A Method To Measure The Focusing Properties (R_Matrix) of a Magnet


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A Method to Measure the Focusing Properties (R-Matrix) of a Magnet*

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Abstract
In this paper we discuss, and study the feasibility of a method that may be used to measure the focusing properties of a magnet. This method may prove valuable when applied to “non-conventional” magnets that deviate from the usual dipole magnets or other multipole magnets (quadrupoles/sextupoles etc.) which are commonly used in a synchrotron or a beam line. In this category of “non-conventional” magnets, fall special magnets, which come under the name “Snakes”, which are being used in synchrotron accelerators[1,2,3,4] to introduce artificial spin resonances to help overcome the intrinsic and/or imperfection spin resonances which appear during the acceleration of polarized beams. This method of measuring the focusing properties of a magnet requires the use of “low energy” and “high rigidity” heavy-ions which may be obtained from the BNL Tandem accelerator. In brief the method consists on, injecting “narrow beamlets” of heavy ions into a magnet and measuring the coordinates, of these “narrow beamlets”, at the entrance and exit of the magnet.
From the measurement of the coordinates of the “narrow beamlets” we can deduce information on the R matrix (first order transfer matrix elements) and higher order matrix elements that define the focusing properties of the magnet.

INTRODUCTION

The common method, which is applied in the magnet division of the Brookhaven National Laboratory (BNL), to determine the focusing properties of a magnet, is to measure the magnetic field of the magnet by using various techniques[5] which provide accurate measurements to determine the focusing properties of a magnet like dipoles or higher order multipoles. The magnetic-field measurement method however is not very accurate when applied to measure the focusing to measure the focusing properties of a Helical Snake[3,4] or of any other magnet that does not lend itself to any of the magnetic measurements technique. In the rest of the paper we discuss the principle of the proposed method the apparatus for the required measurements, and the required accuracy of the measurements.

PRINCIPLE OF THE METHOD

The focusing properties of a magnet are considered known when the coordinates of a particle at the exit of a magnet can be determined, assuming that the coordinates of the particle at the entrance of the magnet are known. The above sentence can be expressed either, schematically in Figure 1, which shows a magnet with the coordinates of a particle at the entrance and exit coordinate systems, or mathematically in equation (1) below.

\[ x_{d}(\text{out}) = \sum R_{p} x_{d}(\text{in}) + \sum W_{6p} x_{6d}(\text{in})x_{d}(\text{in}) + \sum \Sigma 6_{2} T_{1} n_{m} x_{m}(\text{in})x_{m}(\text{in}) + \ldots \quad \text{HOT} \quad (1) \]

The notation in equation (1) is:

\((x_{1}, x_{2}, x_{3}, x_{4}, x_{5}, x_{6}) \leftrightarrow (x, y, y', \delta l, \delta p, p)\)

Where \(x, y, x', y'\) are the lateral \((x, y)\) and angular \((x', y')\) deviations of the particle from the trajectory of the central particle. The quantity \(\delta p\) is the momentum deviation of the particle’s momentum \(p\) from the momentum \(p_{0}\) of the central-particle, and \(\delta l\) is the path length difference of the particle’s path from the path of the central particle.

Figure 1. Schematic diagram of the magnet with the entrance and exit coordinate systems. The beginning of the red arrows show the location of the particle and the direction of the arrow shows the direction of the particle at the entrance and exit coordinate systems.
The coefficients in equation (1) are defined as the partial derivatives of the output coordinates with respect to the input coordinates.

\[ R_{ij} = \frac{\partial x_i(\text{out})}{\partial x_j(\text{in})} \quad \text{1st order} \]
\[ W_{ij} = \frac{\partial x_i(\text{out})}{\partial (x_j(\text{in})x_k(\text{in}))} \quad \text{2nd order} \]
\[ T_{i\ldots p} = \frac{\partial x_i(\text{out})}{\partial (x_{i_1}(\text{in})x_{i_2}(\text{in})x_{i_3}(\text{in}))} \quad \text{3rd order} \]
\[ \ldots \]
\[ T_{i\ldots p} = \frac{\partial x_i(\text{out})}{\partial (x_{i_1}(\text{in})x_{i_2}(\text{in})x_{i_3}(\text{in})x_{i_4}(\text{in}))} \quad \text{n\textsuperscript{th} order} \]

The knowledge of the coefficients \( R_{ij}, W_{ij}, T_{i\ldots p} \) and of the higher order coefficients, determine the focusing properties of any magnet.

