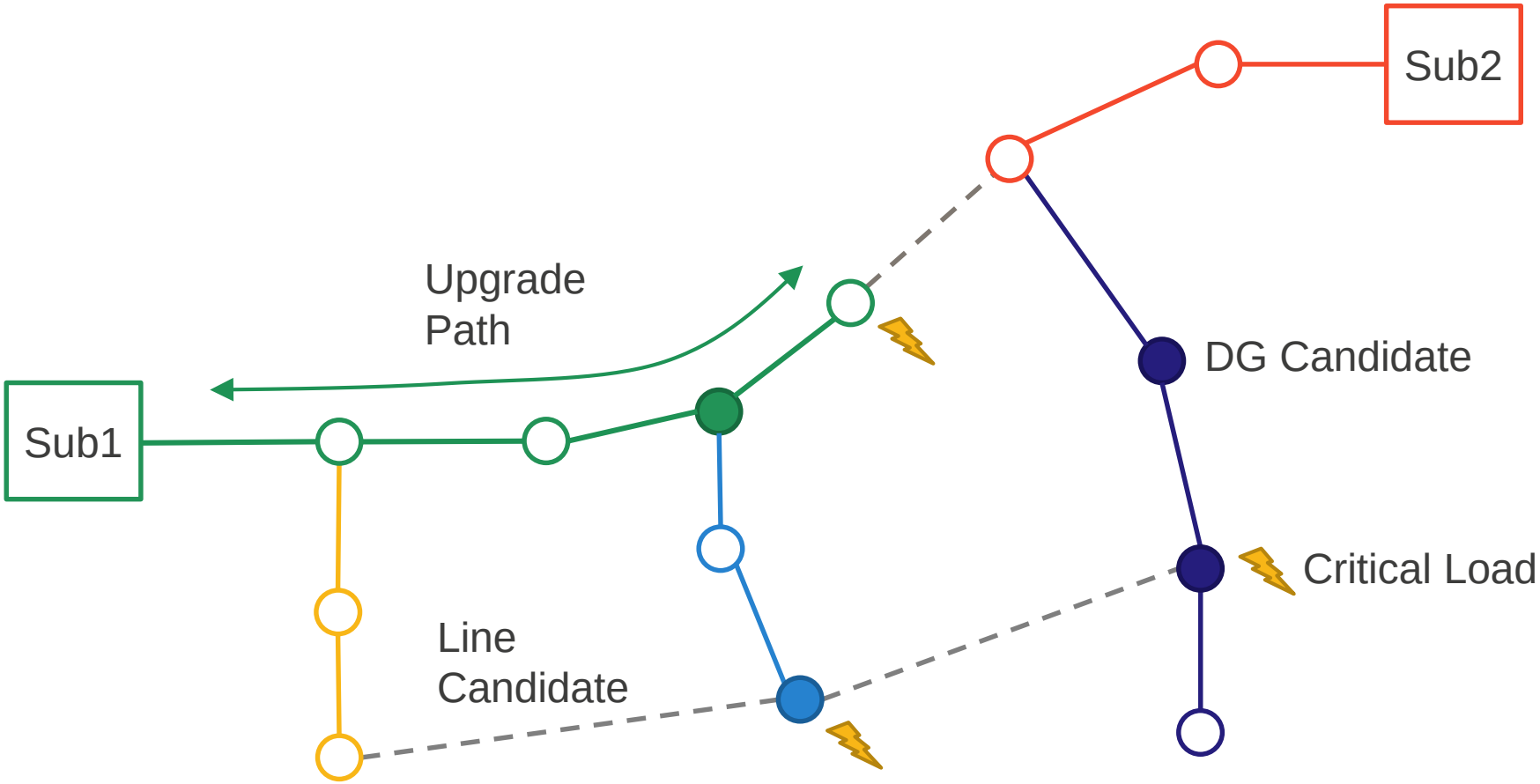
Find a new solution

Iterate until solution is feasible for all scenarios

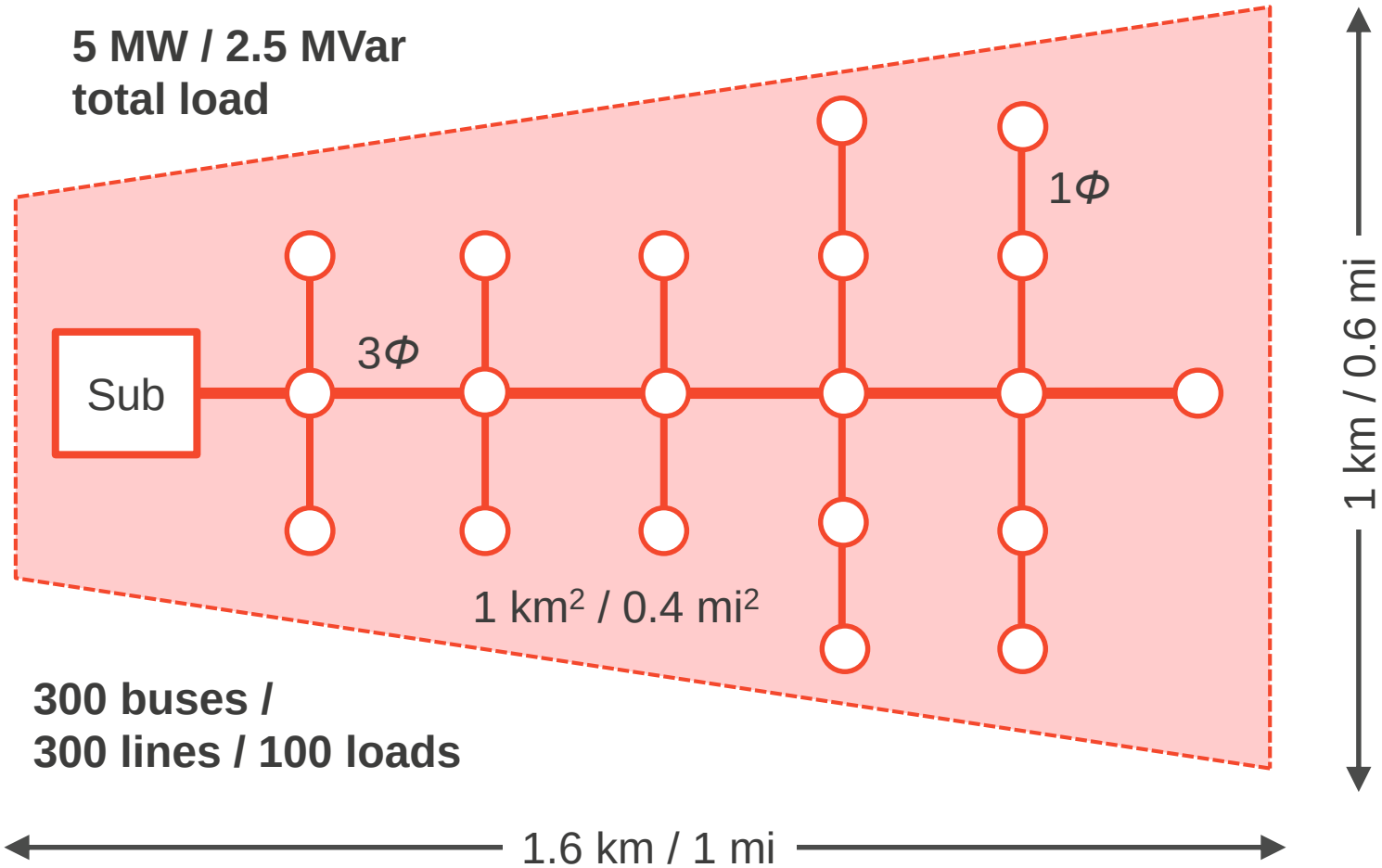
Adaptation to Practical Distribution Systems

- **Objective is to run on a 1,000+ bus practical distribution systems consisting of multiple feeders**
- **Key insights to enable this are:**
  - Multi-commodity or linearized unbalanced powerflow (Low et al)
  - Sparsity in the number of Boolean & integer variables. This requires engineering judgement to select good candidates for new lines, hardening paths and distributed generation placement
  - Pre-compute switching states
  - Hardened paths have zero probability of damage

Candidate Feeder Upgrades



Case Study 1: Small 12.47 kV Feeder



Case Study 1 Results

Damage & Resilience Requirements	
20%	Damage prob.
50% / 98%	Normal / Critical load served
500 kW	Generator increments

Computation	
30 min.	Exact search
20	Disaster scenarios

Results (Exact)	
40	Lines hardened
0	Lines built
1 MW / 1	Total gen. power / # gen.
$3.5M	Capex

Damage & Resilience Requirements	
20%	Damage prob.
60% / 98%	Normal / Critical load served
500 kW	Generator increments

Computation	
3 h (timeout)	Exact search
20	Disaster scenarios

Results (Exact)	
40	Lines hardened
0	Lines built
4 MW / 7	Total gen. power / # gen.
$12.5M	Capex

Case Study 1 Results

Damage & Resilience Requirements	
20%	Damage prob.
60% / 98%	Normal / Critical load served
500 kW	Generator increments

