RIKEN-TODAI
Mini-Workshop on “Topics in Hadron Physics at RHIC”
March 23-24, 2004

Organizers:
H. En’yo, H. Hamagaki, T. Hatsuda, Y. Watanabe, and K. Yazaki

Location:
Nishina Memorial Hall of RIKEN, Wako, Saitama, Japan

RIKEN BNL Research Center
Building 510A, Brookhaven National Laboratory, Upton, NY 11973-5000, USA
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Preface to the Series

The RIKEN BNL Research Center (RBRC) was established in April 1997 at Brookhaven National Laboratory. It is funded by the "Rikagaku Kenkyusho" (RIKEN, The Institute of Physical and Chemical Research) of Japan. The Center is dedicated to the study of strong interactions, including spin physics, lattice QCD, and RHIC physics through the nurturing of a new generation of young physicists.

During the first year, the Center had only a Theory Group. In the second year, an Experimental Group was also established at the Center. At present, there are four Fellows and eight Research Associates in these two groups. During the third year, we started a new Tenure Track Strong Interaction Theory RHIC Physics Fellow Program, with six positions in the first academic year, 1999-2000. This program had increased to include ten theorists and one experimentalist in academic year, 2001-2002. With five fellows having already graduated, the program presently has eleven theorists and three experimentalists. Of these eleven RHIC Physics Fellows, five have been awarded/offered tenured positions, and this will be their final year in the program.

Beginning in 2001 a new RIKEN Spin Program (RSP) category was implemented at RBRC. These appointments are joint positions of RBRC and RIKEN and include the following positions in theory and experiment: RSP Researchers, RSP Research Associates, and Young Researchers, who are mentored by senior RBRC Scientists. A number of RIKEN Jr. Research Associates and Visiting Scientists also contribute to the physics program at the Center.

RBRC has an active workshop program on strong interaction physics with each workshop focused on a specific physics problem. Each workshop speaker is encouraged to select a few of the most important transparencies from his or her presentation, accompanied by a page of explanation. This material is collected at the end of the workshop by the organizer to form proceedings, which can therefore be available within a short time. To date there are sixty proceeding volumes available.

The construction of a 0.6 teraflops parallel processor, dedicated to lattice QCD, begun at the Center on February 19, 1998, was completed on August 28, 1998. A 10 teraflops QCDOC computer in under development and expected to be completed this year.

N. P. Samios, Director
April 1, 2004

*Work performed under the auspices of U.S.D.O.E. Contract No. DE-AC02-98CH10886.
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Contact Information
Introduction

The RIKEN-TODAI Mini-Workshop on “Topics in Hadron Physics at RHIC” was held on March 23rd and 24th, 2004 at the Nishina Memorial Hall of RIKEN, Wako, Saitama, Japan, sponsored by RIKEN (Institute of Physical and Chemical Research) and TODAI (University of Tokyo). The workshop was planned when we learned that two distinguished theorists in hadron physics, Professors L. McLerran and S.H. Lee, would be visiting TODAI and/or RIKEN during the week of March 22-26. We asked them to give key talks at the beginning of the workshop and attend the sessions consisting of talks by young theorists in RIKEN, TODAI and other institutes in Japan and they kindly agreed on both. Considering the JPS meeting scheduled from March 27 through 30, we decided to have a one-and-half-a-day workshop on March 23 and 24.

The purpose of the workshop was to offer young researchers an opportunity to learn the forefront of hadron physics as well as to discuss their own works with the distinguished theorists.

We had a session consisting of two one-hour talks by Profs. McLerran and Lee and three sessions each consisting of four half-an-hour talks by young theorists. The titles of the two main talks are given below.

L. McLerran (RBRC) “What is the experimental evidence for the Color Glass Condensate?”
S. H. Lee (Yonsei Univ.) “Tensor method in pentaquark baryons – Mass, decay modes and ideal mixing –”

Although the workshop was announced only at the beginning of March, we had totally 31 participants including 18 graduate students and young postdocs.

The main talks were both inspiring and pedagogical, discussing the frontiers of hadron physics. The talks by young theorists were also well prepared and excellent. Discussions during the talks were active and fruitful. Profs. McLerran and Lee told us after the workshop that they had been very much impressed not only by the excellent presentations but also by the advanced contents of the talks of young speakers.

We are grateful to RIKEN for the financial support which enabled us to organize this workshop. It was held as an activity related to the collaboration with the RIKEN-BNL Research Center and we thank the director of the Center, Professor N. Samios, for the approval of publishing this proceedings as a volume of the RBRC Workshop Proceedings Series and general support. We are obliged to all the participants for making the workshop exciting and fruitful.

Special thanks are due to Ms.N. Kiyama, who did most of the administrative works and took care of drinks and snacks during the breaks, and Drs. M. Hirai, M. Ohtani and K. Sudoh for general assistance.

Hideto En’yo, Hideki Hamagaki, Tetsuo Hatsuda
Yasushi Watanabe and Koichi Yazaki

RIKEN,
May, 2004
Program of RIKEN-TODAI Mini-workshop

March 23 (Tue)

Session 1  Main lectures
Chair: T. Hatsuda (Tokyo)

10:00-10:10  H. En’yo (RIKEN)
Openning address
10:10-11:10  L. McLerran (RBRC)
"What is the experimental evidence for the Color Glass Condensate ?"
11:10-12:10  S. H. Lee (Yonsei)
"Tensor method for pentaquark baryons"

12:10-13:30  Lunch

Session 2  Various phases in QCD
Chair: K. Iida (RIKEN)

13:30-14:00  H. Abuki (YITP)
"On the role of strange quark mass in QCD phase diagram
--BCS-BEC crossover in color-flavor locked phase ?--"
14:00-14:30  M. Tachibana (RIKEN)
"Novel phases of high density quark matter and their roles in physical systems"
14:30-15:00  M. Ohtani (RIKEN)
"Color ferromagnetism and quantum hall states in quark matter"
15:00-15:30  Y. Hatta (Kyoto)
"Relation between the chiral and deconfinement phase transitions"
15:30-16:00  Coffee

Session 3  Dynamical aspects of the QCD plasma
Chair: H. Fujii (Tokyo)

16:00-16:30  K. Ohnishi (Tokyo)
"Mode coupling theory for the dynamic aspect of the chiral phase transition"
16:30-17:00  T. Ikeda (RBRC)
"The effect of memory on relaxation in \( \phi^4 \) theory"
17:00-17:30  N. Ishii (Titech)
"Glueball properties near QCD phase transition"
17:30-18:00  T. Doi (Titech)
"Thermal effects on quark-gluon mixed condensate from lattice QCD"
March 24 (Wed)

Session 4  Hard QCD, hadron spectroscopy
            Chair: K. Yazaki (RIKEN)

10:00-10:30  T. Takahashi (YITP)
            "Detailed lattice QCD study of the three-quark potentials:
            the ground-state and the excited-state of the static 3Q system"

10:30-11:00  M. Hirai (RIKEN)
            "Polarized parton distributions and their uncertainties"

11:00-11:30  K. Sudoh (RIKEN)
            "Comment on $\pi^0$ double spin asymmetry at RHIC"

11:30-12:00  C. Sasaki (Nagoya)
            "Chiral doubling of heavy-light hadrons in the vector manifestation"

End of workshop
List of Participants

Abuki, Hiroaki (YITP)
Akiba, Yasuyuki (RIKEN)
Doi, Takumi (Tokyo Institute of Technology)
En'yo, Hideto (RIKEN/RBRC)
Fuji, Hirotsugu (University of Tokyo)
Hatsuda, Tetsuo (University of Tokyo)
Hatta, Yoshitaka (Kyoto University)
Hirai, Masanori (RIKEN)
Iida, Kei (RIKEN)
Ikeda, Takashi (RBRC)
Ishii, Noriyoshi (Tokyo Institute of Technology)
Kohama, Akihisa (RIKEN)
Kojo, Toru (Kyoto University)
Lee, Suhoung (Yonsei University)
Mao, Yajun (Peking University)
Matsuura, Taeko (University of Tokyo)
Mawatarai, Kentaro (Kobe University)
Mclerran, Larry (RBRC)
Nisida, Yusuke (University of Tokyo)
Ohnishi, Kazuaki (University of Tokyo)
Ohta, Shigemi (KEK)
Ohatani, Munehisa (RIKEN)
Sasaki, Chihiro (Nagoya University)
Sasaki, Syoichi (University of Tokyo)
Sudoh, Kazutaka (RIKEN)
Suzuki, Katsuhiko (Numazu College of Technology)
Tachibana, Motoki (RIKEN)
Takahashi, Toru (YITP)
Taketani, Atsushi (RIKEN)
Torii, Hisayuki (RIKEN)
Watanabe, Yasushi (RIKEN/RBRC)
Yazaki, Koichi (Tokyo Woman's Christian University/RIKEN)
What Is The Experimental Evidence
For The Color Glass Condensate?

Larry McLerran
What is the Experimental Evidence for the Color Glass Condensate?

Larry McLerran

Nuclear Theory and Riken Brookhaven Research Center
BNL PO Box 5000
Upton NY 11973

Abstract

I discuss the Color Glass Condensate as a theory of the high energy limit of QCD. I discuss the experimental evidence concerning this hypothesis from HERA and RHIC.
What is the Evidence for the Color Glass Condensate?

Motivation:

\[ x G(x,Q^2) \]

\[ Q^2 = 20 \text{ GeV}^2 \]
\[ Q^2 = 200 \text{ GeV}^2 \]
\[ Q^2 = 5 \text{ GeV}^2 \]

\( x \)

\( 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1} \)

\( \frac{G,L,R}{\text{HERA}} \)
\( M,Q \)
\( M,V \)
\( \text{BFKL} \)

Low Energy

Gluon Density Grows

High Energy

Gluon Density Grows

Density high \( \rightarrow \)
Separation small \( \rightarrow \)
\( \alpha_s \ll 1 \)
What is Color Glass Condensate?

- Glue at large $x$ generates glue at small $x$
- Glue at small $x$ is classical field
- Time dilation $\rightarrow$ glassy
- High phase space density $\rightarrow$ condensate

Phase space density $\rho$
- Negative mass term $\sim -\rho$
- Repulsive interactions $\sim \alpha_S \rho^2$

$$\rho = \frac{1}{x^2} \frac{dN}{dx dy dp_T} \sim \frac{1}{\alpha_S}$$

Separation of longitudinal scales
Renormalization Group
Limiting Fragmentation

Total multiplicity is $\sim N_Q + N_T + N_G$
for projectile evaluated at $Q^2 \sim Q^2_{sat}(x_{target})$

Jalilian-Marian
Theory (Not Model) of CGC

Mathematically similar to a spin glass

\[ Z = \int_{X_0} [dA][d\rho] \exp(iS[A, \rho] - \chi[\rho]) \]

- \( \rho \) is the source
- \( X_0 \) is the cutoff in \( x \)
- \( \chi[\rho] \) is the weight function for the fields

M, V; K, B
JIMWLK; F;

\( \chi[\rho] \) satisfies RG equation derived from QCD

JIMWLK equation
Equivalent in limits to DGLAP, BFKL and BK equations

\( \chi[\rho] \) quadratic is MV model

\[ \ln(1/x) \]

\[ \ln(p_T) \]
Why CGC is Important:

It is a new universal type of matter.

Matter:
Separation of gluons is small compared to size.

New:
Can only be probed at high energy and it is produced in heavy ion collisions.

Universal:
Independent of hadron Fundamental
(Universality also applies in scaling region)

It is a theory of:

The origin of glue and sea quarks in hadrons.

The origin and nature of cross sections and particle production

The distribution of valence quantum numbers at small $x$

Provides initial conditions for the QGP as produced in heavy ion collisions.
Infinite Momentum Frame:

In rest frame
Valence quarks and occasional glue and sea quarks

Nucleon is made of states with 3 quarks
3 quarks with one gluon
3 quarks with two gluons

......
3 quarks with 1000 gluons

......

At low energy:
3 quarks and few gluons

At high energy:
Lots of gluons

Description is frame independent:

Mueller-Kovchegov Duality:
Infinite momentum frame multigluon states are important.

Target rest frames:
Coherent multiple scattering.

Dipole Model Included in CGC Framework
Iancu-Mueller
What does the gluon wall look like?

On sheet $x^- = t - z$ is small
Independent of $x^+ = t + z$
$F^{i+} \sim \partial/\partial x^- A^i$ is big.
$F^{i-}$ and $F^{ij}$ are small
$F^{i\pm} \sim E^i \pm \epsilon^{ij} B^j$

$\vec{E} \perp \vec{B} \perp z$
Fields random in color

The density of gluons per unit area:

$$\frac{1}{\pi R^2} \frac{dN}{dy} \sim \frac{1}{\alpha_s} Q_s^2(x)$$
Experimental Evidence:
HERA

\[ \sigma \sim F \left( \frac{Q^2}{Q_{sat}^2(x)} \right) \]

\( Q_{sat}^2(x) \) dependence on \( x \) computed by Mueller and Triantafyllopoulos

First observed by G-B,K,S. Geometric scaling shown to be property of CGC by IIM

For \( Q^2 \ll Q_{sat}^2 \) trivial
Works til \( Q^2 \sim Q_{sat}^2/\Lambda^2 \)
Needs \( x \leq 10^{-2} \)
Experimental Evidence:

$F_2$ for $x < 10^{-2}$ and $Q^2 < 50 \text{ GeV}^2$

Must include computable deviations from geometric scaling

3 parameters: $R, x_0, \lambda$

$$N(x, r_T) = \exp\left\{ -\frac{1}{2\kappa \lambda} \ln(1/x) \ln^2(1/r_T^2 Q_{sat}^2) \right\}$$
Experimental Evidence:

\( F_2 \) for \( x < 10^{-2} \) and \( Q^2 < 50 GeV^2 \)

Early work by Golec-Biernat, Wustoff, Stasto, Kwieceinski, Kowalski, Bartels, Teaney.
Work above by Iancu, Itakura and Munier
Experimental Evidence:

Diffraction and Quasi-Elastic

\[ \gamma \rightarrow q \bar{q} \]

\[ M^2 \]

\[ \text{ZEUS 1994} \]

\[ \theta / \sigma_{\text{tot}} \]

\[ W(\text{GeV}) \]

G-B, W; M, H; K, M; G-B, B; K, T;
Experimental Evidence:
Diffraction and Quasi-Elastic

\[ \gamma^* p \rightarrow \rho p \]
Some sensitivity to \( \rho \) wavefunction
Mueller, Munier and Stasto

\[ \gamma^* p \rightarrow J/\Psi p \]
Teaney and Kowalski
Experimental Evidence:

Qualitative Understanding of Hadronic $\sigma^{tot}$

The total hadronic cross section:

![Graph showing $\sigma_{inel}$ and $\sigma_{elastic}$ cross sections against center of mass energy.]

Nucleon becomes dark at some impact parameter $b$ when the probability to scatter is of order one

The number of gluons increases as

$$(x_0/x_{min})^\lambda$$

$x_{min} \sim \Lambda_{QCD}/E$

The impact parameter distribution is

$exp(-2m_\pi b)$.  

$$(x_0/x_{min})^\lambda exp(-2m_\pi b) \sim 1$$

or

$$\sigma \sim b^2 \sim ln^2(1/x_{min}) \sim ln^2(E)$$

Froissart Bound Saturation
Heavy Ion Collisions

M.K.W; G. M; Krasnitz, Nara, Venugopalan

[Diagram showing energy density over time]

10^7
10^6
10^5
10^4
10^3
10^2
10^1

Energy Density (GeV/fm^3)

Time (fm/c)

Nuclear Matter

Energy Density of Nuclear Matter

Energy Density in Cores of Neutron Stars

Energy Density ~ 10-30 times that inside a proton

Melting Colored Glass

Energy Density at Formation e ~ 10-30 GeV/fm^3

Quark Gluon Plasma

Mixed Hadron Gas and Quark Gluon Plasma
Evidence from Heavy Ions:

Krasnitz and Venugopalan

CGC is only theory which allows consistent computation of multiplicity

\[
\frac{1}{\pi R^2} \frac{dN}{dy d^2p_T} \sim \frac{1}{\alpha_s} \frac{Q_{sat}^4}{p_T^2}
\]

for \( p_T \gg Q_{sat} \)

For \( p_T \leq Q_{sat} \), the distribution softens and becomes integrable:

\[
\frac{dN}{dy} \sim \frac{1}{\alpha_s} Q_{sat}^2
\]
Global Features of Heavy Ion Collisions

\[ \frac{1}{N_{\text{part}}} \frac{dN}{dy} \sim \frac{1}{\alpha_s} \]

CGC initial conditions ala Kharzeev, Levin and Nardi lead to good rapidity and \( p_T \) distributions
Nara and Hirano

Not so much processing of \( dN/dy \)
AA and dA Collisions:
Probing the Nuclear Wavefunction

Heavy Ion Experiments at
central rapidities give
evidence of strong suppression
relative to incoherent scattering

Is it an initial state
effect (CGC)
or final state (QGP)

Gluons and quarks come
from $x \sim 10^{-1}$
Not small $x$

Some features of data look like
geometrical scaling: $K, L, M$

Conclusively established:
Jet quenching at central
rapidity is largely a
final state effect (QGP).
dA Collisions and Shadowing:

What about the rapidity dependence of $R_{dA}$?

Cronin enhancement at nuclear rapidity
Small effect at zero rapidity
Forward rapidity? $x \sim 10^{-3}$.

Quantum evolution plus classical rescattering for CGC give suppression
Centrality and rapidity dependence different
Very rapid evolution in Color Glass Condensate

K, K
A, A, K, S, W
Classical multiple scattering gives Cronin enhancement
More matter at small $x$ bigger Cronin enhancement

G, V, W

16
dA Collisions: Data

Brahms shows Cronin enhancement turn into suppression in deuteron fragmentation region

Brahms shows suppression increase as function of centrality

At lower $p_T$, star shows suppression increase as one goes to fragmentation region of deuteron
dA Data: More

Phobos data at fixed $p_T$ as function of rapidity.

Phenix data for $R_{dAu}$ of $J/\Psi$ as function of centrality for different rapidities.

Phenix $R_{dAu}$ for stopped hadrons and charm decays with $1 \text{ GeV} < p_T < 3 \text{ GeV}$ as function of rapidity and centrality.
dA: What does it mean?

Perturbative QCD for single particle inclusive should work at $p_T$ of several GeV

If Hijing with shadowing fits the forward data it is off by about a factor of two for the central data!

If this is "corrected" for multiple scattering one can arrange it to fit the central region but then it is almost a factor of 2 too large in the forward region!

Accardi; Leval et. al. QM2004

But surely someone will find some shadowing computation

CGC provides a first principles theory of shadowing. It is falsifiable!
Does Color Glass Describe Brahms Data?

Jamal Jalilian-Marian:
Use CGC fits to DIS
and saturation momentum determined from RHIC multiplicity measurements
No free parameter
Summary

"This is nothing short of a major discovery"
"it's going to trigger a real revolution in nuclear physics"
D. Kharzeev

Revolutions:

A re-alignment of relationships between large groups of people

Friends try to kill one another. Sometimes successfully!

Has bad consequences if you are not successful!

Perhaps the most famous scientific revolution:
The Copernican Revolution:

Ptolemaic theory could describe data much better than Copernican long after Copernican was accepted.

Copernicus had a simple intellectual framework.

It unified a large amount of different phenomena.

It was falsifiable.
Tensor Method for Pentaquark Baryons: Mass, Decay Modes and Ideal Mixing

Su Houng Lee
This talk consists of two parts. In the first part, I will start by listing the experiments that reported observing the pentaquark states. Then I will discuss the soliton approach for the pentaquark, which motivated the experimental search for pentaquark states, and also discuss the criticisms related to the validity of the soliton approach. Then I will discuss other approaches on pentaquark states.

In the second part of my talk, I will introduce the tensor method based on SU(3) flavor symmetry. From constructing invariants composed of anti decuplet, decuplet and octet representations, I will rederive the mass formula for the pentaquark states, which follows from the SU(3) symmetry, and also discuss possible interaction terms. I will also discuss why ideal mixing and fall apart decay naturally follows from an effective OZI rule. The simple tensor method provides specific ratios and selection rules from which one can experimentally test and discriminate between octet and decuplet pentaquark states.

Tokyo 2004
Hadronic world

The Quark Idea
(up, down, strange)

(bottom)

(charm)

(top)

1960

1970

1980

1990

2000

2010

1920

1930

1940

1950

1960

1890

...and many more!

N(939)

Σ(1190)

Δ(1115)

Ξ(1320)

Δ(1232)

Σ*(1383)

Ξ*(1532)

Ω(1673)

Θ(1540)

Σ ?

Ξ(1862)

N ?

Tokyo 2004
Experimental Summary

1. LEPS found $\Theta^+$ in K+ n invariant mass: Y=2 state with: uudd bar(s)

2. Class finds no $\Theta^{++}$ in K+ P invariant mass: anti-decuplet

$$3 \otimes 3 \otimes 3 \otimes 3 \otimes \overline{3} = 35 + (3)27 - (2)10 + (4)10 - (8)8 + (3)1.$$  

<table>
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<th>Basic reaction</th>
<th>$M(\Theta^+)$ in MeV</th>
<th>$\Gamma(\Theta^+)$ in MeV</th>
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<td>$\gamma n \rightarrow K^+K^-n$</td>
<td>$1540 \pm 10$</td>
<td>$\leq 25$</td>
<td>LEPS [1]</td>
</tr>
<tr>
<td>$K^+Xe \rightarrow K^0pXe'$</td>
<td>$1539 \pm 2$</td>
<td>$\leq 9$</td>
<td>DIANA [2]</td>
</tr>
<tr>
<td>$\gamma d \rightarrow K^+K^-pn$</td>
<td>$1542 \pm 5$</td>
<td>$\leq 21$</td>
<td>CLAS [3]</td>
</tr>
<tr>
<td>$\gamma p \rightarrow K^+K^0n$</td>
<td>$1540 \pm 4 \pm 2$</td>
<td>$\leq 25$</td>
<td>SAPHIR [4]</td>
</tr>
<tr>
<td>$\gamma p \rightarrow \pi^+K^-K^+n$</td>
<td>$1537 \pm 10$</td>
<td>$\leq 31$</td>
<td>CLAS [5]</td>
</tr>
<tr>
<td>$\nu_\mu(\bar{\nu}_\mu) + A \rightarrow \mu^- (\mu^+) pK^0S X$</td>
<td>$1533 \pm 5$</td>
<td>$\leq 20$</td>
<td>BBCN [6]</td>
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<tr>
<td>$\gamma p \rightarrow \pi^+K^-K^+n$</td>
<td>$1555 \pm 10$</td>
<td>$\leq 26$</td>
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<tr>
<td>$ed \rightarrow pK^0S X$</td>
<td>$1526 \pm 2 \pm 2.5$</td>
<td>$\leq 7.5$</td>
<td>HERMES [8]</td>
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</table>

**TABLE I:** Summary of the experimental data for the $\Theta^+(1540)$ baryon.

3. NA49 hep-ex/0310014: found $\Xi^*(1862)$ in $\Xi^- \pi^- (\text{ddss bar(u)})$ with width $< 18$ MeV

4. H1 hep-ex/04031017: found $\Theta c0(3099)$ in $D^* p$ (uudd bar(c)) with width $= 12 \pm 3$ MeV (not D p, due to experimental diff?)

*Tokyo 2004*


**Solition Model: original prediction** (Diaknov, Petrov, Polyakov 97)

1. SU(3) solition

\[ L_{Kin}(U^{2}) + L_{Skyrme}(U^{4}) + L_{W-Z} \]

\[ U(x,t) = R(t) \begin{pmatrix} \exp(i\pi \cdot r) & 0 \\ 0 & 1 \end{pmatrix} R^\dagger(t) \]

2. Hamiltonian

\[ H^{Rot} = \frac{1}{2I_1} \sum_{A=1}^{3} \hat{J}_{A}^{12} + \frac{1}{2I_2} \sum_{A=4}^{7} \hat{J}_{A}^{12} \]

\[ E_{10} - E_{8} = \frac{3}{2I_1}, \quad E_{10} - E_{8} = \frac{3}{2I_2} \]

\[ J''_8 = -\frac{N_c B}{2\sqrt{3}} \]

\( \rightarrow \) only SU(3) representations containing Hypercharge= \( N_c /3 \) are allowed

moreover, the number of states at S=0 must determine the spin of the representation because \( l=J \) in the SU(2) soliton

3. Anti decuplet mass:

including \( O(m_{sN_c}) \) and \( O(m_{sN_c}^0) \) splitting

splitting within anti-decuplet: fixed by splitting within octet and decuplet and

splitting among representation: fixed by \( N(1710) \) to be a member of anti decuplet

4. Anti decuplet width:

including \( O(1/N_c) \) correction

---

*Tokyo 2004*
Solition Model: comparison with experiment

<table>
<thead>
<tr>
<th></th>
<th>Solition model</th>
<th>experiment</th>
<th>Quark model</th>
<th>SU(3)</th>
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<td>&gt;140</td>
<td>1862</td>
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<td></td>
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<td></td>
<td>Only s = ( \frac{1}{2} ) antidecuplet</td>
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<tr>
<td>Θ_c</td>
<td></td>
<td>2700</td>
<td>3099</td>
<td>&lt;12</td>
</tr>
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</table>

Tokyo 2004
**Solition Model for antidecuplet: why it is an artifact of large \( N_c \)**
(T. Cohen 03)

1. Solution picture is valid at large \( N_c \):
   Semi-classical quantization is valid for slow rotation; i.e., valid for describing excitations of order \( 1/N_c \), so that it does not mix and breakdown with vibrational modes of order 1

2. Lowest representation at large \( N_c \) \( (p,q) \)

   - Octet \( (1, \frac{N_c - 1}{2}) \)
   - Decuplet \( (3, \frac{N_c - 3}{2}) \)
   - Anti decuplet \( (0, \frac{N_c + 3}{2}) \)

   (lowest representation containing \( s = 1 \)) \( Y = \frac{N_c B}{3} + S \)

\[
H_{\text{Rot}} = \frac{1}{2I_1} \sum_{A=1}^{3} \hat{J}^{i2}_A + \frac{1}{2I_2} \sum_{A=4}^{7} \hat{J}^{i2}_A
\]

\[
E_{i0} - E_8 = \frac{3}{2I_1}, \quad E_{10} - E_8 = \frac{3 + N_c}{4I_2}
\]

*Tokyo 2004*
Bound state approaches for SU(3) soliton

1. SU(2) soliton + Kaon \[ L_{\text{Kin}}(U^2) + L_{\text{Skyrme}}(U^4) + L_{W-Z} \]

2. Successful for hyperon (attractive anti-kaon) but repulsion for kaon (no pentaquark) from WZ term

3. But found to give bound state for charmed meson (uudd bar(c)) with mass 2700 MeV.
   But H1 found \[ \Theta_c^0 \rightarrow D^* + p \] at 3099 MeV?


