Space Charge Compensation in Laser Particle Accelerators

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Abstract. Laser particle acceleration (LPA) involves the acceleration of particle beams by electromagnetic waves with relatively short wavelength compared with conventional radio-frequency systems. These short length scales raise the question whether space charge effects may be a limiting factor in LPA performance. This is analyzed in two parts of an accelerator system, the acceleration sections and the drift region of the prebuncher. In the prebuncher, space charge can actually be converted to an advantage for minimizing the energy spread. In the accelerator sections, the laser fields can compensate for space charge forces, but the compensation becomes weaker for high beam energy.

INTRODUCTION

Laser particle acceleration (LPA) concepts resemble radio-frequency (RF) linear accelerators in that a travelling wave is set up to move synchronously with the particle. Efficient acceleration in both LPA and RF systems requires that the spread of the particles along the wave be somewhat smaller than the wavelength, $\lambda = 2\pi c/\omega$ (c is the speed of light in vacuum; ω is the frequency of the driving wave), or else that the particles be organized into narrow bunches separated by λ. However, LPA's differ from RF systems in that the wavelength corresponding to optical frequencies is on the order of 10^{-6} to 10^{-5} m rather than 10^{-2} m. Thus *microbunching* on a very fine scale is necessary. In RF accelerators, the small length scale is the transverse dimension of the particle beam. For a typical high-quality beam of moderate energy (50-100 MeV) the transverse dimension may be 200-500 μm, which is smaller than the needed *microbunching* scale. However, in an LPA with, e.g. $\lambda = 10$ μm, the microbunching scale (somewhat less than 10 μm) is the smaller length scale. The appearance of fine scales raises the issue of space charge effects.

The principal space charge effect in an RF linac is a defocusing tendency since the smallest length scale is the transverse dimension of the beam. However in an LPA the smallest length scale is longitudinal; hence the principal space charge effect is a de-bunching tendency. Moreover, the smaller length scale in an LPA makes space charge inherently more difficult. We consider two elements of an LPA system where space charge debunching may be important. One is in the accelerator sections themselves, and the other is in the drift regions between accelerator sections. The largest drift region is at the upstream end of the system where the initial microbunching is first created.

In this paper we analyze the de-bunching effect of space charge in (1) the prebuncher, and (2) in the acceleration sections. The dynamics of debunching are analyzed using quasi-one-dimensional models; the transverse beam dynamics are analyzed neglecting space charge. These results are applied to the practical case of the STaged ELectron Laser Acceleration (STELLA) experiment (1) on the BNL Accelerator Test Facility.

SPACE CHARGE IN THE PREBUNCHER

The prebuncher may be the system element most vulnerable to space charge because there is no longitudinal force to oppose space charge effects. Therefore, the only thing opposing space charge forces is inertia. Fortunately, the effective "mass" for relative longitudinal motions is $\gamma^3 m_e$ (γ is the relativistic factor and m_e is the electron rest mass). A simple method of prebunching, shown in Fig. 1, is composed of an energy modulator (e.g. an inverse free-electron laser) followed by a drift section. A macrobunch with little energy spread enters the modulator and receives an energy modulation with longitudinal periodicity length ~λ (laser wavelength). Thus, within each segment of length λ, particles behind the segment center move slightly faster,