Computation	
3 h (timeout)	Exact search
20	Disaster scenarios

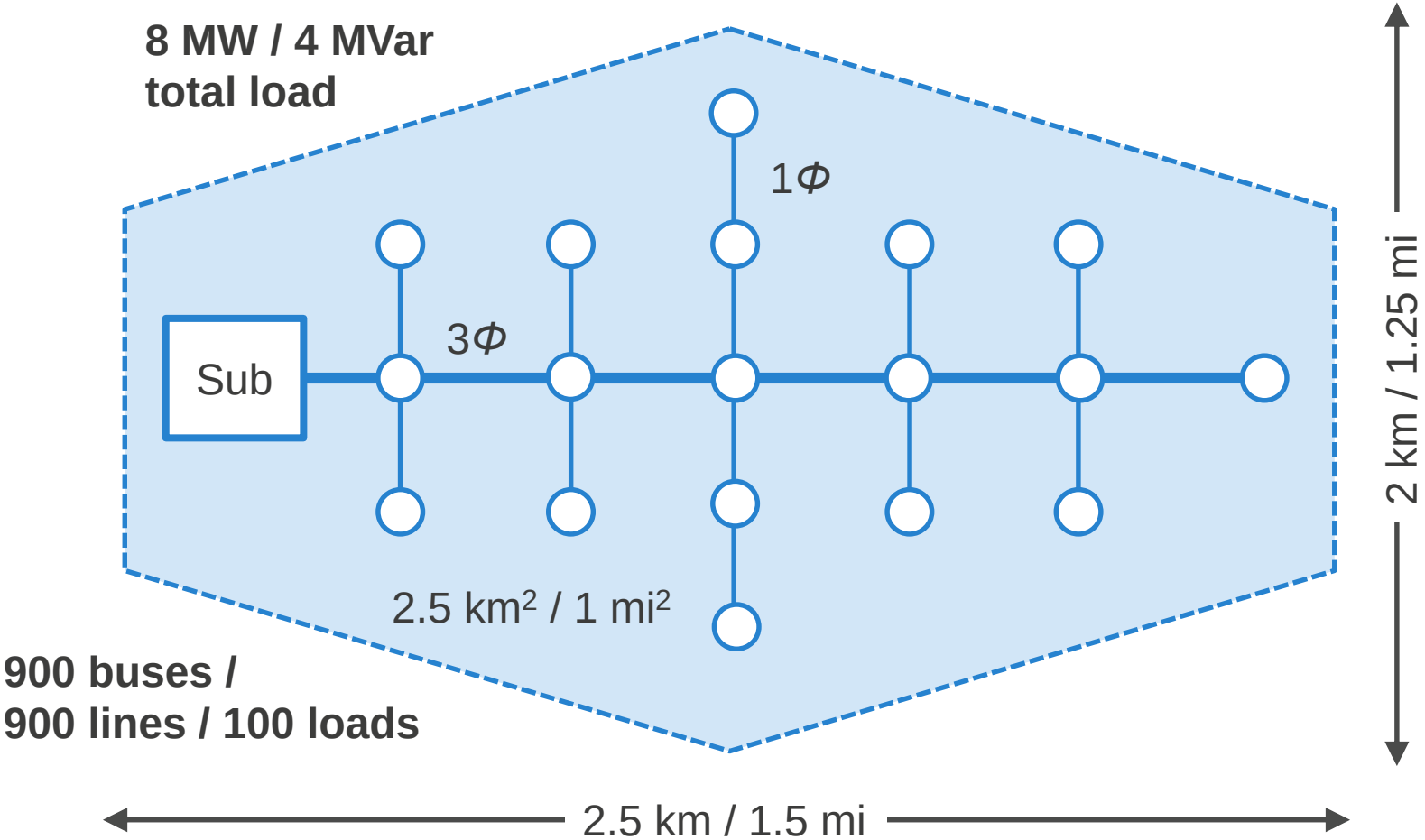
Results (Exact)	
40	Lines hardened
0	Lines built
4 MW / 7	Total gen. power / # gen.
$12.5M	Capex

Damage & Resilience Requirements	
20%	Damage prob.
60% / 98%	Normal / Critical load served
500 kW	Generator increments

Computation	
3 min.	VN search
20	Disaster scenarios

Results (Exact)	
55	Lines hardened
0	Lines built
1.5 MW / 2	Total gen. power / # gen.
$5M	Capex

Case Study 2: Larger 12.47 kV Feeder



Case Study 2 Results

Damage & Resilience Requirements	
20%	Damage prob.
50% / 98%	Normal / Critical load served
500 kW	Generator increments

Computation	
3 h	Exact
20	Disaster scenarios

Results (Exact)	
35	Lines hardened
0	Lines built
0 MW / 0	Total gen. power / # gen.
$500k	Capex

Damage & Resilience Requirements	
20%	Damage prob.
50% / 98%	Normal / Critical load served
500 kW	Generator increments

Computation	
10 min.	VN search
20	Disaster scenarios

Results (Exact)	
40	Lines hardened
0	Lines built
0 MW / 0	Total gen. power / # gen.
$600k	Capex