   1. for heavy-light meson: chiral partners exist: BaBar (D_s*), CLEO, Belle
      \( (0^-,0^+) \ (1^-,1^+) \) with mass 350–400 MeV apart

   2. H1 observed chiral partner of ground state \( \Theta_c(pD) \)

   3. If this is right \( \rightarrow \) there should be ground state charmed pentaquark at 2700 MeV and the theta pentaquark?

*Tokyo 2004*
**Tensor notation of flavor SU(3)**

What are model dependent results and what are the consequences of SU(3)?

1. Fundamental representation \( q^i \rightarrow U^i_j q^j \)

2. Octet representation \( B^i_\alpha \rightarrow U^i_\alpha B^\alpha_\beta U^{\dagger \beta}_j \)

3. Anti-decuplet representation

\[ P_{ijk} \rightarrow P_{abc} U^{\dagger \alpha}_i U^{\dagger \beta}_j U^{\dagger \epsilon}_k, \]

4. Construction of effective Lagrangian

contract each index of representation with delta and epsilon

\[ \delta^j_m U^i_j U^{\dagger \epsilon}_m = \delta^i_l \]

\[ \varepsilon_{ijk} U^i_l U^j_m U^k_n = \varepsilon_{lmn} \]

*Tokyo 2004*
SU(3) mass formulas (m_s)

- QCD mass term
  \[ H_{QCD} = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} q = \bar{q}^i Y_i^j q_j \]

- In Hadronic world, there are no quarks and Hadrons form higher irreducible representation of SU(3)

- Octet representation mass term
  \[ H_{octet} = a \bar{B}^i_b Y^i_l B^l_i + b \bar{B}^i_b Y^i_l B^l_j + c \bar{B}^i_b B^i_j \]

4 masses - 3 parameters
Gell-Mann Okubo mass formula

\[ 3M_\Lambda + M_\Sigma = 2(M_N + M_\Xi) \]

3x1115+1190=2x(939+1320)

4535 MeV = 4518 MeV

Tokyo 2004
SU(3) mass formula

- Decuplet representation mass term

\[ H_{Decuplet} = a \bar{T}^{ijk} Y^l T_{lj} + c \bar{T}^{ijk} T_{ijk} \]

4 masses–2 parameters

\[ M_\Omega - M_\Xi^* = M_\Xi^* - M_\Sigma^* = M_\Sigma^* - M_\Delta \]

140 MeV  150 MeV  150 MeV
Mass formula for anti–decuplet

- $\Theta^+(uudd\bar{s})$ is an isosinglet; a member of anti–decuplet

\[
H_{\text{Anti-decuplet}} = a T^{ijk} Y^l T_{lij} + c T^{ijk} T_{ijk}
\]

Again 2 parameters

\[
M_{\Xi_{10}} - M_{\Sigma_{10}} = M_{\Sigma_{10}} - M_{N_{10}} = M_{N_{10}} - M_{\Theta}
\]

Since $M_{\Theta} = 1540$ MeV, $M_{\Xi_{3/2}} = 1862$ MeV

We will have

\[
M_{N_{10}} = 1647 \text{ MeV}, \quad M_{\Sigma_{10}} = 1755 \text{ MeV}
\]
SU(3) symmetric interaction is not so bad to begin with

\[ L = g_1 B^i_j P^i_j B^i_i + g_2 B^i_j P^i_j B^i_i \]

Nijmegen potential uses SU(3)symmetry for NN, YN interactions.

For Decuplet meson Baryon, only one term is possible

\[ L = g \varepsilon_{ilm} D^{ijk} P^l_j M^m_k \]

\[ \Gamma(\Delta \rightarrow N\pi) = \frac{3G_0^2}{2\pi(M_\Delta + M_N)^2} |\vec{p}|^3 \frac{M_N}{M_\Delta} \cdot \frac{1}{5} = 110 \text{ MeV} vs. 110 \text{ MeV (exp.)}, \]

\[ \Gamma(\Sigma^* \rightarrow \Lambda\pi) = \frac{3G_0^2}{2\pi(M_{\Sigma^*} + M_\Lambda)^2} |\vec{p}|^3 \frac{M_\Lambda}{M_{\Sigma^*}} \cdot \frac{1}{10} = 35 \text{ MeV} vs. 35 \text{ MeV (exp.)}, \]

\[ \Gamma(\Sigma^* \rightarrow \Sigma\pi) = \frac{3G_0^2}{2\pi(M_{\Sigma^*} + M_\Sigma)^2} |\vec{p}|^3 \frac{M_\Sigma}{M_{\Sigma^*}} \cdot \frac{1}{15} = 5.3 \text{ MeV} vs. 4.8 \text{ MeV (exp.)}, \]

\[ \Gamma(\Xi^* \rightarrow \Xi\pi) = \frac{3G_0^2}{2\pi(M_{\Xi^*} + M_\Xi)^2} |\vec{p}|^3 \frac{M_\Xi}{M_{\Xi^*}} \cdot \frac{1}{10} = 8.6 \text{ MeV} vs. 10 \text{ MeV (exp.)}, \]

Tokyo 2004

Diakonov, Petrov, Polyakov (97)
Search for remaining anti-decuplet states

Very small hadronic interaction

Y. Oh, H. Kim, SH Lee (hep-ph/0310117)

\[ L_{\text{10-8-10}} = -ig\varepsilon_{ilm} \overline{T}^{ijk} M^l_j B^m_k \]

\[ \Theta \quad \overline{g} \quad N \quad K \]

\[ \Gamma_{\Xi_{10}} = 3.5 \times \Gamma_{\Theta} < 18 \text{ MeV} \]

Forbidden decay

\[ L_{\text{10-8-10}} = g\varepsilon_{iji}\varepsilon_{mno} \overline{T}^{ijk} M^l_k D^{mno} = 0 \]

N(1710) \rightarrow \Delta \pi

Hence can not be a pure anti-decuplet

Tokyo 2004
Pentaquark Baryons can mix
(Follow the quark picture of Jaffe and Wilczek)

Previous scenario is possible but expect to have a pentaquark octet

\[ 3 \otimes 3 \otimes 3 \otimes 3 \otimes 3 \]
\[ \rightarrow (\bar{3} \oplus 6) \otimes (\bar{3} \oplus 6) \otimes \bar{3} \]
\[ \rightarrow \bar{6} \otimes \bar{3} = 10 + 8 \]
**Mixing in Vector Meson Octet**

Vector meson octet mass formula

\[
\begin{align*}
\omega_8 = & \frac{1}{\sqrt{3}} \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d) - \frac{2}{\sqrt{3}} \bar{s}s, \\
\omega_1 = & \frac{1}{\sqrt{3}} \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d) + \frac{2}{\sqrt{3}} \bar{s}s,
\end{align*}
\]

\[H_{octet} = a \bar{M}_i^j Y_j^l M_l^i + b \bar{M}_i^j Y_j^l M_l^i + c \bar{M}_i^j M_j^i \]

\[3 M_{\omega_8}^2 + M_{\rho}^2 = 4 (M_{K^*}^2) \]

\[\rightarrow M_{\omega_8} = 900 \text{ MeV} \]

\[
\begin{align*}
\omega_8 = & \frac{1}{\sqrt{3}} \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d) - \frac{2}{\sqrt{3}} \bar{s}s, \\
\omega_1 = & \frac{1}{\sqrt{3}} \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d) + \frac{2}{\sqrt{3}} \bar{s}s,
\end{align*}
\]

\[
\begin{align*}
\omega_8 & = \frac{1}{\sqrt{3}} \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d) - \frac{2}{\sqrt{3}} \bar{s}s, \\
\omega_1 & = \frac{1}{\sqrt{3}} \frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d) + \frac{2}{\sqrt{3}} \bar{s}s,
\end{align*}
\]

\[\tan \theta_w = \frac{1}{\sqrt{2}}, \text{ mixing angle from wave function} \]

\[M_{\omega_8}^2 \approx \frac{1}{3} M_{\omega}^2 + \frac{2}{3} M_{\rho}^2, \quad \tan \theta_M \approx \frac{1}{\sqrt{2}}, \text{ mixing angle from mass} \]

**Ideal mixing** : \(\omega_1, \omega_8 \rightarrow \omega, \phi\)

\[\tan^2 (\theta_w - \theta_M) \approx 10^{-3} \]

*Tokyo 2004*
When is Angle from Mass or wave function equal?

1. Angle from diagonalizing mass matrix

\[
H = \begin{pmatrix} E_8 & \Delta \\ \Delta & E_1 \end{pmatrix} \rightarrow \begin{pmatrix} \phi \\ \omega \end{pmatrix} = \begin{pmatrix} \cos \theta_M & \sin \theta_M \\ -\sin \theta_M & \cos \theta_M \end{pmatrix} \begin{pmatrix} \psi_8 \\ \psi_1 \end{pmatrix} \rightarrow \frac{E_8 - E_1}{\Delta} = \frac{\cos^2 \theta_M - \sin^2 \theta_M}{2 \cos \theta_M \sin \theta_M}
\]

2. Angle from wave function

\[
\psi_8 = \cos(\theta_W) \phi - \sin(\theta_W) \omega \\
\psi_1 = \cos(\theta_W) \omega + \sin(\theta_W) \phi, \quad \text{when does } \theta_W = \theta_M \\
\]

Hint from looking at correlator,

\[
\frac{<\psi_8,\psi_8> - <\psi_1,\psi_1>}{<\psi_8,\psi_1>} \rightarrow \frac{(\cos^2 \theta_W - \sin^2 \theta_W)[<\phi,\phi> - <\omega,\omega>]}{2 \cos \theta_W \sin \theta_W [<\phi,\phi> - <\omega,\omega>]},
\]

hence, \(\theta_M = \theta_W\) when \(<\phi,\omega> = 0\) \(\rightarrow\) OZI rule
In terms of quarks, ideal mixing

1. OZI rule

\[ <\bar{u}u, \bar{u}u>, <\bar{d}d, \bar{d}d> \not\rightarrow <\bar{s}s, \bar{s}s> \quad \Rightarrow \quad <\bar{u}u, \bar{s}s> \]

\[ u \quad \Rightarrow \quad u \quad \Rightarrow \quad \bar{u} \quad \Rightarrow \quad \bar{u} \quad \Rightarrow \quad \bar{s} \]

2. No mixing in pseudo scalar

\[ <\bar{u}u, \bar{u}u>, <\bar{d}d, \bar{d}d>, <\bar{s}s, \bar{s}s> \Rightarrow \text{dominated by instantons} \]

\[ u \quad \Rightarrow \quad \bar{u} \quad \Rightarrow \quad \bar{u} \quad \Rightarrow \quad \bar{s} \]
For pentaquark?

1. Very small pseudo scalar component, hence everything can be assumed to be dominated by connected diagrams: But really an assumption

   a) Mass: Ideal mixing (Jaffe, Wilczek)

   b) Decay width: Fall apart decay (Close, Dudek)
Mass in Ideal mixing scenario of pentaquarks

\[ \bar{6} \otimes \bar{3} = 10 \oplus 8 \]

8 masses - 3 - 2 parameters

\[
\begin{align*}
M_{N_q} + 2M_{N_s} &= 2M_\Theta + M_{\Xi_{10}} \\
2M_{\Sigma_q} + M_{\Xi_s} &= M_\Theta + 2M_{\Xi_{10}} \\
3M_{\Lambda_8} &= M_{\Sigma_q} + M_{N_q} + 2M_{\Xi_s} - M_{\Xi_{10}}
\end{align*}
\]

\[ \begin{array}{c c c c c}
10 & 8 & 10+8 \\
\Theta_{10} &=& uudd\bar{s} \\
N_{10} &=& \sqrt{\frac{1}{3}} uudd\bar{d} + \sqrt{\frac{2}{3}} uud\bar{s} \\
\Sigma_{10} &=& \sqrt{\frac{2}{3}} uuds\bar{d} + \sqrt{\frac{1}{3}} uuss\bar{s} \\
\Xi_{10} &=& uuss\bar{d} \\
\end{array} \]

\[ \begin{array}{c c c c c}
& N_q &=& uudddd \\
& \Theta_{10} &=& uudd\bar{s} \\
& \Sigma_q &=& uuds\bar{d} \\
& N_s &=& uud\bar{s}s \\
& \Xi_{10} &=& uuss\bar{d} \\
& \Sigma_s &=& uussss \\
\end{array} \]

Tokyo 2004
1. One possible scenario

\[ M_{\Xi_8} = M_{\Xi_{10}} = 1862 \text{ MeV} \]

\[ M_{\Lambda_8} = M_{\Sigma_q} \]

\[ M_{N_q} = 1440 \text{ MeV} \]

then

\[ M_{N_s} = 1751 \text{ MeV} \]

\[ M_{\Lambda_8} = M_{\Sigma_q} = 1651 \text{ MeV} \]

\[ M_{\Sigma_s} = 1962 \text{ MeV} \]
Tensor method for decay modes for ideally mixed pentaquarks

hep-ph/040235, SHL, H. Kim, Y. Oh

$$6 \otimes \bar{3} = 10 \oplus 8, \quad S^{ij} \otimes \bar{q}^k = T^{ijk} \oplus S^{[ij,k]}$$

For the octet field,

$$S^{[ij,k]} = \varepsilon^{ijk} P^i_l + \varepsilon^{lik} P^i_l$$

$$L_{888} = (d + f)P^i_j M^l_i B^j_l + (d - f)P^i_j B^l_i M^j_i$$

$$L = -ig\varepsilon_{ilm}(\bar{T}^{ijk} + S^{[ij,k]} ) M^l_j B^m_k \rightarrow f / d = 1/3$$

where $$S^{[ij,k]} = \varepsilon^{ijk} P^i_l + [i \leftrightarrow j]$$
Decay modes for Ideally mixed pentaquarks

All the pentaquarks are Narrow: Close, Dudek hep-ph/0401192, hep-ph/040235, Y. Oh, H. Kim, SHL

The difference between octet and anti decuplet decays are just additive

\[
L = -ig(\Sigma^0 KN + \Sigma^0 \pi \Sigma) - ig(\Sigma^A KN - \Sigma^A \pi \Sigma)
\]
Detection of octet Pentaquark Baryons?

\[ 3 \otimes 3 \otimes 3 \otimes 3 \otimes 3 \]
\[ \rightarrow (\bar{3} \oplus 6) \otimes (\bar{3} \oplus 6) \otimes \bar{3} \]
\[ \rightarrow \bar{6} \otimes \bar{3} = 10 + 8 \]

NA49 observed Pentaquark from

\[ \Xi^0_{10}(1862) \rightarrow \Xi^-(1321) + \pi^+ \]

Future experiments could observe octet Pentaquark from

\[ \Xi^0_8(1862 \pm \delta) \rightarrow ? \rightarrow \Xi^*_{10}(1530) + \pi^+ , \text{ with large width but ,} \]
\[ \Xi^0_{10}(1862) \otimes \Xi^-_{10}(1530) + \pi^+ \]

Octet pentaquark exists without large mixing with anti decuplet pentaquark

If none is observed, look at branching ratios into octet decay and see if there is octet component

Tokyo 2004
### Pentaquark anti-decuplet decay

<table>
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<tr>
<th></th>
<th>( \Theta^+ )</th>
<th>( \tilde{N}^+ )</th>
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<td>( K^- p )</td>
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### Pentaquark octet decay

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<td>( \eta \Xi^0 )</td>
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<td>( K^0 \Sigma^0 )</td>
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<td>( K^0 \Xi^- )</td>
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Ideally mixed Pentaquark Nucleon and sigma decay modes, which are expected to be also narrow

<table>
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<tr>
<th>$\Sigma_{qq}$</th>
<th>$\Sigma_{ss}$</th>
<th>$\Sigma_{qq}$</th>
<th>$\Sigma_{ss}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+ \Sigma^0$</td>
<td>$\sqrt{\frac{3}{2}}$</td>
<td>$K^+ \Xi^0$</td>
<td>$\sqrt{6}$</td>
</tr>
<tr>
<td>$\pi^0 \Sigma^0$</td>
<td>$-\sqrt{\frac{3}{2}}$</td>
<td>$\eta_{qg} + \Sigma^+$</td>
<td>$-\sqrt{6}$</td>
</tr>
<tr>
<td>$\eta_{qg} \Sigma^+$</td>
<td>$\sqrt{\frac{3}{2}}$</td>
<td>$\eta_{qg} \Sigma^-$</td>
<td>$\sqrt{\frac{3}{2}}$</td>
</tr>
<tr>
<td>$\pi^+ \Lambda$</td>
<td>$-\sqrt{\frac{3}{2}}$</td>
<td>$\eta_{qg} \Sigma^-$</td>
<td>$\sqrt{\frac{3}{2}}$</td>
</tr>
<tr>
<td>$\bar{K}^0 \bar{p}$</td>
<td>$-\sqrt{3}$</td>
<td>$K^- n$</td>
<td>$-\sqrt{3}$</td>
</tr>
</tbody>
</table>

Tokyo 2004
Summary

1. If one finds s=3/2 anti-deuplet pentaquark $\rightarrow$ soliton picture is ruled out.

2. If one finds charmed pentaquark at 2700 MeV $\rightarrow$ supports bound state approach for solitons and chiral doubling

3. If octet pentaquark is observed $\rightarrow$ strongly supports diquark picture.

4. Should look for remaining potentially narrow states. Starting with no mixing.

5. By looking at $\Xi_0$ I decay in more detail, can learn if there is other octet pentaquark states and some idea about ideal mixing.

Tokyo 2004
On The Role of Strange Quark Mass In QCD Phase Diagram
--BCS-BEC Crossover in Color-Flavor Locked Phase?--

Hiroaki Abuki
On the role of strange quark mass in QCD phase diagram

- BCS-BEC crossover in color-flavor locked phase? -

Hiroaki Abuki

Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto, Japan

Abstract

We revisited the unlocking transition from the color-flavor locked (CFL) phase to the two-flavor pairing (2SC) phase paying a special attention to the multiplicity of the gap parameters. By deriving the exact formula of the effective potential for 5-gap parameters, we show that the CFL phase is much robust against the increase of the strange quark mass than is predicted by the kinematical criterion for the unlocking. As temperature is decreased, the CFL phase gets stronger against the strange quark mass. However, we could not conclude at which point the CFL state gets unlocked to the 2SC state just on $T = 0$ for some numerical difficulties and also for the limit of the adopted model. Even a possibility of surviving CFL pairs for all finite value of strange quark mass manifests itself at zero temperature. In this case, the 2SC phase may be realized only as the asymptotic limit of the distorted CFL state as $M_s \to \infty$. In contrast to this ambiguous situation at zero temperature, we can find a clear 2nd order phase transition from the CFL to the 2SC at finite temperature. At low temperature, we found the distorted CFL state even for the region $M_s > \mu_s$ and more detailed analyses indicate a power law scaling relation between the strange quark density $\rho_s$ and the unlocking critical temperature $T_c$ in this region: $T_c \sim \rho_s^\alpha$. These two facts strongly suggest that the CFL state undergoes the smooth crossover from the weak coupling BCS-like state to the Bose-Einstein condensed state of bound the CFL diquarks as one increases (decreases) strange quark mass (density).
On the role of strange quark mass in QCD phase diagram

BCS-BEC crossover in Color-Flavor Locked Phase?

Miroaki Abuki (YITP, Kyoto Univ., Japan)

based on H. Abuki, arXiv:hep/0401245

1. Quark pairing phenomena in QCD
2. 2SC pairing and CFL pairing
3. Dynamical effect of strange mass
4. Unlocking transition revisited
5. Asymptotic Smooth Unlocking?
6. BCS-BEC crossover in the CFL phase?
7. Summary and future problems
Quark pairing phases in QCD

Hadrons (confined)

End point

QGP

2SC?

CFL?

308MeV ∼400MeV? CFL onset point

Neutron star core? Strange quark star?

2004年3月23日
Riken-Tokyo Mini-WS04, H. Abuki


(1) 2SC

Bailin-Love, '80
Iwasaki-Iwado, '95, etc...

Order:
\[ \langle q_i^a q_j^b \rangle_{2CS} = \Delta \varepsilon^{a b, c} \varepsilon_{s, i j} \]

Rank: 4

Gapless modes:
5 (8, 8, 8, 8, \overline{10}, \overline{10})

Symmetry:
\[ SU(2)_c \times SU(2)_f \]

Gaps, Quasiquarks and Multiplets:
\[ \Delta, SU(2)_f \times SU(2)_c \text{ doublet}^2 (4) \]
\[ = \begin{pmatrix} 2_{(r, b)} & 2_{(u, d)} \end{pmatrix} \]

Correlation works between 4 quarks out of 9

(ii) pure CFL

M. Alford, K. Rajagopal, F. Wilczek,

\[ \langle q_i^a q_j^b \rangle_{CFL} = \Delta_A \varepsilon^{a b, c} \varepsilon_{s, i j} \]

9 (full rank)

\[ SU(3)_{C + V} : \delta_i^a \text{ remains invariant} \]

\[ \Delta_i = 2 \Delta_A, \; SU(3)_{C + V} \text{ singlet} (1) \]
\[ \Delta_8 = -\Delta_A, \; SU(3)_{C + V} \text{ octet} (8) \]

Correlation works among all 9 quarks
The role of the strange quark mass and unlocking transition

pure CFL

\[ \langle q_i^a q_j^b \rangle_{\text{CFL}} = \Delta A \varepsilon^{abL} \varepsilon_{Lij} \quad \rightarrow \quad \langle q_i^a q_j^b \rangle_{\text{2SC}} = \Delta \varepsilon_{abg} \varepsilon_{s,ij} \]

\[ M_s^2 / \mu = 0 \]

pure 2SC

\[ M_s^2 / \mu = \infty \]

Unlocking transition?

1. Is there sharp phase separation?
2. If there actually is a phase transition
   i. the critical value of \( [M_s^2 / \mu]_{\text{crit.}} \)?
   ii. the order of phase transition?
3. The nature, mechanism of unlocking?

2004年3月23日
Riken-Tokyo Mini-WS04, H. Abuki
Trials in Past:
M. Alford et al., Nucl. Phys. B558,'99;

\[ \text{Gap (GeV)} \]

\begin{align*}
0.12 & \quad CFV \quad 2 \equiv C \\
0.1 & \quad b = -e
\end{align*}

\[ T = 0 \quad \mu = 400 \]

\[ \text{Axial(8)} \quad \text{Vector(8)} \]

\[ \text{unlocking!} \]

\[ \text{From: M. Alford et al, NPB558} \]

1. Is there sharp phase separation? : Yes
2. If there actually is a phase transition
   i. The critical value of \( \left[ M_s^2 / \mu \right] \), crit.? : \( \left[ M_s^2 / \mu \right] \), crit. \( \sim \Delta_8 \)
   ii. the order of phase transition? : 1st order

2004年3月23日  Riken-Tokyo Mini-WS04, H. Abuki
Trials in Past:

3. The nature, mechanism of unlocking?

: kinematical one rather than dynamical

\[ \langle u s - s u \rangle_{\text{CFL}} = \langle d s - s d \rangle_{\text{CFL}} \to 0 \]

\[ p_F^{u,d} - p_F^s \lesssim \frac{M_8^2}{4\mu} \left\{ \begin{array}{ll}
\leq \Delta_8 & : \text{CFL (smallest gap in CFL)} \\
\geq \Delta_8 & : \text{Unlocked into 2SC}
\end{array} \right. \]

Kinematical criterion works well! However...
Does Color-Flavor Unlocking need to be revisited?


   ☑ 1. Based on the expansion in \( R = M_s^2 / 2p_F \)
   ☑ 2. Constructing effective potential


   ☑ 1. Non-perturbative treatment of \( M_s \)
   ☑ 2. Constructing effective potential

   (Wrong construction!!)

Can we really neglect the dynamical effect of strange quark mass even after eliminating these ambiguities?