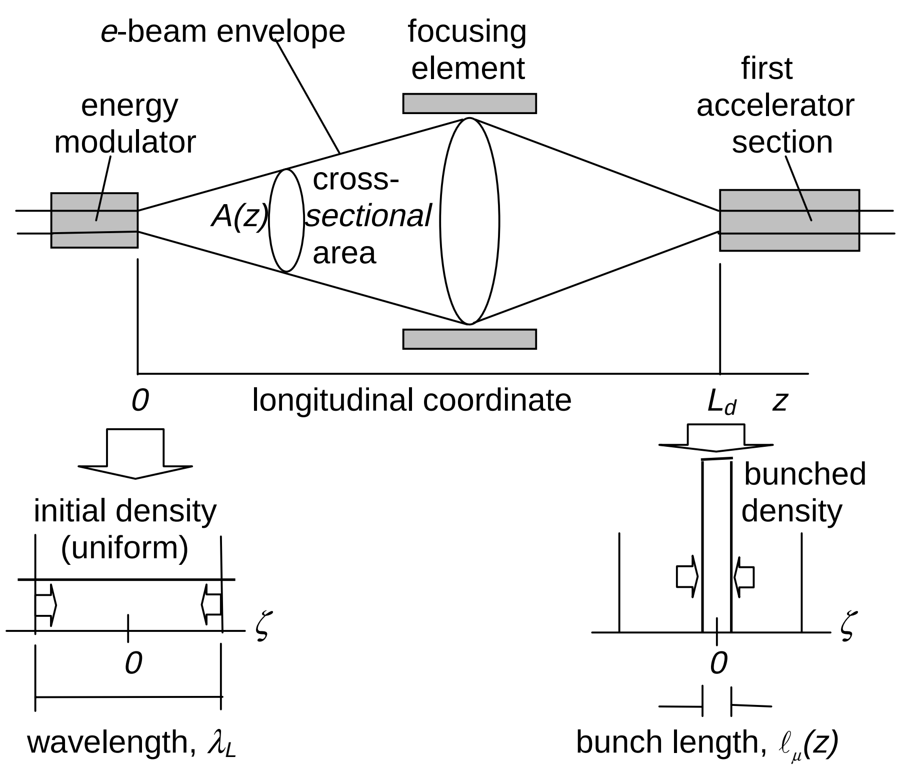

FIGURE 1. Idealized prebunching arrangement.

while those ahead move slightly slower. Then in the drift section the faster particles catch up while the slower slip back, gathering them into microbunches.

In this section we show that (1) space charge forces can be compensated by adjusting the strength of the energy modulator to allow effective microbunching, (2) a second level of compensation can reduce the energy spread. The latter is a positive byproduct of space charge: a coherent energy modulation is required to produce the initial bunching; space charge effects can reduce or eliminate this energy spread. It may be possible to both generate short microbunches and minimize the energy spread.

Estimate of space charge debunching

It is helpful to begin with a simple scaling estimate. Longitudinal forces from space charge cause oscillations at the relativistically-corrected plasma frequency, $\omega_p = (\mu_0 c^2 e^2 n/m_e \gamma^3)^{1/2}$, where μ_0 is the free-space permeability; n, $-e$ are the electron density and charge. Space charge effects become significant if the transit time through the drift region, L_d/c, (L_d is the drift section length) is comparable to or exceeds the plasma oscillation time, $1/\omega_p$. i.e. if $\omega_p L_d/c \sim 1$. Accordingly, define the space charge parameter:

$$\sigma_{SC} \quad \equiv \quad \omega_p^2 L_d^2 \big/ c^2 \; ; \tag{1}$$

if $\sigma_{SC} \sim 1$ or more, space charge will be important in the prebuncher. Since $\sigma_{SC} \propto n$, a practical expression for it is found once the density is identified. The macrobunch properties are charge Q_M, length ℓ_M, and normalized emittance, ε_N. Note that in an LPA, the macrobunch length ℓ_M (usually in mm) is much longer than the microbunch length (< $\lambda/2$). The charge is related to the density by $Q_M = enA\ell_M$, where A is the cross-sectional area of the beam. Suppose $A = \pi\varepsilon_N L_d/\gamma$, which is the characteristic area of a focused beam over a drift distance L_d: this area is proportional to the geometric mean of the waist area and the far-field area at a distance L_d from the waist. Combining these gives the density $n = \gamma Q_M/\pi e \varepsilon_N L_d \ell_M$. Then

$$\sigma_{SC} \quad = \quad \frac{4 r_e L_d}{\gamma^2 \ell_M \varepsilon_N} \frac{Q_M}{e} \quad = \quad 7.04\times10^{-5} \frac{L_d(m) Q_M(nC)}{\gamma^2 \ell_M(m) \varepsilon_N(\pi m - rad)} \tag{2}$$

where $r_e = e^2\mu_0/4\pi m_e$ is the classical electron radius.