Development Roadmap

Existing

- Commodity flow constraints: neglect voltage & reactive power flow
- C++ HPC implementation
- Test on feeder made of IEEE 37-bus test case copied 3-times

In Progress

- Extension to distribution systems with 1000+ buses and multiple substations
- Java HPC implementation
- Pulls system data from DEW commercial software

Open-source code release

AWS

- Implementation within Amazon Web Services
- User interface via web-based GIS frontend
- Access via RESTful API for research use

NCRECA OMF

- Implementation as study module within OMF web-based analysis framework

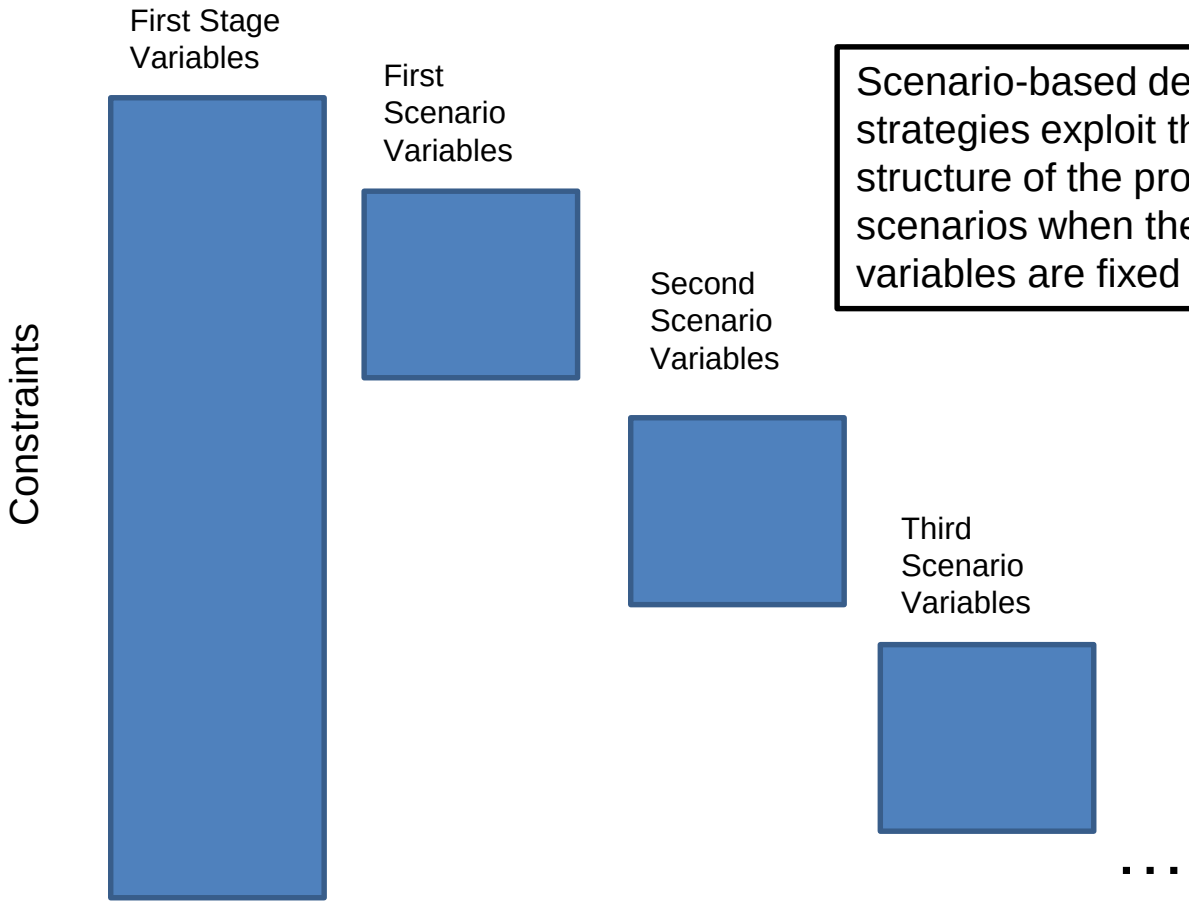
Thanks & Questions

Problem Formulation & Solution Methods

Implementation of "Withstand"—Algorithm

- Baseline Standard
  - CPLEX 12.6—commercial mixed integer program solver
- Decomposition Algorithms (cutting planes)
  - Danzig-Wolfe
  - Benders
  - Disjunctive
  - Logic
  - Scenario ← Biggest computational gains

Implementation of "Withstand"—Decomposition Algorithm



Scenario-based decomposition strategies exploit the separable structure of the problem over scenarios when the first stage variables are fixed