2004年3月23日
Riken-Tokyo Mini-WS04, H. Abuki
What about dynamical effect of strange quark mass?

\[
\langle q_i^a q_j^b \rangle_{CFL} = \Delta A \varepsilon^{abL} \varepsilon_{Lij} + \Delta S S_{ij}^{ab} = \frac{1}{2} \sum_{\alpha, \beta = 1}^{9} \Delta_{\alpha \beta} (\lambda_\alpha)_{ai} (\lambda_\beta)_{bj}
\]

\[
\Delta_{\alpha \beta} = \begin{pmatrix}
\Delta_8 E_8 \\
\Delta_1
\end{pmatrix}
: \text{Pure CFL (Chiral limit)}
\]

\[
\Rightarrow \Delta_{\alpha \beta} = \begin{pmatrix}
\Delta_3^3 E_3 \\
\Delta_2^2 E_4 \\
\Delta_1^1
\end{pmatrix}
: \text{Distorted CFL (} m_s \leq m_s^c \text{)}
\]

\[
\Rightarrow \Delta_{\alpha \beta} = \begin{pmatrix}
\Delta E_3 \\
0 E_4 \\
-\Delta / 3 & -\sqrt{2}\Delta / 3 \\
-\sqrt{2}\Delta / 3 & -2\Delta / 3
\end{pmatrix}
: 2SC (m_s \geq m_s^c)
\]

9=3+2^2 +1+1

4=3+1 =2+2
\[ \Delta_{\alpha\beta} = \begin{pmatrix} \Delta_8^3 E_3 \\ \Delta_8^2 E_4 \\ \Delta_8 \Delta \\ \Delta \Delta_1 \end{pmatrix} \quad \text{Minimal interpolating State between 2SC and pure CFL} \]

\[ R \begin{pmatrix} \Delta_8^1 & \Delta \\ \Delta & \Delta_1 \end{pmatrix} R^T = \begin{pmatrix} \chi_1 & 0 \\ 0 & \chi_2 \end{pmatrix} \text{with } |\chi_1| > |\chi_2| \]

States are distinguished by value of \((\Delta_8^3, \Delta_8^2, \chi_1, \chi_2)\)

<table>
<thead>
<tr>
<th>\Delta_{\alpha\beta} \text{ value}</th>
<th>\text{Gauge Symmetry}</th>
<th>\text{States}</th>
</tr>
</thead>
<tbody>
<tr>
<td>\Delta_8^3 = \Delta_8^2 = 0, \chi_1 = 0</td>
<td>SU(2)_L \times \text{SU}(2)_R \times U(1)_Y</td>
<td>\text{GUT}</td>
</tr>
<tr>
<td>\Delta_8^3 = \chi_1, \Delta_8^2 = \chi_2 = 0</td>
<td>\text{SU}(2)_L \times \text{SU}(2)_R \times \text{SU}(2)_Y \times U(1)_X</td>
<td>\text{2SC}</td>
</tr>
</tbody>
</table>

2004年3月23日
Riken-Tokyo Mini-WS04, H. Abuki
Effective potential
for multi-gap parameters

Pauli trick has to be carefully done in the case that many couplings enters in gap parameter space

\[ g^2 \frac{\partial \Omega_{\text{eff}}}{\partial \Delta_i} \neq \left[ \Delta_i - g^2 K_i [\tilde{\Delta}] \right] \]

Which is wrong point in Ref: M. Alford et al., NPB558, '99;

\[ g^2 \frac{\partial \Omega_{\text{eff}}}{\partial \Delta_i} = \sum C^{ij} \left[ \Delta_j - g^2 K_j [\tilde{\Delta}] \right] : \text{linear combination} \]

\( C^{ij} \) relate condensates \( \phi \sim \langle qq \rangle \) and gap parameters \( \Delta \) as

symbolically \( g^2 \phi^i = \sum C^{ij} \Delta_j, \quad (g^2 \text{tr}(S(\Sigma)\Sigma) \sim C^{ij} \Delta_i \Delta_j) \)

\[ \Omega_{\text{eff}}(\tilde{\Delta}) = \frac{1}{2g^2} C^{ij} \Delta_i \Delta_j - \int_0^1 dt \ C^{ij} \Delta_i K_j [t\tilde{\Delta}] \]

2004年3月23日
Riken-Tokyo Mini-WS04, H. Abuki
1. Asymptotic smooth Unlocking: No sharp unlocking for finite value of $M_s$.
2. Continuous 2nd order transition at some critical value: $M_s^c > \mu$. 

- Critical mass in kinematical criterion

- Parameters: $T = 0\text{MeV}$, $\mu = 400\text{MeV}$
Effective potential
in multi-gap parameter space

$M_s = 400\,\text{MeV}$

$\mu = 400\,\text{MeV}, T = 0\,\text{MeV}$

[MeV/fm$^3$]
- 1 2SC is saddle point: always a solution of gap equation
- 4
- 7
- 10
- 13
- 16
- 19
- 22
- 25
- 28
- 31
- 34
- 37
- 40
- 43
- 46

Distorted CFL state moves towards the 2SC state and Condensation energy gets reduced CFL state disappears into 2SC state as $M_s \rightarrow \mu$

$2^{nd}$ order transition?
Phase Diagram in $(T, M_s)$ plane

(1) $M_s^{\text{unlock}} > \mu$
(2) $T_c^{\text{unlock}} \sim \rho_s^{\alpha}$

$\sqrt{\mu \Delta g_{M_s=0,T=0}}$

$M_s$ [MeV]

$M_s = \mu$

$\mu = 400\text{MeV}$
What is BCS/BEC crossover?

In non-relativistic case: $\text{BEC}$ (strong, dilute limit) $\leftrightarrow \text{BCS}$ (weak, dense)

Weak coupling gap equation:

$$\Delta(p) = \sum_k V(p,k) \left[ 1 - 2n_F(\epsilon_k) \right] \frac{\Delta(k)}{2\epsilon_k[\Delta]}, \text{ with }$$

$$\begin{align*}
\epsilon_k[\Delta] &\equiv \sqrt{E_k^2 + \Delta(k)^2} \\
E_k(\nu) &= \left( \hbar^2 k^2 / 2m - \mu \right) \\
f_F(\epsilon) &= 1 / (1 + e^{\epsilon/T})
\end{align*}$$

is equivalent to the following wave equation:

$$\frac{p^2}{m} \varphi(p) - (1 - 2n_p) \sum_k V(p,k) \varphi(p) = 2\mu \varphi(p), \text{ with }$$

$$\begin{align*}
\varphi(p) &= \langle a_p a_{-p} \rangle_{\text{BCS}} \\
n(p) &= \langle a_p^+ a_p \rangle_{\text{BCS}}
\end{align*}$$

This is (Pauli-blocked) 2-body Schodinger (BS) equation. If $2\mu < 0$ in dilute limit $\rho \to 0$, this signals appearance of bound state! (BEC)
What is BCS/BEC crossover?


\[ T / \varepsilon_F \]

\[
T_{\text{Dissoc.}} \sim \frac{E_b}{\left[ \ln \left( \frac{E_b}{\varepsilon_F} \right) \right]^{2/3}}
\]

Melo et al., PRL71, ’93

**BCS type scaling:**
\[ T_C \sim e^{-1/k_F|a_s|} \]

Depends on Coupling itself

2nd order

\[ T_C \sim \rho^{2/3} \]

BEC type:
Depends on Density

Possible relation to precursory pseudo gap phase?

Phase diagram for $M_s = 400 \text{ MeV}$

- **QGP**
- **2SC**
- **Dissociation?**
- **BEC of CFL diquarks?**
- **CFL**
- "True" unlocking line
- Unlocking line in kinematical criterion
- 2nd order!
- "Stiff" CFL pairs!

$M_s = 400 \text{ [MeV]}$

H. Abuki, hep-ph/0401245

2004年2月19日
Summary

Color-Flavor Unlocking is revisited in more complete way than was taken in work in past

1. Is there sharp phase separation?
2. If there actually is a phase transition, the critical value of \([M_s^2/\mu]_{\text{crit.}}\)? the order of phase transition?
3. The nature, mechanism of unlocking?

(1a) Unlocking is of continuous 2\textsuperscript{nd} order at finite T

(1b) Asymptotic smooth change realizes for T=0?

(2) CFL is tough (stiff) against Ms!

\[ [M_s^2/\mu]_{\text{crit.}}(T=0) > \mu \] and \( T_{\text{c,unlock}} \sim \rho_s^\alpha \)

(3) An interesting possibility of BCS-BEC crossover in the relativistic CFL superfluid
Future problems

1. Imposing Electric & Color Neutralities:
   This is shown to strongly disfavor naive 2SC state!
   M. Alford and K. Rajagopal, JHEP206. 031,’02: Model independent analysis

2. Chiral condensate and in medium Strange quark mass.

3. Study the BCS-BEC crossover in more detail.
   (1) The interaction between bosons
   (2) Time-dependent diquark fluctuation modes?
       & Gross-Pitaevski equation in BEC region?
   (3) Discuss the thermodynamic properties in crossover
       in detail, and seek for Astrophysical consequences...
Fermi surface and Cooper instability

Our starting point: there is Large Fermi sphere
- Pauli-Blocking
- Low Energy Modes with Large Density of State

Results in non-perturbative IR dynamics

\[ \frac{-1}{g^2} + \bullet = 1PI \text{ vertex} \]
\[ \Gamma^{(2)}(p) = -\Delta_F^{-1}(p). \]
low \( p \) RPA mode is \textit{tachyonic} at \( T=0 \)

Cooper instability!
leads to reorganization of Fermi ball
into BCS state with non-vanishing \( ee \) condensate

BCS mechanism \( \Delta \sim \omega_D e^{-\frac{1}{Ng^2}} \)
2004年2月19日
Attraction in color-antitriplet channel

\[ \sum_{\alpha} (\lambda_{\alpha} / 2)_{ab} (\lambda_{\alpha} / 2)_{cd} = -\frac{N+1}{2N} (\delta_{ab}\delta_{cd} - \delta_{ad}\delta_{bc}) + \frac{N-1}{2N} (\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc}) \]

\( \bar{3}_c \) (anti-triplet) \hspace{1cm} 6_c \) (sextet)

Attraction! \hspace{1cm} Repulsive

Dynamical Formation of \( \langle q_a q_b \rangle \approx \varepsilon_{abc} \Delta^c \)?


- Ginzburg-Landau Approach
- Phenomenological IR cut off
- Small gap (a few hundred keV)

2004年2月19日
Proposed pairing patterns in QCD (1)

2SC (2 flavor ansatz)

1. **Color** Attractive anti-symmetric channel
   
   \( \varepsilon_{abc} : \) Color anti-triplet

2. **Dirac** attractive in total \( J=0 \) (8 Dirac operator) and aligned chirality \( (m/\mu \ll 0) \) (4 Dirac operator)
   
   \( \{ C \gamma_5, C \gamma_5 \gamma_0 \vec{\gamma} \cdot \hat{q} \} : \) Parity (+) Favored by instanton

   \( \{ C, C \gamma_0 \vec{\gamma} \cdot \hat{q} \} : \) Parity (−) ??? ☹️ ???

3. **Flavor** Anti-symmetric (purely from Pauli principle)
   
   \( \varepsilon_{ij} : SU(2)_f \) flavor singlet
Proposed pairing patterns in QCD (2)

CFL (Color-Flavor Locking)


1. Dirac structure is the same as in 2SC
\[ \{ C\gamma_5, C\gamma_5\gamma_0\vec{\gamma} \cdot \hat{q} \}: J = 0^+ \text{ and aligned chirality} \]
anti-symmetric with respect to spinor indices

2. CFL ansatz for Flavor and Color structure
\[ \langle q^a_i q^b_j \rangle_{\text{CFL}} = \Delta_A \epsilon^{abl} \epsilon_{lij} + \Delta_S (\delta_i^a \delta_j^b + \delta_j^a \delta_i^b) \]
\[ (\bar{3}_c \times \bar{3}_f) \quad (6_c \times 6_f) \]
(Attractive) (Repulsive) in naïve OGE level
symmetric under \((a, i) \Leftrightarrow (b, j)\) Pauli principle is OK.

2004年2月19日

c.f. \(B\)-phase in He\(^3\)
Symmetry of CFL

\[ \langle q^a_{Li} q^b_{Lj} \rangle_{\text{CFL}} = \sum_{I} \Delta_A \epsilon^{abl}_{\ell i j} = - \langle q^a_{Ri} q^b_{Rj} \rangle_{\text{CFL}} \]

Invariant under \( SU(3)_{C+L} \) \( SU(3)_{C+R} \)

\[ SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B \rightarrow SU(3)_{C+V} \]
\[ \Rightarrow U(1)_{\text{EM}} \]
\[ \Rightarrow \bar{U}(1)_{\text{EM}} \]

- color symmetry is broken: 8 massive gluons
- chiral symmetry is broken: octet NG bosons
- baryon number is broken: superfluid mode (H)
Analogy with He$^3$ system

$$\langle \psi_\alpha(\vec{q})\psi_\beta(-\vec{q}) \rangle = \sum_{a,i} \hat{d}_i \left[ \sigma^a \sigma^2 \right]_{\alpha\beta} d^i_a$$

- General $L=1, S=1$ state
- with $3 \times 3$ matrix $d^i_a$ ($a$: spin, $i$: spatial)

**B-phase (BW state; Balian-Wertheraner)**

$$d^i_a = \Delta_{BW} \delta^i_a$$ (S and L are locked!!)

- Gap is equal on the entire Fermi surface

**A-phase (ABM state; Anderson-Morel)**

$$d^i_a = \Delta_{ABM} \delta^3_a (e_1 + ie_2)_i$$

- Gap is zero at South and North poles

2004年3月23日 Riken-Tokyo Mini-WS04, H. Abuki
Solution of NJL gap equation: What about the unlocking for $T = 0$?
Solution of NJL gap equation: What about the unlocking for $T \neq 0$?
\[
\Delta_{\alpha\beta} = \begin{pmatrix}
\Delta_8^3 E_3 \\
\Delta_8^2 E_4 \\
\Delta^1_8 \\
\Delta \\
\Delta^1_1
\end{pmatrix} : \text{Minimal interpolating State between 2SC and pure CFL}
\]

\[
\begin{align*}
\Delta_8^3 &= \frac{1}{12} \left( 5 K_{83} \begin{pmatrix} \Delta_8^3 \end{pmatrix} - 2 K_{11} \begin{pmatrix} \Delta_8^1, \Delta, \Delta^1_1 \end{pmatrix} - K_{81} \begin{pmatrix} \Delta^1_8, \Delta, \Delta^1_1 \end{pmatrix} - 2\sqrt{2} K \begin{pmatrix} \Delta^1_8, \Delta, \Delta^1_1 \end{pmatrix} \right) \\
\Delta_8^2 &= \frac{1}{12} \left( 2 K_{82} \begin{pmatrix} \Delta_8^2 \end{pmatrix} - 2 K_{11} \begin{pmatrix} \Delta_8^1, \Delta, \Delta^1_1 \end{pmatrix} + 2 K_{81} \begin{pmatrix} \Delta^1_8, \Delta, \Delta^1_1 \end{pmatrix} + \sqrt{2} K \begin{pmatrix} \Delta^1_8, \Delta, \Delta^1_1 \end{pmatrix} \right) \\
\Delta_8^1 &= \frac{1}{12} \left( -K_{81} \begin{pmatrix} \Delta_8^1, \Delta, \Delta^1_1 \end{pmatrix} - 2 K_{11} \begin{pmatrix} \Delta_8^1, \Delta, \Delta^1_1 \end{pmatrix} + 2\sqrt{2} K \begin{pmatrix} \Delta^1_8, \Delta, \Delta^1_1 \end{pmatrix} - 3 K_{83} \begin{pmatrix} \Delta_8^3 \end{pmatrix} + 8 K_{82} \begin{pmatrix} \Delta_8^2 \end{pmatrix} \right) \\
\Delta^1 &= \frac{1}{6} \left( -K_{81} \begin{pmatrix} \Delta_8^1, \Delta, \Delta^1_1 \end{pmatrix} - 3 K_{83} \begin{pmatrix} \Delta_8^3 \end{pmatrix} - 4 K_{82} \begin{pmatrix} \Delta_8^2 \end{pmatrix} \right) \\
\Delta &= \frac{1}{12} \left( -2 K \begin{pmatrix} \Delta_8^1, \Delta, \Delta^1_1 \end{pmatrix} + \sqrt{2} K_{81} \begin{pmatrix} \Delta_8^1, \Delta, \Delta^1_1 \end{pmatrix} + \sqrt{2} \left( 2 K_{82} \begin{pmatrix} \Delta_8^2 \end{pmatrix} - 3 K_{83} \begin{pmatrix} \Delta_8^3 \end{pmatrix} \right) \right)
\end{align*}
\]
Detail of kernel

\[ K_{83}[\Delta] = \int \frac{d^3q}{(2\pi)^3} \frac{1}{4} \left( \tanh \left( \frac{E_{0+}[\Delta]}{2T} \right) \frac{\Delta}{E_{0+}[\Delta]} + \tanh \left( \frac{E_{0-}[\Delta]}{2T} \right) \frac{\Delta}{E_{0-}[\Delta]} \right) g^2(q,k) \left[ g_{\mu\nu} D_{\mu\nu}(q-k) \right] \]

\[ K_{82}[\Delta] = \int \frac{d^3q}{(2\pi)^3} \frac{1}{8} \left( + \tanh \left( \frac{E_{0+}[\Delta]}{2T} \right) \frac{\Delta}{E_{0+}[\Delta]} + \tanh \left( \frac{E_{0-}[\Delta]}{2T} \right) \frac{\Delta}{E_{0-}[\Delta]} \right) g^2(q,k) \left[ g_{\mu\nu} D_{\mu\nu}(q-k) \right] \]

\[ K_{81}[\Delta_1,\Delta_2,\Delta_3] = \{ \text{ultra super complicated form...} \} \]

\[ K_{11}[\Delta_1,\Delta_2,\Delta_3] = \{ \text{ultra super complicated form...} \} \]

\[ K[\Delta_1,\Delta_2,\Delta_3] = \{ \text{ultra super complicated form...} \} \]

\[ E_{M\pm}[\Delta] = \sqrt{\left( \sqrt{q^2 + M^2} \pm \mu \right)^2 + \Delta^2} : \text{Quasi-Quark and Quasi-AntiQuark energy} \]
Derivation of effective potential

\[ \frac{\partial \Omega_{\text{eff}}}{\partial g^2} = \frac{1}{g^2} \left\langle H_{\text{int}} \right\rangle = \frac{1}{2g^2} \text{tr} \left[ S(\Sigma) \Sigma \right] \text{ with definition} \]

\[ \Sigma_{\alpha\beta} = g^2 T_{\gamma\alpha} S^{\gamma\delta} T_{\delta\beta}, \quad S = \begin{pmatrix} \phi^1 E_3 \\ \phi^2 E_4 \\ \phi^3 & \phi^4 \\ \phi^4 & \phi^5 \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Delta_1 E_3 \\ \Delta_2 E_4 \\ \Delta_3 & \Delta_4 \\ \Delta_4 & \Delta_5 \end{pmatrix} \]

\[ T_{\gamma\alpha} \text{ is quark-gluon vertex in color-flavor mixed base. We have} \]

\[ \frac{\partial \Omega_{\text{eff}}}{\partial g^2} = \frac{1}{2g^4} C^{ij} \Delta_i \Delta_j, \quad \text{with} \quad C = \begin{pmatrix} 3 & 0 & -3 & -6\sqrt{2} & -6 \\
0 & -8 & 8 & 4\sqrt{2} & -8 \\
-3 & 8 & -5 & 2\sqrt{2} & -2 \\
-6\sqrt{2} & 4\sqrt{2} & 2\sqrt{2} & -12 & 0 \\
-6 & -8 & -2 & 0 & -4 \end{pmatrix} \]
For $M_s = 250$ [MeV]

$M_s = 250$ MeV
$\mu = 400$ MeV
Asymptotic Smooth Unlocking or Continuous 2\textsuperscript{nd} order transition?

\[ \Delta_{82} \text{ [MeV]} \]

\[ M_s \text{ [MeV]} \]

2004年3月23日

Riken-Tokyo Mini-WS04, H. Abuki
What happens when we go towards low density?

Pair "wavefunction": \( \varphi(\vec{r}) = \langle q(t, \vec{r}) q(t, 0) \rangle \rightarrow N \frac{\sin(\mu r)}{(\mu r)^{3/2}} e^{-\Delta r} \)

What about internal structure of Cooper pairs?

---

Improved ladder Schwinger-Dyson approach
What is BCS/BEC crossover?

What about the relativistic fermion case?

Weak coupling gap equation:

\[ \Delta(p) = \sum_k V(p,k) \left[ (1 - 2n_p^+) \frac{\Delta(k)}{2\epsilon_k^+[\Delta]} + (1 - 2n_p^-) \frac{\Delta(k)}{2\epsilon_k^-[\Delta]} \right] \]

with

\[ \epsilon_k^\pm[\Delta] \equiv \sqrt{E_k^\pm^2 + \Delta(k)^2} \]

\[ E_k^\pm = \sqrt{k^2 + m^2} \mp \mu \]

can be converted into:

\[ \begin{align*}
  \left[ -\mu + \sqrt{p^2 + m^2} \right] \varphi_+(p) &= \sum_k \left( 1 - 2n_p^+ \right) V(p,k) \left[ \varphi_+(k) + \varphi_-(k) \right] \\
  \left[ -\mu - \sqrt{p^2 + m^2} \right] \varphi_+(p) &= \sum_k \left( 1 - 2n_p^- \right) V(p,k) \left[ \varphi_+(k) + \varphi_-(k) \right]
\end{align*} \]

which is regarded as Pauli-blocked Dirac eigenvalue equation

\[ \left( i\tilde{\alpha} \cdot \tilde{\nabla} + m\beta - (1 - 2n) \hat{V} \right) \varphi(x) = \mu \varphi(x) \quad \text{with} \quad \tilde{\alpha} = \gamma_0 \tilde{\gamma}, \quad \beta = \gamma_0 \]
S-wave scattering length and Phase shift


\[ q \cot \delta(q) = -\frac{1}{a_s} + O(q^2) \]

\[
\frac{m}{4\pi a_s} = -\frac{1}{g_s^2} + \sum \frac{m}{k^2}
\]
Novel Phases of High Density Quark Matter and Their Roles in Physical Systems

M. Tachibana
Novel phases of high density quark matter and their roles in physical systems *

M. Tachibana†

Abstract

In this paper, we investigate several interesting aspects of high density quark matter, particularly emphasizing on remarkable phase structures of color superconductivity and their possible roles in physical systems. Firstly thermal color superconducting phase transitions in high density three-flavor quark matter are investigated in the Ginzburg-Landau approach. Effects of nonzero strange quark mass, electric and color charge neutrality, and direct instantons are considered. Weak coupling calculations show that an interplay between the mass and electric neutrality effects near the critical temperature gives rise to three successive second-order phase transitions as the temperature increases. Secondly we show calculations of neutrino interaction rates such as mean free path and emissivity in color-flavor locked quark matter based on the effective field theory approach, which may play an important role when we consider the issue on neutron star cooling.

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1 Introduction

Unraveling the phase structure at high baryon density is one of the most challenging problems in quantum chromodynamics (QCD). Among others, color superconductivity in cold dense quark matter has been discussed from various viewpoints [1, 2]. In relation to real systems such as newly born compact stars in stellar collapse, it is important to study the color superconductivity not only as a function of the quark chemical potential \( \mu \) but also as a function of the temperature \( T \). This is because the possible presence of color superconducting quark matter in a star affects the star’s thermal evolution [3]. In this paper, we show some interesting aspects of color superconductivity, especially emphasizing on novel phase structures associated with realistic systems. The content of this paper is as follows. In section 2, we describe thermal phase transition based on the Ginzburg-Landau approach. In section 3 and 4, effective field theory in the CFL phase is introduced, computing the weak interaction rates associated with neutron star cooling.

2 Melting pattern of diquark condensates

In this section, we adopt the Ginzburg-Landau (GL) approach near the transition temperatures, which was previously used to study the massless three-flavor case [4, 5, 6] and is a more advantageous framework to weak coupling calculations than other mean-field approaches [7, 8]. In a realistic situation, the GL potential acquires the following corrections. First of all, nonzero \( m_s \) affects the potential through the \( s \) quark propagator [8] in such a way as to lower the temperature at which a diquark condensate with \( s \) quarks dissolves. This is because the pairing interaction due to one-gluon exchange is effectively diminished by \( m_s \) if the pair contains the \( s \) quark. Secondly, when quark matter with nonzero \( m_s \) is beta equilibrated and neutralized by electrons near the transition temperatures, the chemical potentials between \( d, s \) quarks and \( u \) quarks differ. Through this chemical potential difference, the GL potential acquires another \( m_s \) dependence, which is the essential origin of the dSC phase. Thirdly, the instanton contribution gives an \( m_s \) dependence to the GL potential through the effective four-fermion interaction proportional to \( m_s \). We finally note that in weak coupling, color neutrality makes negligible difference in the GL potential near the transition temperatures [4].

We assume that diquark pairing takes place in the color-flavor anti-
symmetric channel with $J^P = 0^+$, which is predicted to be the most attractive channel in weak coupling [9]. In this case, the pairing gap of a quark of color $b$ and flavor $j$ with that of color $c$ and flavor $k$ has the form $\phi_{bcjk} = \epsilon_{abc} e_{ijk} (d_a)^i$ [4]. Here the $3 \times 3$ matrix $(d_a)^i$ transforms as a vector under $G = SU(3)_C \times SU(3)_{L+R} \times U(1)_B$ and belongs to the $(3^*, 3^*)$ representation of $SU(3)_C \times SU(3)_{L+R}$.

For a homogeneous system of massless quarks ($m_{ud,s} = 0$), the GL potential is invariant under $SU(3)_C \times SU(3)_{L+R}$ and valid near the critical temperature, $T_c$, common to all states belonging to the channel considered here. This potential reads [4, 10]

$$S = \bar{\alpha} \sum_a |d_a|^2 + \beta_1 (\sum_a |d_a|^2)^2 + \beta_2 \sum_{a\, b} |d_a^* \cdot d_b|^2,$$

where $(d_a)^i = (d_a^u, d_a^d, d_a^s)$ and the inner product is taken for flavor indices. In the weak coupling regime, the coefficients are [4]

$$\beta_1 = \beta_2 = \frac{7\zeta(3)}{8(\pi T_c)^2} N(\mu) \equiv \beta, \quad \bar{\alpha} = 4N(\mu)t \equiv \alpha_0 t,$$

where $N(\mu) = \mu^2/2\pi^2$ is the density of states at the Fermi surface, and $t = (T - T_c)/T_c$ is the reduced temperature. With the parameters (2), one finds a second order phase transition at $T = T_c$ from the CFL phase ($d_a^i \propto \delta_a^i$) to the normal phase ($d_a^i = 0$) in mean-field theory [4].