Quasi-one dimensional (1D) model

Since the thickness of each microbunch ℓ_μ ($< \lambda/2$) is much smaller than the transverse beam dimension, the longitudinal motion in response to space charge effects is approximately 1D. We therefore adopt the 1D model, which has been checked elsewhere (2). Divide the beam into segments of length λ, and consider electrons within a single segment. Define a longitudinal coordinate in the moving frame of the segment, $\zeta \equiv z - z_0(t)$, where z_0 is the segment center and $|\zeta| \le \lambda/2$ within the segment. Identify ζ with individual electrons in the segment. The initial energy modulation is sinusoidal in ζ. We approximate this crudely as a saw-tooth form, $\gamma = \bar{\gamma} - \Delta\gamma(2\zeta/\lambda)$, with corresponding velocity modulation $\Delta u_z = c(\Delta\gamma/\gamma^3)\cdot 2\zeta/\lambda$, where $\bar{\gamma}$ denotes the mean value, and $\Delta\gamma$ is the amplitude of the energy spread. This is the 1D reduction of the spheroidal bunch model (3). The sawtooth model admits a self-similar solution. The drifting electrons gather into a microbunch, where $\pm\ell_\mu(t)/2$ is the position of the outermost electrons. Then within the bunch, $|\zeta| \le \ell_\mu/2$:

$$n = n_i A_i \lambda / A\ell_\mu \,; \quad u_z = (\zeta/\ell_\mu) d\ell_\mu/dt \,; \quad E_z = -(m_e \lambda \omega_p^2 A_i / eA)\zeta/\ell_\mu \,. \quad \text{(3a, b, c)}$$

As in Fig. 1, $A(z)$ is the beam cross-sectional area; and A_i, n_i denote initial values. Hereafter we adopt $\tilde{t} \equiv ct/L_d$ as the dimensionless time variable, with $0 \le \tilde{t} \le 1$ in the drift section.

The amount of microbunching (which varies with $\tilde{t}$) is expressed as a phase spread, $\psi_\mu \equiv 2\pi\ell_\mu/\lambda$. Before bunching begins, $\psi_\mu = 2\pi$, and the goal is to achieve as close to ideal microbunching ($\psi_\mu = 0$) as possible. Then the equation of motion for $\psi_\mu(\tilde{t})$ and its initial conditions are

$$d^2\psi_\mu / d\tilde{t}^2 = 2\pi\sigma_{SC} A_i / A(\tilde{t}),$$
$$\psi_\mu(0) = 2\pi, \qquad (\psi'_\mu)_i \equiv (d\psi_\mu / d\tilde{t})_{\tilde{t}=0} = -4\pi L_d \Delta\gamma_i / \lambda\bar{\gamma}^3 \,. \quad (4)$$

Nominally only half the modulated electrons are in a phase range for which they gather into a bunch; electrons in the other half tend to spread out. Therefore, in the space charge parameter (Eqs. 1, 2) *half* the macrobunch charge should be used.