Design Network—Optimization

$$\text{minimize} \quad \sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N, k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$$

$$\text{s.t.} \quad -\mathrm{x}_{ij}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le x_{ij}^s Q_{ij}^k$$

$$-\left(1 - \tau_{ij}^s\right) Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \left(1 - \tau_{ij}^s\right) Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$$x_{ij}^s \le x_{ij},\ \tau_{ij}^s \le \tau_{ij},\ t_{ij}^s \le t_{ij},\ z_{ij}^{sk} \le z_i^k,\ u_i^s \le u_i$$

$$z_i^k \le M_i^k u_i,\ x_{ij}^s = t_{ij}^s,\ x_{ij}^s \le \overline{x}_{ij}^s,\ \tau_{ij}^s \le x_{ij}^s$$

$$3 - x_{ij}^s - \overline{\tau}_{ij}^s \ge \tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1$$

$$\mathrm{l}_\mathrm{i}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\mathrm{g}_\mathrm{i}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + \left(1 - \overline{\tau}_{ij}\right)\right) \le |s| - 1$$

$$\sum_{i\in CL, k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL, k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L, k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L, k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Key Features

- Least cost design for a set of scenarios
- Three-phase real power flows
- Enforces radial operations
- Enforces phase balance
- Discrete variables for load shedding (per scenario), line switching (per scenario), capital construction (first stage)

- Understand the boundaries of tractability
- Optimality vs. computation tradeoff

Optimization Model: Multi-Commodity Flow

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\,, \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \; x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-\left(1 - \tau_{ij}^s\right) Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \left(1 - \tau_{ij}^s\right) Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_{\mathrm{i}}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

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$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\; x_{ij}^s \le \overline{x}_{ij}^s$$

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Optimization Model: Multi-Commodity Flow

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

Minimize expansion cost

s.t. $x_{ij}^s \le x_{ij}, \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

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Optimization Model: Multi-Commodity Flow

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$$x, y, \tau, u, t \in \{0,1\}$$

Auxiliary variables for linking first and second stage. Useful for decomposition.

Optimization Model: Multi-Commodity Flow

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$$x, y, \tau, u, t \in \{0,1\}$$

Line capacity constraints. Capacity is 0 when line is unavailable or open.

Optimization Model: Multi-Commodity Flow

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

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Phase imbalance tolerance

$x_{ij}^s = t_{ij}^s$ when x is damaged

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$$\sum_{ij\in s}\left(\overline{x}_{ij}^s + (1-\overline{\tau}_{ij})\right) \le |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model: Multi-Commodity Flow

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}, \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-(1-\tau_{ij}^s)\, Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le (1-\tau_{ij}^s)\, Q_{ij}^k$$

$$-\beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

Links damaged lines with hardening variables

$$\mathrm{l}_\mathrm{i}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\mathrm{g}_\mathrm{i}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s}\left(\overline{x}_{ij}^s + (1-\overline{\tau}_{ij})\right) \le |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model: Multi-Commodity Flow

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\,, \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \; x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-\left(1 - \tau_{ij}^s\right) Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \left(1 - \tau_{ij}^s\right) Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\boxed{\mathrm{l}_\mathrm{i}^{\mathrm{ks}} = y_i^s d_i^k}$$

Load switching

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\mathrm{g}_\mathrm{i}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + \left(1 - \overline{\tau}_{ij}\right)\right) \le |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\; x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model: Multi-Commodity Flow

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\,, \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \; x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-\left(1 - \tau_{ij}^s\right) Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \left(1 - \tau_{ij}^s\right) Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_\mathrm{i}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

Power produced

$$\mathrm{g}_\mathrm{i}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + \left(1 - \overline{\tau}_{ij}\right)\right) \le |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\; x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model: Multi-Commodity Flow

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\,, \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \; x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-\left(1 - \tau_{ij}^s\right) Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \left(1 - \tau_{ij}^s\right) Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_{\mathrm{i}}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\boxed{\mathrm{g}_{\mathrm{i}}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0}$$

Nodal flow balance

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + \left(1 - \overline{\tau}_{ij}\right)\right) \le |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\; x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model: Multi-Commodity Flow

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\,, \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \; x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-\left(1 - \tau_{ij}^s\right) Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \left(1 - \tau_{ij}^s\right) Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_{\mathrm{i}}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\mathrm{g}_{\mathrm{i}}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$\boxed{0 \le z_i^{ks} \le u_i^s Z_i^k}$$

Links generation construction and capacity

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + \left(1 - \overline{\tau}_{ij}\right)\right) \le |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\; x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model: Multi-Commodity Flow

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}, \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-\left(1-\tau_{ij}^s\right) Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \left(1-\tau_{ij}^s\right) Q_{ij}^k$$

$$-\beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij}\frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_\mathrm{i}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\mathrm{g}_\mathrm{i}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s}\left(\overline{x}_{ij}^s + \left(1-\overline{\tau}_{ij}\right)\right) \le |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\ x_{ij}^s \le \overline{x}_{ij}^s$$