Let us now consider the effect of a nonzero $m_s$ in the quark propagator on the GL potential. We assume $m_{ud} = 0$ for simplicity and consider the high density regime, $m_s \ll \mu$. Near $T_c$ the leading effect of $m_s$ is to modify the quadratic term in the GL potential (1). The corrections to the quartic terms are subleading and negligible. Since $m_s$ affects only $us$ and $ds$ pairings, the correction to the quadratic term has the form

$$\epsilon \sum_a (|d_a^u|^2 + |d_a^d|^2) = \epsilon \sum_a (|d_a|^2 - |d_a^s|^2).$$

Note that this correction induces an asymmetry in the flavor structure of the CFL phase.

In weak coupling, $\epsilon$ can be calculated by including $m_s$ in the Nambu–Gor'kov quark propagator when evaluating the thermodynamic potential. Following Ref. [4], we expand the thermodynamic potential not only in $d_a^i$ but also in $m_s$ up to $O(m_s^2)$, and obtain

$$\epsilon \simeq \alpha_0 \frac{m_s^2}{4\mu^2} \ln \left( \frac{\mu}{T_c} \right) \sim 2\alpha_0 \sigma.$$
Here the dimensionless parameter $\sigma$ is given by

$$\sigma = \left( \frac{3\pi^2}{8\sqrt{2}} \right) \frac{m_a^2}{g\mu^2},$$  \tag{5}$$

where $g$ is the QCD coupling constant. As long as $\sigma \ll 1$, which is relevant at asymptotically high density, the following GL analysis near $T_c$ is valid. In the latter estimate in Eq. (4) we use the leading-order result in $g$, $\ln(T_c/\mu) \sim -3\pi^2/\sqrt{2g}$ \cite{9}. This behavior of $T_c$ originates from the long-range color magnetic interaction which prevails in the relativistic regime. As a result of the modification by $m_s$ to $\ln(T_c/\mu)$, $\epsilon$ has a positive value such that $ud$ pairing is favored over $us$ and $ds$ pairings. Consequently, the CFL phase becomes asymmetric in flavor space and its critical temperature is lowered, leading to the appearance of the 2SC phase ($d_s^s \propto \delta^{us}$) just below $T_c$ \cite{8}. Note that Eq. (3) has no effect on the 2SC phase. We also note that $T_c$ itself is modified by $m_s$ through the modification of the normal medium as $T_c(1 + \mathcal{O}(g\sigma))$.

We turn to discuss effects of charge neutrality which also depend on $m_s$ as mentioned above. Under beta equilibrium and charge neutrality, the electron chemical potential $\mu_e$ and the shift $\delta\mu_i$ of the chemical potential of flavor $i$ from the average ($\mu$) are related by $\delta\mu_i = -q_i\mu_e$, with electric charge $q_i$. We see from Ref. \cite{4} that the thermodynamic-potential correction due to $\delta\mu_i$ has the form

$$\eta \left( \frac{1}{3} \sum_a |d_a|^2 - \sum_a |d_a^a|^2 \right).$$ \tag{6}$$

In weak coupling, where one may regard normal quark matter and electrons as noninteracting Fermi gases, $\mu_e$ is related to $m_s$ as $\mu_e = m_s^2/4\mu$. This estimate is valid in the vicinity of $T_c$ where corrections to $\mu_e$ by a finite pairing gap affect only the quartic terms in the GL potential. By combining this estimate with the weak coupling expression for $\eta/\mu_e$ given in Ref. \cite{4}, we obtain

$$\eta \simeq \alpha_0 \frac{m_s^2}{8\mu^2} \ln \left( \frac{\mu}{T_c} \right) \sim \alpha_0 \sigma.$$ \tag{7}$$

Since $\eta > 0$, $ds$ pairing is more favorable than $ud$ and $us$ pairings. This feature stems from the modification by $\delta\mu_i$ to the exponential factor of $T_c$, $\exp[-3\pi^2/\sqrt{2g}]$, which tends to increase (decrease) the critical temperature for $ij$ pairing when $\delta\mu_i + \delta\mu_j > 0(< 0)$. 

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We consider color neutrality of the system as well. In contrast to the case at \( T = 0 \), however, it affects only the quartic terms in the GL potential through possible chemical potential differences between colors [4, 11], and in weak coupling its magnitude is suppressed by \( \mathcal{O}((T_c/g\mu)^2) \) compared to the leading quartic terms. Thus color neutrality has no essential consequence to the phase transitions considered in this Letter. A major difference between corrections of the charge neutrality and the color neutrality is that the former shifts the quark chemical potentials even in the normal phase, while the latter works only when the pairing occurs. This is why the former is more important than the latter near \( T_c \).

The direct instanton at nonzero \( m_\sigma \), which induces an effective four-fermion interaction between \( u \) and \( d \) quarks [12], leads to a quadratic term in the GL potential, \( \zeta \sum_\alpha |d_{\alpha}^e|^2 \). An explicit weak coupling calculation shows that \( \zeta \sim -a_0(m_\sigma/\mu)(\Lambda_{QCD}/\mu)^9(1/g)^{14} \). The negative sign indicates that the instanton favors \( ud \) pairing as does one-gluon exchange [see Eq. (3)], but the magnitude of \( \zeta \) is highly suppressed at high densities. Hereafter we will thus ignore instanton effects.

Since the two effects of nonzero \( m_\sigma \), characterized by Eqs. (3) and (6), favor \( ud \) pairing and \( ds \) pairing, respectively, the finite temperature transition from the CFL to the normal phase at \( m_\sigma = 0 \) is expected to be significantly modified. In fact, successive color-flavor unlocking transitions take place instead of a simultaneous unlocking of all color-flavor combinations. To describe this hierarchical thermal unlocking, it is convenient to introduce a parameterization,

\[
\Delta_\alpha^e = \begin{pmatrix}
\Delta_1 & 0 & 0 \\
0 & \Delta_2 & 0 \\
0 & 0 & \Delta_3
\end{pmatrix},
\]  

where \( \Delta_{1,2,3} \) are assumed without loss of generality to be real. We also name the phases for later convenience as

\[
\begin{align*}
\Delta_{1,2,3} \neq 0 & : \text{mCFL}, \\
\Delta_1 = 0, \quad \Delta_{2,3} \neq 0 & : \text{uSC}, \\
\Delta_2 = 0, \quad \Delta_{1,3} \neq 0 & : \text{dSC}, \\
\Delta_{1,2} = 0, \quad \Delta_3 \neq 0 & : \text{2SC},
\end{align*}
\]

where \( \text{dSC} (\text{uSC}) \) stands for superconductivity in which for \( d (u) \) quarks all three colors are involved in the pairing.
Figure 1: Transition temperatures of the three-flavor color superconductor in weak coupling: (a) all quarks are massless; (b) nonzero $m_s$ in the quark propagator is considered; (c) electric charge neutrality is further imposed. The numbers attached to the arrows are in units of $\sigma T_c$.

In terms of the parameterization (8), the GL potential with corrections of $O(m_s^2)$ to the quadratic term, Eqs. (3) and (6), reads

$$S = \bar{\alpha}'(\Delta_1^2 + \Delta_2^2 + \Delta_3^2) - \epsilon \Delta_1^2 - \eta \Delta_1^2$$

$$+ \beta_1(\Delta_1^2 + \Delta_2^2 + \Delta_3^2)^2 + \beta_2(\Delta_1^4 + \Delta_2^4 + \Delta_3^4),$$

where $\bar{\alpha}' = \bar{\alpha} + \epsilon + \frac{\eta}{3}$.

We proceed to analyze the phase structure dictated by Eq. (10) with the weak coupling parameters (2), (4), and (7) up to leading order in $g$. In Figs. 1 and 2 the results obtained by solving the coupled algebraic equations, $\partial S/\partial \Delta_{1,2,3} = 0$, are summarized. Figure 1(a) shows the second-order phase transition, CFL $\rightarrow$ normal for $m_s = 0$. Figures 1(b,c) represent how the phase transitions and their critical temperatures bifurcate as we introduce (b) effects of a nonzero $m_s$ in the quark propagator and then (c) effects of charge neutrality. In case (b), two second-order phase transitions arise, mCFL (with $\Delta_1 = \Delta_2$) $\rightarrow$ 2SC at $T = T_1^c \equiv (1 - 4\sigma) T_c$ and 2SC $\rightarrow$ normal at $T = T_c$. In case (c), there arises three successive second-order phase transitions, mCFL $\rightarrow$ dSC at $T = T_1^c$, dSC $\rightarrow$ 2SC at $T = T_2^c$, and 2SC $\rightarrow$ normal at $T = T_3^c$. Shown in Fig. 2 is the $T$-dependence of the gaps $\Delta_{1,2,3}$ for case (c). All the gaps are continuous functions of $T$, but their slopes are discontinuous at the critical points, which reflects the second order nature of the transitions in the mean-field treatment of Eq. (10).

We may understand the bifurcation of the transition temperatures in weak
Figure 2: A schematic illustration of the gaps squared as a function of $T$.

coupling as follows. In the massless case (a), $T_c$ is degenerate between the CFL and 2SC phases, the chemical potential is common to all three flavors and colors, and the CFL phase is more favorable than the 2SC phase below $T_c$. As one goes from (a) to (b), nonzero $m_s$ sets in, which tends to suppress the pairing interactions including the $s$ quark. The critical temperature for the CFL phase is then lowered, which allows the 2SC phase to appear at temperatures between $T_c^s$ and $T_c$. As one goes from (b) to (c), charge neutrality sets in, which acts to decrease the chemical potential of $u$ quarks by $(2/3)\mu_e$ and increase that of $d$ and $s$ quarks by $(1/3)\mu_e$. Since $\mu_e > 0$, the average chemical potential of quarks involved in $ds$ pairing increases, while those in $ud$ and $us$ pairings decrease equally. Accordingly, the transition temperatures further change from $T_c$ to $T_c^{II}$ and from $T_c^s$ to $T_c^I$ and $T_c^{II}$.

Now we examine in more detail how the color-flavor unlocking in case (c) proceeds with increasing $T$ from the region below $T_c^I$.

(i) Just below $T_c^I$, we have a CFL-like phase, but the three gaps take different values, with an order $\Delta_3 > \Delta_1 > \Delta_2 \neq 0$ (the mCFL phase). The reason why this order is realized can be understood from the GL potential (10). The $\epsilon$-term and $\eta$-term in Eq. (10) tend to destabilize $us$ pairing ($\Delta_2$) relative to $ud$ pairing ($\Delta_3$) and $ds$ pairing ($\Delta_1$). Since $\epsilon > \eta (> 0)$, $ds$ pairing is destabilized more effectively than $ud$ pairing. The value of each gap in the mCFL phase reads

$$\Delta_3^2 = \frac{\alpha_0}{8\beta} \left( \frac{T_c - T}{T_c} + \frac{8}{3} \sigma \right),$$

(11)
The mCFL phase has only a global symmetry $U(1)_{C+L+R} \times U(1)_{C+L+R}$ in contrast to the global symmetry $SU(3)_{C+L+R}$ in the CFL phase with $m_{u,d,s} = 0$. There are no gapless quark excitations in both mCFL and CFL phases. As $T$ increases, the first unlocking transition, the unlocking of $\Delta_2$ (the pairing between $Bu$ and $Rs$ quarks), takes place at the critical temperature,

$$T_c^I = \left(1 - \frac{16}{3} \sigma \right) T_c.$$  \hspace{1cm} (14)

(ii) For $T_c^I < T < T_c^{II}$, $\Delta_2 = 0$ and

$$\Delta_3^2 = \frac{\alpha_0}{6 \beta} \left( \frac{T_c - T}{T_c} + \frac{2 \sigma}{3} \right),$$ \hspace{1cm} (15)

$$\Delta_1^2 = \frac{\alpha_0}{6 \beta} \left( \frac{T_c - T}{T_c} - \frac{7 \sigma}{3} \right).$$ \hspace{1cm} (16)

In this phase, we have only $ud$ and $ds$ pairings (the dSC phase), and there is a manifest symmetry, $U(1)_{C+L+R} \times U(1)_{C+L+R} \times U(1)_{C+V+B} \times U(1)_{C+V+B}$. By diagonalizing the inverse quark propagator in the Nambu-Gor’kov representation, we find three gapless quark excitations in the color-flavor combinations: $Bu$, $Rs$, and a linear combination of $Ru$ and $Bs$. At $T = T_c^{II}$, the second unlocking transition, the unlocking of $\Delta_1$ (the pairing between $Gs$ and $Bd$ quarks), takes place at the critical temperature,

$$T_c^{II} = \left(1 - \frac{7 \sigma}{3} \right) T_c.$$ \hspace{1cm} (17)

(iii) For $T_c^{II} < T < T_c^{III}$, one finds the 2SC phase, which has only $ud$ pairing with

$$\Delta_3^2 = \frac{\alpha_0}{4 \beta} \left( \frac{T_c - T}{T_c} - \frac{1}{3} \sigma \right).$$ \hspace{1cm} (18)

The 2SC phase has a symmetry $SU(2)_C \times SU(2)_{L+R} \times U(1)_{C+B} \times U(1)_{L+R+B}$ [6]. In this phase the $s$ quark and $B$ quark excitations are gapless. The
final unlocking transition where $\Delta_3$ (the pairing between $Rd$ and $Gu$ quarks) vanishes occurs at

$$T_{c}^{\text{III}} = \left(1 - \frac{1}{3}\sigma\right) T_c.$$  \hspace{1cm} (19)

Above $T_{c}^{\text{III}}$, the system is in the normal phase.

In Table I, we summarize the symmetry and the gapless quark modes in each phase discussed above. The number of gluons having nonzero Meissner masses, which is related to the remaining color symmetry, is also shown [14]. We note that more gapless quark modes may appear if the system is in the close vicinity of $T_{c}^{\text{I}}, T_{c}^{\text{II}},$ and $T_{c}^{\text{III}}$ where $\Delta_2, \Delta_1,$ and $\Delta_3$ are less than $\sim m_s^2/\mu$ [13].

So far, we have studied the phase transitions in the mean-field level. In weak coupling, as shown in Ref. [6] in the massless limit, thermally fluctuating gauge fields could change the order of the transitions described in Figs. 1 and 2. A detailed account on this effect will be reported elsewhere [14]; here we recapitulate the important results. First, the second order transition, $\text{mCFL} \rightarrow \text{dSC}$, remains second order even in the presence of the thermal gluon fluctuations. This is because all eight gluons are Meissner screened at $T = T_{c}^{\text{I}}$ and thus cannot induce a cubic term with respect to the order parameter in the GL potential. On the other hand, the transitions, $\text{dSC} \rightarrow 2\text{SC}$ and $2\text{SC} \rightarrow$ normal, become weak first order since some gluons, which

<table>
<thead>
<tr>
<th>Symmetry</th>
<th>Gapless quark modes</th>
<th>Number of massive gluons</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{mCFL}$</td>
<td>$[U(1)]^2$</td>
<td>none</td>
</tr>
<tr>
<td>$\text{dSC}$</td>
<td>$[U(1)]^4$</td>
<td>$Bu, Rs$\newline($Ru, Bs$)</td>
</tr>
<tr>
<td>$2\text{SC}$</td>
<td>$(SU(2))^2 \times [U(1)]^2$</td>
<td>$Bu, Bd, Bs, Rs, Gs$</td>
</tr>
</tbody>
</table>

Table I: The symmetry, the gapless quark modes, and the number of Meissner screened gluons in the $\text{mCFL}$, $\text{dSC}$, and $2\text{SC}$ phases. The gapless quark mode $(Ru, Bs)$ denotes the linear combination of $Ru$ and $Bs$ quarks.
are massless in the high temperature phase, become Meissner screened in the low temperature phase (Table I).

In summary, we have investigated color-flavor unlockings at finite temperatures taking into account the strange quark mass and charge neutrality in the GL approach. We find three successive unlocking transitions, mCFL → dSC → 2SC → normal, occurring in weak coupling. Most remarkably, the dSC phase appears between the mCFL and 2SC phases. In this phase all eight gluons are Meissner screened and the three quark excitations are gapless. The question of how the phase structure of neutral quark matter obtained near $T_c$ is connected to that at $T = 0$, which approaches the CFL phase in the limit of high density, is an interesting open problem.

3 Effective Field Theory in CFL phase

In this section, we introduce the effective lagrangian in the CFL phase in order to calculate the weak interaction rates related to neutron star cooling. As was already mentioned, in the CFL phase, all nine quarks are massive and all the gluons acquire the Meissner mass. Due to this, the dominant degrees of freedom at low energies such as several tens of MeV are Namubu-Goldstone bosons associated with chiral symmetry breaking as well as U(1) baryon symmetry breaking. There are several articles that describe in detail the effective theory for the Goldstone bosons in Color-Flavor-Locked quark matter [19, 20, 21]. It is possible to parametrize low energy excitations about the CFL ground state in terms of the two fields $B = H/\sqrt{2\hat{f}_H}$ and $\Sigma = e^{2\pi f_{\pi}}$, representing the Goldstone bosons of broken baryon number $H$ and of broken chiral symmetry, the octet $\pi$. Then the leading terms of the effective Lagrangian describing the octet Goldstone boson field $\pi$ are given by

$$\mathcal{L} = \frac{1}{4} f_{\pi}^2 \left[ T \nabla_0 \Sigma \nabla_0 \Sigma^\dagger - v^2 \bar{\nabla} \Sigma \bar{\nabla} \Sigma^\dagger \right] + f_{\pi}^2 \left[ \frac{a}{2} Tr \tilde{M} (\Sigma + \Sigma^\dagger) + \frac{\bar{x}}{2} Tr M (\Sigma + \Sigma^\dagger) \right].$$

The decay constant $f_{\pi}$ has been computed previously [21] and is proportional to the quark chemical potential $\mu$. $\nabla_0$ includes the Bedaque-Schafer terms [22] and $\tilde{M} = |M| M^{-1}$, where $M = \text{diag}(m_u, m_d, m_s)$. The velocity factor $v = 1/\sqrt{3}$ and $a = 3\sqrt{3}/f_{\pi}^2$ [21], where $\Delta$ is the gap which is around 100 MeV at $\mu = 500$ MeV.
To incorporate weak interactions, we gauge the chiral Lagrangian in the usual way by replacing the covariant derivative by [23]

\[ D_\mu \Sigma = \nabla_\mu \Sigma - \frac{ig}{\sqrt{2}} (W_\mu^+ \tau^+ + W_\mu^- \tau^-) \Sigma - \frac{ig}{\cos \theta_W} Z_\mu (\tau_3 \Sigma - \sin^2 \theta_W [Q, \Sigma]). \]  

(21)

The fields \( W_\mu^\pm, Z_\mu \) describe weak gauge bosons. The charge matrix is diagonal \( Q = \text{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}) \) as well as weak-isospin matrix \( \tau_3 = \frac{1}{2} \text{diag}(1, -1, -1) \) whereas \( \tau^+ \) and \( \tau^- \) are the isospin raising and lowering operators which incorporate Cabbibo mixing.

For momenta small compared to \( f_\pi \) we can expand the nonlinear chiral Lagrangian to classify diagrams as the first order (proportional to \( f_\pi \)) and the second order (independent of \( f_\pi \)). The amplitudes for the leading order processes are given by

\[
A_{\pi^0 \rightarrow \nu \bar{\nu}} = \frac{G_F}{\sqrt{2}} f_\pi \tilde{p}_\mu j_\mu^Z, \quad A_{\pi^\pm \rightarrow e\nu} = G_F f_\pi \cos \theta_C \tilde{p}_\mu j_\mu^W, \quad A_{K^\pm \rightarrow e\nu} = G_F f_\pi \sin \theta_C \tilde{p}_\mu j_\mu^W
\]  

(22)

where \( \tilde{p}_\mu = (E, \nu^2 \bar{p}) \) is the modified four-momentum of Goldstone boson and \( j_\mu^W \) and \( j_\mu^Z \) describe the charged and neutral leptonic currents, respectively. \( \theta_C \) is the Cabbibo mixing angle. Note that the meson "four-momenta" that appear in the matrix element do not correspond to the on-shell four momenta of the mesons. This is because the covariant derivative contains the in-medium velocity and for the case of kaons, the energy shift arising from the Bedaque-Schafer term.

In addition to the octet Goldstone bosons, the massless Goldstone boson associated with \( U(1)_B \) breaking couples to the weak neutral current. Amplitude for processes involving the \( U(1)_B \) Goldstone boson \( H \) and the neutral current is given by

\[
A_{H \rightarrow \nu \bar{\nu}} = \frac{4}{\sqrt{3}} G_F f_H \tilde{p}_\mu j_\mu^Z, \quad \text{(23)}
\]

The decay constant for the \( U(1)_B \) Goldstone boson has also been computed in earlier work and is given by \( f_H = 3 \mu^2 / (8 \pi^2) \) [21]. Using these amplitudes we shall evaluate some interaction rates associated with thermal evolution of the early born neutron star, i.e., neutrino opacity and neutrino emissivity in the next section.
4 Neutrino Rates

4.1 Neutrino Opacity

Figure 3 shows the resulting neutrino mean free path in a CFL meson plasma as a function of temperature. In the CFL medium, novel Cherenkov like processes such as $\nu \rightarrow H \nu, \nu \rightarrow \pi^0 \nu$ and $\nu_a \rightarrow \pi^\pm e^\mp$ are allowed owing to the fact that mesons can have a space like dispersion relation [3]. Especially the massless $U(1)_B$ Goldstone boson $H$ is space like for all momenta. Thus the processes involving $H$ boson become dominant. Since these processes do not have mesons in the initial state they can occur at zero temperature. The neutrino mean free path due to the reaction $\nu \rightarrow H \nu$ can be calculated analytically and is given by

$$\frac{1}{\lambda_{\nu \rightarrow H\nu}(E_\nu)} = \frac{256}{45\pi} \frac{[v(1-v)^2(1+\frac{v^2}{4})]}{(1+v)^2} G_F^2 f_H^2 E_\nu^3. \quad (24)$$

Although eq.(24) has no intrinsic temperature dependence, it arises by setting $E_\nu = \pi T$, which corresponds to mean energy of neutrinos in thermal equilibrium.

![Figure 3: Neutrino mean free path in a CFL meson plasma as a function of temperature](image1)

![Figure 4: Rate of energy loss due to neutrino emitting reactions](image2)
4.2 Neutrino Emissivity

The rate of energy loss due to neutrino emitting reactions is shown in Figure 4. Charged current decays of pions and kaons, and the novel neutral current decay of $\pi^0$ are the leading one body processes. In vacuum, the amplitudes for similar processes are proportional to the lepton mass due to angular momentum conservation. However, in the CFL phase, due to the lack of Lorentz invariance, the decay of finite momentum pions and kaons is not suppressed by the electron mass. Thus even the process such as $\pi^0 \to \nu \bar{\nu}$ could happen.

Decay processes can occur only when the meson four momentum is time like. Thus, the one body decay of $H$ boson is forbidden and only low momentum pions and kaons can participate in the decay process. This accounts for the saturation of the one body decay contribution to the emissivity with increasing temperature seen in Figure 4. At high temperature, processes involving two mesons in the initial states become important. This is because these processes do not have momentum threshold unlike those of one body decays.

In summary, we have shown that novel processes in which mesons are either emitted or absorbed from neutrinos occur in the CFL plasma and contribute to neutrino opacity. Absorption of thermal, massless $H$ bosons is the dominant reaction contributing to the opacity for temperatures in the range $T = 1\text{--}30 \text{ MeV}$. We find that the mean free path for thermal neutrinos at $T = 10 \text{ MeV}$ is of the order of 10 meters and at $T = 5 \text{ MeV}$ it is similar to 100 meters. In the table below we compare the neutrino mean free path in CFL matter with those in nuclear matter [24] and unpaired quark matter [25] under similar conditions.

<table>
<thead>
<tr>
<th>phase</th>
<th>process</th>
<th>$\lambda(T=5 \text{ MeV})$</th>
<th>$\lambda(T=30 \text{ MeV})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear Matter</td>
<td>$\nu n \to \nu n$</td>
<td>200 m</td>
<td>1 cm</td>
</tr>
<tr>
<td></td>
<td>$\nu_e n \to e^- p$</td>
<td>2 m</td>
<td>4 cm</td>
</tr>
<tr>
<td>Unpaired Quarks</td>
<td>$\nu q \to \nu q$</td>
<td>350 m</td>
<td>1.6 m</td>
</tr>
<tr>
<td></td>
<td>$\nu_e d \to e^- u$</td>
<td>120 m</td>
<td>4 m</td>
</tr>
<tr>
<td>CFL</td>
<td>$\lambda_{3B}$</td>
<td>100 m</td>
<td>70 cm</td>
</tr>
<tr>
<td></td>
<td>$\nu \phi \to \nu \phi$</td>
<td>$\geq 10 \text{ km}$</td>
<td>4 m</td>
</tr>
</tbody>
</table>

The findings presented in the table indicate that neutrino mean free path in CFL matter is similar to or shorter than in unpaired quark matter for the whole temperature range. This surprising result arises solely due to
the novel processes involving Cherenkov absorption and radiation of CFL Goldstone bosons. Despite the energy gap in the quark excitation spectrum the opacity remains large and there is no exponential suppression of neutrino cross section even at low temperature. This is in sharp contrast to earlier findings in the two flavor superconducting phase [26]. However, since we expect the specific heat of the CFL phase $C_V \sim T^3$ to be small compared to that of unpaired quark matter, the cooling rates could still differ and needs to be investigated. The early evolution of a newly born neutron star is, in general, a complex process which depends on several macroscopic ingredients and microscopic conditions. In order to gauge how color superconductivity in the neutron star core will impact observable aspects of early evolution the rates computed in this work as well as the thermodynamic properties of the CFL phase need to be included in detailed numerical simulations of core collapse supernova. This is the only reliable means to bridge the gap between theoretical expectation of color superconductivity and observable aspects of core collapse supernova.