Consider the beam optics in the drift section. The prebuncher (Fig. 1) is divided into two drift regions: before "1" and after "2" the focuser ("triplet"). The width $w(z)$ of a drifting beam is $w^2 = W_{1,2}^2 + 4\theta_{1,2}^2(z - Z_{1,2})^2$. In each region, W, Z are the waist width and position, and θ is the asymptotic divergence angle. The normalized emittance is $\varepsilon_N = \gamma W\theta/2$. For a circular beam, the area $A = \pi w^2/4$ is

$$A(\tilde{t})/A_w \quad = \quad 1 + \left(A_\varepsilon/A_w\right)^2 \left(\tilde{t} - Z/L_d\right)^2 \qquad (5)$$

where $A_w \equiv \pi W^2/4$, and $A_\varepsilon \equiv \pi\varepsilon_N L_d/\bar{\gamma}$. Regions 1 and 2 have the same A_ε but different A_w, and areas match at the "idealized" triplet location, $\tilde{t} = z_m/L_d$.

These equations have an analytic solution of the form in terms of two factors *J, J'* that depend on the beam optics,

$$\left(\psi'_\mu\right)_f \;=\; \left(\psi'_\mu\right)_i + 2\pi\,\sigma_{SC} J', \qquad \left(\psi_\mu\right)_f \;=\; 2\pi + \left(\psi'_\mu\right)_i + \pi\,\sigma_{SC} J \qquad (6a,\ b)$$

For a constant area beam $A = A_i$: $J, J' = 1$. Focusing accentuates space charge effects so that $J, J' > 1$ and depend on the beam optics. Both are largest when the second focus lies *before* the end of the drift section because the second beam waist lies within the drift region. Their analytic forms are given in Ref. (2)

Space charge affects both bunching and energy spread. From Eq. 6b the bunching $(\psi_\mu)_f$ can made zero (ideal case) by making $(\psi_\mu')_i$ more negative. This is done by increasing the initial energy modulation. This also reduces the final energy spread, proportional to $(\psi_\mu')_f$ (Eq. 6a). By adjusting the focusing geometry (triplet and second focus locations), which modifies *J, J'*, one might simultaneously achieve maximal bunching $[(\psi_\mu)_f \to 0]$ *and* eliminate the energy spread introduced by the prebuncher $[(\psi_\mu')_f \to 0]$.

Practical example

We apply this theory to estimate space charge effects in the STELLA experiment, which includes a prebuncher (inverse free electron laser) and accelerator section (inverse Cerenkov acceleration), both driven by a 10.6 μm wavelength laser. Table I lists the beam optics parameters in the prebuncher; for this example the space charge parameter is $\sigma_{SC} = 0.55$ (accounting for the factor of two effective reduction in the charge). Consider an example where the initial modulation and second focus location are chosen for dual compensation, i.e. maximal bunching and minimal final energy spread. Suppose the second waist is located at $Z_2/L_d = 0.985$, i.e. just inside the downstream end of the drift section. Then in this idealized model the final bunching and energy spread are perfect, i.e. $(\psi_\mu)_f = 0$ and $(\psi_\mu')_f = 0$, respectively. Dual compensation, (spatial *and* energy spread reduction) can only be achieved if the macrobunch charge is large enough.

This analysis is based on the 1D model. The reduction factor accounting for two-dimensional effects is $F_{2D} \approx 1/(1+1.1\gamma\ell_\mu/w)$. At the triplet ($z \approx z_m$, $\ell_\mu \approx 0.6\lambda$), $F_{2D} \approx$ 2/3; and at the second waist ($z \approx 0.98L_d$, $\ell_\mu \approx \lambda/10$), $F_{2D} \approx 2/3$. Thus the 1D model *overestimates* the space charge effect by a factor of ~ 3/2. Accounting for 2D effects, this would corresponds to a macrobunch charge of $Q_M \approx 0.225$ nC.

TABLE 1. Beam optics parameters in STELLA

Relativistic parameter	$\gamma = 80$	Drift section length	$L_d = 2$ m
Normalized emittance	$\varepsilon_N = 1$ (π mm-mrad)	First waist position	$Z_I/L_d = -0.125$
Macrobunch charge	$Q_M = 0.15$ nC	Focuser position	$z_m/L_d = 0.6$
Macrobunch length	$\ell_M = 3$ mm		

SPACE CHARGE IN THE ACCELERATOR

In the acceleration sections, the electromagnetic fields that accelerate and focus the beam produce bunching forces. These can counteract space charge debunching effect. However, since in a practical LPA system there will be many acceleration sections, the inertial effect is unimportant. Thus the question is whether the laser-induced bunching is adequate to compensate for space-charge debunching. This question is again addressed using a quasi-1D, sharp-boundary bunch model.