Enforces radial operation

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Optimization Model: Multi-Commodity Flow

minimize $\sum_{ij\in E} c_{ij}x_{ij} + \sum_{i,j\in E} \kappa_{ij}\tau_{ij} + \sum_{i\in N,k\in p_i} \zeta_i^k z_i^k + \sum_{i\in N} \mu_i u_i + \sum_{ij\in E} \alpha_{ij} t_{ij}$

s.t. $x_{ij}^s \le x_{ij}\,, \tau_{ij}^s \le \tau_{ij}, t_{ij}^s \le t_{ij}, z_i^{ks} \le z_i^k, u_i^s \le u_i$

$$-x_{ij0}^s Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{sk} \le \; x_{ij1}^s Q_{ij}^k$$

$$x_{ij0}^s + x_{ij1}^s \le x_{ij}^s$$

$$-\left(1 - \tau_{ij}^s\right) Q_{ij}^k \le \sum_{k\in p_{ij}} f_{ij}^{ks} \le \left(1 - \tau_{ij}^s\right) Q_{ij}^k$$

$$-\beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le f_{ij}^{k's} - \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|} \le \beta_{ij} \frac{\sum_{k\in p_{i,j}} f_{ij}^{ks}}{|p_{ij}|}$$

$x_{ij}^s = t_{ij}^s$ when x is damaged

$$\mathrm{l}_\mathrm{i}^{\mathrm{ks}} = y_i^s d_i^k$$

$$0 \le g_i^{sk} \le z_i^{ks} + g_i^{k^+}$$

$$\mathrm{g}_\mathrm{i}^{\mathrm{ks}} - l_i^{ks} - \sum_{j\in N} f_{ij}^{ks} = 0$$

$$0 \le z_i^{ks} \le u_i^s Z_i^k$$

$$\sum_{ij\in s} \left(\overline{x}_{ij}^s + \left(1 - \overline{\tau}_{ij}\right)\right) \le |s| - 1$$

$$\tau_{ij}^s \ge x_{ij}^s + \overline{\tau}_{ij}^s - 1,\; x_{ij}^s \le \overline{x}_{ij}^s$$

$$\sum_{i\in CL,k\in p_i} l_i^{ks} \ge \lambda \sum_{i\in CL,k\in p_i} d_i^k$$

$$\sum_{i\in N\setminus L,k\in p_i} l_i^{ks} \ge \gamma \sum_{i\in N\setminus L,k\in p_i} d_i^k$$

$$x, y, \tau, u, t \in \{0,1\}$$

Resilience criteria—minimum amount of load served
Is generalized to a chance constraint

Optimization Model: LinDist Flow

- **Baran and Wu 89, Gan and Low 14**
- **Assumption**
  - Derivation of a linearized model of the power flow physics
  - Adds voltage to the models
- **Justification**
  - Situations where multi-commodity flow assumptions do not hold
    - Loopy operations
    - Capacitor placement
    - Non-firm generation
    - Contrived cases
      - Long lines are cheaper than short lines
      - Initial network does not satisfy voltage constraints

Optimization Model: LinDist Flow

$$v_j^a = v_i^a - 2(r_{ij}^{aa} P_{ij}^a + x_{ij}^{aa} Q_{ij}^a + \overline{r}_{ij}^{ab} P_{ij}^b + \overline{x}_{ij}^{ab} Q_{ij}^b + \underline{r}_{ij}^{ac} P_{ij}^c + \underline{x}_{ij}^{ac} Q_{ij}^c)$$

$$v_j^b = v_i^b - 2(\underline{r}_{ij}^{ba} P_{ij}^a + \underline{x}_{ij}^{ba} Q_{ij}^a + r_{ij}^{bb} P_{ij}^b + x_{ij}^{bb} Q_{ij}^b + \overline{r}_{ij}^{bc} P_{ij}^c + \overline{x}_{ij}^{bc} Q_{ij}^c)$$

$$v_j^c = v_i^c - 2(\overline{r}_{ij}^{ca} P_{ij}^a + \overline{x}_{ij}^{ca} Q_{ij}^a + \underline{r}_{ij}^{cb} P_{ij}^b + \underline{x}_{ij}^{cb} Q_{ij}^b + r_{ij}^{cc} P_{ij}^c + x_{ij}^{cc} Q_{ij}^c)$$

$$\mathrm{g(q)_i^{ks}} - l(q)_i^{ks} - \sum_{j \in N} f(q)_{ij}^{ks} = 0$$

$$v_-^k \leq v^k \leq v_+^k$$

Optimization Model: LinDist Flow

$$v_j^a = v_i^a - 2(r_{ij}^{aa} P_{ij}^a + x_{ij}^{aa} Q_{ij}^a + \overline{r}_{ij}^{ab} P_{ij}^b + \overline{x}_{ij}^{ab} Q_{ij}^b + \underline{r}_{ij}^{ac} P_{ij}^c + \underline{x}_{ij}^{ac} Q_{ij}^c)$$