Acknowledgments

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References


Color Ferromagnetism and Quantum Hall State of Gluon

M. Ohtani
Color Ferromagnetism and Quantum Hall State of Gluon

M. OHTANI (RIKEN), A. IWAZAKI (Nishogakusha-U), O. MORIMATSU (KEK), T. NISHIKAWA (KEK)

Abstract
In non-Abelian gauge theory, the perturbative vacuum is known to be unstable against condensation of a color magnetic field. The spontaneously-generated color magnetic field can be set to be in maximal Abelian subalgebra and off-diagonal gluons involve unstable modes. Since the most unstable modes are uniform in the direction parallel to the magnetic field, the system is effectively reduced to two dimensional one accompanied by the magnetic field, which is analogous to the quantum Hall system of electrons. We propose that these unstable modes compose a quantum Hall state in quark matter to realize a stable color ferromagnetic state.

We show that this state is gapped state and absolutely stable by studying the excitation energy of fluctuation around the state and also the vortex excitation. This state must be supplied with color charge by quark matter and turn out to be energetically favored than color super-conducting phase below a critical chemical potential. These fact implies that the color ferromagnetic state along with quantum Hall state of gluon may arise between hadron phase and color super-conducting phase.

Topics in Hadron Physics, Mar 23 @ RIKEN
Color Ferromagnetic State

Spontaneous generation of color magnetic field, $\langle B \rangle \neq 0$
with a quantum Hall state of the gluons
Savvidy state in SU(2) gauge theory

\[ \mathcal{L}_{\text{gluon}} = -\frac{1}{4} \tilde{F}_{\mu \nu}^2 = -\frac{1}{4} (\partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu})^2 - \frac{1}{2} (\partial_{\mu} - igA_{\mu}) \Phi_{\nu} \right)^2 \\
+ ig(\partial^\mu A^\nu - \partial^\nu A^\mu) \Phi^*_{\mu} \Phi_{\nu} + g^2 (\Phi_{\mu} \Phi^*_{\nu} - \Phi^*_{\mu} \Phi_{\nu})^2 \]

color magnetic field: \( A_\mu = A_\mu^3 \), (color) charged vector field: \( \Phi_\mu = \frac{A_\mu^1 + iA_\mu^2}{\sqrt{2}} \)

\[ V(B) = \frac{1}{2} B^2 + \frac{11g^2 B^2}{48\pi^2} \left( \ln \frac{gB}{A^2} - \frac{1}{2} \right) - \frac{ig^2 B^2}{8\pi} \]

\( \langle B \rangle \neq 0 \)

Savvidy PL B 71 (1977) 133

Savvidy state is unstable
due to \( \text{Im} V(B) \neq 0 \)

Nielsen and Olesen, NP B 144(1978)376
\[ D^\mu D_\mu \Phi^\nu + 2i g (\partial^\mu A^\nu - \partial^\nu A^\mu) \Phi_\mu = 0 \]

\[ E_{\pm}^2(n, k) = 2gB \left( n + \frac{1}{2} - s_z \right) + k^2 \quad \text{"Landau Level"} \]

\[ (n=0, 1, 2, ... \ ; s_z = \pm 1: \text{spin} \ ; k = \text{momentum} \parallel B) \]

\[ \text{Unstable modes: Lowest Landau Level, } n=0 \text{ with spin parallel to } B \]

\[ E_{-}^2(n = 0, \ |k| < \sqrt{gB}) = k^2 - gB < 0 \]

The instability of the Savvidy vacuum

\[ \Leftarrow \text{ unstable modes of } \Phi_\mu \text{ under the magnetic field } B \]

\[ \text{The most unstable modes are uniform in the direction } \parallel B \quad (\because k = 0) \]

with \( \text{(degeneracy for each } n \text{/ area}) = gB/2\pi \)
We extract from $\mathcal{L}_{\text{gluon}}$ only the most unstable modes: $k=0 \iff$ uniform in the direction $\parallel B$. \quad \Rightarrow \quad 2+1\text{dim.}

$$\mathcal{L}_{2+1} = \left| (\partial_\mu - igA_\mu)\phi \right|^2 + 2gB \left| \phi \right|^2 - \frac{g^2}{2l_3} \left| \phi \right|^4$$

where $\phi \equiv (\Phi_1 - i\Phi_2) \sqrt{\frac{l_3}{2}},$

"$l_3$" length scale of a domain

With LLL condition: $(D_1 + iD_2)\phi = 0$

The negative mass term $-2gB$

(If $A_\mu = 0$, potential from $\mathcal{L}_{2+1}$ has the typical double well.)

$\Rightarrow \langle \phi \rangle = 0$ is unstable.
Our proposal

Gluons in $B^{\text{color}}$

Unstable modes ($k = 0$)
of gluons in $B^{\text{color}}$
- effectively 2d
- occupy LL

Electrons in $B$

2d electrons
in semiconductors
under strong $B$
- occupy LL

Absolutely stable state:
Quantum Hall state (QHS)
- finite gap
- incompressible
  
  cf. superconductor
If $A_\mu = 0$, typical double well potential

The unstable state $\langle \phi \rangle = 0$ decays into the stable state $\langle \phi \rangle = \nu$

The spatially uniform mode ($k = 0$) condenses.

Under $B \neq 0$, the unstable mode couples with $A_\mu (x)$:

$$\mathcal{L}_{2+1} = |(\partial_\mu - igA_\mu)\phi|^2 + 2gB|\phi|^2 - \frac{g^2}{2l_3} |\phi|^4$$

There is no such uniform state as $\langle \phi \rangle = \nu$.

Chern-Simons theory: useful for 2-dim systems
Chern-Simons theory

Chern-Simons field $a_u \rightarrow$ fictitious flux $\rightarrow$ particle statistics in 2-dim
the quantum Hall states are described by Chern-Simons gauge theory.

For electrons

$$\mathcal{L} = \Phi^\dagger (i \partial_t - eA_0 - ea_0) \Phi - \frac{1}{2M} \left| (-i \nabla + eA + ea) \Phi \right|^2 + \frac{\epsilon^{\mu \nu \lambda} e^2}{4\pi m} a_\mu \partial_\nu a_\lambda + ...$$

$$\Rightarrow \quad m \Phi^\dagger \Phi = \frac{\nabla \times a}{2\pi/e}; \text{ permutation phase: } \exp \left[ \frac{i}{2} \oint e a \cdot dx \right] \rightarrow \exp(i m \pi)$$

$m :$ odd integer for fermion

Chern-Simons flux

$$\begin{array}{c}
\text{electron} \\
= \\
\text{bosonized electron}
\end{array}$$
Chern-Simons gauge theory for the Quantum Hall states of gluons.

Chern-Simons flux \( a_\mu \)

\((\# \text{ of flux: even for boson})\)

original gluons \( \equiv \) composite boson \( \phi_c \)

\[ L_{cs} = (\partial_\mu - igA_\mu + ia_\mu)\phi_c|^2 + 2gB|\phi_c|^2 - \frac{g^2}{2l_3}|\phi_c|^4 + \frac{1}{4\pi m}\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda ; \ m : \text{even} \]

\[ \star \text{ Quantum Hall states: } \langle \phi_c \rangle = v_c, \ a_i = gA_i \]

\((\text{Chern-Simons gauge field cancels the magnetic field.})\)

\[ \nabla \times \cdot a \]

bosons feel totally
no gauge flux to form
a uniform & stable state
Quantum Hall state in quark matter

- The state $\langle \phi_c \rangle = \nu_c$, $a_i = gA_i$ corresponds to QHS with filling $1/m \leftarrow$ Hall conductance $= \frac{1}{m} \frac{g^2}{h}$

- The color charge density of the condensed gluons: $\rho_c = \frac{gB}{2\pi m}$

↓

Quark matter is a supplier of the color charge.

"In the quark matter, Color ferromagnetic state is stabilized by the formation of gluons quantum Hall state."
Color ferro and color superconductivity


\[
\begin{align*}
I & : \Delta = 0, \sigma = 0 \quad \cdots \text{QGP} \\
II & : \Delta = 0, \sigma \neq 0 \quad \cdots \text{Hadron} \\
III & : \Delta \neq 0, \sigma = 0 \quad \cdots \text{Color Super}
\end{align*}
\]

\[gH = 0.4 \text{ GeV}^2\]

color magnetic field \(\uparrow\) against

color super cond.
Comparison of the quark energy in various states — where the ferromagnetic state is realized in the phase diagram

free quark gas

(for any $\mu$)

$\uparrow$ (for $\mu < \sqrt{2gB}$)

$\downarrow$ (for $\mu > \sqrt{2gB}$)

BCS state of quarks

if gap energy $\sim O(100\text{MeV})$

$\Rightarrow \mu_c = 200 \sim 500 \text{ MeV}$

Color ferromagnetic state can be realized between hadron phase and color super conducting phase
QHS of $\Theta^-$: quasi-particles w/ a fractional charge
$\Rightarrow$ vortex excitation in QHS of gluon

\[ |\phi|^2 \]

\[ \nabla \rightarrow g^2 \rightarrow \]

pair creation of vortices for low density
Implications of color ferro with QHS

- Comparison with electron QHS:
  - quantum Hall plateau, Anderson localization by Impurities,
  - quasi particles with a rational (fractional) charge, incompressibility
  - → Quark dynamics
  - → EoS in N…

- Real observable magnetic field is induced in quark matter owing to the rotational motions of quarks.

"Such a phase of the quark matter produced by heavy ion collisions may be detected by observing the strong magnetic field"
Summary

- Color ferromagnetic state can exist in quark matter matter accompanying a quantum Hall state of gluons.
- The state lies between hadron phase and color superconducting phase.
- (Anti-)Vortex excitations will break the state for low density.
- If the state is realized in the quark matter produced produced by heavy ion collisions, real magnetic field field might be observed.
Relation Between the Chiral and Deconfinement Phase Transitions

Yoshitaka Hatta
Relation between the chiral and deconfinement phase transitions

Yoshitaka Hatta
(Kyoto U. and RIKEN)

Abstract

We argue that the chiral phase transition and the deconfinement phase transitions in finite temperature QCD are continuously connected by the glueball-sigma mixing.
QCD in the non-perturbative regime

\[ \Lambda_{QCD} < 4\pi f_\pi, m_\rho \]
chiral phase transition

\[ m = 0 \quad \text{chiral symmetry} \quad q \rightarrow e^{i \gamma_5 t_F \theta^a} q \]

quark condensate

\[ \langle \overline{q} q \rangle \]

\[ N_f = 2 \quad \text{second order} \]
\[ N_f \geq 3 \quad \text{first order} \]
deconfinement phase transition

\[ m = \infty \]

center symmetry

\[ \Omega(\beta, x) = e^{\frac{2\pi i}{N_C} n} \Omega(0, x) \]

Polyakov loop

\[ < L > \]

\[ = < P \exp(-ig \int_0^\beta A_4(x, \tau) d\tau) > \]

\[ N_C = 2 \quad \text{second order} \]

\[ N_C = 3 \quad \text{first order} \]
Lattice result: an empirical fact

For all values of $m$,

$$T_D(m) \approx T_C(m)$$

Deconfinement phase transition knows chiral phase transition. Why?
Preceding works

Basic idea: What is responsible for the breaking of center symmetry is the constituent quark mass.

Gocksch & Ogilvie, '84
Digal, Laermann & Satz, '00
Fukushima, '02~

Jump of the Polyakov loop (deconfinement?) is a consequence of the chiral phase transition.

Fukushima, hep-ph/0209311
End-point — existence of a massless excitation

\[ L' = L - \langle L \rangle_D \]

\[ \sigma' = \sigma - \langle \sigma \rangle_C \]

Gavin, Gocksch & Pisarski, '94
$SU_c(2)$ pure glue theory --- second order phase transition

Consider the specific heat

$$C_V \sim \frac{d^2\Omega}{dT^2}$$

$$\sim |t|^{-\alpha} \quad (\alpha \approx 0.1) \quad t = (T - T_c)/T_c$$

$$\sim \langle \sum_p \Box \Box \rangle$$

$$\sim \int d^d x \langle L^2(x)L^2(0) \rangle_c$$
Explicit structure of the divergence

\[ \int d^d x < L^2(x) L^2(0) >_c \]

\[ \sim \int d^d x \frac{\exp(-m_G |x|)}{|x|^{2(d-1/\nu)}} \sim |t|^{-\alpha} \]

scaling dimension

\[ m_G \sim |t|^{\nu} \quad (\sim \xi^{-1}) \]

The screening mass of the electric glueball goes to zero.

Lattice measurement \quad Datta & Gupta, '98
At $m = \infty$, a Polyakov loop and a glueball are distinct.

For $m < \infty$, a Polyakov loop is like a glueball.

$$G \sim |L|^2 = |L' + \langle L \rangle_D|^2$$

For $m < \infty$, a Polyakov loop is just one of many interpolating fields of the electric glueball.
\[ \int d^d x < L'(x)L'(0)>_c \sim \int d^d x < \bar{q}q(x)\bar{q}q(0)>_c \]
\[ \sim \int d^d x < G(x)G(0)>_c \sim |t|^{-\gamma} \]
A bridge over the phase diagram

\[ \phi = G \cos \theta + \sigma \sin \theta \]

\[ \phi' = -G \sin \theta + \sigma \cos \theta \]

\[ \theta \approx 0 \]

\[ \phi = G \cos \theta + \sigma \sin \theta \]

\[ \theta \approx \frac{\pi}{2} \]
'Level repulsion' scenario

$\phi' \approx G$

$\phi \approx \sigma$

$\theta \approx \frac{\pi}{2}$

$\phi' \approx G$

$\phi \approx \sigma$

$\theta \approx 0$

Single field describes the (crossover) phase transition for all values of $m$.

Naturally,

$T_D(m) = T_C(m)$
Zero temperature QCD

Mass (GeV)

$G$

$\sigma$

level repulsion!

$\sim m_G/2$
Level repulsion
A global phenomenon!

$\phi$ mass

$\sigma$ m

$\sim m_G/2$

G

Arch bridge
Summary

2\textsuperscript{nd} order $\implies$ vanishing of a certain screening mass
Crossover $\implies$ dropping of a certain screening mass

The two phase transitions are just different limits of a single (crossover) phase transition driven by the lightest screening state $\phi$.

The glueball-sigma mixing and level repulsion provides not only a unified description of the QCD phase transitions, but also a consistent characterization of the entire phase diagram in the $(T, m)$ plane.
Mode Coupling Theory for the Dynamic Aspect of the Chiral Phase Transition

K. Ohnishi
Mode coupling theory for the dynamic aspect of the chiral phase transition

K. Ohnishi (Komaba, Univ. of Tokyo) with
K. Fukushima (MIT)
K. Ohta (Komaba, Univ. of Tokyo)

Abstract
We reanalyze the dynamic aspect of the chiral phase transition by means of the mode coupling theory. We point out the difference of the dynamic behaviors between the chiral phase transition and the antiferromagnet, which have been considered to belong to the same dynamic universality class. We also suggest reconsideration of Halperin and Hohenberg's rules for the classification of the dynamic universality class. By application of the mode coupling theory to the chiral phase transition, we derive the kinetic equation for it and find the dynamic critical exponent to be $z = 1 - \eta/2 \cong 0.98$. The value is contrasted with $z = d/2 = 1.5$ of the antiferromagnet. The difference explicitly shows the different dynamic behavior of the two systems.

Content
1. Introduction
2. Dynamic universality class of the chiral phase transition
3. Mode coupling theory for the linear sigma model
4. Summary
1. Introduction

We are interested in the non-equilibrium or dynamic property of the 2nd order chiral phase transition.

Dynamic critical phenomena

- Critical slowing down
- Softening of propagating modes etc.

Degrees of freedom for describing the non-equilibrium behavior

\[ \cdots \quad \text{Slow variables} \quad \left\{ \begin{array}{l} \text{Order parameter} \\ \text{Conserved quantities} \end{array} \right\} \]

Ex. The magnetization \( M \) and the energy \( E \) are the slow variables for the ferromagnet.
The fluctuations of slow variables ⇒ Slow modes

Slow mode \[\cdots\] \{ Diffusive mode
                    \ Propagating mode

Propagating mode \[\cdots\] spin wave, sound wave, particle mode etc.

Slow modes appear in the (dynamic) spectral function as narrow peaks.

\[ S(\omega)/\omega \quad \quad q \ll 1 \text{ finite and fixed} \]

\begin{align*}
\text{Propagating} & \quad \Rightarrow \quad \text{Narrowing} \\
\quad & \quad \Rightarrow \quad \text{Diffusive} \\
\quad & \quad \Rightarrow \quad \text{Propagating}
\end{align*}

\begin{align*}
\text{Diffusive mode} & \quad : \quad \text{Pole with only the imaginary part.} \\
\text{Propagating mode} & \quad : \quad \text{Pole with the imaginary part as well as the real part (frequency } \omega_0).\end{align*}
Example — Heisenberg Ferromagnet —

Slow variables ⋯
\[
\begin{align*}
\text{Magnetization (order parameter)} & \quad \vec{M} = (M_x, M_y, M_z) \\
\text{Energy (conserved quantity)} & \quad E
\end{align*}
\]

- Ordered phase \((T < T_c)\)

\[
\begin{align*}
M_x \\
M_y \\
M_z \\
E
\end{align*}
\]

- Propagating mode (Spin wave)
- Diffusive mode

- Disordered phase \((T_c < T)\)

\[
\begin{align*}
M_x & \quad \text{Diffusive mode} \\
M_y & \quad \text{Diffusive mode} \\
M_z & \quad \text{Diffusive mode} \\
E & \quad \text{Diffusive mode}
\end{align*}
\]

- Dynamic universality class

How to classify critical points:
\(\Leftarrow\) Halperin & Hohenberg (Rev.Mod.Phys. 49 (1977) 435)

(i) Whether or not the order parameter is a conserved quantity.

(ii) What kinds of conserved quantity are contained.

Observed critical points (ferromagnet, antiferromagnet, liquid-gas, superfluid etc) have been comprehensively classified.
2. Dynamic universality class of the chiral phase transition

Chiral phase transition belongs to the same dynamic universality class as that of the antiferromagnet.

$\Leftarrow$ Rajagopal & Wilczek (NPB 399 (1993) 395)

- Slow variables for the both systems

<table>
<thead>
<tr>
<th></th>
<th>antiferromagnet</th>
<th>chiral</th>
</tr>
</thead>
<tbody>
<tr>
<td>order parameter</td>
<td>staggered magnetization</td>
<td>meson field $\phi$</td>
</tr>
<tr>
<td>conserved quantity</td>
<td>magnetization $M$</td>
<td>chiral charge $Q$</td>
</tr>
<tr>
<td></td>
<td>energy $E$</td>
<td>energy $E$</td>
</tr>
<tr>
<td></td>
<td>momentum $P^i$</td>
<td>momentum $P^i$</td>
</tr>
</tbody>
</table>

Rajagopal and Wilczek have analyzed the chiral phase transition by using the kinetic equation of the antiferromagnet. They have obtained the dynamic critical exponent $z = d/2 = 3/2$.

However this is not the case.

- Slow modes in the disordered phase:

$$\begin{cases}
\text{Staggered mag.} & \cdots \text{diffusive mode} \\
\text{Meson field} & \cdots \text{propagating mode (particle mode)}
\end{cases}$$

Different slow modes apparently indicate the different dynamic behavior of the two systems.

The two systems cannot be in the same dynamic universality class.

$\Rightarrow$ The prescription of Halperin and Hohenberg for classifying the dynamic universality class does not work.
3. Mode coupling theory for the linear sigma model

- Mode coupling theory (Kawasaki, Ann.Phys.61 (1970) 1)

Kinetic equation for the slow variables $A_j$:

$$\frac{d}{dt} A_j(t) = \sum_l \left( i\omega_{jl} - \frac{k_B L^0_{jl}}{\chi_l} \right) A_l(t) + \frac{i}{2} \sum_{lm} \Omega_{j;lm} (\chi_l \chi_m)^{-\frac{1}{2}} (A_l A_m - \langle A_l A_m \rangle) + f_j$$

where

$$\omega_{jl} = -i \left( \dot{A}_j, A_l^\dagger \right) / \chi_l = -k_B T \left\langle \left[ A_j, A_l^\dagger \right] \right\rangle / \chi_l$$

$$\Omega_{j;lm} = -k_B T \left\{ \left\langle \left[ A_j, A_l^\dagger A_m^\dagger \right] \right\rangle - \left\langle \left[ A_j, A_p^\dagger \right] \right\rangle \chi_p^{-1} \left( A_p, A_l^\dagger A_m^\dagger \right) \right\} / (\chi_l \chi_m)^{\frac{1}{2}}$$

$\omega_{jl}$ gives the frequency matrix for the slow modes.

$$\begin{cases} 
\omega = 0 \rightarrow \text{diffusive mode} \\
\omega \neq 0 \rightarrow \text{propagating mode}
\end{cases}$$

- O(2) linear sigma model

$$\mathcal{L} = \frac{1}{2} \left[ (\partial_\mu \phi_1)^2 + (\partial_\mu \phi_2)^2 \right] - \frac{1}{2} \mu^2 \left( \phi_1^2 + \phi_2^2 \right) - \frac{\lambda}{4} \left( \phi_1^2 + \phi_2^2 \right)^2$$

$$\phi_a(x, t) = \frac{1}{\sqrt{V}} \sum_k \frac{1}{\sqrt{2\omega_{ak}}} \left( a_k^a(t) + a_{-k}^a(t) \right) e^{ik \cdot x}$$

$$\pi_a(x, t) = \frac{1}{\sqrt{V}} \sum_k (-i) \sqrt{\omega_{ak} / 2} \left( a_k^a(t) - a_{-k}^a(t) \right) e^{ik \cdot x}$$

Commutation relation

$$\left[ \phi^a(x, t), \pi^b(y, t) \right] = i\delta_{ab} \delta(x - y),$$

$$\left[ \phi^a(x, t) \phi^b(y, t) \right] = \left[ \pi^a(x, t), \pi^b(y, t) \right] = 0$$
Slow variables $A_j$

\[ \phi^a(t) = \frac{1}{\sqrt{V}} \int d^3 x e^{-i \vec{q} \cdot \vec{x}} (\phi^a(x,t) - \langle \phi^a \rangle) \]

\[ Q_q(t) = \frac{1}{\sqrt{V}} \int d^3 x e^{-i \vec{q} \cdot \vec{x}} \epsilon^{ab} \pi^a \phi^b(x,t) \]

\[ E_q(t) = \frac{1}{\sqrt{V}} \int d^3 x e^{-i \vec{q} \cdot \vec{x}} \left( \frac{1}{2} (\partial_t \phi^a)^2 + \frac{1}{2} (\nabla \phi^a)^2 + \frac{1}{2} \mu^2 (\phi^a)^2 + \frac{\lambda}{4} (\phi^a)^4 \right) (x,t) \]

\[ P^i_q(t) = \frac{1}{\sqrt{V}} \int d^3 x e^{-i \vec{q} \cdot \vec{x}} (-\pi^a \nabla^i \phi^a) (x,t) \]

From these slow variables, the propagating mode for the meson mode does not appear.

\[ \implies \text{Canonical momentum } \pi \text{ is necessary.} \]

The finite commutation relation $[\phi(x,t), \pi(y,t)] = i \delta(x-y)$ gives the finite frequency.

Frequency matrix in the disordered phase ($T > T_c$)

\[
\omega_{jl} = \begin{pmatrix}
-\frac{k_B T}{2 \omega_{1q} \chi_{1q}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{k_B T}{2 \omega_{1q} \chi_{1q}} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{k_B T}{2 \omega_{2q} \chi_{2q}} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{k_B T}{2 \omega_{2q} \chi_{2q}} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{k_B T}{2 \omega_{2q} \chi_{2q}} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{D}{\chi_{eq}} q^i \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]

\[ \frac{k_B T}{2 \omega_{aq} \chi_{aq}} = \omega_{aq} \implies \omega_{aq} = \sqrt{q^2 + m^2} \implies 2 \chi_{aq} = \frac{k_B T}{q^2 + m^2} \]

Meson modes have appeared.
The microscopic equation of motion for the meson field

The Heisenberg equation for the meson field:

\[ \frac{d}{dt} \phi(\vec{x}, t) = [\phi(\vec{x}, t), H] \]
\[ \frac{d}{dt} \pi(\vec{x}, t) = [\pi(\vec{x}, t), H] \]

⇒ \( \phi \) and \( \pi \) are necessary in order to describe the meson dynamics.

cf. Antiferromagnet

The Heisenberg equation is for the spin variables:

\[ i \frac{d}{dt} s^\alpha_i = [s^\alpha_i, H] \quad (\alpha = x, y, z) \]
\[ H = \sum_{jl} J_{ij} \vec{s}_j \cdot \vec{s}_l \]

⇒ The three staggered magnetization \( N_{x,y,z} \) are the appropriate degrees of freedom for the antiferromagnet.

We can derive the full kinetic equation for the \( O(4) \) sigma model.

The dynamic critical exponent \( z = 0.98 \) is obtained.

\( \Leftrightarrow z = d/2 \) for the antiferromagnet
4. Summary

- The chiral phase transition and the antiferromagnet are not in the same dynamic universality class.
  The slow modes are different in the two systems in spite of perfect coincidence of the slow variables (the order parameter and conserved quantities).
  ⇒ Halperin and Hohenberg's prescription for classification of the dynamic universality class does not work.

- We reanalyzed the chiral phase transition by means of the mode coupling theory.
  ⇒ The canonical momentum $\pi$ is necessary in order for the meson mode (particle mode) to appear.