Generic laser fields and Lorentz force

Our interest is in the laser fields in the *e*-beam path , i.e. very near the axis. A radially polarized, axisymmetric laser can be expressed as a superposition of modes described by the Bessel functions J_0 and J_1. The propagation vector of each mode **k** has the same amplitude $|\mathbf{k}| = \omega N/c$, but different angle θ to the z axis, (N is the index of refraction of the medium). Generally the dominant modes are clustered near a single angle θ_L. Thus, near the axis the laser wave can be approximated as a single Bessel mode with $\theta \approx \theta_L$. Its nonzero components are

$$E_r = \frac{-E_0}{tan\,\theta_L} J_1(k_r r) sin\,\psi \,,\; E_z = E_0\, J_0(k_r r) cos\,\psi \,,\; B_\theta = \frac{-E_0 N}{c\, sin\,\theta_L} J_1(k_r r) sin\,\psi \qquad \text{(7a, b, c)}$$

where $E_0 = const$; the components of **k** are $k_r = Nk_L\, sin\,\theta_L$ and $k_z = Nk_L\, cos\,\theta_L$ with $k_L = \omega/c = 2\pi/\lambda_L$ (λ_L is the vacuum wavelength). The phase function is $\psi = \omega_L t - \kappa z = k_L(ct - N\, cos\,\theta_L z)$. For later use we define the reference electron (subscript R) as one which runs along the z-axis and is perfectly synchronous ($\psi = const$). Since z = $\beta ct + const$ (for constant relativistic parameter β), the synchronism condition is $N\beta_R\, cos\,\theta_L = 1$. If the *e*-beam path is small enough to lie well within the “Bessel spot” of the J_0 function, $k_r r << 2.405$, then the small argument approximations can be used:

$$E_r \approx -E_0 \frac{k_z r}{2} sin\psi\,, \qquad E_z = E_0 \cdot cos\,\psi\,, \qquad cB_\theta = -E_0 \cdot \frac{N^2 k_L r}{2} sin\psi \qquad \text{(8a, b, c)}$$

Equations of motion

The equations of motion for a relativistic electron are

$$\frac{d}{c\,dt}\left(m_e c^2 \gamma \boldsymbol{\beta}\right) = \mathbf{F}_L\,, \qquad \frac{d\mathbf{r}}{c\,dt} = \boldsymbol{\beta} \qquad \text{(9a, b)}$$

where $\mathbf{r}$ is its position and velocity, and $\boldsymbol{\beta} \equiv \mathbf{v}/c$, $\beta = (1-1/\gamma^2)^{1/2}$. The Lorentz force, $\mathbf{F}_L = -e(\mathbf{E}+\boldsymbol{\beta}\times c\mathbf{B})$, retaining first order components, is

$$F_{Lx} = -eE_0 \frac{Nk_L x}{2} sin\psi\left(-cos\,\theta_L + N\beta_z\right), \qquad F_{Lz} = -eE_0\,cos\,\psi \qquad \text{(10a, b)}$$

where F_{Ly} is the same as F_{Lx} with x replaced by y. The phase acts as the longitudinal coordinate of an electron. Its evolution is governed by

$$\frac{d\psi}{c\,dt} = k_L\left(1 - N\beta_z\,cos\,\theta_L\right) \qquad \text{(11)}$$