$$v_j^b = v_i^b - 2(\underline{r}_{ij}^{ba} P_{ij}^a + \underline{x}_{ij}^{ba} Q_{ij}^a + r_{ij}^{bb} P_{ij}^b + x_{ij}^{bb} Q_{ij}^b + \overline{r}_{ij}^{bc} P_{ij}^c + \overline{x}_{ij}^{bc} Q_{ij}^c)$$

$$v_j^c = v_i^c - 2(\overline{r}_{ij}^{ca} P_{ij}^a + \overline{x}_{ij}^{ca} Q_{ij}^a + \underline{r}_{ij}^{cb} P_{ij}^b + \underline{x}_{ij}^{cb} Q_{ij}^b + r_{ij}^{cc} P_{ij}^c + x_{ij}^{cc} Q_{ij}^c)$$

$$\mathrm{g(q)}_{\mathrm{i}}^{\mathrm{ks}} - l(q)_i^{ks} - \sum_{j \in N} f(q)_{ij}^{ks} = 0$$

$$v_-^k \leq v^k \leq v_+^k$$

Ohm's Law approximation

Optimization Model: LinDist Flow

$$v_j^a = v_i^a - 2(r_{ij}^{aa} P_{ij}^a + x_{ij}^{aa} Q_{ij}^a + \overline{r}_{ij}^{ab} P_{ij}^b + \overline{x}_{ij}^{ab} Q_{ij}^b + \underline{r}_{ij}^{ac} P_{ij}^c + \underline{x}_{ij}^{ac} Q_{ij}^c)$$

$$v_j^b = v_i^b - 2(\underline{r}_{ij}^{ba} P_{ij}^a + \underline{x}_{ij}^{ba} Q_{ij}^a + r_{ij}^{bb} P_{ij}^b + x_{ij}^{bb} Q_{ij}^b + \overline{r}_{ij}^{bc} P_{ij}^c + \overline{x}_{ij}^{bc} Q_{ij}^c)$$

$$v_j^c = v_i^c - 2(\overline{r}_{ij}^{ca} P_{ij}^a + \overline{x}_{ij}^{ca} Q_{ij}^a + \underline{r}_{ij}^{cb} P_{ij}^b + \underline{x}_{ij}^{cb} Q_{ij}^b + r_{ij}^{cc} P_{ij}^c + x_{ij}^{cc} Q_{ij}^c)$$

$$\mathrm{g(q)}_{\mathrm{i}}^{\mathrm{ks}} - l(q)_i^{ks} - \sum_{j \in N} f(q)_{ij}^{ks} = 0$$

$$v_-^k \leq v^k \leq v_+^k$$

Reactive power conservation

Optimization Model: LinDist Flow

$$v_j^a = v_i^a - 2(r_{ij}^{aa} P_{ij}^a + x_{ij}^{aa} Q_{ij}^a + \overline{r}_{ij}^{ab} P_{ij}^b + \overline{x}_{ij}^{ab} Q_{ij}^b + \underline{r}_{ij}^{ac} P_{ij}^c + \underline{x}_{ij}^{ac} Q_{ij}^c)$$

$$v_j^b = v_i^b - 2(\underline{r}_{ij}^{ba} P_{ij}^a + \underline{x}_{ij}^{ba} Q_{ij}^a + r_{ij}^{bb} P_{ij}^b + x_{ij}^{bb} Q_{ij}^b + \overline{r}_{ij}^{bc} P_{ij}^c + \overline{x}_{ij}^{bc} Q_{ij}^c)$$

$$v_j^c = v_i^c - 2(\overline{r}_{ij}^{ca} P_{ij}^a + \overline{x}_{ij}^{ca} Q_{ij}^a + \underline{r}_{ij}^{cb} P_{ij}^b + \underline{x}_{ij}^{cb} Q_{ij}^b + r_{ij}^{cc} P_{ij}^c + x_{ij}^{cc} Q_{ij}^c)$$

$$\mathrm{g(q)_i^{ks}} - l(q)_i^{ks} - \sum_{j \in N} f(q)_{ij}^{ks} = 0$$

$$v_-^k \leq v^k \leq v_+^k$$

Voltage limits

Algorithm Overview

- **Exact Algorithms**
  - CPLEX 12.6
  - Scenario-based Decomposition
- **Heuristics**
  - Greedy
    - Union of single scenario solutions
    - Based on industry algorithms
  - Variable Neighborhood Search
    - Ruin and Recreate—hybrid of exact methods and local search
    - Iteratively relax variable assignments (ruin)
    - Use exact method to find optimal variable assignments for relaxed variables, given the fixed partial solution (recreate)