- It is necessary to analyze the critical points that involves the particle mode, such as
  - Critical end point
  - Tricritical point
  - Color superconductor
  - Confinement-deconfinement transition
  - Elector-weak transition

because they are not classified into any dynamic universality classes that have been considered in "Rev.Mod.Phys. 49 (1977) 435 Halperin & Hohenberg."
The Effect of Memory on Relation in $\phi^4$ Theory

Takashi Ikeda
The effect of memory on relaxation in $\phi^4$ theory

Takashi Ikeda
(RIKEN BNL Research Center)

Abstract

We derive a kinetic equation including a memory effect in $\phi^4$ theory by using a generalized Kadanoff-Baym ansatz. Solving the kinetic equations with and without the memory effect, we found that, while relaxation of a single particle excitation is unaffected by that effect in the weak coupling regime, that effect leads to faster relaxation in the moderately large coupling and to slower relaxation in the strong coupling.
Motivation

- Nonequilibrium physics
  - challenging problem in relativistic many-body theory
  - motivated by Heavy-ion collisions, early universe, etc......

Generalized kinetic equations beyond Boltzmann eq.
are necessary for
strongly correlated and/or dense plasmas!

- (Classical) Boltzmann equations;
  - Including scattering effect, but independent collisions
  - strongly correlated plasmas
Strategy

- Starting point: Kadanoff-Baym (KB) equations
  - Designed to study nonequilibrium phenomena
  - One-particle distribution function, mean field, scattering effect, memory effect, dynamical spectral function

KB equations

- Equal time limit
  - Generalized KB ansatz for two-time Green's functions (PRB34(1986), P. Lipavsky et al.)
    - Quasiparticle approximation
      - Kinetic equations including memory effect
        - (non-Markovian collision term)
          - Neglecting memory effect
            - Boltzmann-type equation with Markovian collision term
Model: $\phi^4$ theory

- Lagrangian: $\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - g^2 \phi^4$

- In 3+1 dimensions and spatially homogeneous case
- Selfenergies as in thermal equilibrium, temperature $T$
- Single particle excitation

Self-energies from 3-loop order of 2PI effective action

- Leading order
  \[ \frac{M_T}{g^2} = M_T \]
  generating effective mass $M_T$
  by solving gap eq. self-consistently
  (Mean field)

- Next-to Leading order
  \[ \frac{g^2}{g^2} = \sum \]
  leading to damping of excitation
  (Scattering effect)
**Thermal Mass**

Gap equation

\[ M_T^2 = 12g^2 \int \frac{d^3k}{(2\pi)^3} \frac{N_{\text{eq}}(\omega_k)}{\omega_k} + \frac{3g^2M_T^2}{4\pi^2} \left( \ln \frac{M_T^2}{\mu^2} - 1 \right) \]

\( \omega_p^2 = p^2 + M_T^2 \), modified MS scheme

High temperature expansion

(Leading order of HTL)

\[ M_T^2 = g^2 T^2 \]

Result agrees with PRD63 065003 by J.-P.Blaizot et al.
From KB eq. to Boltzmann eq.

KB eq. in equal-time limit

\[ \frac{x_0 = y_0 = t}{s = x_0 - y_0} \]

\[
2 \frac{\partial_t \partial_s G^< (p, s, t)}{s = 0} = -i \int_{t_0}^{t} dz_0 [ \Sigma^> (p, t, z_0) G^< (p, z_0, t) - \Sigma^< (p, t, z_0) G^> (p, z_0, t) \\
+ G^< (p, t, z_0) \Sigma^> (p, z_0, t) - G^> (p, t, z_0) \Sigma^< (p, z_0, t)]
\]

Reconstruction Problem:

- **Kadanoff-Baym (KB) ansatz:** \( G^> (k, t) = \rho (k, t) N^> (k, t), \quad (N^< = N, \quad N^> = 1+N) \)
  
  lead to inconsistencies in time argument of memory effect

- **Generalized KB ansatz:** \( G^> (p, x; x_0, y_0) = 1/i [G_R (p, x; x_0, y_0) N^> (p, x; y_0) \\
- N^> (p, x; x_0) G_A (p, x; x_0, y_0)] \)

  (PRB34(1986), P. Lipavsky et al.)

  take into account memory effect properly

using quasiparticle approximation: \( \rho (p, p_0) = 2\pi \epsilon (p_0) \delta (p_0^2 - p^2 - M_T^2) \)
From KB eq. to Boltzmann eq. (cont'd)

Kinetic equation with memory effect for $\delta N (= N - N_{eq})$

$$\frac{\partial}{\partial t} \delta N(p,t) = 2 \int_{t_0}^{t} dz_0 \left( \frac{\partial}{\partial z_0} \gamma(p,t-z_0) \right) \delta N(p,z_0)$$

Non-Markovian collision term

Time-dependent damping rate

Neglecting memory effect

$N(p,z_0) \rightarrow N(p,t)$

Finite-initial time generalization of Boltzmann eq.

$$\frac{\partial}{\partial t} \delta N(p,t) = -2 \gamma(p,t-t_0) \delta N(p,t)$$

Markovian collision term

Memory effect

Complete collision limit $t_0 \rightarrow -\infty$

(Classical) on-shell Boltzmann eq.

$$\frac{\partial}{\partial t} \delta N(p,t) = -2 \gamma_\infty(p) \delta N(p,t)$$

exponential damping!
Time-dependent Damping Rate

\[ \gamma(p,t) \equiv \frac{1}{2\omega_p} \int \frac{dp_0}{2\pi} \left[ \Sigma^{<}(p) - \Sigma^{>}(p) \right] \frac{\sin(p_0 - \omega_p)t}{p_0 - \omega_p} \]

\[ \text{On-shell limit} \]

(Fermi's golden rule)

- In the early time... Oscillation
  More violently for stronger coupling

- In the late time
  Approach to some constant value

\[ \gamma_{\infty}(p=0) = \frac{1}{4M_T} \left[ \Sigma^{<}(p,M_T) - \Sigma^{>}(p,M_T) \right] \]

Damping rate in on-shell Boltzmann eq.
Damping Processes

\[ g(2\pi T) = 0.69 \]

\[ \frac{\partial}{\partial t} \delta N(p, t) = 2 \int_0^t dz_0 \left( \frac{\partial}{\partial z_0} \gamma(p, t - z_0) \right) \delta N(p, z_0) \]

Solve these equations &
compare solutions

\[ \frac{\partial}{\partial t} \delta N(p, t) = -2 \gamma(p, t) \delta N(p, t) \]

Markovian solution
Decay of Single Particle Excitation: I

- Weak coupling regime

\[ g(2\pi T) = 0.105 \]

- In complete agreement
- Exponential damping

Why?

Relaxation time \( tT \approx 4300 \)

- Time \( tT \approx 20 \): oscillation of \( \gamma(t) \) ends
  (separation of scales)

In this case, kinetic equations essentially coincide with on-shell Boltzmann eq.

Memory effect doesn't influence relaxation in weak coupling regime.
Decay of Single Particle Excitation: II

- Moderately large coupling regime

\[ \frac{\delta N(p=0,t)}{\delta N(p=0,0)} \]

\[ g(2\pi T) = 0.69 \]
\[ g(2\pi T) = 0.91 \]
\[ g(2\pi T) = 1.02 \]

Memory effect leads to faster relaxation!!

Consistent with results of KB equations in 2+1 dimensions.

Note: relaxation with memory effect seems to represent exponential damping in late times.
Decay of Single Particle Excitation: III

- Strong coupling regime

Memory effect leads to:
- Oscillation around thermal value
- Slower relaxation of amplitude

This never occurs without memory effect. Damping rate is always positive!
Summary and Outlooks

- Construct kinetic equation including memory effect from KB eqs. by generalized Kadanoff-Baym ansatz.

- Memory effect doesn't influence relaxation in weak coupling regime, due to separation of scales.

- As coupling increases, memory effect leads to:
  - Faster relaxation for moderately large coupling
  - Oscillating around thermal value and slower relaxation for strong coupling

- Outlooks:...  
  - Totally nonequilibrium situation
  - Directly solve KB equation
  - Fermionic system (NJL model)
  - Non-abelian gauge theory
Glueball Properties
Near the QCD Phase Transition

Noriyoshi Ishii
Glueball properties near the QCD phase transition

Noriyoshi Ishii (TITECH)
in collaboration with
Hideo Sukanuma (TITECH)

We have reported glueball properties near the critical temperature $T_c$ of QCD phase transition using quenched SU(3) anisotropic lattice QCD Monte Carlo calculation. We have first reviewed the effective model results, which support the pole mass reduction of the scalar glueball near $T_c$. We have then constructed the glueball correlators from 5,000-9,900 gauge field configurations generated with the quenched lattice QCD with $\beta = 6.25$ and the renormalized anisotropy $\gamma_g = a_s / a_T = 4$ at finite temperature. In addition to the ordinary pole-mass analysis, where the narrowness of the glueball peak is assumed, we have analyzed the glueball correlators with an advanced analysis, which can take into account the effects of thermal width of the glueball peak. We have found that the scalar glueball receives a huge thermal effect near $T_c$, and that this thermal effect can be understand as a significant thermal width broadening of the glueball peak as $\Gamma(T_c) \approx 300 \text{ MeV}$ together with a modest reduction in the peak center as $\Delta \omega_0(T_c) \approx 100 \text{ MeV}$. We finally make a comment on the origin of such a large thermal width and a possible physical implication of our results.
Glueball properties near the QCD phase transition

Noriyoshi Ishii (TITECH)
in collaboration with
Hideo Sukanuma (TITECH)

Plan of the talk

1. Motivation / Background
2. Lattice QCD formalism
3. Numerical Results
4. Summary / Discussion

START
Motivation / Background

At $T > 0$, QCD vacuum is expected to change its structure

Low Temperature $\rightarrow$ Tc $\rightarrow$ High Temperature

Confinement (hadron) phase
- color confinement
- spontaneous chiral symmetry breaking ($\Rightarrow$ massive constituent quarks)

The onset of phase transition
- Partial restoration of the chiral symmetry
- Reduction of the string tension

QGP Phase
- deconfined quarks and gluons
- chiral symmetry restoration (massless current quark)

Hadron = bound state of quarks & gluons

Properties (mass, width, etc.) of some hadrons are expected change as a result of the change of QCD vacuum,
- charmonium
- light q qbar
- glueball

$T = 0.87T_c$ $\Rightarrow$ $T = 0.93T_c$

- T. Hatsuda et al., PRL55, 158 (85)
- T. Hashimoto et al., PRL57, 2123 (86)
- T. Hatsuda et al., PRD47, 1225 (93)
- H. Ichie et al., PRD52, 2944 (95)
Lattice studies of thermal hadrons near Tc

Lattice QCD simulations are performed to observe these phenomena.

1. QCD-TARO Collaboration, PRD63,054501('01) ↔ light q qbar
2. T.Umeda et al., Int. J. Mod. Phys. A16,2215('01) ↔ charmonium

Unfortunately, no significant changes are observed below Tc.

We are interested in glueball at finite temperature, due to

1. The significance of the nonperturbative gluon condensate $\langle GG \rangle$ suggests condensation of the 0++ glueball in the vacuum like Higgs particle.
   $\longrightarrow$ the 0++ glueball may be sensitive to the change of QCD vacuum.

2. The large difference between the glueball mass and Tc in quenched QCD.
Glueballs

\[ L = -\text{tr} \, G_{\mu\nu} G^{\mu\nu} + \sum \bar{q}_f (i \gamma^\mu D_\mu - m_f) q_f \]

\[ D_\mu = \partial_\mu - igA_\mu T^a \]

\[ G^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc} A^b_\mu A^c_\nu \]

Glueballs are the hadrons which mainly consist of gluons only. Their existence is a prediction of QCD.

Non-Abelian nature of the gauge group

self-interaction of gluons

glueball = bound state of gluons

\[ \text{glueball} \]
Theoretical studies of glueball

There have been a lot of theoretical studies on the glueball:

✓ MIT bag model
T.Barnes et al., NPB224(’83)241., etc.

✓ Constituent gluon model
D.Hom et al., PRD17(’78)898., etc.

✓ Flux tube model
N.Isgur et al., PLB147(’84)169., etc.

✓ Instanton liquid model
T.Schaefer et al., PRL75(’95)1707.

✓ QCD sum rule

✓ Lattice QCD

  ➢ quenched level
C.J.Morningstar et al., PRD66(’99)034509.
J.Sexton et al., PRL75(’95)4563., etc.

  ➢ full QCD
K.M.Bitar et al., PRD44(’91)2090.
A.Hart et al., PRD65(’02)034502., etc.

FIG. 8: The mass spectrum of glueballs in the pure SU(3) gauge theory. The masses are given in terms of the hadron scale $\mu$, along the left vertical axis, and in terms of GeV along the right vertical axis, minimizing $\sim 10$ MeV. The mass information indicated by the vertical arrows of the boxes do not include the momentum in setting $\mu$. The location of states whose interpretation requires further study are indicated by the dashed hollow boxes.

Except for minor variations, quenched lattice QCD predicts

\[
m(0^{++}) \approx 1500 - 1700 \text{ MeV} \\
m(2^{++}) \approx 2000 - 2400 \text{ MeV}
\]
The glueball in experiment

The glueball can mix with flavor-singlet q qbar.

Careful considerations are necessary to determine which flavor-singlet meson is a glueball!

We need criterion to determine which one is a glueball:

- Glueballs are created in the glue-rich process
- Glueballs should be exotic mesons
- Their decay width should be narrower than ordinary mesons (OZI rule)
- Flavor-blind decay
- Glueballs should not decay into two photons.

Serious glueball candidates:

\[ f_0(1500), f_0(1710) \]
The large difference between the glueball mass and $T_c$ in quenched QCD

In quenched QCD,

$$T_c \approx 260 - 280 \text{ MeV} \ll m_G \approx 1500 - 1700 \text{ MeV}$$

⇒ Thermal excitation of the glueball is suppressed in the confinement phase by a very strong statistical factor as

$$\exp \left(-\sqrt{m_G^2 + \vec{p}^2} / T \right) \approx e^{-m_G/T_c} \approx 0.00207$$

The glueball density is very small even near $T_c$:

$$\rho(T_c) = N(T_c) / V = \int \frac{d^3k}{(2\pi)^3} \frac{1}{e^{\sqrt{m_G^2 + k^2} / T_c} - 1} \approx \frac{1}{(4.0 \text{ fm})^3}$$

This density is considered to be TOO SMALL:

• The glueball is considered to have a rather small size!
  $$r \leq 0.2 - 0.4 \text{ fm}$$

• The interactions among glueballs are expected to be very short-ranged, since the interaction is mediated by rather heavy glueball ($m_G > 1.5 \text{ GeV}$) exchanges.
One possible answer would be

**glueball (pole) mass reduction** (and/or **swelling of the glueball size**)

near the critical temperature $T_C$

In fact, the glueball mass reduction near $T_C$ is suggested by

H. Ichie et al., PRD52, 2944 (1995)

based on an effective model studies (dual Ginzburg-Landau model).

In this talk, we report the anisotropic lattice QCD study of the thermal glueball near the critical temperature.

**References:**

- N. Ishii et al., PRD66,094506(2002)
- N. Ishii et al., EPJA17,77(2003)
Dual Ginzburg-Landau (DGL) model
(an effective model of confinement based on the dual Higgs Mechanism)

\[ L = -\frac{1}{4} \left( \partial_\mu \vec{B}_\nu - \partial_\nu \vec{B}_\mu \right)^2 + \sum_{\alpha=1}^{3} \left[ (i\partial_\mu - g\vec{E}_\alpha \cdot \vec{B}_\mu) \chi_\alpha \right] \left[ -\lambda \left( |\chi_\alpha|^2 - v^2 \right)^2 \right] \]

- \( B_\mu = (B^3_\mu, B^8_\mu) \) dual gauge field
- \( \chi_\alpha (\alpha = 1, 2, 3) \) QCD monopole field
- \( \epsilon_1 = (1, 0), \epsilon_2 = (-\frac{1}{2}, -\frac{\sqrt{3}}{2}), \epsilon_3 = (-\frac{1}{2}, \frac{\sqrt{3}}{2}) \)

Higgs phase of DGL
Electric flux is squeezed due to dual Meissner mechanism

confinement phase of QCD

**Effective Potential**

**Vacuum expectation value**

**The glueball mass reduction near \( T_c \)**

A particular combination:
\[ \phi = \chi_1 + \chi_2 + \chi_3 \]
satisfies gauge-invariance condition, and becomes observed as a physical particle, i.e., the 0++ glueball

cf. H. Ichie et al., PRD60, 077501 ('99)

\[ m_G^2 = 4\lambda \chi^2 \]

30-40% pole mass reduction of glueball near \( T_c \)

**Important:** We use the DGL argument only in the section of motivation. Our main results are nothing to do with it.
Lattice QCD formalism

\[ L = -\text{tr} G_{\mu\nu} G^{\mu\nu} + \sum_\text{fermions} \bar{q}_i \left( i\gamma^\mu \not{D}_\mu - m_f \right) q_i \]

\[ D_\mu = \partial_\mu - ig A_\mu^a T^a \]

\[ G_{\mu\nu}^a \equiv \partial_\mu A_\nu^a + \partial_\nu A_\mu^a + ig f^{abc} A_\mu^b A_\nu^c \]

Weak coupling at short distance (asymptotic freedom)
But strong coupling at long distance!

discretization

\[ S = S_G + S_F \]

\[ S_G = - \frac{1}{2g^2} \sum_{x,\mu\nu} \text{tr} \left( U_{\mu\nu}(x) \right) \]

\[ U_{\mu\nu}(x) = U_\mu^\dagger(x) U_\nu(x + \hat{\mu}) U_{\mu\nu}^+(x + \hat{\nu}) U_\nu^\dagger(x) \]

\[ S_F : \text{Fermionic part} \]

\[ S_G = - \frac{1}{2} \sum \text{tr} \left( \exp \left( ia^2 g^2 G_{\mu\nu}(x) \right) \right) = \frac{1}{4} \int d^4 x G_{\mu\nu}^a G^{a\mu\nu} \]

site
quark fields live on sites.

link

\[ U_\mu(x) = \exp \left[ i a g A_\mu^a(x) T^a \right] \in SU(3)_c \]
The glueball correlation

\[ \phi(\tau) \equiv \mathcal{G}_{ij}^a \mathcal{G}_{ji}^a \]

The glueball temporal correlator:

\[ G(\tau) \equiv Z(T)^{-1} \text{Tr} \left( e^{-\mu_0 c_{\text{vac}} T} \phi(\tau)\phi(0) \right) \]

The correlator

Lattice QCD Monte Carlo calculation provides numerical data of the correlator.

The spectral function:

\[ \rho(\omega) = \sum_{m,n} \frac{|\langle n | \phi | m \rangle|^2}{Z(\beta)} e^{-\beta E_m} \times 2\pi \delta(\omega - \Delta E_{mn}) - \delta(\omega + \Delta E_{mn}) \]

The spectral function

The spectral representation

\[ G(\tau) = \int \frac{d\omega}{2\pi} \frac{\cosh[\omega(1/\beta - \tau)]}{\sinh[\beta\omega/2]} \rho(\omega) \]

• We adopt some Ansatz on \( \rho(\omega) \) and fit the lattice QCD data of \( G(\tau) \) to extract the physical observables such as mass, width, etc.

• Maximal Entropy Method to extract \( \rho(\omega) \) from \( G(\tau) \) directly. (no Ansatz is needed)
Narrow peak ansatz

If we assume the **narrowness of the peak**, the spectral function can be approximated by introducing a temperature dependent "pole mass" \( m(T) \).

\[
\rho(\omega) \approx 2\pi A [\delta(\omega - m(T)) - \delta(\omega + m(T))]
\]

At \( T > 0 \), each peak acquires a **thermal width** through the interaction with the heat bath. \( \neq \text{decay width} \)

It is desirable to respect the presence of thermal width at \( T > 0 \).
Ansatz with thermal width

- What is the appropriate functional form?
  - With increasing $T (>0)$, bound state poles of $G_R(\omega)$ are moving off the real $\omega$ axis into complex $\omega$ plane.

  \[
  \rho(\omega) = -2\text{Im}(G_R(\omega)) = 2\pi A \left[ \delta_R(\omega-\omega_0) - \delta_R(\omega+\omega_0) \right] + \cdots
  \]

  Lorentzian at $\omega_0$ with width $\Gamma$
  \[
  \delta_R(\omega-\omega_0) = \frac{1}{\pi} \text{Im} \left[ \frac{1}{\omega - \omega_0 + i\Gamma} \right] = \frac{1}{\pi} \frac{\Gamma}{(\omega - \omega_0)^2 + \Gamma^2}
  \]

- The appropriate functional form is "Breit-Wigner type".
  \[
  g(t) = \int_0^\infty \frac{d\omega}{2\pi} \frac{\cosh[\omega(\beta/2-t)]}{\sinh[\beta\omega/2]} 
  \times 2\pi A \left[ \delta_R(\omega-\omega_0) - \delta_R(\omega+\omega_0) \right] 
  \]

  $A$, $\Gamma$, $\omega$ fit parameters
What happens if the thermal width is broad?

The spectral representation

\[ G(t) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\rho(\omega)}{2 \sinh(\beta \omega/2)} \cosh[\omega(\beta/2 - t)] \]

\( G(t) \) is an average of \( \cosh[\omega(\beta/2 - t)] \) with the weight \( \frac{\rho(\omega)}{2 \sinh(\beta \omega/2)} \).

In the narrow peak ansatz, the weight is approximated by the \( \delta \) function.

The role of the denominator \( 2 \sinh(\beta \omega/2) \equiv e^{\beta \omega/2} \):

- Small \( \omega \) region \( \leftrightarrow \) enhanced,
- Large \( \omega \) region \( \leftrightarrow \) suppressed.

The peak position is shifted to below!
Numerical Results

The lattice parameter setup:

1. Gauge config. by anisotropic Wilson action with $\beta_{\text{lat}} = 2N_c / g^2 = 6.25$

2. Lattice spacings are determined from the string tension as $\sqrt{\sigma} = 440 \text{MeV}$

$$a_s = 0.084\text{fm}, \ a_t = 0.021\text{fm} \quad \rightarrow \quad \text{renormalized lattice anisotropy} \quad a_s / a_t = 4$$

3. The Lattice size: $20^3 \times N_t (N_t = 24, \ldots, 72) \leftrightarrow T = 390, \ldots, 130 \text{MeV}$

4. For each $T$, we pick up gauge configs every 100 sweeps after skipping 20,000 sweeps for thermalization.

5. Number of gauge configs: 5,500-9,900 (bin size: 100)

6. We adopt APE smearing to improve the glueball operator to have a better overlap to the lowest glueball peak.

7. The critical temperature is determined from the Polyakov loop susceptibility as

$$T_c \approx 280 \text{MeV}$$
The glueball correlator at low temperature (T=130MeV)

Spatially extended operator is used to enhance the lowest-lying glueball peak.

The both ansatz fit the lattice QCD data very well.

It is natural, since the thermal width is still narrow at low temperature.
The effective mass, the effective center and the effective width

The effective mass

\[
\frac{G(t)}{G(t+1)} = \frac{\cosh[(t - \beta/2)m_{\text{eff}}(t)]}{\cosh[(t + 1 - \beta/2)m_{\text{eff}}(t)]}
\]

The existence of the plateau is a necessary condition for the single pole saturation.

The effective center and the effective width

\[
\frac{G(t)}{G(t+1)} = \frac{g(t, \omega_{\text{eff}}(t), \Gamma_{\text{eff}}(t))}{g(t+1, \omega_{\text{eff}}(t), \Gamma_{\text{eff}}(t))}
\]
\[
\frac{G(t+1)}{G(t+2)} = \frac{g(t+1, \omega_{\text{eff}}(t), \Gamma_{\text{eff}}(t))}{g(t+2, \omega_{\text{eff}}(t), \Gamma_{\text{eff}}(t))}
\]

The existence of the simultaneous plateau is a necessary condition for the single peak saturation.

\[
g(t, \omega_0, \Gamma) = \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\cosh[\omega(\beta/2-t)]}{2\sinh(\beta\omega/2)} \times 2\pi \left[ \delta_R(\omega - \omega_0) - \delta_R(\omega + \omega_0) \right]
\]
The glueball correlator at high temperature \((T=253\text{MeV} < T_c)\)

\[
\frac{G(t)}{G(0)}
\]

Spatially extended operator is used to enhance the lowest-lying glueball peak.

The **narrow-peak ansatz** fails to fit the lattice QCD data around \(t=0\).

The **Breit-Wigner ansatz** fits the lattice QCD data in the whole region rather well.
The smearing dependence ($T < T_c$)

From the operator size of the best smearing operator at various $T$, we find no significant thermal effect on the glueball size below $T_c$. $r \approx 0.43 \text{ fm}$ for all $T < T_c$. 

The lowest peak contribution seems to be maximally enhanced in this region.
The glueball correlator above $T_c$ ($T=390$ MeV)

Spatially extended operator is used to enhance the lowest-lying glueball peak.
The peak center

The thermal width

The polemass from narrow peak ansatz

- The Breit-Wigner ansatz
  - thermal width broadening near Tc
  - $\Gamma(T_c) \approx 300 \text{ MeV}$
  - modest reduction in peak center
    $\Delta a \lesssim 100 \text{ MeV}$

- The narrow peak ansatz
  - polemass reduction near Tc
    $\Delta m \approx 300 \text{ MeV}$
The low lying glueball peak(0++)

\[ \frac{\rho(\omega)}{G(0)} \text{[GeV}^1\text{]} \]

The thermal width is seen to become broader with increasing temperature.
Maximum Entropy Method

A method to directly obtain the spectral function without adopting a particular Ansatz.

"Maximize":

\[ Q = \alpha S - \frac{1}{2} \chi^2 \]

where

\[ \chi^2 = \sum_{\tau,l} \left( G(\tau_l) - G_A(\tau_l) \right) \left( c_\tau \right) \]

Shannon-Jaynes entropy:

\[ S \equiv \int_0^\infty d\omega \left[ A(\omega) - m(\omega) - A(\omega) \log \left( \frac{A(\omega)}{m(\omega)} \right) \right] \]

\[ K(\tau,\omega) = \frac{\cosh \left[ \omega (\beta / 2 - \tau) \right]}{\cosh \left[ \beta \omega / 2 \right]} \]

\[ G_A(\tau) = \int_0^\infty d\omega K(\tau,\omega) A(\omega) \]

\[ m(\omega) \propto \omega^4 \exp(-\omega^2/4) \]

Graphical representation with data points for different temperatures.
Summary / Discussion

1. We have constructed the **temporal $0^+$ glueball correlators** at finite temperature using 5,500-9,900 gauge configurations generated by SU(3) anisotropic lattice QCD at quenched level.