In general an electron path will differ from the reference electron ($\beta_z = \beta_R$). We introduce perturbed quantities that represent the difference between an electron and the reference. Also consider the acceleration gradient $W' = dW/dz$, where $W = m_e c^2 \gamma$ is the electron energy. The acceleration gradient for the reference electron, using Eqs. 9a, 10b, and the identity $d(\beta\gamma) = d\gamma/\beta$, is

$$W_R' = eE_0\,cos\,\psi_R \qquad \text{(12)}$$

Consider the perturbed quantities β_x, β_y, $\tilde{\beta}_z = \beta - \beta_R$, $\tilde{\psi} = \psi - \psi_R$. The equation of motion in one transverse direction (x) is then governed by

$$\frac{1}{\gamma_R}\frac{d}{c\,dt}\left(\gamma_R \frac{dx}{c\,dt}\right) + k_\beta^2 x = 0 \qquad \text{(13)}$$

where the factor, $-\gamma_R^3 mc^2 / k_L$, has been divided out. The “wave number” k_β,

$$k_\beta^2 \quad = \quad \frac{W_R'}{W}\frac{\theta_L^2 N k_L}{2} tan\psi_R \tag{14}$$

is the length scale for the oscillatory "betatron" motion. If the change in γ_R can be ignored, the transverse motion has the form $x = x_{max}\, cos\left(k_\beta ct + \varphi\right)$, where x_{max} and φ are constants. Consider parameters relevant to the STELLA experiment: W_R' = 100 MeV/m; W = 50 MeV; θ_L = 20 mrad; $N \approx 1$; λ_L = 10 µm; and $\psi_R = 0.85\pi$. Then the betatron "wave number" is $k_\beta = 11.3\ \mathrm{m}^{-1}$.

Although Eq. (13) is for a single particle, it traces a figure in phase space that coincides with that of a shell distribution. Therefore, an emittance can be found. The normalized emittance is $1/\pi$ times the area of the closed curve that the particle traces in x-p_x phase space, where $p_x = \gamma dx/dz$. The semi-major axes of the phase-space ellipse are x_{max} and $\gamma k_\beta x_{max}$: thus $\varepsilon_N = \gamma k_\beta x_{max}^2$. We are interested in the filled-in distribution for which the shell corresponding to x_{max} is the phase space boundary. The emittance for this "top hat" distribution, averaging over all particles, is half that of the bounding shell: $\varepsilon_N = \gamma k_\beta x_{max}^2 / 2$. For axisymmetry (equal emittance in x and y) the cross-sectional area of the beam is $A = \pi x_{max}^2 = 2\pi\, \varepsilon_N / \gamma k_\beta$.

Consider the longitudinal particle dynamics accounting only for the laser force. Expand Eq. (11) to separate reference and perturbed parts. If $\beta_R \approx 1$, then

$$\frac{1}{\gamma_R^3}\frac{d}{c\,dt}\left(\gamma_R^3 \frac{d\tilde{\psi}}{c\,dt}\right) + k_{||}^2 \tilde{\psi} \quad = \quad 0 \tag{15}$$

where again the factor, $-\gamma_R^3 mc^2 / k_L$, has been divided out. The "wave number" $k_{||}$,

$$k_{||}^2 \quad = \quad \frac{W_R'}{W}\frac{k_L}{\gamma_R^2} tan\psi_R \tag{16}$$

is the length scale for longitudinal oscillation, and represents the bunching effect of the laser. If $\theta_L >> 1/\gamma_R$, the longitudinal oscillation is much slower than the transverse (betatron), i.e. $k_{||} << k_\beta$.