Scenario Based Decomposition (Review)

$$ResilientDesign(S)$$

Solve over all damage scenarios

$$s \leftarrow chooseScenario(S)$$

Select 1 scenario

$$\sigma \rightarrow solveMIP(s)$$

Design network for damage scenario 1

$$while\ \left(\sim Feasible(\sigma, S \backslash s)\right)$$

Is solution feasible for remaining scenarios

$$s \rightarrow s \cup chooseScenario(S \backslash s)$$

$$\sigma \rightarrow solveMIP(s)$$

If NOT, add an infeasible scenario to the set under consideration

Find a new solution

Iterate until solution is feasible for all scenarios

Outperformed other decomposition strategies—second stage influences feasibility, not optimality. Continuous investment variables also adds difficulty

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

Solve the LP relaxation

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \dots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \dots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$j \leftarrow j + 1, k \leftarrow k - \frac{step}{2}$

return σ^*

Intuition: LP relaxation guides the search procedure

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \dots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \dots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$j \leftarrow j + 1, k \leftarrow k - \frac{step}{2}$

return σ^*

Count differences between current best solution and relaxation

Intuition: *n* is a parameter used to control the size of the neighborhood. Larger differences between the LP relaxation and the incumbent solution indicate that a larger neighborhood should be considered.

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \dots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \dots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$j \leftarrow j + 1, k \leftarrow k - \frac{step}{2}$

return σ^*

Order variables by difference from LP relaxation

Intuition: Variables whose assignments differ from the LP relaxation have more potential to improve the incumbent solution.

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \dots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime$ and $j < maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \dots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$j \leftarrow j + 1, k \leftarrow k - \frac{step}{2}$

return σ^*

Compute best solution in neighborhood J(1…k)

Intuition: Ruin and recreate

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \dots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \dots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$j \leftarrow j + 1, k \leftarrow k - \frac{step}{2}$

return σ^*

Update best solution

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \dots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \dots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$j \leftarrow j + 1, k \leftarrow k - \frac{step}{2}$

return σ^*

Intuition: When a better solution is not found, increase the size of the neighborhood

Increase neighborhood size

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \dots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \dots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$j \leftarrow j + 1, k \leftarrow k - \frac{step}{2}$

return σ^*

Shuffle ordering after restart

Intuition: Consider different sets of variables to relax

Variable Neighborhood Search

$ResilientDesign(S, maxTime, maxRestarts, maxIterations)$

$\sigma^{LP} \leftarrow Solve(P^{LP}), \sigma^* \leftarrow \sigma', restart \leftarrow false$

while $(t < maxTime$ and $i < maxRestarts)$

$j \leftarrow 0$

$n \leftarrow |x \in X : |\sigma^*(x) - \sigma^{LP}(x)| \neq 0|$

$J \leftarrow \langle \pi_1, \pi_2, \dots \pi_{|J|} \rangle \in X : |\sigma^*(\pi_i) - \sigma^{LP}(\pi_i)| \leq |\sigma^*(\pi_{i+1}) - \sigma^{LP}(\pi_{i+1})|$

if $(restart = true)$

$i \leftarrow i + 1$

$step \leftarrow \frac{4n}{d}, k \leftarrow |X| - step$

$shuffle(J)$

else

$step \leftarrow \frac{n}{d}, k \leftarrow |X| - step$

while $(t < maxTime\ and\ j \leq maxIterations)$

$\sigma' \leftarrow Solve(P(\sigma^*, J(1, \dots k))$

if $\left(f(\sigma') < f(\sigma^*)\right)$

$\sigma^* \leftarrow \sigma', i \leftarrow 0, restart \leftarrow false, j \leftarrow maxIterations$

else

$j \leftarrow j + 1, k \leftarrow k - \frac{step}{2}$

return σ^*

Adjust neighborhood parameters

Intuition: Neighborhood size is based on differences between LP relaxation and incumbent solution.

Case Study 1 Results

Damage & Resilience Requirements	
20%	Damage prob.
50% / 98%	Normal / Critical load served
500 kW	Generator increments

Computation	
3-5 min.	Greedy search
30 min.	Exact search
20	Disaster scenarios

Results (Exact)	
40	Lines hardened
0	Lines built
1 MW / 1	Total gen. power / # gen.
$17M	Capex

Damage & Resilience Requirements	
20%	Damage prob.
60% / 98%	Normal / Critical load served
500 kW	Generator increments

Computation	
3-5 min.	Greedy search
3 h	Exact search
20	Disaster scenarios

Results (Exact)	
40	Lines hardened
0	Lines built
4 MW / 7	Total gen. power / # gen.
$26M	Capex