2. We have performed an advanced analysis adopting **Breit-Wigner ansatz** for the spectral function in order to take into account the possible appearance of the thermal width at finite temperature.
   a. A significant thermal width broadening near $T_c$: $\Gamma(T_c) \cong 300 \text{ MeV}$
   b. A modest reduction in peak center near $T_c$: $\Delta \omega_0(T_c) \cong 100 \text{ MeV}$
   c. No significant thermal effects in the glueball size.

3. What is the origin of such a huge thermal width?
   a. It is not thermally excited lowest-lying glueballs. (Too small glueball density even at $T_c$)
   b. The contributions of bound state glueballs listed in C.J.Morningstar et al., PRD60,034509('99) is estimated to be not significant. (They are too heavy.)
   c. We have to resort to **higher energy resonance modes** trusting in the sufficient increase in the number of modes. $\mapsto$ Hagedom's picture, Resonance gas model, ...?

4. Future Plans:
   Glueball correlations above $T_c$, Experimental observability, Extension to the unquenched QCD calculations, etc.
Contributions from higher resonances

\[ Z = \sum_n e^{-E_n/T} \]

\[ = \int_0^\infty dE e^{-E/T} D(E) \]

\[ D(E) \equiv \sum_n \delta(E - E_n) \] is the density of states.

If \( D(E) \) increases rapid enough so that it can compensate the statistical suppression factor \( e^{-E/T} \), higher energy states have a chance to contribute to the thermodynamic quantity.

For example,

\[ D(E) \approx e^{bE} \quad \Rightarrow \quad T_C = 1/b \]
Glueball spectrum in quenched SU(3) lattice QCD

Most systematic result of glueball spectra from the quenched lattice QCD

FIG. 8. The mass spectrum of glueballs in the pure SU(3) gauge theory. The masses are given in terms of the hadronic scale $r_0$ along the left vertical axis and in terms of GeV along the right vertical axis (assuming $r_0^{-1} = 410$ MeV). The mass uncertainties indicated by the vertical extents of the boxes do not include the uncertainty in setting $r_0$. The locations of states whose interpretation requires further study are indicated by the dashed hollow boxes.

Thermal Effects on Quark-Gluon Mixed Condensate $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$
From Lattice QCD

Takumi Doi
Thermal effects on quark-gluon mixed condensate $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$
from lattice QCD

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and Hideo Suganuma

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The thermal effects on the quark-gluon mixed condensate $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ are investigated in SU(3)$_c$ lattice QCD at the quenched level. We discuss the importance of $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$, as it behaves as another chiral order parameter and has the large contribution to the operator product expansion in QCD. We calculate $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle$ as well as $\langle \bar{q}q\rangle$ in the chiral limit for $0 \lesssim T \lesssim 500\text{MeV}$, using the lattices at $\beta = 6.0, 6.1, 6.2$ in high statistics. Except for the sharp decrease of both the condensates around $T_c \simeq 280\text{MeV}$, the thermal effects are found to be weak below $T_c$. We also find that the ratio $m_0^2 \equiv g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q\rangle/\langle \bar{q}q\rangle$ is almost independent of the temperature even in the very vicinity of $T_c$, which indicates nontrivial similarity in the critical behaviors of the two different condensates.
1 Contents

- Motivation
  The condensates and hadron physics
  Condensates represent non-perturbative nature of QCD
  - Quark gluon mixed condensate $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$
    Physical importance of the mixed condensate $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$
    \( \triangleright \) chiral order parameter
    \( \triangleright \) strong relation to hadron phenomenology

- The Lattice formalism
  SU(3)$_c$ lattice at the quenched level
  KS fermion

- Lattice results at finite temperature
  - chiral restoration of the QCD vacuum

- Summary and Outlook
2 Motivation

QCD \iff hadron physics

How to understand?

The condensate is key concept.

The condensates embody the non-perturbative properties of QCD.

e.g.
\[
\langle \bar{q}q \rangle \iff \text{spontaneous chiral-symmetry breaking}
\]
\[
\langle G_{\mu\nu}G^{\mu\nu} \rangle \iff \text{trace anomaly}
\]

These vacuum structures of QCD are reflected on the hadronic properties.
I

Schematic correspondence between hadrons and condensates

Physical parameters are under control
(Temperature, quark mass, ...)

Lattice QCD

QCD sum rule

(Schematic correspondence between hadrons and condensates)

Condensates
$\langle \bar{q}q \rangle, \langle GG \rangle$
$\langle q\sigma Gq \rangle$

Hadrons
$N, \Delta, \pi$

QCD
3 Quark-gluon mixed condensate

\[ g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \]

- Chirality flipping operator

\[
g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle = g\langle \bar{q}_L (\sigma_{\mu\nu}G_{\mu\nu}) q_R \rangle + g\langle \bar{q}_R (\sigma_{\mu\nu}G_{\mu\nu}) q_L \rangle
\]

Another order parameter of spontaneous chiral symmetry breaking
Direct correlation between quarks and gluons

g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \text{ represents the color-octet components of quark-antiquark pairs in the vacuum.}

\begin{align*}
g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle & \iff (q\bar{q}) \otimes (gG_{\mu\nu}\sigma_{\mu\nu}) \\
& \quad \text{color 8} \quad \text{color 8}
\end{align*}

\begin{align*}
\langle \bar{q}q \rangle & \iff (q\bar{q}) \\
& \quad \text{color 1}
\end{align*}

g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \text{ and } \langle \bar{q}q \rangle \text{ characterizes different aspect of the QCD vacuum.}
Large contribution to hadron spectrum

QCD sum rules for baryon masses

B.L. Ioffe,

W-Y.P. Hwang and K.-C. Yang,
Phys. Rev. D49 (1994) 460

\[ \lambda_N^2 m_N e^{-m_N^2/M^2} \]
\[ = \frac{1}{(2\pi)^2} M^4 \langle -\bar{q}q \rangle + 0 + \cdots \]

\[ \lambda_{\Delta}^2 m_{\Delta} e^{-m_{\Delta}^2/M^2} \]
\[ = \frac{4}{3(2\pi)^2} M^4 \langle -\bar{q}q \rangle - \frac{2}{3(2\pi)^2} M^2 \langle -g\sigma_{\mu\nu}G_{\mu\nu}q \rangle + \cdots \]
QCD sum rule for penta-quark system $\Theta^+$

J. Sugiyama, T. Doi and M. Oka,

$$\eta(x) = \epsilon^{abc} \epsilon^{def} \epsilon^{cfg} \{u_a^TCd_b\} \{u_d^TC\gamma_5d_e\}C\bar{s}_g^T$$

(a) \hspace{1cm} (b)

(c) \hspace{1cm} (d)

$$m_0^2(s) \equiv g\langle \bar{s}\sigma \cdot Gs \rangle / \langle \bar{s}s \rangle$$

is a key parameter to determine the parity of $\Theta^+$.  

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Quark-gluon mixed condensate
\[ g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \]

- The order parameter of chiral symmetry breaking
- Direct correlation between quarks and gluons
- Large contribution to hadron spectrum

Thermal effects on \[ g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \]
is interesting to investigate !!

Physics at RHIC (Finite temperature QCD)

In spite of the importance of this quark-gluon mixed condensate \[ g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \], there is only preliminary (but pioneering) lattice result.


- KS-fermion, \( 8^4, \beta = 5.7(a \simeq 0.18[fm]) \),
- only 1 point \( \times \) 5 configs
- [ only at zero temperature ]
4 Our Lattice formalism

Keeping chiral symmetry is important because $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ is a chiral order parameter.

\[\downarrow \quad \downarrow\]

We use KS-fermion.

KS fermion

Doublings are assigned as SU($N_f=4$) flavors

Explicit chiral symmetry is preserved.
• KS fermion

\[ S_F = \frac{1}{2} \sum_{s,\mu} \eta_\mu(s) \bar{\chi}(s) \left[ U_\mu(s)\chi(s + \mu) - U_\mu^+(s - \mu)\chi(s - \mu) \right] + ma \sum_s \bar{\chi}(s)\chi(s) \]

\( \chi, \bar{\chi} \): single-component Grassmann fields

SU\((N_f = 4)\) spinor field \( q \) is composed of \( \chi \) as

\[ q_i^f(x) = \frac{1}{8} \sum_\rho (\gamma_1^{\rho_1}\gamma_2^{\rho_2}\gamma_3^{\rho_3}\gamma_4^{\rho_4})_{if} \chi(x + \rho) \]

\[ \rho = (\rho_1, \rho_2, \rho_3, \rho_4), \quad \rho_\mu \in \{0, 1\} \]

Spinor DoF \((4) \times \) Flavor DoF \( SU(N_f = 4) \)

are assigned to the \( 2^4 = 16 \) sites on the hypercube.

\( S_F \) is invariant under

\[ q \rightarrow e^{i\alpha(\gamma_5 \otimes \gamma_5^f)} q \]

\[ \bar{q} \rightarrow \bar{q} e^{i\alpha(\gamma_5 \otimes \gamma_5^f)} \]

Explicit chiral symmetry is preserved.
\[ \langle \bar{q}q \rangle \equiv - \left\langle \text{Tr} [ S_F(x,x) ] \right\rangle_{\text{average}} \]

\[ g \langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle \equiv - \left\langle \text{Tr}_{\text{color}} [ g G_{\mu\nu} \text{Tr}_{\text{spinor}} [ \sigma_{\mu\nu} S_F(x,x) ] ] \right\rangle_{\text{average}} \]

In terms of the KS fermion,

\[ a^3 \langle \bar{q}q \rangle = - \frac{1}{28} \sum_{\rho} \text{Tr} [ \langle \chi(x+\rho) \bar{\chi}(x+\rho) \rangle ] \]

\[ \text{Tr} [ \bar{q}q ] \]

"local"
In terms of the KS fermion,

\[ a^5 g \left< \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \right> = - \left< \text{Tr} \left[ g G_{\mu\nu} \right] \text{Tr} \left[ \sigma_{\mu\nu} S_F(x, x) \right] \right> \text{average} \]

\[
\begin{align*}
\frac{1}{2^8} \sum_{\mu, \nu, \rho} \text{Tr} \left[ U'_{\mu, \nu}(x + \rho) \left< \chi(x + \rho + \mu + \nu) \bar{\chi}(x + \rho) \right> \right] \\
\sigma_{\mu\nu} G_{\mu\nu}^\text{lat}(x + \rho) 
\end{align*}
\]

\[
\text{Tr} \left[ \sigma_{\mu\nu} q\bar{q} \times G_{\mu\nu} \right]
\]

"local"
We generate gauge configurations with the standard Wilson action under the quenched approximation. We take 16 point sources ($\beta = 6.0$) or 2 point sources ($\beta = 6.1, 6.2$) to calculate the condensates.
Finite temperature lattice QCD

(a) $16^3 \times 16, 12, 10, 8, 6, 4$ at $\beta = 6.0$

(\(a \simeq 0.1[\text{fm}]\))

16 points $\times$ 100 configs $\Rightarrow$ 1600 data

(b) $20^3 \times 20, 12, 10, (8), 6$ at $\beta = 6.1$

(\(a \simeq 0.09[\text{fm}]\))

2 points $\times$ 100 configs $\Rightarrow$ 200 data

(c) $24^3 \times 24, 16, 12, (10), 8$ at $\beta = 6.2$

(\(a \simeq 0.07[\text{fm}]\))

2 points $\times$ 100 configs $\Rightarrow$ 200 data

(b')(c') For $T \sim T_c$, $20^3 \times 8$ ($\beta = 6.1$)

and $24^3 \times 10$ ($\beta = 6.2$),

2 points $\times$ 1000 configs $\Rightarrow$ 2000 data

large & fine lattices, and large statistics!

First study of $g\langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle$ at finite temperature
(jackknife errors are hidden in the symbols)
Results for finite temperature

\[ \frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_{T=0}} \]

\[ \frac{g \langle \bar{q}\sigma Gq \rangle_T}{g \langle \bar{q}\sigma Gq \rangle_{T=0}} \]

\( T \) (MeV)

\( T' \) (MeV)
\[ m_0^2 \equiv g \langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle / \langle \bar{q} q \rangle \]

\[
\frac{m_0^2(T)}{m_0^2(T = 0)} = \frac{g \langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle_T / g \langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle_{T = 0}}{\langle \bar{q} q \rangle_T / \langle \bar{q} q \rangle_{T = 0}}
\]

\[ m_0^2 \equiv g \langle \bar{q} \sigma_{\mu \nu} G_{\mu \nu} q \rangle / \langle \bar{q} q \rangle \]

is almost independent of the temperature.

(Very nontrivial results!)

Universality in the critical behavior of the chiral order parameters?
Electric/Magnetic components of \( g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \)

\[
R_{E/B}(T) \equiv \frac{\sum_i g\langle \bar{q}\sigma_{4i}G_{4i}q \rangle_T}{\sum_{j<k} g\langle \bar{q}\sigma_{jk}G_{jk}q \rangle_T}
\]

**Common critical behavior!**
Relation to the chiral zero modes:

\[
\langle \bar{q}q \rangle \quad \leftrightarrow \quad \frac{1}{V} \int d\lambda' \frac{m \rho(\lambda')}{\lambda'^2 + m^2}
\]

\[
g\langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle \quad \leftrightarrow \quad \frac{1}{V} \int d\lambda' \frac{m \rho(\lambda')}{\lambda'^2 + m^2} \langle \lambda' \sigma_{\mu\nu} G_{\mu\nu} | \lambda' \rangle
\]

\[
iD |\lambda\rangle = \lambda |\lambda\rangle
\]

\[\rho(\lambda)\]: the spectral density

\[
g\langle \bar{q} \sigma_{\mu\nu} G_{\mu\nu} q \rangle:
\]

Critical behavior is mainly governed by \( \rho(\lambda) \)

The correlation between gluon field and zero modes

\[
\langle \lambda | \sigma_{\mu\nu} G'_{\mu\nu} | \lambda \rangle \bigg|_{\lambda=0}
\]

is insensitive to the temperature.
5 Summary and Outlook

- We have investigated the thermal effects on the quark-gluon mixed condensate \( g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle \) using KS-fermion in SU(3) lattice.

0 \( \lesssim T \lesssim 500\text{MeV} \)

\[
\begin{align*}
16^3 \times 16, 12, 10, 8, 6, 4 \text{ lattice, } \beta &= 6.0, \\
&16 \text{ points } \times 100 \text{ configs} = 1600 \text{ data} \\
20^3 \times 20, 12, 10, (8), 6 \text{ lattice, } \beta &= 6.1, \\
&2 \text{ points } \times 100 \text{ configs} = 200 \text{ data} \\
24^3 \times 24, 16, 12, (10), 8 \text{ lattice, } \beta &= 6.2, \\
&2 \text{ points } \times 100 \text{ configs} = 200 \text{ data} \\
20^3 \times 8 (\beta = 6.1) \text{ and } 24^3 \times 10 (\beta = 6.2) \text{ lattice,} \\
&2 \text{ points } \times 1000 \text{ configs} = 2000 \text{ data}
\end{align*}
\]
Chiral restoration has been studied in terms of $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ as well as $\langle \bar{q}q \rangle$.

The thermal effects of $g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle$ and $\langle \bar{q}q \rangle$ have been found to be remarkably weak for $T \lesssim 0.9 T_c$.

We have obtained $m_0^2 = g\langle \bar{q}\sigma_{\mu\nu}G_{\mu\nu}q \rangle/\langle \bar{q}q \rangle$ is almost independent of temperature.

We have found $g\langle \bar{q}\sigma_{4i}G_{4i}q \rangle$ and $g\langle \bar{q}\sigma_{jk}G_{jk}q \rangle$ also have the same critical behavior.

New aspects of the chiral structure of QCD?

- Universality of the chiral order parameters
- The correlation between gluon and chiral zero modes

hep-lat/0402005 (2004).

**Future plan:**

- Full QCD calculation (dynamical quark effects) (in progress)
- Complementary use of
  - Lattice QCD and the QCD sum rule
  (Hadron spectroscopy at finite temperature)
Detailed Lattice QCD Study of the Three-quark Potentials: The Ground-state and the Excited-state of the Static 3Q System

Toru T. Takahashi
RIKEN-TODA Mini-Workshop on
"Topics in Hadron Physics at RHIC"

Detailed lattice QCD study of the three-quark potentials:
the ground-state and the excited-state of the static 3Q system

Toru T. Takahashi (Yukawa Institute for Theoretical Physics)
with H. Suganuma (Tokyo Institute of Technology)

We study the ground-state three-quark (3Q) potential and the excited-state 3Q potential using SU(3) lattice QCD. From the accurate and thorough calculation for more than 300 different patterns of 3Q systems, the static ground-state 3Q potential is found to be well described by the Coulomb plus Y-type linear potential (Y-ansatz) within 1%-level deviation. With lattice QCD, we calculate also the excited-state potential in the 3Q system, and find the gluonic excitation energy to be about 1 GeV. This large gluonic-excitation energy would play an essential role to the success of the quark model for the low-lying hadrons in terms of the absence of the gluonic mode. Finally, we give a functional form which reproduces the gluonic-excitation energy in the 3Q system.
Introduction

Interested in hadron dynamics

QCD is the fundamental theory of the Dynamics.

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2} \text{tr} F_{\mu\nu} F^{\mu\nu} + \sum_f \bar{\psi}(i\gamma^\mu D_\mu - m_f) \psi$$
Lattice QCD to Hadron Physics

In practical,
Coupling $g \gg 1$ at the hadronic scale.

\underline{Perturbative calculation}

The large coupling constant leads to rich nonperturbative nature

- chiral symmetry breaking
- color confinement

→Lattice QCD calculations

Reliable nonperturbative method
Lattice QCD

Lattice QCD calculation is a powerful tool for the hadron spectroscopy.

\[ \uparrow \downarrow \]

The calculation itself says almost nothing about the internal structure of hadrons.

It is important to investigate the interquark potential using lattice QCD.
Interquark Potentials

The quark-antiquark potential is well known.

\[ V_{Q\bar{Q}} = -\frac{A}{r} + \sigma r + C \]

Coulomb type term \quad \text{Linear confinement term}

Color confinement & MESON properties

The flux-tube formation among quarks can be observed using lattice QCD.
Flux-tube formation in the ground-state quark-antiquark system

Figure in hep-ph/09809263
Prof. Nora Brambilla,
(figure provided by Prof. G. S. Bali)
3Q potentials

3Q potential is important for the BARYONS.

However,
in spite of its importance,

Until the end of the 20th century,
The 3Q potential form was not settled.
3Q potentials

Roughly, there were 2 candidates for long-distance behavior of the 3Q potential form.

2-body sum
\[ L_P = a + b + c \]

3-body
\[ L_{\text{min}} = \overline{AP} + \overline{BP} + \overline{CP} \]

For its simplicity, used in quark model calculations

Supported by …

Center vortex model

Dual superconductor picture of the QCD vacuum \[ T_8 \]

There appear 2 abelian charges.

(However, in hep-th/0305101, he re-calculated and found that Y-law is supported.)
3Q potentials

Lattice QCD studies

Sommer and Wosiek,

Thacker, Eichten and Sexton,
Lattice '87

C.Alexandrou et.al.

Even lattice QCD studies were controversial.

Contaminations of excited-state components

→ Smearing technique to enhance the G.S. component

Too few different patterns of quark configurations

→ 300 different patterns of quark configurations
3Q potentials

We have studied 3Q potential, which is responsible for baryon properties.

We have found that the 3Q potential obeys the following form.

\[ V_{3Q} = - \sum A \frac{1}{|r_i - r_j|} + \sigma L_{\text{min}} + C \]

2-body Coulomb 3-body linear

\( L_{\text{min}} \) is the length of Y-shaped flux-tube.

- universality of the string tension
- reproduction of OGE result

Flux-tube formation in the ground-state 3Q System

Action density in the static 3 quark system in the Abelian-projected QCD.

H.Ichie, V.Bornyakov, T.Streuer, G.Schierholtz

Gluonic Excitations of the 3Q System

Excitation induced by the massless GLUONS

Massive
Due to χSB

Massless
Gauge-boson

Hybrid Hadrons

In spite of its masslessness, we find no HYBRIDS in the low-lying hadron spectra.

Simple quark model well works. ONLY

→ Lattice QCD study about the excited-state potential
Excitation modes of the gluons
Gluonic Excitations of the 3Q System

$\beta = 5.8$

$\beta = 6.0$ (finer lattice)

$\Delta E$ is large in comparison with the excitation of quark origin.

$\rightarrow$ It leads to the absence of the hybrid hadrons in the low-lying mass spectra.

$\rightarrow$ This supports the success of the quark model, which does not have the explicit gluonic degrees of freedom and has gluons as an instantaneous interaction among quarks.
Gluonic Excitations of the 3Q System

What is it?

Vibrational modes of the flux tube?

H.Ichie, V.Bornyakov, T.Streuer, G.Schierholz

Figure in hep-ph/09809263
Prof.Nora Brambilla,
(provided by Prof.G.S.Bali)
Gluonic Excitations of the 3Q System

Static Quark-Antiquark system

K.J.juge, J.kuti, C.Morningstar

Naïve estimation using NG-string action

\[ \int_0^{R/a_s} \int_0^{R/a_s} dz \sqrt{x_i \left( \frac{e_i^{\mu}}{e_i^{\nu}} \beta / \mu \right) + \Sigma \Delta} \]

Fails to reproduce the excitation
at the hadronic scale

Finite radius effect....?

More complicated in the 3Q system
Gluonic Excitations of the 3Q System

Phenomenologically,
we have found the functional form which describe $\Delta E$
in the static 3 quark system.

$$\Delta E = \frac{A}{L_Y} + C$$

$$L_Y = \frac{1}{2}$$
Gluonic Excitations of the 3Q System

$\beta = 5.8$

$\beta = 6.0$ (finer lattice)

This form can reproduce $\Delta E_{3Q}$ with $\chi^2/N_{DF} \sim 1$ for about 100

Patterns of 3 quark spatial configurations
Gluonic Excitations of the 3Q System

We stress that this result can be applied to the 3Q system where 0 ~ Lmin ~ 1.5 fm. The infrared asymptotic behavior should be investigated carefully.

\[ \Delta E = \frac{A}{L_Y} + O \]

\[ L_Y = \frac{1}{2} | \langle \rangle | \]

This may represent global oscillations of the 3 quark system.