Consider the effect of space charge on longitudinal motion. The space charge force is $\mathbf{F} = -e\mathbf{E}$, where the space charge field satisfies Poisson's equation, $\nabla\cdot\mathbf{E} = -en/\varepsilon_0$. In the quasi-1D case $F_z = \gamma^3 mc^2 \left(\omega_p / c\right)^2 \left(z - z_R\right)$. Using the perturbed phase, $\tilde{\psi} = -k_L\left(z - z_R\right)$, the space-charge force to be included on the right side of Eq. (15) is

$$F_z \quad = \quad \left(\omega_p / c\right)^2 \tilde{\psi} \tag{17}$$

where again, $-\gamma_R^3 mc^2/k_L$, has been factored out. The plasma frequency can be expressed in terms of reference quantities. Given fixed cross-sectional area, continuity implies $n\ell_\mu = n_0\lambda_L$, where n_0 is the unbunched density and ℓ_μ is the full width (longitudinal) of a microbunch; all electrons in the microbunch lie in the range $-\ell_\mu/2 \le z-z_R \le \ell_\mu/2$. The unbunched case has $\ell_\mu = \lambda_L$. Define $\pm\psi_\mu$ as the phase of the extremal electrons, $z-z_R = \pm\ell_\mu/2$: then $n = n_0\cdot\pi/\psi_\mu$. With $Q_M = en_0A\ell_M$, we have $(\omega_p/c)^2 = \pi k_{SC}^2/\psi_\mu$, where the constant associated with space charge oscillations is

$$k_{SC}^2 \quad = \quad \frac{2r_e k_\beta}{\gamma_R^2 \ell_M \varepsilon_N}\cdot\frac{Q_M}{e} \tag{18}$$

Adding the space-charge force in the equation of motion, Eq. (15),

$$\frac{1}{\gamma_R^3}\frac{d}{c\,dt}\left(\gamma_R^3\frac{d\tilde{\psi}}{c\,dt}\right) + k_{||}^2\tilde{\psi} \quad = \quad \pi k_{SC}^2\frac{\tilde{\psi}}{\psi_\mu} \tag{19}$$

Solve this for the outermost electrons $\tilde{\psi} = \pm\psi_\mu$:

$$\frac{1}{\gamma_R^3}\frac{d}{c\,dt}\left(\gamma_R^3\frac{d\psi_\mu}{c\,dt}\right) + k_{||}^2\psi_\mu \quad = \quad \pm\pi k_{SC}^2 \tag{20}$$

The solution has two parts: oscillatory (homogeneous solution) and quasi-steady (inhomogeneous); our interest is in the latter. The quasi-steady phase spread is

$$\left|2\psi_\mu\right| \quad = \quad 2\pi\frac{k_{SC}^2}{k_{||}^2} \quad = \quad 2\pi\frac{r_e N\theta_L^2}{\ell_M\varepsilon_N k_\beta}\cdot\frac{Q_M}{e} \tag{21}$$

Practical example

Consider the parameters relevant to the STELLA experiment: $W_R' = 100$ Mev/m; $W = 50$ MeV; $\theta_L = 20$ mrad; $N \approx 1$; $\lambda_L = 10$ μm; $\psi_R = 0.85\pi$; $Q_M = 0.15$ nC; $\ell_M = 3$ mm; and $\varepsilon_N = 10^{-6}$(π m-rad), the quasisteady phase of the outermost electrons is $|2\psi_\mu|/\pi = 0.06$. Thus, for these conditions the phase spread by space charge effects is relatively small. However, this becomes more difficult for high-energy accelerators. The scaling of phase spread with important parameters is

$$\left|2\psi_\mu\right| \quad \propto \quad \left(\frac{W\lambda_L}{W_R'}\right)^{1/2}\cdot\frac{\theta_L Q_M}{\ell_M\varepsilon_N}$$

This suggests that for high energy (W) the space charge phase spread will become significant. This is a consequence of the following: the bunching effect of the laser, $k_{||}^2 \propto 1/W\gamma^2 \propto 1/\gamma^3$, decreases faster with increasing energy than the debunching effect of space charge, $k_{SC}^2 \propto k_\beta/\gamma^2 \propto 1/\gamma^{5/2}$. This tendency is mitigated if the acceleration gradient (W') is also increased.

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