In the “paraxial ray approximation”, which assumes, that the momentum deviation \( \delta p \) of the particle is much smaller than momentum \( p_0 \) of the central particle, and that the lateral coordinates \( x,y \) in both entrance and exit coordinate systems are much smaller than the radius of curvature \( \rho \) of the particle moving in the magnetic field \( B \) of the magnet, most of the higher than the 1\textsuperscript{st} order terms which appear in equation (1) \( (W_{ij}, T_{i\ldots p} \ldots \text{etc}) \) are usually negligible. Therefore, the equation (1) above can be written

\[ x(\text{out}) = \sum R_{ij} x_j(\text{in}) + \text{"few HOT"}. \quad (2) \]

In equation (2) the expression “few HOT” includes the higher order term which are comparable in magnitude to the first order terms. In summary, the first order coefficients \( R_{ij} \), and the “most important” higher order coefficients appearing in equations (1,2) can be computed by measuring the coordinates \((x,x',y,y')_{\text{exit}}\) of an “adequate” number of particles at the exit of the magnet and the corresponding coordinates \((x',x,y,y')_{\text{in}}\) of the particles at the entrance coordinate system. By solving equation (2) we can determine the first order \( R_{ij} \) terms as well as the “most important” higher order coefficients.

**DESCRIPTION OF THE APPARATUS**

A schematic diagram of the proposed apparatus which can be used to measure the focusing properties of a magnet is shown in figure 2.

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1 The radius of curvature \( \rho \) is defined in the equation \( B\cdot p = k p / q \) (\( B \) is the field of the magnet, \( p \) is the momentum, and \( q \) is the charge state of the ion, \( k \) is a constant which depends on the units).

2 The size of the holes depends on the magnet to be measured. For the Partial Snake magnet the diameter of the holes should be no greater than 0.5 mm.
location of each “pin-hole image” on the visual flags (foils).
A schematic diagram of the detection system is shown in Fig. 3. An alternative ion position measuring system which utilizes thin wires is discussed in ref. [5]. The direction of the rays $(x',y')_m$ at the entrance of the magnet can be determined from the coordinates $(x,y)_m$ of the rays at the entrance, as defined by the “1st ion-position defining device”, and the coordinates $(x,y)_{out}$ of the rays at the exit as measured by the “2nd ion-position measuring device” when the field of the magnet is off.

- The third device “3rd ion-position measuring device” is located at the exit of the magnet at a specified distance, from the location of “2nd ion-position measuring device” otherwise it is identical to the “2nd ion-position measuring device”. Position measurements of the ions taken from the 2nd and 3rd “ion-position measuring devices” will determine the direction of the ions at the exit of the magnet $(x',y')_{out}$.
- The fourth device is the “ray-direction defining” magnet which is a dipole magnet that can change the direction $(x',y')_m$ of the rays at the entrance of the magnet and also serves to select the charge-state to be used when using heavy ions.

Figure 3. Schematic diagram of an ion-position measuring system to be used in the position measurement of the pencil-like ion beams which will enter and exit the magnet.

**COMPUTER SIMULATIONS**

The method to measure the focusing properties of a magnet was tested using computer simulations on a 3D model of the warm helical snake [3] which is installed in AGS. The raytracing into the magnet was performed with the computer code SPRAY [6]. The required 3D magnetic fields of the magnet were computed using the computer code opera [7] on a 3D model of the magnet [4]. Error analysis of the proposed method as well as comparison of the method with the magnetic measurements method has been performed and appears in [4].

**SUMMARY**

A method to determine experimentally the focusing properties of a magnet has been discussed. This method determines the first order matrix elements $(R_{matrix})$ of a magnet and the most “significant” higher order matrix elements. The error in the determination of the first order matrix elements is “reasonably small” for the measured $R_{matrix}$ to be used in various calculations which may involve the focusing properties of the measured magnet.

**REFERENCES**

[7] Vector Field Inc.