For further study, we need modeling of the coupled vibrations of the Y-type Flux-tube. At that time, this Lattice QCD result will be useful.
Summary

Ground-state of the static 3Q system

\[ V_{3Q} = - \sum_{(g.s.)} \frac{A}{|q_i^a - q_j^a|} + 0 L_{\text{min}} + C \]

2-body Coulomb  3-body linear

\[ L_{\text{min}} \]: the length of Y-shaped flux tube

- universality of the string tension
- reproduction of OGE result

Summary

Excited-state of the static 3Q system

\[ \Delta E = \frac{A}{L_Y} + C \]  
(excitation energy)

\( L_Y \): the length of vibrating flux tube?

\[ L_Y = \frac{1}{2} \left| \begin{array}{ccc} Q_1 & P & Q_3 \\ Q_2 & P_2 & Q_3 \\ P_3 & P_1 & P \end{array} \right| \]

\[ E_{1st} - E_{gs} > 1 \text{ GeV} \quad \text{Phys.Rev.Lett:90(2003)182001} \]

Gluonic excitations do not contribute to the low-lying hadron spectra
Polarized Parton Distributions and Their Uncertainties

M. Hirai
Polarized parton distributions and their uncertainties

M. Hirai
(Asymmetry Analysis Collaboration)

Radiation Laboratory, RIKEN

I discussed new AAC analysis of the polarized parton distributions by using current polarized deep inelastic scattering data and uncertainty estimation for these distributions by the Hessian method. Moreover, I discussed determination of the polarized gluon distribution from prompt photon data which will be measured by the RHIC experiments.

http://spin.riken.bnl.gov/aac/

Mar 24 2004, RIKEN
Polarized deep inelastic scattering

- Cross section

\[
\frac{d^2\sigma}{d\Omega \, dE'} = \frac{\alpha^2}{2Mq^4} \frac{E'}{E} L_{\mu\nu} W^{\mu\nu}
\]

- Hadron tensor (asymmetric part)

\[
W_A^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} \frac{q_\rho}{\nu} \left\{ s_\sigma M^2 v G_1(v, Q^2) + \left[ s_\sigma - \frac{(s \cdot q) P_\sigma}{M v} \right] M v^2 G_2(v, Q^2) \right\}
\]

- \( Q^2 \to \infty \), \( x \) fixed

\[
M^2 v G_1(v, Q^2) \to g_1(x), \quad M v^2 G_2(v, Q^2) \to g_2(x)
\]
Polarized structure function $g_1$

- Structure function $g_1$

$$g_1(x) = \frac{1}{2} \sum_{i=1}^{n_f} e_i^2 \left[ \Delta C_q(x) \otimes (\Delta q_i(x) + \Delta q_i(x)) + \Delta C_g(x) \otimes \Delta g(x) \right]$$

$\Delta C_q, \Delta C_g$ (coefficient function)

$\Delta q(x) = q^+(x) - q^-(x), \quad q(x) = q^+(x) + q^-(x)$

$$(f \otimes g) = \int \frac{dy}{y} f \left( \frac{x}{y} \right) g(y)$$

- DGLAP evolution equation

$$\frac{d}{d \ln Q^2} \begin{pmatrix} \Delta \Sigma(x, Q^2) \\ \Delta g(x, Q^2) \end{pmatrix} = \begin{pmatrix} \Delta P_{qq}(x, \alpha_s) & \Delta P_{qg}(x, \alpha_s) \\ \Delta P_{gq}(x, \alpha_s) & \Delta P_{gg}(x, \alpha_s) \end{pmatrix} \otimes \begin{pmatrix} \Delta \Sigma(x, Q^2) \\ \Delta g(x, Q^2) \end{pmatrix}$$

$$\left( \Delta \Sigma = \Delta u^+ + \Delta d^+ + \Delta s^+ \right)$$
Spin asymmetry $A_1(x, Q^2)$

$$A = \frac{d\sigma(\uparrow \downarrow) - d\sigma(\uparrow \uparrow)}{d\sigma(\uparrow \downarrow) + d\sigma(\uparrow \uparrow)}$$

$$\approx \frac{g_1}{F_1} = g_1 \frac{2x(1+R)}{F_2}$$

$$R(x, Q^2) = \frac{\sigma_T}{\sigma_L}$$


- Polarized DIS experiments
  - Proton: E130, E143, EMC, SMC, HERMES, E155
  - Deuteron: E143, E155, SMC
  - Neutron: E142, E154, HERMES, JLab

Total data 399 points
($Q^2 > 1\text{GeV}^2$)
Uncertainty estimation

- Functional form of polarized PDF
  \[ \Delta f_i(x, Q_0^2) = N x^\alpha (1 - \gamma x)^\beta f_i(x, Q_0^2), \quad i = u_v, d_v, q, g \]

- Hessian method
  \[ [\Delta \Delta f(x)]^2 = \Delta \chi^2 \sum_{i,j} \frac{\partial \Delta f(x)}{\partial a_i} H_{ij}^{-1} \frac{\partial \Delta f(x)}{\partial a_j} \]

- Hessian matrix \( H_{ij} \)
  - parameter error
  - correlation between each parameters

- \( \Delta \chi^2 \) is relevant to the confidence level of uncertainty, and depend on \( \chi^2 \) distribution \( K(s) \) with \( N \) degree of freedom
  - \( \int_0^{\Delta \chi^2} K(N, s) \, ds = 0.683 \) \( (N: \) the number of parameter)  
  - AAC03, \( N=11 \): \( \Delta \chi^2 = 12.64 \)
Result

- **Constraint condition**
  - Antiquark SU(3)$_f$ symmetry: $\Delta \bar{u}(x) = \Delta \bar{d}(x) = \Delta \bar{s}(x) = \Delta \bar{q}(x)$
  - Positivity condition: $|\Delta f(x)| \leq f(x)$

- **Total $\chi^2$**
  - $\chi^2/(d.o.f.) = 346.5 \ (0.893)$

- **1st moment**
  \[
  (\Delta \Sigma = \Delta u + \Delta d + 6\Delta q)
  \]

<table>
<thead>
<tr>
<th></th>
<th>$\Delta g$</th>
<th>$\Delta \Sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAC03</td>
<td>$0.499 \pm 1.268$</td>
<td>$0.213 \pm 0.138$</td>
</tr>
<tr>
<td>AAC00</td>
<td>$0.532 \pm 1.848$</td>
<td>$0.241 \pm 0.220$</td>
</tr>
</tbody>
</table>


- Unpol PDF: GRV98
- $Q_0^2 = 1$ GeV$^2$
- $N_f = 3$
- $\Lambda_{QCD} = 299$ MeV
- Data: 399
Polarized PDFs from DIS data (AAC03)

- PDF uncertainties reduced by including precise data
- Well determined valence quark distribution
  - Small uncertainty of $\Delta u_v, \Delta d_v$
- Antiquark uncertainty is significantly reduced
  - $g_1^p \propto 4\Delta u_v + \Delta d_v + 12\Delta \bar{q}$
- Undetermined $\Delta g(x)$
  - Large uncertainty
  - Indirect contribution to $g_1^p$
  - Correlated with $\Delta g(x)$
Correlation between $\Delta \bar{q}(x)$ and $\Delta g(x)$

- Fixed $\Delta g(x) = 0$
  - $\chi^2/(d.o.f.) = 355.0 (0.915)$

- $\Delta q_v(x)$ uncertainties are scarcely changed

- Antiquark uncertainty is reduced
  - Strong correlation with $\Delta g(x)$

- Well determined $\Delta g(x)$
  - Eliminating correlation effect
  - Flavor decomposition $\Delta \bar{q}(x)$
Spin asymmetry $A_1^p$

- Precise data improve asymmetry uncertainty
- Good agreements with polarized DIS data
  - Quark and antiquark distributions are constrained
  - Gluon distribution need other constraint

$Q^2 = 5 \text{ GeV}^2$
Prompt photon production

- Polarized prompt photon cross section
  - $qg \rightarrow \gamma q, q\bar{q} \rightarrow \gamma g$

\[
\frac{d\Delta \sigma^{pp \rightarrow \gamma X}}{dp_T} = \sum_{a,b} \int_{\eta_{bin}} d\eta \int_{\xi_1}^{\xi_2} dx_1 \int_{\xi_2}^{\xi_1} dx_2 \Delta f_a^A(x_1, \mu) \Delta f_b^B(x_2, \mu) \frac{d\Delta \hat{\sigma}^{ab}}{dp_T d\eta}(x_1, x_2, \sqrt{s}, \eta, p_T, \mu)
\]

\[
\left( \xi_1 = \frac{x_r e^\eta}{2 - x_r e^{-\eta}}, \quad \xi_2 = \frac{x_1 x_r e^{-\eta}}{2 x_r - x_r e^\eta}, \quad x_r = \frac{2 p_T}{\sqrt{s}}, \quad \mu = p_T \right)
\]

- Fragmentation contribution is neglected
  - Isolation cut (Reduction of $\pi^0 \rightarrow \gamma \gamma$, collinear photon)

- Spin asymmetry and statistical error

\[
A_{\ell\ell}^\gamma = \frac{d\Delta \sigma^\gamma}{dp_T}, \quad \delta A_{\ell\ell}^\gamma = \frac{1}{P_1 P_2 \sqrt{L_{int}} \sigma^\gamma \varepsilon}
\]

\[
\begin{align*}
P : \text{beam polarization} \\
L_{int} : \text{integrated luminosity} \\
\sigma^\gamma : \text{unpolarized cross section} \\
\varepsilon : \text{detection efficiency (} \varepsilon \leq 1) \\
\end{align*}
\]
Uncertainty of $A_{LL}^{\gamma}(P_T)$

- Large uncertainty of $A_{LL}^{\gamma}$ is mainly composed of gluon uncertainty $\delta \Delta g(x)$
  - Small uncertainty except $\delta \Delta g(x)$

- Weak constraint on quark and antiquark distributions
  - Off $\delta \Delta g(x)$ uncertainty is composed of quark and antiquark uncertainty
  - The off $\delta \Delta g(x)$ is smaller than expected statistical errors

- Prompt photon data mainly constricts the gluon uncertainty
Comparison with statistical error

- Overhaul factor for gluon uncertainty $\delta \Delta g(x)$
  - $1/18$ ($P_T=10\sim20$ GeV)

- Constricting $\delta \Delta g(x)$ in different $x_T$ ($=2P_T/\sqrt{s}$ ) region
  - $x_T = 0.1\sim0.2$ ($\sqrt{s}=200$ GeV)
  - $x_T = 0.04\sim0.08$ ($\sqrt{s}=500$ GeV)

- Gluon contribution: $\Delta g = \int_0^1 dx \Delta g(x)$
  - Covering wide range of $x_{BJ}$

- $L_{int} = 320$ pb$^{-1}$ (200 GeV)
- 800 pb$^{-1}$ (500 GeV)

- $|\eta| < 0.35$
- $P=0.7$
- $\Delta \phi = \pi$
$\Delta g(x)$ uncertainty from prompt photon

- **AAC03**
  - $P_T = 10$ GeV
  - $|\eta| < 0.35$

- DIS
  - $\delta \Delta g/18$
  - $\sqrt{s} = 200$ GeV

- $A_{LL}$ vs. $P_T$ for DIS
Summary

- Polarized PDF from polarized DIS data
  - $\Delta u_\nu(x)$, $\Delta d_\nu(x)$ are determined well
  - $\Delta \Sigma = 0.213 \pm 0.138$ ($Q^2 = 1$ GeV$^2$)
  - $\Delta g(x)$ could not be constrained

- $\Delta g(x)$ uncertainty can be reduced by prompt photon data
  - $A_{LL}^\gamma$ uncertainty is composed of the $\Delta g$ uncertainty
  - Current $A_{LL}^\gamma$ uncertainty quite larger than expected statistical errors

- Comparison $A_{LL}^\gamma$ uncertainty with expected statistical errors
  - Effective region ($P_T = 10\sim20$ GeV)
  - $X_T = 0.1\sim0.2$ ($\sqrt{s} = 200$ GeV), 0.04$\sim$0.08 ($\sqrt{s} = 500$ GeV)
Comment on $\pi^0$ Double Spin Asymmetry at RHIC

K. Sudoh
Comment on $\pi^0$ double spin asymmetry at RHIC

K. Sudoh (RIKEN)
RIKEN-TODAI Mini-Workshop
24 March 2004
hep-ph/0403102, with M. Hirai

Abstract
Double spin asymmetry for $\pi^0$ production has been measured by the PHENIX. The preliminary data indicate negative asymmetry at low $p_T$, whereas the theoretical prediction is positive asymmetry.

We study effects of polarized gluon distribution on the spin asymmetry, and suggest the possibility to obtain sizable negative asymmetry in larger $p_T$ regions.
Contents

I. Introduction

II. Inclusive $\pi^0$ Production in $\bar{p}p$ Collisions

\[ \bar{p} + \bar{p} \rightarrow \pi^0 + X \]

III. Correlation to $\Delta g(x)$

IV. Summary
I. Introduction

- Uncertainty of the polarized gluon distributions $\Delta g(x)$ is still large, since the gluon contribution appears in NLO in polarized DIS.
  
  ➤ RHIC has started!!

- The $\pi^0$ spin asymmetry $A_{LL}^{\pi}$ has been measured by the PHENIX.
  - The data suggest significant negative asymmetry!!!
Negative Asymmetry

- Comparison $A_{LL}^\pi$ with the PHENIX data
  \[ \sqrt{s} = 200 \text{ GeV}, \quad |\eta| \leq 0.38 \]
- pQCD prediction indicates positive asymmetry.

A. Bazilevsky, talk at Spin-03, Dubna

Inclusive $\pi^0$ Production $@$RHIC

- Subprocesses for $\pi^0$ production
  - $O(\alpha_s^2)$ 2→2 tree-level channels in LO
    
    $$gg \rightarrow q(g)X, \quad qg \rightarrow q(g)X, \quad qq \rightarrow qX,$$
    $$q\bar{q} \rightarrow q(g,q')X, \quad qq' \rightarrow qX, \quad q\bar{q}' \rightarrow qX$$
  - Dominant contribution:
    $$gg \rightarrow q(g)X \quad : \text{in small } p_T$$
    $$qg \rightarrow q(g)X \quad : \text{in large } p_T$$

- $\pi^0$ spin asymmetry is sensitive to $\Delta g(x)$.

  Determined by a balance between $gg$ and $qg$ contributions.
Double Spin Asymmetry

Longitudinal Double Spin Asymmetry:

\[ A_{LL}^0 = \frac{[d\sigma_{++} - d\sigma_{+-}]/dp_T}{[d\sigma_{++} + d\sigma_{+-}]/dp_T} = \frac{d\Delta\sigma/dp_T}{d\sigma/dp_T} \]

\[ \frac{d\Delta\sigma}{dp_T} \equiv \frac{1}{2} \left[ \frac{d\sigma_{++}}{dp_T} - \frac{d\sigma_{+-}}{dp_T} \right] \]

\[ = \sum_{a,b,c} \int d\eta \int dx_a \int dx_b \int dz \Delta f_a(x_a, Q^2) \Delta f_b(x_b, Q^2) \frac{d\Delta\hat{\sigma}}{dp_T d\eta} D_c^\pi(z, Q^2) \]

PDF and FF: Determined using experimental data

Amplitude: Calculable in pQCD

Where does the inconsistency come from?
Where does ambiguity come from?

- **Fragmentation function: $D_\pi^m(z)$**
  - Unpolarized $\pi^0$ cross section by the PHENIX is consistent with pQCD calculation.
    \[ \sqrt{s} = 200 \text{ GeV}, \quad |\eta| \leq 0.35 \]
  - Determined from $e^+e^-$ data

- **Pol. quark distribution: $\Delta q(x)$**
  - Flavor blind reaction
  - Depends on the sum $\Delta q(x) + \Delta \bar{q}(x)$.

Ambiguity comes from $\Delta g(x)$
Modification of $\Delta g(x)$

Can we obtain negative asymmetry by modifying $\Delta g(x)$?

3 configurations of $\Delta g(x)$ within their uncertainty

- **Scen.1** standard $\Delta g$
  - global analysis with pol. DIS
  - $A_{LL}$ does not become negative.

- **Scen.2** $\Delta g$ with a node
  - possibility $gg<0$ ($\Delta g(x_a) \times \Delta g(x_b)<0$)
  - $|x_a-x_b|$ is small in $\eta<0.38$

- **Scen.3** negative $\Delta g$
  - $gg>0$, $qg<0$
  - $|\Delta g(x)|$ is small in low $x$
  - $|\Delta g(x)|$ is large in high $x$

$\Delta g(x)$ has $Q^2$ dependence by DGLAP equation.
$A_{LL}^{\pi}$ with modified $\Delta g(x)$

- **AAC $\Delta g(x)$**
  - Both $gg$ and $qg > 0$
  - $A_{LL}$ is positive and increases with $p_T$
- **Sample-1**
  - In low $p_T$
    - Both $gg$ and $qg < 0$
    - Slight negative $A_{LL}$ in $\eta < 0.38$
  - In large $p_T$
    - Both $gg$ and $qg > 0$
    - $A_{LL}$ becomes large positive
- **Sample-2**
  - $gg > 0$, $qg < 0$, $A_{LL} < 0$ in whole $p_T$
  - $|A_{LL}|$ becomes larger to negative

Difficult to obtain sizable asymmetry in low $p_T$, since partonic cross section itself is too small.
Uncertainty of $A_{LL}^{\pi}$

- Comparison $A_{LL}$ uncertainty with errors of PHENIX data
- $A_{LL}$ uncertainty comes from the gluon uncertainty
- The same constriction as the polarized DIS data
IV. Summary

• Correlation between $\pi^0$ spin asymmetry and $\Delta g(x)$ is studied.

  ★ It is extremely difficult to derive sizable $A_{LL}$ in a few % level in low $p_T$.

  – Slight negative asymmetry at low $p_T$ can be obtained.
  – It is premature to conclude that pQCD is not applicable to $\pi^0$ asymmetry.
  – Existence of $\Delta g(x)$ which keeps $A_{LL}$ to be negative in whole $p_T$.

• PHENIX data motivate us to modify $\Delta g(x)$ drastically?
  – Negative asymmetry suggests negative polarization of $\Delta g(x)$.

Precise data in wide $p_T$ regions is needed to determine the gluon spin content.
Chiral Doubling of Heavy-Light Hadrons in the Vector Manifestation

Chihiro Sasaki
Chiral Doubling of Heavy-Light Hadrons in the Vector Manifestation

Chihiro SASAKI

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Abstract

Starting with a hidden local symmetry Lagrangian at the vector manifestation fixed point that incorporates heavy-quark symmetry, we calculate the splitting of chiral doublers of heavy-light mesons proposed by Nowak, Rho and Zahed [Phys. Rev. D 48, 4370 (1993)] and Bardeen and Hill [Phys. Rev. D 49, 409 (1994)], and show, in the three-flavor chiral limit, that the splitting is proportional to the light-quark condensate $\langle \bar{q}q \rangle$ and comes out to be $\sim 0.37 \text{ GeV} \sim \frac{1}{3}m_N$ where $m_N$ is the nucleon mass. This is to be compared with the recently measured BaBar result for the splitting between $D_s^+(2316.8: 0^+)$ and $D_s^+(0^-)$ of 348 MeV and the CLEO result for that between $D_s^{*+}(2463: 1^+)$ and $D_s^{*+}(1^-)$ of 352 MeV.

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1This talk is based on the work done in hep-ph/0312182 by M. Harada, M. Rho and C. Sasaki.
Chiral Doubling of Heavy-Light Hadrons in the Vector Manifestation

Chihiro SASAKI (Nagoya University)

based on
Contents

1. Introduction
   - Heavy quark symmetry & chiral symmetry, “Chiral doubling”
   - Our approach

2. Wigner Realization of Chiral Symmetry

3. Our Effective Field Theory

4. Matching to the Operator Product Expansion

5. Quantum Correction

6. Mass Splitting

7. Summary
Introduction

D-meson: heavy-light system
(c) (u,d,s)

"Light-quark cloud" surrounds the heavy-quark.

- Heavy quark sector ... Heavy quark symmetry
  $M_Q$ (heavy quark mass) $\rightarrow \infty$ limit $\Rightarrow$ Spin-flavor symmetry:
  Dynamics of the heavy quark $Q$ is independent of its mass and spin.

  Ground state: $D(0^-), D(1^-) \Leftarrow$ spin partner ($s_{\text{light}} = \frac{1}{2}$)

- Light quark sector ... Chiral symmetry
Recent discoveries ($c\bar{s}$-system)

$D(0^+) : 2317 \text{ MeV} \quad \text{[BaBar (03)]}$

$D(1^+) : 2463 \text{ MeV} \quad \text{[CLEO (03)]}$

\[
\begin{pmatrix}
D(0^-) : 1969 \text{ MeV} \\
D(1^-) : 2112 \text{ MeV}
\end{pmatrix}
\] : known

*mass shifts* \[
\begin{aligned}
m(0^+) - m(0^-) \\
m(1^+) - m(1^-)
\end{aligned}
\] \(\sim 350 \text{ MeV}\)
Prediction: "Chiral doubling"

\[ D(0^+) \text{ (or } D(1^+) \text{)} \text{ is chiral partner of } D(0^-) \text{ (or } D(1^-) \text{)}. \]
\[ m(0^+) - m(0^-) \quad m(1^+) - m(1^-) \quad \sim m_{\text{constituent}} \]

[Nowak, Rho and Zahed (92), Bardeen and Hill (93).]

\[ \cdots \text{ based on the manifestation of chiral symmetry restoration} \]
\[ \text{à la linear sigma model} \]

\[ \begin{array}{cc}
D(0^-) & D(1^-) \\
D(0^+) & D(1^+) \\
\end{array} \]

\[ \Rightarrow \text{ Heavy quark symmetry} \]

\[ \text{Chiral symmetry} \]
• Our approach:

Chiral symmetric phase $\Rightarrow$ Chiral broken phase

• The main assumption:
  At the chiral symmetry restoration point, the vector manifestation (VM) is realized.
  Massless $\rho$-meson becomes chiral partner of $\pi$.

• formulated in the framework of hidden local symmetry (HLS) theory.
  $\cdots$ dynamical d.o.f. $= \pi, \rho$

• characterized by the fixed point of RGEs.

[Harada and Yamawaki (01).]

\* We start from the Lagrangian at the VM fixed point and include the effect of chiral symmetry breaking.
Wigner Realization of Chiral Symmetry

Chiral representations of low-lying mesons (zero helicity)

- In broken phase, the eigenstate of the chiral representation under $SU(3)_L \times SU(3)_R$ symmetry does not agree with the mass eigenstate.

\[
|s\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle, \\
|\pi\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle \sin \psi + |(1, 8) \oplus (8, 1)\rangle \cos \psi, \\
|V\rangle = |(1, 8) \oplus (8, 1)\rangle, \\
|A_1\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle \cos \psi - |(1, 8) \oplus (8, 1)\rangle \sin \psi.
\]

mixing angle $\psi \simeq 45^\circ$

F. J. Gilman and H. Harari, PR 165, 1803 (1968);
• Chiral symmetry restoration
  → Representation mixing is dissolved at critical point.
  → There are 2 possibilities for pattern of chiral symmetry restoration.

One possibility \((\cos \psi \to 0)\) | Another possibility \((\sin \psi \to 0)\) |
---|---|
\(|\pi\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle,\) & \(|\pi\rangle = |(1, 8) \oplus (8, 1)\rangle,\) \\
\(|s\rangle = |(3, 3^*) \oplus (3^*, 3)\rangle.\) & \(|V\rangle = |(1, 8) \oplus (8, 1)\rangle.\)

· · · Standard scenario · · · Vector Manifestation
Our Effective Field Theory

- Restoration point of chiral symmetry:
  - Light sector
    \[ M_\rho = 0 : \text{Massless } \rho\text{-meson is chiral partner of } \pi. \]
  - Heavy sector
    \[ \Delta M \equiv M(0^+) - M(0^-) = M(1^+) - M(1^-) = 0 : \]
    \[ D(0^+) \text{ (or } D(1^+)) \text{ is chiral partner of } D(0^-) \text{ (or } D(1^-)). \]

- Terms expressing chiral symmetry breaking:
  - Light sector
    \[ M_\rho \neq 0 : \text{Massive } \rho\text{-meson} \]
  - Heavy sector
    Chiral symmetry breaking generates \( \Delta M \neq 0. \)
Concept of the Matching in the Wilsonian Sense

- The bare Lagrangian of the EFT is defined at a suitable matching scale $\Lambda$.

$\Rightarrow$ We obtain the bare Lagrangian of the EFT after integrating out the high energy modes.

i.e., the quarks and gluons above $\Lambda$
Matching to the Operator Product Expansion

\[ \Delta_{SP}(Q^2) \equiv G_S(Q^2) - G_P(Q^2) \]

where

\[ G_S(Q^2) : \text{Scalar correlator for } D(0^+) \]
\[ G_P(Q^2) : \text{Pseudoscalar correlator for } D(0^-) \]

**EFT sector**

\[ \Delta_{SP}^{(EFT)}(Q^2) = \frac{3F_D^2 M_D^2}{M_D^2 + Q^2} \left( M(0^+) - M(0^-) \right) \]

**OPE sector** [Narison (03)]

\[ \Delta_{SP}^{(OPE)}(Q^2) = -\frac{2m_H^3}{m_H^2 + Q^2} \langle \bar{q}q \rangle \]
Equating $\Delta_{SP}^{(\text{EFT})}$ and $\Delta_{SP}^{(\text{OPE})}$ at the matching scale $\Lambda$.

Bare mass splitting

$$\Delta M(\Lambda) \equiv M(0^+) - M(0^-) = -\frac{2\langle \bar{q}q \rangle}{3 F_D^2}$$

Parameters: estimated by using QCD sum rule [Narison (88)]

$D$-meson decay constant $F_D = 0.205 \text{ GeV}$

Light quark condensate $\langle \bar{q}q \rangle = -(0.243 \text{ GeV})^3$

$$\Delta M_{\text{bare}} = \Delta M(\Lambda) = 0.23 \text{ GeV}$$

$\Downarrow$

Enhanced by quantum correction of $\rho$-meson
Quantum Correction

\[ \Pi_{pp} \]

\[ \Pi_{ss} \]

\( \pi \)

\( \rho \)

\( \Lambda', \Lambda', \Lambda, \ln \Lambda \)

\( \Lambda', \ln \Lambda \)

\( \Lambda', \Lambda', \Lambda, \ln \Lambda \)

\( \Lambda', \ln \Lambda \)

\( \Lambda', \Lambda', \Lambda, \ln \Lambda \)

\( \Lambda', \ln \Lambda \)
\[ \Delta M = \Delta M_{\text{bare}} \times C_{\text{quantum}} \propto \langle \bar{q}q \rangle \]

\[ = \Delta M_{\text{bare}} \exp \left[ -C_2(N_f) \frac{g^2}{2\pi^2} (1 - 2k - k^2) \ln \frac{\Lambda}{\mu} \right] \]

Parameters, e.g.,
\[ C_2(N_f = 3) = 4/3, \]
\[ g = 6.27 \quad \text{[← HLS gauge coupling constant]} \]
\[ k = 0.59 \quad \text{[← extracted from } D^* \rightarrow D\pi\text{]} \]
\[ \mu = m_\rho = 771 \text{ MeV}, \]
\[ \Lambda = 1.1 \text{ GeV}. \]

\[ \Rightarrow \quad C_{\text{quantum}} \simeq 1.6 \]

The quantum effect increases \( \Delta M_{\text{bare}} \) by \(~ 60\%\).
Mass Splitting

$$\Delta M_{\text{bare}} = 0.23 \text{ GeV}$$

↓ enhanced by $\rho$-loop effect

$$\Delta M = 0.37 \text{ GeV} \sim m_{\text{constituent}}$$

Good agreement with experiment
Summary

• We started from the premise that at the critical point, the VM is realized.

• We included the effect of chiral symmetry breaking and matched our EFT to the OPE.

  – $\Delta M_{\text{bare}} \propto \langle \bar{q} q \rangle$
  
  – Due to the quantum effect of $\rho$-meson, $\Delta M_{\text{bare}}$ was enhanced by about 60%.

  $\Rightarrow \Delta M \propto \langle \bar{q} q \rangle$
• This suggests that
  – identifying the chiral symmetry restoration as the VM
  – the chiral doubling as a signal of spontaneous chiral symmetry breaking

  are mutually consistent.

• What is the characteristic feature of the VM?
  ⇒ Decay process
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Volume 60 – Lattice QCD at Finite Temperature and Density – BNL-72083-2004
Volume 58 – RHIC Spin Collaboration Meeting XX – BNL-71900-2004
Volume 57 – High pt Physics at RHIC, December 2-6, 2003 – BNL-72069-2004
Volume 56 – RBRC Scientific Review Committee Meeting – BNL-71899-2003
Volume 52 – RIKEN School on QCD “Topics on the Proton” – BNL-71694-2003
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